DeltaTick: Applying Calculus to the Real World through Behavioral Modeling

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Abstract Certainly one of the most powerful and important modeling languages of our time is the Calculus. But research consistently shows that students do not understand how the variables in calculus-based mathematical models relate to aspects of the systems that those models are supposed to represent. Because of this, students never access the true power of calculus: its suitability to model a wide variety of real-world systems across domains. In this paper, we describe the motivation and theoretical foundations for the DeltaTick and HotLink Replay applications, an effort to address these difficulties by a) enabling students to model a wide variety of systems in the world that change over time by defining the behaviors of that system, and b) making explicit how a system's behavior relates to the mathematical trends that behavior creates. These applications employ the visualization and codification of behavior rules within the NetLogo agent-based modeling environment (Wilensky, 1999), rather than mathematical symbols, as their primary building blocks. As such, they provide an alternative to traditional mathematical techniques for exploring and solving advanced modeling problems, as well as exploring the major underlying concepts of calculus.

Introduction

"Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures – thereby losing sight of both the mathematics and of its practical value." (Hughes-Hallett et al 1994)

As Hughes-Hallet notes, many complicated problems can be solved easily using the tools of calculus – not only in science, technology, engineering and mathematics (STEM), and importantly also in the social, behavioral, and economic (SBE) sciences. But by focusing only on analytic procedures, calculus educators have largely failed to help students understand why or how those procedures work. While it is clear that recent reforms in calculus education have addressed some of these issues by helping students to better understand the concepts that underlie calculus rather than simply the procedures and methods involved, researchers have yet to show that students are learning how to apply these concepts generatively to describe, model, and explore real-world phenomena (Ganter, 2001). At the same time, there are an increasing number of calls from science and industry for students to understand calculus in an interdisciplinary, applied, and generative manner (NCTM, 1989; AAAS, 1990). Because calculus is an important tool for so many different domains and topics, students without a calculus background can only explore a limited, highly simplified set of problems and ideas, and students are rarely exposed to the true power of calculus: its suitability to model a wide variety of real-world systems across domains.

In this paper, we provide the motivational background and brief description of the DeltaTick and HotLink Replay applications, designed to address these issues by a) enabling students to model a wide variety of systems in the world that change over time by defining the behaviors of that system, and b) making explicit the links between that system's behavior and the mathematical trends that behavior creates. These applications employ the visualization and codification of behavior rules within the NetLogo agent-based modeling environment (Wilensky, 1999), rather than mathematical symbols, as their primary building blocks. As such, they provide an alternative to traditional mathematical techniques for exploring and solving advanced modeling problems, enabling even students without a calculus background to explore those topics, while at the same time introducing students to many of the major underlying concepts of calculus.

The DeltaTick project builds on work that complements typical algebra-based representations of calculus concepts such as rate of change, integration, and differentiation with alternative – and in some ways, more accessible – representations using computational media. It extends this work by representing these concepts in the context of *systems* – that is, scientific and social phenomena for which many components and behaviors in a system contribute to a single measurable quantity that changes over time. In the following sections, we will describe these representational shifts and the applications we are developing in more detail, and then illustrate how a number of important concepts in calculus can be explored – and how interesting systems can be modeled and studied – using this alternative media.

Background

The DeltaTick project leverages the power of computers for enabling new ways to represent the mathematics of change, and is rooted in design-based research on representational infrastructure shift (diSessa, 2000; Papert, 1996; Kaput, Noss & Hoyles, 2002), the “restructuration” of domains (Wilensky & Papert, 2006), and the computational and pedagogical affordances of agent-based modeling (Wilensky & Resnick, 1999; Wilensky & Reisman, 2006) to provide an alternative method for exploring and creating models of the real-world systems. In this section, we briefly review this literature, provide a description of what we mean by a *system* in the “calculus of systems”, and conclude with an illustration of how the notion of *rate of change* in calculus and differential equations can be fruitfully “restructurated” as a collection of the behaviors executed by elements or agents in the case of a predator-prey system

Restructurations and Shifts in Representational Infrastructure

Computation enables dynamic encoding of information that was previously only encoded using static media (diSessa, 2000; Papert, 1996; Kaput, Noss & Hoyles, 2002; Wilensky, 2006). Over the past 20+ years, researchers have explored how the ability to codify and execute rules over time using computers can enable students to explore the important ideas underlying the mathematics of change. A number of computational learning environments have been developed that leverage students’ intuitive thinking about the mathematics of change – primarily in the context of one- and two-dimensional motion (Nemirovsky, Tierney & Wright, 1998; Roschelle, Kaput & Stroup, 2000), but also in the context of other simple phenomena such as banking or other dynamical systems (Wilhelm & Confrey, 2003; Carlson, Jacobs, Coe, Larsen & Hsu, 2002; Herbert & Pierce, 2008). These programs enable students to visualize, collect, or even produce their own meaningful quantitative patterns of change over time – all while relating these systems with graphs, number tables, and other more “mathematical” representations of these quantities.

Although these computer-based models look very different from their typical equation-based counterparts, both models encode what can be very similar information about how the system changes over time. This is an example of what Wilensky and Papert (2006) refer to as *restructurations:* changes in the encoding of disciplinary knowledge as the result of changes in the technology that is used to encode that knowledge. The language of calculus was developed using static media, so that if one is interested in how a certain rate of change affects the trend of a quantity over time, they must use symbolic “tricks” like integration to *mathematically* predict that trend. Now that computers enable the simple externalization and rapid execution of rules (that themselves define rates of change), we can replace those symbolic tricks with the writing and execution of rules that *computationally* predict the trend.

Wilensky and Papert note that a given knowledge “structuration” reflects the technologies and techniques available to a given community when that knowledge was created. As such, the classical or common encoding of knowledge for a given domain is not necessarily the *best*, *most complete*, or *most accessible* way of presenting it. It might be that as new technologies and techniques emerge, new knowledge encodings grant more people access and facility of use of that knowledge – even though at the same time, those different encodings may reduce other affordances provided by their alternatives, such as efficiency or precision.

Agent-Based Modeling and the NetLogo Modeling Environment

The DeltaTick project differs from other environments that leverage the representational affordances of computers, because we use *agent-based modeling* as a means to represent different kinds of systems that change over time. Agent-based modeling is a computational technique in which a system comprised of a number of homogenous elements or *agents* each execute relatively simple individual-based rules, producing often unexpected or interesting aggregate, system-level patterns. Whereas typical calculus-based treatments of many systems involve an equation representing the quantitative patterns of aggregate measures of phenomena (such as the pressure of a gas, ratio of chemicals in a solution, or trends in a population), agent-based models involve programmatic instructions representing the behavior of individual elements of the system that produce those measures (such as the Netwonian motion of individual gas particles, the reactions of individual and pairs of molecules in a chemical solution, or reproduction and death of individuals in a population).

Agent-based modeling has been increasing in popularity for not only educational, but also scientific purposes, and a number of environments for creating agent-based models exist. One such agent-based modeling environment, NetLogo (Wilensky, 1999) was developed with the explicit goal to be “low-threshold and high-ceiling” – making it very simple for beginners to build models, but also possessing enough power to allow for the development of highly sophisticated models. It has been shown that the restructuration of a number of domains in mathematics and science (such as biology, Wilensky & Resnick, 1999; chemistry, Levy, Kim & Wilensky, 2004;

materials science, Blikstein & Wilensky, 2006; and electromagnetism, Sengupta & Wilensky, 2008) within the NetLogo environment can help to make understanding and analyzing these systems accessible to many more students. We believe such models – especially if explicitly related to more typical calculus-based notions of concepts such as rate of change – may also provide an accessible means through which the mathematics those systems (namely, calculus-based equations) can be introduced to students. Both the DeltaTick and HotLink Replay applications have been developed to work with NetLogo, but to explicitly emphasize the relationships between this form of modeling and that mathematical models typically used to explore quantitative patterns of change over time.

Defining a Calculus of Systems

While a great deal of research has been done investigating student understanding of change over time, little is known about how they understand change over time in *systems* – that is, in phenomena for which calculus is used to model change over time of single values that are affected by a number of elements and behaviors. For example, a single population measure that changes over time is influenced by the reproductive and resource-based behavior of each member of the population, and different behaviors (such as reproduction, or the absence of reproduction due to a lack of resources, or death) can change the population measure in different wants. When we use the term "calculus of systems", we are specifically referring to phenomena which always involves multiple components or agents, and which can also involve multiple behaviors for each of those agents (shaded in the table below) – but not systems that only involve one or a small number of agents.

		Number of Agents	
		One	*Many*
Number of Behaviors	*One*	Example: One-dimensional motion Agent: Moving object Behavior: motion in single direction	Example: Simple exponential (Malthusian) population growth Agents: members of population Behavior: reproduction
Number of Behaviors	*Many*	Example: Projectile Motion Agent: Projectile Behaviors: inertia, downward acceleration due to gravity	Example: Lotka-Volterra predator-prey system Agents: members of population Behaviors: reproduction, predation, death

Different classes of phenomena that can be modeled using Calculus. We are interested in classes that involve many agents (shaded).

These situations comprise a particularly interesting and important class of phenomena for students to explore and understand for a number of reasons. First, a large and increasing number of important concepts in science, technology, engineering, and mathematics; as well as in the social and behavioral sciences, involve a number of components (atoms, particles, people, markets, and so on) that all interact to produce some outcome of interest. Second, enabling students to connect ideas in calculus to these more complex systems increases the number of real-world and personally meaningful phenomena that they are able to explore and model. Finally, by better understanding how seemingly different scientific and social systems might produce similar or different quantitative trends over time, students might have more access and encouragement to draw interdisciplinary links between topics they learn in school and/or find personally interesting. If students do possess qualitative resources for reasoning about mathematical patterns and relationships in complex systems similar to those that they exhibit when reasoning about simple change-over-time situations, this points to exciting implications for calculus and science education at earlier levels of education.

Macro-Level Rates of Change as the Result of Micro-Level Behaviors

Finally, we would like to provide an example of how a relatively complex calculus-based representation of a system that involves multiple rates of change can be re-conceptualized in terms of the behaviors that produce those rates of change, and thus, the quantitative patterns of interest that we wish to model. Consider, for example, the mathematical models used to describe a typical predator-prey system (the Lotka-Volterra equations), here involving wolves (*W*) and sheep (*S*):

$$\frac{dS}{dt} = b_S N_S - k_1 N_S N_W \qquad \frac{dW}{dt} = k_W N_S N_W - d_2 N_W$$

A detailed description of the meaning of these equations is included in Wilensky & Reisman (2006), but for our purposes it suffices to state that these equations describe how wolf and sheep populations change over time as a result of the sheep birth rate (b_S), the predation rate at which wolves consume sheep (k_1), and the death rate of wolves (d_2), multiplied by the total number of sheep and wolves in the system (N_S and N_W).

Another way to conceptualize this system is not in terms of rates, but in terms of the relevant behaviors of a wolf or sheep, and the effects those behaviors can have on the population of each. For example, each wolf can reproduce or die, which leads to an increase or decrease in the wolf population. Since wolves prey on sheep, a wolf death also affects the likelihood that a sheep will be eaten, and so on. Each of these behaviors can be mapped onto the corresponding rates of change that are included in the traditional differential equations featured above, and serve to explain the mechanisms through which those rates of change manifest: clearly connecting the mathematics of the system with the system itself.

This alternative method for representing rate-based mathematical models of systems resembles systems dynamics models (such as STELLA; Peterson & Richmond, 1992), but includes a number of differences. First, it is defined at the *agent* level rather than the *aggregate* level: in other words, rather than describing a collection of elements of a system (such as wolf and sheep in this example, or a collection of atoms in a gas, and so on). Second, the basic building blocks of these systems are *behaviors* that each of those agents produce, rather than rates of change as is the case in systems dynamics models – providing students with more information to build an integrated understanding of both the mechanisms and mathematical trends of systems of interest.

DeltaTick and HotLink Replay: Facilitating Construction of Rate-Based Systems Modeling

DeltaTick: Building the Model. To use DeltaTick, users first define the actors or agents in a system. They are then are asked to define how the system changes over time as a result of the behaviors of those agents - specifically, how certain properties (such as the number, size, position, and so on) of agents change per discrete unit or ``tick'' of time. Although users can create the behaviors they would like to assign to agents from "scratch", DeltaTick comes equipped with libraries of behaviors that can be simple dragged onto agents for common models that might be of interest to users: for example, the "population growth" library includes behaviors that can be added that fundamentally change the shape of the curve produced by the model: reproduce-clone, wander, die-randomly, enough-space?, partner-here?, and so forth. Once added, the user can also modify these included behaviors to their own preferred specifications.

DeltaTick allows this construction to occur within a freeform drag-and-drop interface, and makes explicit to users which values of interest (that is, which aggregate-level outcome measures, such as population in the example above) are affected by which behaviors. When a model has been created, it can be saved and loaded into the NetLogo modeling environment, or run and recorded directly by the HotLink Replay application.

HotLink Replay: Running the Model. Once the model is defined, users are able to load it into the HotLink Replay system and execute it over several ticks of time. The model will produce both a visual and graphical representation of the phenomena, so that the student is able to directly relate what goes on in terms of behavior to the graph that characterizes that behavior quantitatively. After the model stops running, the student will be able to move back and forth over the duration of the model, evaluating important features of the graph (such as inflection points, limits, maxima and minima that may occur in different models) and comparing them to the visualized behavior of the model that produced those features.

Though conceptually very simple, this approach to constructing models of, and exploring, systems that change over time affords a great deal of flexibility in terms of what can be modeled, and how complex a model can be. In fact, the two examples used above -- of continually compounding interest and population growth -- are not typically presented to students until well after the first year of calculus. The basic underlying principle of this system is that ideas of derivative, integral, change, and so on can be presented within, and understood in the context of, real-world systems that change over time.

Using DeltaTick, a number of important underlying concepts of calculus are re-conceptualized in the context of an agent-based model that emphasizes the behaviors, rather than the quantitative trends, of systems. A model developed can be envisioned as an equation, for which different parameters can be inputted and which can be executed for a given period of time in order to compute results. An individual "tick", or iteration, of the model can be conceptualized as the differential: embodying a single collection of behaviors that contribute to the changes to quantities of interest for the smallest available unit of time. Finally, the integral or accumulation is obtained by running a model for several ticks; allowing for the repeated execution of those behaviors than affect outcomes of interest and build upon quantities derived from the previous execution.

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