A way of computer use in mathematics teaching -The effectiveness that visualization brings-

Shuichi Yamamoto and Naonori Ishii

College of Science and Technology, Nihon University, Funabashi, Chiba, 274-8501, Japan

syama@penta.ge.cst.nihon-u.ac.jp

Abstract

We report a class of the mathematics in which an animation technology (calculating and plotting capabilities) of the software *Mathematica* is utilized. This class is taught for university students in a computer laboratory during a second semester.

It is our purpose to make a student realize the usefulness and the importance of mathematics easily through visualization. In addition, we hope that students will acquire a new power of mathematics needed in the 21st century. For several years, we have continued this kind of class, and have continued to investigate the effectiveness that our teaching method (especially visualization) brings in the understanding of the mathematics.

In this paper, we present some of this teaching method, which is performed in our class. From the questionnaire survey, it is found that our teaching method not only convinces students that the mathematics is useful or important but also deepens the mathematic understanding of students more.

Introduction

Mathematics plays evidently an important role in the 21st century. However, it is very difficult for students to realize the significance of learning mathematics in a traditional class. We think that computer use gives students the motivation for learning mathematics. In particular, the remarkable evolution of animation technology enables students to visualize a lot of formulas and mathematical concepts. Through this visualization, we can lead students to realize the significance of learning mathematics. In addition, it is expected that students will acquire new mathematical abilities needed in the 21st century.

We report a class taught for university students in a computer laboratory during a second semester, in which animation technology (calculating and plotting capabilities) of the software *Mathematica* is used.

The characteristic of our class is as follows:

(1) We first teach the contents of the program that students will execute on *Mathematica*. Our class is designed so that students can learn the program step by step. In addition to watching a picture on the monitor of a computer, the learning of this program helps students understand the mathematical concept.

(2) We present the mathematical properties which we cannot explain on a blackboard, but which we are able to explain more easily if using a computer. This makes students realize the utility of mathematics better.

(3) We present the problems of mathematics which students can solve on the monitor of a computer by using mathematical concepts without a pencil and a paper. It is important for students to solve these kinds of problems in order to promote their conceptual understanding of mathematics.

In this paper, we introduce a teaching method of integration and give examples of the above (2) and (3). We also report the student evaluation for our teaching.

A teaching method of integration

In our university, most students can calculate the integral of a function such as x^2, but they do not know what the integration represents. Our aim is to teach what the following formulas mean.

$$\int x^2 dx = \frac{x^3}{3} + C, \quad \int_0^1 x^2 dx = \frac{1}{3}.$$

For the purpose, the students are provided the following two examples.

Example 1: Compute the Riemann sum which gets close to the definite integral $\int_0^1 x^2 dx$.

Example 2: Observe how a primitive function $x^3/3$ is obtained from a function x^2 on the closed interval $[0,1]$.

In Example 1, let $f(x) = x^2$ be a function defined on the closed interval $[0,1]$ and for the sake of simplicity, we consider the Riemann sum

$$S(n) = \sum_{i=0}^{n-1} f(\frac{i}{n}) \times \frac{1}{n}.$$

According to the program, the students define the Riemann sum $S[n]$ on *Mathematica*. First, let students evaluate $S[10], S[20], S[50], S[100]$ using calculating capability of *Mathematica*. Next, for a larger n, let students evaluate $S[n]$. As n gets larger and larger, the students observe that the value of $S[n]$ gets closer and closer to the definite integral of x^2 over $[0,1]$

$$\int_0^1 x^2 dx = \frac{1}{3}.$$

In order to make students understand the concept of the definite integral, it seems to be important for students to compute the Riemann sum directly.

In Example 2, let $f(x)$ be the same function as in Example 1. We consider the accumulation function defined by $f(x)$ to be a primitive function of $f(x)$. By definition, the accumulation function $F(x)$ is given by

$$F(x) = \int_0^x t^2 dt.$$

In order to observe how $F(x)$ is derived from $f(x)$, we put $x_k = k/n$ and approximate the value $F(x_k)$, the Riemann sum

$$y_k = \sum_{i=0}^{k-1} f(\frac{i}{n}) \times \frac{1}{n}.$$

Using *Mathematica*, we produce an animation of line graphs, which are created by connecting the points (x_k, y_k) ($k = 0,1,...,n-1$) in numerical order, and run. Then students find a curve which the line graph gets close to as $n \to \infty$, and visualize this curve as the graph of the primitive function of $f(x)$.

At the last of the class which we did on 19th December in 2007, our teaching on the integral calculus was inspected by the questionnaire surveys to 30 students. The questions and results are as follows.

Question 1: Do you think our method which makes use of a computer is a better way than the traditional approach which

utilizes a textbook?

The result is Table 1 which shows the indicated scales and the number of students who agreed to the scale.

Table1

Strongly think so.	Think so.	Neutral.	Do not think so.	Do not think so at all.
6	18	6	0	0

Question 2: Why do you think so? Choose a suitable reason (multiple answers are possible).

The result is Table 2 which shows the chosen reasons and the number of students who agreed to the items

Table 2

I can imagine the contents by watching a graph and an animation.	22
It is easy to understand the contents because studying step by step through a program.	9
The typing operations make me concentrate in learning.	3

Trigonometric functions and connections with waves

In the class that trigonometric functions are taught, only many formulas are emphasized and the role of these formulas is not explained so much. It is effective from an educational view point that students learn the connections between trigonometric functions and waves and discover the role of formulas for trigonometric functions. For example, what kind of wave motion phenomena does the function $\sin x \cos t$ give by changing the value of t ? What kind of behavior does the addition formula $\sin(x+t) = \sin x \cos t + \cos x \sin t$ give by changing t ? By helping students visualize the changes, which can hardly explain with a blackboard and chalk, they realize the significance of using mathematics.

The students learn the following.

Example 3: Observe how the graph of the sum of $\sin x$ and $\cos x$ is drawn.

Example 4: Observe how the graph of $\sin(x+t)$ shifts when t moves.

Example 5: Observe how the graph of $\sin x \cos t$ shifts when t moves.

Example 6: Observe how the graph of $\sin x \cos t + \cos x \sin t$ shifts when t moves.
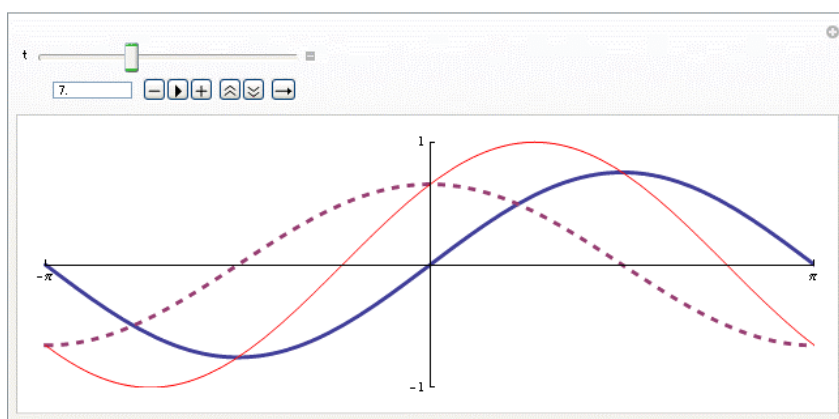

Figure1: An animation of graphs of $\sin x \cos t$, $\cos x \sin t$ and $\sin x \cos t + \cos x \sin t$ when t moves.

The graph in Figure1 shows the case in which $t = 7$. The students can observe the movement by clicking a triangle key.

The animation in Figure 1 shows graphically two waves shifting up and down combined to become the wave shifting to the left. From these observations, students realize that the trigonometric function plays an important role in wave theory. Moreover, students can understand the formula visually in connection with the behavior of waves.

Conceptual understanding of derivatives

The students learn the relationship between the shape of the graph of a function and the sign of its derivative. The graph of the derivative $f'(x)$ represents the instantaneous growth rate of $f(x)$ at a point x. When the value of $f'(x)$ is negative, the graph of $f(x)$ is going down. When the value of $f'(x)$ is positive, the graph of $f(x)$ is going up. Here we consider the following problem which is found in [3].

Problem 1: Figure 2 shows the graphs of two functions. One is the graph of the derivative of the other. Determine which is which and explain your reasoning.
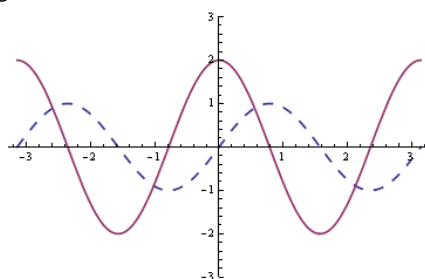

Figure 2: Graphs of a function and its derivative.

The solid line represents $f(x)$ and the dotted line represents $f'(x)$, but no equations are given, so students have to rely on their graphical knowledge of functions and their derivatives.

We requested the students who took the traditional class to solve this problem in 2007. Most students at the time were not going to use the relationship between the shape of the graph of a function and the sign of its derivative for solving the problem. The answers of students who chose these functions correctly were based on their knowledge that the derivative of $\sin x$ is $\cos x$.

We believe it is important in the new century to raise student's ability to solve problems through the graphical knowledge of functions and their derivatives. We attempt to raise this ability by using animation technology. In our class, the following examples and problem are given for students.

Example 7: Let $f(x) = x(x-t)(x-2)$. When the vale of t moves from -1 to 2, observe the relationship between the shape of the graphs of $f(x)$ and the sign of $f'(x)$ by watching the graphs of $f(x)$ and $f'(x)$.

Example 8: Let $g(x) = \sin tx$. When the value of t moves from 0 to 3, observe the relationship between the shape of the graphs of $g(x)$ and the sign of $g'(x)$ by watching the graphs of $g(x)$ and $g'(x)$.

The students who studied the above examples come to solve the following problem by the graphical knowledge of functions and their derivatives.

Problem 2: Figure 3 shows the graphs of two functions. One is the graph of the derivative of the other. Determine which is which and explain your reasoning.   **Figure 3: Graphs of a function and its derivative**.


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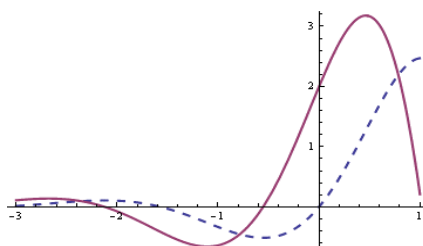
The performance of the students and summary

We conducted a student survey at the end of the semester. The investigation items are as follows.

1: The feeling that mathematics is more useful increased.

2: I brought myself to study mathematics further.

3: I realized the importance of mathematics more strongly.

4: Being able to visualize mathematical concepts and properties, I have understood mathematics deeper.

5: Getting free from memorizing formulas, I could enjoy learning mathematics.

6: Others

The following Table 3 shows the performance of the 55 students who took our class in 2008.

Table 3: The performance of the students
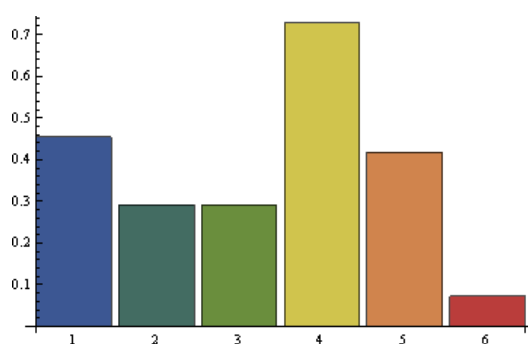
In Table 3, the numbers of the horizontal axis correspond to those of the investigation items above and the vertical axis shows the ratio of the student who agreed to the item. Students could choose the multiple items.

From Table 3, we found that about 74% students felt that the mathematics was useful or important and about 73% students understood mathematics more deeply by the assistance of visualization. Most students believe that to learn mathematics is the same as memorizing ways of solving given problems. We want to pay attention to the fact that about 40% students could learn mathematics enjoyably. Our teaching method appears to produce improvement in the attitudes of students learning mathematics.

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