Creative mathematical activity of the students in the model of differentiated teaching in Russian Federation

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Abstract. In this paper, creative mathematical activities of school pupils in conditions of the differentiated teaching in Russian Federation are described. Various forms of differentiated teaching (internal – level, external – profile) are characterized. Ways of using entertaining problems for detecting and fostering mathematical abilities are revealed. New course of geometry for differentiated teaching is introduced.

Creativity and differentiated teaching.

Creativity is one of the most important concepts in mathematics education. The creative approach is understood as certain abilities and readiness of a person for creating something new. The purpose of educational process at school is the education of a person who would use creatively approach for solving scientific or practical problems, would think independently and critically, a person able to develop and protect her/his opinions and beliefs, regularly acquire new knowledge by self-education, increases her/his qualifications, creatively use them for changing real world.

Among various school subjects, the mathematics is the best one for promoting the development of creative thinking, for encouraging the creative approach to the life in pupils.

In this paper we will consider the ways of promoting creativity in conditions of differentiated teaching.

There are various kinds of the differentiated teaching. Researchers distinguish the following kinds of the differentiation of teaching: internal (by level), external (profile differentiation), wide differentiation, diagnostical differentiation and continuous differentiation.

The internal, or level, differentiation of teaching is very important, as its means and methods penetrate entire teaching process at school.

G.V.Dorofeyev et al. (1990) interpret the level differentiation as such system of teaching at which each pupil, having acquired some minimum of the general educational preparation (being universal and providing opportunity of adaptation in constantly changing living conditions), gets the right and the guaranteed opportunity to pay primary attention to those directions which to the greatest degree answer her/his inclinations. Thus the level differentiation is expressed as follows: being taught in one classroom, under the same syllabus and the textbook, pupils can acquire the knowledge at various levels. Note that such internal differentiation is present also in all forms of external differentiation because at the level of the "selected" pupils their individuality also plays the important, and it is simply impossible to neglect it.

The question about level differentiation is closely connected with a problem of planning of obligatory results of teaching and with educational standards.

In our view, the elaboration of precise and practically usable standards for all school subjects still requires a lot of efforts.

It is possible to study and introduce various means of internal differentiation of teaching. However, the main complication here is connected with the coordination of mass forms of teaching and individual character of developments of pupils, of processes of mastering and application of knowledge.

Thus, the "internal differentiation" comes to the foreground. This kind of the differentiation of teaching is carried out in conditions of usual daily learning in a classroom. It is focused on all pupils and based leaning on their individual possibilities, needs and abilities.

Textbooks on the mathematics used in XX-th century, did not take into account mechanisms of level differentiation in educational process. They textbooks taking into account such mechanisms start to appear now. For example, authors include texts for individual reading (Gusev, 1999 and 2002, Sharygin and Erganjieva, 1992), much effort is applied for the differentiation of system of problems, etc.

The external, or profile differentiation of teaching in secondary school assumes promoting pupil's opportunity to receive education in various directions, under different curricula and syllabuses. A version of profile differentiation is profound studying of subjects. Note that this direction of profile

differentiation in our country has the wide experience, described in a lot of interesting books. The prominent mathematician, academician M.L.Lavrentyev wrote about the profile differentiation: “In fact the teacher teaches not an abstract pupil, but a quite concrete person with certain inclinations and abilities. I think, that it is useful as early as from the seventh grade to begin the differentiation of teaching on interests and propensities, that is to offer to pupils several optional courses... (Kobzev and Gorbachev, 1981).

Recognizing and fostering the creativity by solving entertaining problems.

An efficient way for recognizing and fostering the pupils’ mathematical abilities is solving entertaining problems, as their contents is usually clear for pupils at initial stages of studying mathematics. In the structure of these problems the exhibition of such parameters of mathematical abilities as sharpness, ingenuity, curiosity and inquisitiveness is incorporated. The mankind accumulated the huge quantity of problems useful as a material for revealing mathematical abilities, for the satisfaction of demands of the pupils possessing these abilities, and in general for the exhibition of the fascination of mathematics As a rule, these are not problems usually solved at school at the base level of mathematical education. It is a pity that these interesting, fascinating problems are insufficiently included in textbooks of this base level.

When there appeared the necessity to teach and learn mathematics, the mankind first of all addressed to entertaining problems and to mysterious stories: “To learn playing” was the first methodical instruction. This approach very much fits to the modern elements of the theory of motivation of teaching.

The popular author of books about mathematics Y.I.Perelman (1973) considered one of features of the entertaining science which, in its opinion, consists in that “its means do not exclude work of mind, rather, on the contrary, stimulate the work of thought”. Really, “brainwork is closely connected with acquisition of knowledge, and the entertaining science does not struggle to release from it at all. It helps only to make this work interesting, and, consequently, also pleasant, trying to deny a thousand-year old motto about “a bitter root of learning” (Ibid.).

Unfortunately, in practice of school it is not stipulated, to solve problems of entertaining character directly at the lesson (there is no direct indication in the curriculum, there are no recommendations in the educational literature, there is almost no appropriate material in textbooks). The teacher can independently decide to use or not to use such tasks, but “in fact for the majority of the people interested in mathematics, the first impressions of this science are connected with problems or books of entertaining character” (Ibid.).

In modern works of psychologists and researchers in mathematics education devoted to the study of cogitative mental activities in the process of acquisition of mathematical knowledge, not only the certain positive attitude to the entertaining mathematical tasks is expressed, but also the attempt to give the psychological and-pedagogical characteristics of various kinds of non-standard problems is made. The psychological characteristic of an entertaining mathematical material (problems-puzzles) can be found in S.L.Rubinshteyn's works about the process of thinking. Describing the role of the processes of the analysis and synthesis in the solving of entertaining problems, S.L.Rubinshteyn (1958) argues that “so-called problems-puzzles are not the special funny thing, standing separately from the general laws of thinking... They are connected with the general laws of thinking”. Defining thus the nature of these problems, S.L.Rubinshteyn emphasizes the similarity of puzzles to creative tasks because both are composed on the basis of knowledge of laws of thinking, and he also draws attention to the fact that the essential conditions leading to the solution in puzzles are usually disguised by the non-essential circumstances: «... The puzzle arises when its statement specially emphasizes circumstances for insignificant its solution so that essential conditions of a problem become disguised, covered by insignificant, external circumstances» (Ibid.).

Characterizing the psychological side of the process of solving puzzles, S.L.Rubinshteyn emphasizes the role of the analysis in their solution, the role of a guess as the organic part of the process of thinking. On the basis of his experiments S.L.Rubinshteyn's revealed the “secret” of the occurrence of a guess during the solving. The guess is usually preceded by the careful analysis,

distinguishing the essential conditions in a problem: «... In essence, a guess is a product of the analysis staying behind it» (Ibid.). Thus, in the opinion of S.L.Rubinshteyn, the solution of problems-puzzles occurs as a result of the precise analysis of their conditions during which search of the ways of solving is carried out.

Among few works devoted to entertaining and non-standard problems, worth of attention is a series of experimental studies directed by A.N.Leontyev (1954).

A.N.Leontyev put a problem of finding of a specific part of mental activity consisting of the emergence of a guess, of the idea of the solution. Experimental works performed under his direction were aimed at finding out the conditions under which "the experience of the pupil leads him to the correct solution based on the guess" (Ibid.).

In the research devoted to the analysis of problem solving, G.Polya (1957) distinguishes "problems on finding" and problems on proof [177]. The purpose of problems of the first kind is to determine an unknown element of a problem. G.Polya considers puzzles as problems on finding. For solving them he recommends to use the general rule for all problems on finding: to understand a problem, to choose, recollect auxiliary problem, to solve a part of a problem, to keep only a part of conditions, omitting the rest.

B.L.Kordemsky (1958) emphasizes special significance of problems-puzzles in developing essential elements of mathematical thinking in pupils, promoting the aspiration to search independently ways and means of solving a problem; fostering pupils' ingenuity, flexibility and criticality of mind. Thus, problems of entertaining character can serve as the tool for revealing parameters of mathematical abilities of pupils and a good mean for stimulating pupils' interest to studying mathematics.

Taking into account the wide variety of various kinds of entertaining, joyful problems, for the maintenance of their purposeful and effective use some classification of entertaining problems would be useful. Consider some classifications available in the methodical and mathematical literature.

G. Lenhauer (1940), telling about the hall of mathematical entertainments in Saint Petersburg, shows sets of various entertaining mathematical problems. They are divided into following groups:

Problems not demanding very little mathematical knowledge and based on the ingenuity and a guess.

1. Problems demanding elementary mathematical knowledge or forcing to recollect knowledge once received at school.

3. Questions and problems which aimed at checking mathematical knowledge of a pupil. These are mainly unexpected comparisons and conclusions, sometimes paradoxes etc. This group is divided into three series according to the age of pupils.

4. A series for enthusiasts of difficult and witty mathematical problems. These problems demand for their solving decent mathematical preparation, however not exceeding the volume of a school course.

5. Problems for children of 8-12 years.

6. Problems-jokes, mathematical focuses and entertainments.

M. Gardner (1978) has divided problems into six categories: combinatorial, geometric, number-theoretical, logical, procedural and verbal.

He notes that these categories of problems are not disjoint, they inevitably intersect with each other.

B. Kordemsky (1958) distinguishes two categories of problems for out-of-class activities.

1. Problems close to the school course of mathematics, but of the increased difficulty – this is the kind of problems usually offered at mathematical Olympiads.

2. Problems of the type of mathematical entertainments. B.L.Kordemsky wrote about the second category: "The second category of problems for out-of-class activities (very mixed in their contents) does not have the direct connection with the school program and does not assume the strong mathematical preparation" (Ibid.)

Logical and geometrical problems can be successfully used for revealing and developing the mathematical abilities of pupils. From our experiments, we deduced following conclusions:

1. It is important to reveal pupils' skills to make conclusions from the problem's conditions.

2. It is important to track and develop the activities of the type "synthesis through the analysis". Pupils, making conclusions, are guided by an ultimate goal this process goes. However, this process goes chaotically, it is not subordinated by the general idea, but only uses rather standard analysis.

3. It is important to find out, whether the pupil is able to plan from the very beginning the strategy of the solution - to put forward an idea and to apply non-standard methods and means to the solution.

4. Often enough, planning the strategy of the solution, pupils choose already known, often used ideas. It is interesting to track a birth of a non-standard idea (during the process of the analysis through synthesis).

5. It is important to develop the pupil's skill to use analogies.

New course of geometry.

In our new geometrical course for the secondary school (Gusev, 1999), the process of development of means of thinking activity is carried out through a specially selected system of exercises. Thus the directedness of this system of exercises on shaping of analytic and synthetic activity is determined not only by the contents of these exercises, but also by structuredness of questions, tasks and pieces of knowledge distributed in six groups, each of which has precisely determined purposes. The system of exercises begins with two groups of problems: [↑] - exercises on development of skills to make correct conclusions of a condition of a task (to get corollaries) – this is a kind of synthetic activity; [↓] - exercise on finding out the reasons of a property of an object, i.e. the detection of characteristics of an object: this is one of the foundations of analytical activities. These two groups of exercises constitute the foundation of all thinking processes, all reasonings and proofs.

In each section of a text-book, there are other two groups of tasks, designated by characters [T] and [I]. [T] is a group of tasks, which have somewhat higher level of difficulty, namely tasks aiming in the development of the creativity in pupils. The sign [T] includes the character T which is the first character in a word «creativity» (tvorchestvo), and one may name these tasks "creative".

[I] designates the last group of problems (or tasks), the number of which will be not so much; they require more effort for the solution and assume doing some small research (sometimes this research can connect various themes of a course and even of various courses). Such problem tasks can not be completely solved in a classroom, and assume serious homework (sometimes, of several pupils). The character I is the first character in a word «research» (Issledovanie), and the meaning of these problems is "research" tasks.

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