

## CLEARNESS AS A PRINCIPLE OF THE TEACHING OF MATHEMATICS

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**Abstract** In this paper, the psychological aspects of clearness in teaching mathematics are considered and some suggestions for the achieve the clearness are given

Speaking about the basic principle of teaching of mathematics in universities, B.V.Gnedenko (1981, p.57) indicated that it is necessary “to teach in such way that the students could clearly imagine the origin of basic concepts...” He emphasized he importance of “the transparency, new vision”, «consideration from new points of view” (ibid., p.120) in the teaching of mathematics. Used by scientists - mathematicians, teachers and psychologists verbs: “to find out”, “to explain”, “to clear up”, “to understand” - express the characteristic for the teachers methodical actions aimed at the achievement of clearness in the teaching of mathematics. Considering the importance of lectures, B.V.Gnedenko noted that “most important is to understand the essence of the subject matter, to find out its nature”, “to elucidate” the central idea of the discourse (bid, p. 134). A.A. Stolyar (1974, p.73) indicated the necessity of clearness of symbolic representations of the language of graphs. Explanation is one of the basic stages of the process of teaching. Clearness is one of the principles of the teaching of mathematics.

The clearness of mathematics for students is a characteristic of their perception of mathematics, and also a characteristic of expression of a mathematical thought of a teacher, of his explanation, of his speech. The clearness in the teaching of mathematics is even more difficultly achievable in comparison with other educational disciplines because of its abstractness, of the presence of various forms, ways and languages of representation of information.

How to provide the clearness of mathematics for the students? It is possible to deduce from the statements of A.Ya. Hinchin (1963, p.3) that we teachers:

little care about the clearness about the purpose mathematical elegance of expressions and statements;

too frequently... do not see the necessity... to find out the connection of a theorem or a concept with other, earlier acquired concepts, propositions, problems

should provide the presence “of clear view of the role and place of various parts of the studied theory “in the consciousness of students”;

should not allow “the mixing and jumps, resulting in mess and mistakes in reasoning” (ibid., p. 143).

Note that mixing, jumps and mess, absence or lack of precise branching in reasonings of the teacher create the ambiguity of mathematics in minds of students. This phenomenon is characterized by following complaints of the students: “a complete fog”; “I look at my notes and can not understand anything”; “it is impossible to understand”; “I do not understand what is connected with what”; “a complete mess”, “the confusion in my head”; “everything has mixed up and confused”; “everything is fallen down and does not make sense”; “a mixture of formulas”; “a heap of theorems in my poor head” etc.

In Russian vocabulary the clearness acts as a synonym of definiteness, sharpness - alongside with such characteristics as: exact expression, intelligibility, articulateness, transparency etc. Among meanings of the word “clear” are: “light, not shaded”; “well seen, heard or perceived, understood”; “good organized, precise”.

The clearness of students’ perception of the contents of mathematics is determined by the transparency of its representation (e.g. on a blackboard); by the absence of “noise”, i.e. insignificant symbols, of excessive variety of notations, of the interference of symbolical systems (vector, coordinate etc.).

The requirement of clearness of speech is always urgent. The great attention in rhetoric was given to this from ancient times.

The speech is called clear, if it is perceived without difficulties.

The clearness of speech of the teacher of mathematics usually is connected with its accuracy (inambiguity of mathematical terms, adequacy of used words to their meanings, taking into account the character and volume of the speaking competence (vocabulary) of students (the strict selection of the necessary minimum of terms), explanation of new terms (with the help of comparison, visual analogy, translation to the simple language with the help of synonyms, organization of a context etc.), exception of verbosity, superfluous words and long periods of silence.

How is the character and volume of the vocabulary of the students taken into account in the teaching of mathematics?

Many teachers with wide experience of work can formulate the answer to this question. The more various concepts (not contained in the vocabulary previously), and concepts of abstract character are encountered by the students, the less is the clearness of such speech. A.Ya. Hinchin (1963) indicated also to the necessity of literary elaborateness, accuracy of expressions.

“In order to become clear, a new concept should be precisely expressed, distinct from known concepts, correlated to a wider concept, precisely structured, considered in the logical relations with other concepts (equivalence, contrariness, equivalence, subordination, contradiction etc.).

One of the basic principles of teaching is its scientific character, and it is not reduced only to the requirements of strictness of the language of a discourse, clearness of concepts, consistency, completeness and provability of statements. It assumes that the arrangement of a material should correspond to the psychological features of the cognitive activity of the students. The known classification of cognitive actions includes:

actions of perception (to notice, to distinguish, to identify, to compare, to determine the degree of explicitness of an attribute etc.);

actions of imagination (to mentally transform an object etc.);

- Logical actions (reasoning, conclusions, generalization etc.);

What can do a teaching of mathematics in order to make the subject matter, its rules clearly perceivable, easily noticeable, precisely distinguishable, recognizable in other contexts etc. What efforts can a teacher undertake for to achieve clearness of a material to the students? She/he “explains”, “finds out”, “clears up”. Each of these methodical actions has the semantical peculiarity. To explain means to represent, to transfer clearly, completely, fully, to make clear as a whole. To find out means to establish together with students the essence and the nature of subject matter, to reveal it, to outline, to represent sharply etc. To clear up means to make unclear clear.

It is known that a person perceives those objects on which her/his attention is attracted. Therefore, the clearness of perception in many respects depends on the organization of attention during teaching. In psychology of attention the metaphor of “searchlight of attention” is used as a device that consequently lights up different parts of some area. This metaphor can be rather fruitfully used can be enough fruitfully used for the design of the methodical toolkit for the teaching of mathematics at universities.

Neuropsychological studies (General psychology, 2006) show that the shift of the “searchlight of attention” consists of three operations:

- 1) distraction, release,
- 2) actually movement,
- 3) attraction, “catching” of attention.

Quite often during the learning, unfortunately, occurs the distraction just from the object of cognition, and the “searchlight of attention” is directed not on the notes of a lecture or on the textbook but on other things - a window, a fiction book, a newspaper, a crossword puzzle etc. If the

attention is released from the object of cognition, the clearness of perception is impossible.

However this initial moment in teaching quite often is neglected.

It is known from the psychology of attention that the beam of the “searchlight of attention” is indivisible. Depending on a task the “illuminated” part of a visual field either is narrowed, or is expanded, but never split. It means that in the teaching of mathematics, the teacher should think not only about the mathematical content and tasks, but also about the statement of cognitive tasks to students on each moment of teaching. For example, the statement of a cognitive task can be expressed in the formulation of the following cognitive purposes:

determine distinctive attributes of one concept in comparison with another;

establish the basic parameters and basic dependence between concepts used in the formulation of a theorem;

determine conditions of a theorem and its statement etc.

The teaching should have epistemological support adapted to the cognitive of activities of students, their attention, perception, thinking. Just the cognitive task in many respects determines the narrowing or expansion of a the “illuminated” (by the “searchlight of attention”) area of a visual field, degree of the distinctness of objects in that area, i.e. clearness.

In the recent history of psychology of attention there is a phenomenon of “blindness by the inattention”, i.e. functional blindness, which consists in inability of the observer to apprehend clearly distinguishable stimulus, if her/his attention is engaged in the analysis of other stimulus presented simultaneously with the given or prior to it. What the student will notice and what will she/he not notice if several target objects will be presented in the visual field simultaneously or with an insignificant interval of time? On what the attention will be focused, if “searchlight of attention” “is not split”? What interval of time of presentation of objects will be better for the clearness of their vision? Based on psychology of attention mathematics to develop, it is necessary to develop appropriate methods of teaching of mathematics for more effective organization of attention of the students at for the maintenance of clearness.

The second operation in moving of the “searchlight of attention” is actual movement. Research of B.G.Ananyev (1977) has shown a role of the relation “horizontal – vertical” in the cognitive movement, in the shift of attention. It is the necessary to investigate how the “movement” of attention in a visual field takes place.

The third operation is the attraction or “the catching” of attention. Quite often teacher in the process of teaching draws the attention of the students not to the central moments, but to minor, collateral things, withdrawing thus the “searchlight of attention aside, fixing it not on the essence, which remains not clear. It is necessary to find out how to determine and keep the central moment in a topic, not replacing it by superfluous illustrations, how long it is possible to keep attention of the students on one question, without loss of concentration.

Clearness of perception, the size of the “illuminated areas” perceived by the student, the characteristics of her/his “searchlight of attention” depend on the organization of her/his cognitive activity by the teacher, from the efficiency of the management of students’ attention. The order of presentation of cognitive objects, their size, grouping, arrangement in the visual field, sharpness, distinctness (visual and semantic) can influence the clearness of perception. The order of the arrangement can be determined by the cognitive (or motivational) importance of objects for students, the size of objects, their semantic connections, possible degree of their familiarity to students.

The features of visual perception (recognition) of mathematical objects (sizes, parameters, dependencies, formulas etc.). Students and the teacher are main subjects of the process of study and from their mutual understanding each other in many respects the productivity of the study depends. It is necessary to distinguish the clearness of the explanations of the teacher and the clearness of the representation in students’ minds.

Four relations are possible: the teacher explains clearly and the student understands clearly, the teacher explains clearly, but the student perceives unclear representation, the teacher explains not clearly, but the student perceives a clear representation, the teacher explains not clearly and the student represents the subject matter not clearly.

In the first case the scheme of explanations is strongly close to the scheme of the perception by the student.

In the second case the scheme of the explanation does not fit the student's scheme of the perception. The orientation of students' attention is not developed enough for making unclear clear independently.

In the third case the student clears up the information communicated by the teacher with the help of her/his own hidden orientation of attention that is superiority to the scheme of a discourse of the teacher). Such students manage their attention, they can also sometimes better than the teacher reconstruct the missing organization of visual attention in a visual field. At last, the fourth case, most adverse for the teaching - the student can not or does not try to make unclear clear. The reasons of that case may be the following:

loss of educational motivation, rejection of a perceived material because of its ambiguity;  
absence of the students' own strict orientation of attention (anyway it developed enough for making unclear clear), though the educational motivation is not lost;  
insufficient development of student's skills of the independent analysis and search (i.e. skills of mental activities);  
absence of student's of methodological knowledge, which would allow her/him to have some general invariant, some kind of the scheme of perception.

What are the criteria of clearness? From above-stated, one can deduce the following features:

Distinctness;

Dissolution into parts (for example, of the course of reasoning);

Demarcation (for example, of the different symbolical systems: algebraic, vector, geometrical, trigonometrical);

Adequate orientation of attention, completeness of its scope for the basic objects;

Proper order of concepts and designations,

Sharpness, structure etc.

Important in the consideration of clearness is the question of its levels. According to the theory of learning activity, the mastering of knowledge occurs through the interiorization of knowledge externally developed in material or materialized form. On this basis it is appropriate to consider four levels of clearness of a mathematical contents for students:

The external clearness (possibly even only local), that means that externally everything is clear (precise, distinct), but the interiorization is complicated, the opportunities of translation of knowledge into the internal side are limited;

Clearness of the own scheme of mastering of that material, which is stated,

Internal clearness of the acquired material;

Clearness of expression (reproduction, interpretation and use) of the acquired knowledge.

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