TO TEACH COMBINATORICS, USING SELECTED PROBLEMS

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Abstract: In 1972, professor *Grigore Moisil*, the most famous Romanian academician for *Mathematics*, said about *Combinatorics*, that it is "an opportunity of a renewed gladness", because "each problem in the domain asks for its solving, an expenditure without any economy of the human intelligence". More, the research methods, used in *Combinatorics*, are different from a problem to the other! This is the explanation for the existence of my actual paper, in which I propose *to teach Combinatorics, using selected problems*. **MS classification**: 05A05, 97D50.

INTRODUCTION

Combinatorics, a distinct and special branch of *Mathematics*, appeared as a consequence to solve *counting problems*. The reasoning, used in it, is less analytic, because this is made in a primordial way, like as in *Metalogic*.

We explained in [2] and [3], and now we shall reiterate, that a rigorous and educated thinking in *Mathematics* must begin learning *Enumerative Combinatorics* and then continuing with *Combinatorial Probabilities*, because they represent together, the simplest, but the very refined description of the daily life. More, we must not forget that today, by its new branches as: *Graph Theory*, *Matroid Theory*, *Code Theory* etc., *Combinatorics* is the most dynamic domain in *Mathematics*, yearly having the biggest number of proposed and solved conjectures.

In the actual Romania, *Discrete Mathematics*, and therefore *Combinatorics*, are ignored effectively. So, the precarious education of the students, in this fundamental domain, is reduced only to wield simple algebraic relations, usually known as *"formulas"* and which never permit them, to use correctly, the *arrangements*, the *combinations*, the *permutations* or the *number of the functions* defined on finite integer sets.

We remember here (see [2], [7]), that for a given set $S=\{x_1, x_2, \ldots, x_n\}$, with the *cardinal* $|S| = n$, we have:

i) the *permutations* are different between themselves, *only by the order* of the elements, their number being $n!$;

ii) the *arrangements* of the groups with m elements from n, are different between themselves, *by the order and the type* of the elements, their number being $A_n^m, n \geq m$;

iii) the *combinations* as the *number of the subsets in S*, having m elements, are different themselves *only by the type* of the elements, their number being $C_n^m = \binom{n}{m}$, $n \geq m$;

iv) the *number, of the functions* $f : S=\{x_1, x_2, x_3,\ldots, x_n\} \rightarrow T=\{y_1, y_2, y_3,\ldots, y_m\}$, is m^n.

When we begin to teach *Combinatorics*, we must insist in front of our students, about the *essential role of the words used in the texts* of this domain. Only in this way, we realize the achievement of the best knowledge in *Mathematics*!

For our study in teaching *Combinatorics*, the start point was relating to every student, even if in the high-school course, who might wield without no difficulty, all the notions of this domain. To arrive to our aim, I used a very popular Romanian *Collection Book for Algebra* (see [1]), which in the last 30 years, is the most recommended to the students in the high-school course. Although its exercises and problems are chosen to train the students in any branch of *Algebra*, I noticed astonishingly, that for *Combinatorics*, there are a lot of problems, bad enounced or bad solved …

RESULTS

At the 10-th chapter of [1], in *Problem 25*, we found the same error as in 1981 edition (appeared at „Didactică şi Pedagogică" Printing House). The text of the quoted problem is the next:

(1) "For a game, 3 boys and 5 girls are organized in 2 teams, heaving 4 persons each. In how many manners are organized these teams ? Find the number of the situations, if in each team with 4 persons, we must have only a boy".

We notice for the solution, that 2 teams heaving 4 persons each, will use the subsets with 4 elements, from a set with cardinal 8. So, the first team is chosen in C_8^4 ways and the second in C_4^4 ways. It follows, that for our 2 teams hawing 4 persons each, there are $C_8^4 \cdot C_4^4 = 70$ possibilities.

Normally, the second part of the problem (1) is a *nonsense*! Indeed, if in each team there is only 1 boy, for the first team we need 1 boy and 3 girls, and for the second we have 1 girl less, respecting the given conditions! From here, we state that for the second part of (1), the correct enunciation must be:

(2) "In how many manners are organized 2 teams having 4 persons each, if in any of them, it must be at least, 1 boy ?"

The first team, containing 3 girls and 1 boy, will be chosen in $C_5^3 \cdot C_3^1 = 30$ manners and the second team, hawing 2 girls and 2 boys, will be chosen in $C_2^2 \cdot C_2^2 = 1$ ways. Finally, we decide that the solution of (2) is given by the product $\left(C_5^3 \cdot C_3^1\right)\left(C_2^2 \cdot C_2^2\right) = 30$.

In [4], I proved as a generalization of (2), that $2n-1$ boys and $2n+1$ girls can form in $(4n)!/((2n)!)^2$ possibilities, 2 teams having each, $2n$ persons. If we impose that in each team to be at least 1 boy, the possibilities are given by:

$$\text{(3)} \qquad \sum_{k=1}^{n-1} C_{2n-1}^{k} C_{2n+1}^{2n-k} = \left(C_{4n}^{2n}/2\right) - (2n+1),$$

a sum which asks a *strong work* to compute it!

We shall go on, our incursion in teaching *Combinatorics*, correcting the solution of *Problem 7* of the 10-th chapter from [1], in which the number m^n, of the functions $f : S \to T$, is badly used! The text of this quoted problem is:

(4) "Find the cardinal of the set containing 5 digit numbers which could appear using the even digits from the set $\{0,2,4,6,8\}$."

The solution of (4) is based on the following two cardinals:

$$\text{(5)} \qquad \begin{aligned} &\left|\{g/g : \{d_1, d_2, d_3, d_4, d_5\} \to \{0,2,4,6,8\}\right| = 5^5, \\ &\left|\{h/h : \{d_2, d_3, d_4, d_5\} \to \{0,2,4,6,8\}\right| = 5^4, \end{aligned}$$

where d_1, d_2, d_3, d_4, d_5 are digits and where, for the functions *h*, we must exclude the situations which begin by $d_1 = 0$. Now, the number asked in (4), is $5^5 - 5^4 = 4 \cdot 5^4 = 2500$.

I wanted to correct the error for the solution of (4), appeared in [1], because through this manner, I understand to sustain one of the first *counting principle* appeared in *Mathematics* and having a very strong importance in *Algebra*, *Combinatorics* and *Discrete Probabilities*. Emphasizing the great utility of this *counting principle* which must remain *well stamped upon the mind* of every person interested in *Mathematics*, I proposed in [5], as a good training, the following generalization of (4):

(6)

"i) Find the cardinal $E(n), n \in \mathrm{N}^*$, of the numbers with *n* digits, formed only with the even digits.

ii) Find the cardinal $O(n), n \in \mathrm{N}^*$, of the numbers with *n* digits, formed only with the odd digits.

iii) Find $n \in \mathrm{N}^*$, so that $E(n) + O(n)$ would be a perfect square.

iv) For the cardinal $N(n), n \in \mathrm{N}^*$, of the number with *n* digits, formed with all digits from $\{0,1,\mathrm{K},9\}$, find $(N(n), O(n) + E(n))$.

v) Find $n \in \mathrm{N}^*$, so that $\dfrac{N(n)}{O(n) - E(n)} \equiv 0$ (mod 144)."

For the interested reader, we note here, the answers of (6):

(7) $E(n)=4\cdot 5^{n-1}$, $O(n)=5^{n}$, $E(n)+O(n)$ is a perfect square if $n=2k+1, k\in \mathrm{N}$, $(N(n), E(n)+O(n))=2^{n-1}, \dfrac{N(n)}{O(n)-N(n)}\equiv 0$ (mod 144) only if $n=4k+1$, $k\in \mathrm{N}$.

In the end of this proposed way *to teach Combinatorics using different problems*, I come with another example (see [6]) relating to the number of the functions $f: S\to T$:

(8) "i) Find the set $A=\left\{n\in \mathrm{N}^{*} \Big/ \dfrac{n}{n-2001}\in \mathrm{N}\right\}$.

ii) Find the number of the functions, *f : B→ B* where :

$B=\left\{n\in \mathrm{N} \Big/ \dfrac{n^{2001}}{n-2001}\in \mathrm{N}\right\}$."

The solution of (8) is based on:

(9) $A=\{1972,1978,1998,2000,2002,2004,2024,2030,4002\}$,

and on the fact that for *B*, after the division, we must impose the conditions:

(10) $\dfrac{2001^{2001}}{\mathrm{n}-2001}=\dfrac{3^{2001}\cdot 23^{2001}\cdot 29^{2001}}{\mathrm{n}-2001}\in \mathrm{N}\Leftrightarrow \mathrm{n}-2001=3^{\alpha}\cdot 23^{\beta}\cdot 29^{\delta}\Leftrightarrow$

$\Leftrightarrow \alpha,\beta,\gamma\in\{0,1,\mathrm{K},2001\}\Leftrightarrow |\mathrm{B}|=2002^{3}.$

It occurs that the number, of the functions *f : B → B*, is : $2002^{3\bullet 2002^{3}}$

We invite the interested reader to see [2], for other properties and problems about the *surjective* or *injective* functions defined on discrete and finite sets.

CONCLUSIONS

Each notion in *Combinatorics* must be taught by the intermediate of a well chosen problem, exactly as I tried accomplishing in the anterior paragraph. This is the unique way in which we can incite the curiosity of the students, either in the high-school course, or in the university course, all for a difficult domain, which is very different from the others in *Mathematics*. Finally, I underline that immediately, after the moment when a good understanding was established in the mind of our students, for these new notions, which must be well fixed, we also apply for … *selected problems in Combinatorics* …

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