Problem-centred teaching and modelling as bridges to the 21st century in primary school mathematics classrooms

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Abstract

Moving mathematics classrooms away from traditional teaching is essential for preparing students for the 21st century. Rote learning of decontextualised rules and procedures as emphasized in traditional curricula and teaching approaches have proven to be unsuitable for the development of higher order thinking. The 'dream' is to have skills (that employer seek for the 21st century) such as being able to make sense of complex systems or working within diverse teams on projects [2: p. 316] fostered in mathematics classrooms, even at a primary school level. In this paper it will be shown that the problem solving perspective that modelling emphasizes includes competencies and skills that are essential in developing authentic mathematical thinking and understanding. Results of a study on modelling competencies [1] will be presented to highlight the growth of a problem solving mode of thinking. We will therefore explain that modelling achieves important aims for mathematics education in 21st century. Modelling fosters students' abilities to actualise existing (but not yet explicit) knowledge and intuitions; to make inventions; to make sense and assign meanings; and to interact mathematically [10: p. 176], thereby developing authentic mathematical thinking. The aim is to provide a perspective that shows how modelling meets the challenge of changing mathematics classrooms.

Introduction

Students need to learn mathematics with understanding since 'things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things. Things learned with understanding are the most useful things to know in a changing and unpredictable world' [11: p.1]. Adaptability and flexibility in using mathematical knowledge is particularly important when students solve contextual problems. Mathematical problem solving has many faces and requires definition. Schroeder and Lester's [3: p. 32, 33] three main descriptions of problem solving are used for this paper. In a traditional sense, problem solving means solving 'word' problems as an extension of routine computational exercises. This can be seen as teaching *for* problem solving – teaching of procedures takes place first and then problems specifically related to the taught concepts are solved. In some progressive programs, students are taught *about* problem solving and are taught to employ various methods or heuristics as options when faced with a problem (e.g. drawing a table or graph etc). When students learn *via or through* problem solving, problems are used to teach important mathematical concepts. When students interact with modelling problems, they solve the problems in their own way with mental tools that they already have available to them. The teacher facilitates by connecting different ideas that allow students more sophisticated understandings through these connections. It is by solving problems first and then building by connections between student ideas and representations that students become adaptable and flexible and move toward a problem solving mode of thinking. Modelling allows students to learn via problem solving and can be appreciated as a significant mathematics teaching and learning opportunity.

The Problem-centred approach and modelling

Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier & Wearne [4] build a problem-centred approach on Dewey's principles of reflective inquiry. They work from the assumption that understanding is the goal of mathematics education. Students solve problems at the outset of a mathematics lesson and the process of solving, collaborating, negotiating and sharing leaves 'behind important residue' [4: p. 18]. As expressed by Human [5: p. 303] problems are used as vehicles for developing mathematical knowledge and proficiency together with teacher-led social interaction and classroom discourse. The problems are opportunities for students explore mathematics and come up with reasonable methods for solutions [11: p. 8]. The role of the teacher and the student changes which means that the classroom takes on a different culture. It is in the problem-centred classroom culture where the real benefits to student learning lie. Students have regular opportunity to discuss, evaluate, explain, and justify their interpretations and solutions [6: p. 6]. This can only be achieved if the teacher allows this discussion to take place without presenting or demonstrating set procedures to solve problems. It is this change in focus in the classroom away from 'teacher thoughts' to 'student thoughts' [13: p. 5] that epitomizes student-centred methods such as the problem-centred approach and modelling. The classroom culture now takes on what Brousseau [7: p. 30] termed an 'adidactical situation'. The teacher, in an adidactical situation, does not attempt to tell the students all. Brousseau also explains that the 'devolution' [7: p. 230] of a problem is fundamental in adidactical situations. This happens when the teacher provokes adaptation in the students by the choice of problems put to them. The problems must also be such that the student accepts them and wants to solve them. The teacher refrains from suggesting the knowledge, methods or procedures he/she is expecting or wanting to see. The teacher seeks to transfer part or all of the responsibility of solving the problem on the student. It is this adaptation to the adidactical situation that allows students to learn meaningfully. It is this adaptation to a problem-centred approach that allows a growth of mathematical understanding, adaptability and flexibility. This in turn promotes a problem solving mode of thinking that is so necessary in the 21st century workplace.

The problem-centred approach in mathematics education allows us to understand modelling and its place in effective mathematics teaching and learning. Modelling goes beyond problem solving in that the important questions of *when* and *why* problems are solved as well as *whose* thoughts ideas and constructs are used when solving problems. In defining what a problem solving capacity or mode of thinking entails, the constructs of [10] are used. When students solve problems, they should be provided with opportunities to: actualise existing (but not yet explicit) knowledge and intuitions; make inventions; make sense and assign meanings, and interact mathematically [10: p. 176]. These four constructs encompass what it means to solve problem with understanding and flexibility. It is often difficult for teachers to elicit existing knowledge from students since there is an array of different understandings and levels of thinking in a single classroom. Modelling allows students to verbalise their current ways of thinking and improve on these ways of thinking. In making inventions, students are able to use their own ways of thinking in constructing a response to the problem. Modelling tasks allow students to produce meaningful solutions that keep the context of the problem in sight. While students work collaboratively on modelling tasks they do make sense and assign meaning since they have to communicate their thinking and ideas while interacting mathematically with each other in order to make progress. These four constructs underline what it means to develop a problem solving mode of thinking since they encompass student understanding, adaptability and flexibility in solving problems. It also underlines what students need to learn meaningful mathematics in the 21st century.

Mathematical modelling goes beyond problem solving since students 'create a system of relationships' [12: p. 110] from the given situation that can be generalised and reused. Although students are solving problems when modelling, a modelling approach means that students must display a wider and deeper understanding of the problem. Modelling goes beyond problem solving because students structure and control the problem – not only solve it. The aim of this paper is to show that modelling tasks allow students and teachers access to significant problem solving that bridges student understanding and student problem solving abilities. The development of a problem solving mode of thinking that results from student involvement with modelling tasks is presented in this paper. Problem solving and modelling problems specifically hold a reciprocal interdisciplinary relationship with other knowledge fields. Modelling problems for mathematics classrooms are applicable to and can be sourced from fields outside mathematics such as engineering, architecture, commerce and medicine to name a few.

The study

The main study [1] investigated the development of modelling competencies in grade 7 students working in groups. Partial results will be presented in this paper. Twelve grade 7 students were selected to work in three groups of four students in each group. The results of only one of the groups working on the first (of three) task are presented in this paper. This group comprised students whose mathematics results in a traditional setting the previous year were considered "weak". The groups solved three model-eliciting tasks over a period of 12 weeks in weekly sessions of about one hour. The results from this group's discussions around the task – Big Foot is presented.

Task 1: Big Foot taken from [9: p. 123].
Example of footprint (size 24) given to students. Groups had to find the height/size of this person and also provide a 'toolkit' on how to find anyone's height/size from their footprint. Supporting material: rulers, tape measures, calculators

Table 1: Task Instruction for Big Foot

Students solved and presented their solution as a group with minimal teacher/researcher intervention. These students had not been exposed to a problem-centred approach nor had they solved modelling tasks before. Each group presented their solution to the other groups and students were encouraged to question each other's models. The contact sessions were audio recorded and transcribed. Transcriptions were coded for each competency for the main study and coded again for the results presented in this paper. The competencies identified for the main study were: understanding, simplifying, mathematising, working mathematically, interpreting, validating, presenting, using informal knowledge, planning and monitoring, a sense of direction, student beliefs and arguing. How students develop and refine a problem solving mode of thinking is highlighted in this paper. The four constructs [10: p. 176] were used to code the data from the transcriptions and to structure the discussion in the next section. This assisted in establishing to what extent students working in groups were engaging a problem-centred paradigm when solving model-eliciting problems.

Results

The results presented are from the group's solutions processes for Task 1: Big Foot. This was their very first modelling task so it exhibits the impact modelling tasks have on students thinking. Furthermore it highlights the mathematical learning opportunities that are implicit in a modelling task. In the transcripts R stands for researcher.

Actualising existing knowledge and intuitions
This group had an intuitive idea that there was a universal foot to height ratio although this took place in the second session. The first session was taken up by a seeming avoidance of mathematising the task. Once they had decided to take action on their own intuitions they were successful in producing a model for this task.

M: Ok wait, why don't I take my foot and divide it by my height, times a 100
R: why do you times it by 100?
M: Because that is how you find your percentage so we can find out ...I am saying that when we do his (Big Foot) *then we must get the same...*

The students introduced the idea of a percentage on their own accord, but it later transpired that they used the 'times by 100' to remove the decimal number that resulted from their division. This group also had an intuitive idea that Big Foot had to be very tall and they were able to use this to interpret and validate their progress which allowed them to make progress in their solution process. They had taken a number of measurements including across their hips which they called their 'width'.

M: 58 (a group member's height) *divided by 2 is 28.* Similar to his width - they measured 27 as this person's 'width'.)
N: So he is 30 inches!
M: No he can't be, that's too short.

If student do bring their own ideas and constructs forth and they act on these ideas it is clear to see how 'making inventions' is possible. This would not be possible if students are offered methods or procedures by the teacher.

Making inventions
This group 'invented' their model to assist them in resolving Big Foot.
M: I divided my feet to my height and I timesed it by 100 and I got 15.
N: Yes...
M: So now I have to try get 16 times by what to get 15 again because it is a human. Then that will be right.
On their presentation sheet (see Fig 1) they had written:
The solution is to take his foot size and divide it by an estimated number, multiply that by 100. The result should be 15-20.
Although they did not see a connection here between multiplying and dividing (surprising for this year of their schooling), they invented a way around this of 'estimating' the multiplicand so that the result would be 16. Once this group had found the foot length to height ratio of all their group members, they had four different (although very close) ratios. They then realized they needed more data and tried more people. After trying three more people they found that 16 seemed to be a common ratio. Although they never used the term 'mode', this is a construct that they 'invented' by understanding that they needed this from their set of data. When questioned:
M: R is 16 and N is 16. Then we must use 16.
R: why did you decide that N and I have the right measurements?
M: Because you are the most.

Making sense and assigning meaning
After calculating that 15 was one of the group member's foot/height ratio, they continued to work through the rest of the group, other people in the room as well as continuing this at home and with other students at school. They were clearly in control of this 'method' or model

although it was not an elegant approach it was meaningful and they were able to assign meaning to other areas of the model.

M: Divided by (known height) *and times 100 and let hope it equals something nearby 15 and 20.*
N: 62 divided by 11, ag no sorry the other way; 11 divided by 62 times 100 is
M: Yes I told you. 17. So I equal 15 and you 17.
M: OK it (the quotient) *might be 15, 16 or 17. So he* (Big Foot) *might be: 98, 97 or 96.*

When looking at their presentation sheet- they understood that if the ratio was 15, then their estimated height was too short, or it the ratio was 19, then their estimated height was too tall. They were able to assign meaning to a fairly complicated model which is surprising since they achieved lower mathematics results than average in a traditional setting.
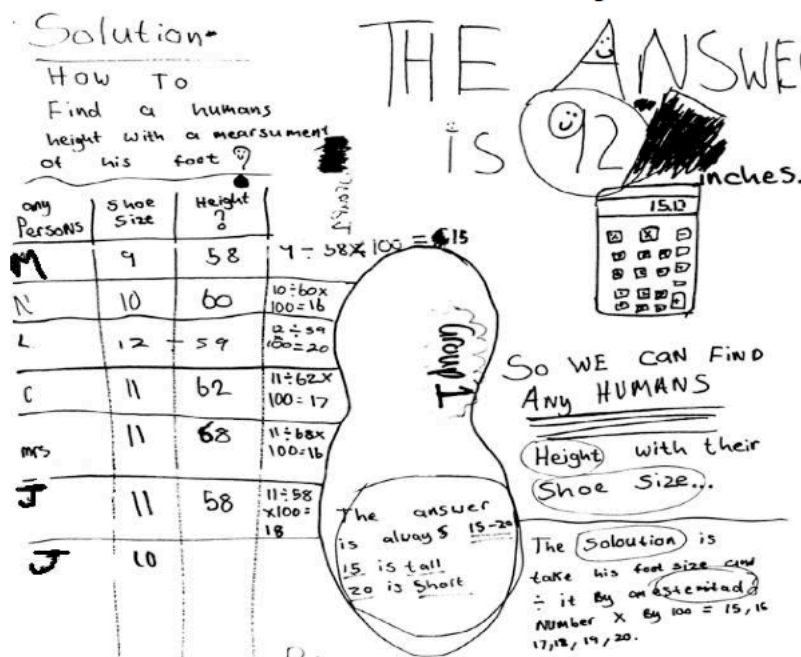

Fig 1: Group presentation sheet

Interacting mathematically

The following excerpt from the transcripts for this group shows how one group member explains a fairly inelegant yet complicated model for Big Foot to the other members.

M: Look, I take your foot (length) *right; the foot is 12* (inches), *then I divide it by any estimated number, like I will take, a number will come in my head and I will divide it* (by the foot length) *and then multiply by 100. Probably* (the result will be) *over 20 or below 15. If it's below 15, it means the person is taller, if it's over 20 it means the person is a bit shorter. Then you estimate a bit lower until you get 15, 16 or 17.*

Conclusion

The confluence of the problem-centred environment and modelling tasks present mathematics education with a 'developmental space' for the learning of essential, meaningful mathematics [1: p. 37].

The data presented in [6] suggested that a problem-centered instructional approach in which the teacher and students engage in discourse that has mathematical meaning as its theme is feasible in the public school classroom [6: p. 25]. The results of [8] suggest that a problem-centered approach together with a change in teacher beliefs is a viable for reforming mathematics classrooms. Furthermore, a modelling approach assists in developing student competencies in

problem solving, modelling and mathematics. Modelling tasks present an arena for teaching and learning that assists teachers in understanding a problem-centred approach and to simultaneously apply these principles in teaching. Modelling tasks can be used successfully by teachers and students unfamiliar to problem solving or a problem-centred approach.

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