# Identifying Modelling Tasks

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**Abstract**
The Comenius Network Project "Developing Quality in Mathematics Education II" funded by the European Commission consists of partners from schools, universities and teacher training centres from eleven European countries. One advantage of the project is the mutual exchange between teachers, teacher trainers and researchers in developing learning material. To support the teachers most effectively the researchers asked the teachers what they wanted the researchers to do. The answer was also a question: How can we identify (good) modelling tasks? A discussion ensued in the research group of this project which resulted in a list of descriptors characterising modelling tasks. This paper focuses on the theoretical background of mathematical modelling and will thereby substantiate the list of descriptors for modelling tasks.

**Introduction**

The work in the Comenius Network "Developing Quality in Mathematics Education II" has one main focus on the development and evaluation of modelling tasks. The idea was that teachers and researchers would develop such tasks in mutual exchange. This is currently taking place. One way of doing this is that the teachers develop tasks and the researchers analyse them theoretically to discuss whether the task is a modelling task or not. To make this easier for the researchers they agreed on a list of descriptors to characterise modelling tasks.

To make the descriptors for the list more explicit different theories about modelling (Blomhøj, Jensen, 2006; Blum, Leiss, Borromeo Ferri, 2006; Greefrath, 2007) which underlie the developed descriptors will be discussed. Further on the descriptors will be compared to lists of modelling competencies from Blum and Kaiser (according to Maaß, 2006) and Ikeda and Stephens (1998), and discussed in this paper.

First three different modelling circles will be described. The last one was the basis for the list of descriptors, thus they will be explained afterwards.

The descriptors will then be compared to different theories about modelling competencies.

In the third part the idea of a checklist for teachers based on the list of descriptors will be presented and discussed. This checklist shall help teachers, especially those at the very beginning of their teaching, to identify and create their own modelling tasks. In the first approach teachers agreed that such a checklist is helpful.

## Different models of mathematical modelling

The basis of mathematical modelling is always a real life situation with which pupils have to deal with mathematically. In literature many different models about mathematical modelling can be found.

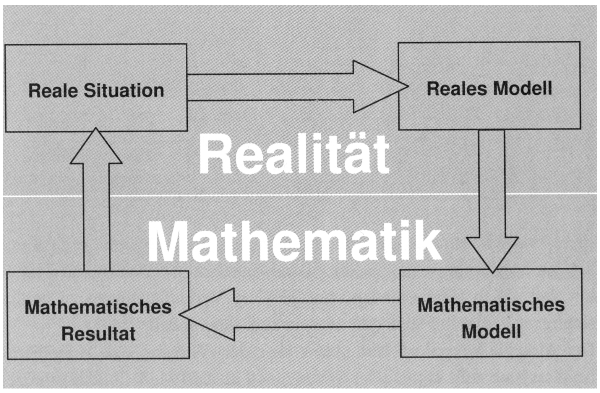


Fig.1: Modelling Circle Greefrath (2007)

The first model that will be presented is from Greefrath (2007): It starts with a real situation (Reale Situation). This is not the whole reality from which a situation must be chosen, but an already structured situation from real life (Realität). This should be transformed into a real model (Reales Modell). This real model is a simplified and structured version of the real situation.

This can now be transformed more easily into a mathematical model (Mathematisches Modell) than the initial real situation. The mathematical model should now lead to a mathematical result (Mathematisches Resultat) which has to be set in relation to the real situation. The starting point is a real situation which obviously must be chosen by someone (e.g. the teacher or the pupils) to deal with mathematically. The transformations between the four stages are not named in this model and are unidirectional.

A second model for the mathematical modelling process can be found in Borromeo Ferri, Leiss and Blum (2006). This model is not the first one developed by Blum, however, it is the current one.

This model starts with a real situation (Realsituation) which means the same as the "Reale Situation" in Greefrath's model. From this real life situation, a model (Situationsmodell) results by constructing and understanding real life (1). This model must be structured in the next step to get a simplified model of the real situation (Reales Modell).This simplified model can now be mathematised into a mathematical modell (Math Modell). With this step you go from the real world (Rest der Welt) into mathematics. By doing some mathematical calculations a mathematical result will be produced. In the fifth step you have to interpret these results to get real results, which may fit to the starting real life situation. Checking if they really fit to the situation is the next step. In the seventh step the results are presented. This model includes the description of the transformations from one stage to another. The arrows representing the transformation point all in the same direction. What can also be seen very well in this model is that the modelling circle is the connection between the real world (Rest der Welt) and mathematics (Mathematik).

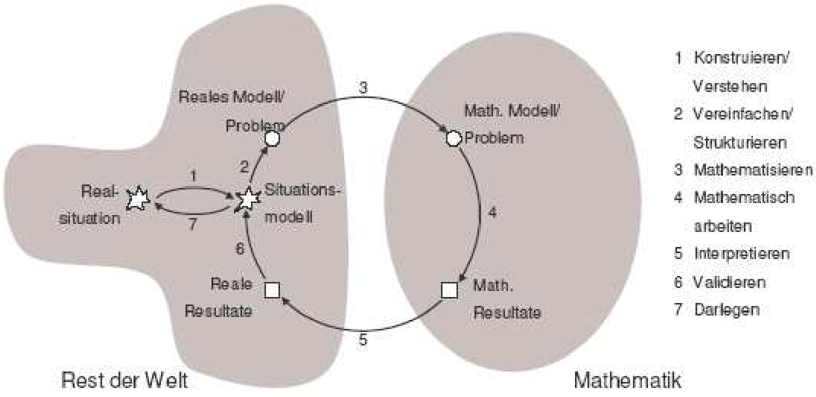


Fig. 2: Modelling Circle Blum (2006)

A third model, which was the basis of the first discussion during the first project meeting, is the model of the mathematical modelling process by Blømhoj & Jensen (2006) This model is very similar to that from Blum. The main difference is that the perceived reality (real life) is part of the circle. From this perceived reality, the motivation to deal mathematically with a Domain of Inquiry results. This Domain of Inquiry is comparable to the real situation in both other models. If you have a look at mathematical modelling lessons in school this step has already been done by the teacher. But this must be something the pupils shall learn, too. The following stages in this model of the mathematical modelling process are similar to those of Blum's model. However, at the "end" of the circle there is another difference to Blum's model. Blum includes the presentation of the results, which is not a part of this modelling circle. Another difference is that the arrows in this model point in both directions. This shows what Borromeo Ferri found out in 2006 a bit more clearly: students do not follow modelling circles in a linear way, but you can find all stages in a complete and finished modelling process.

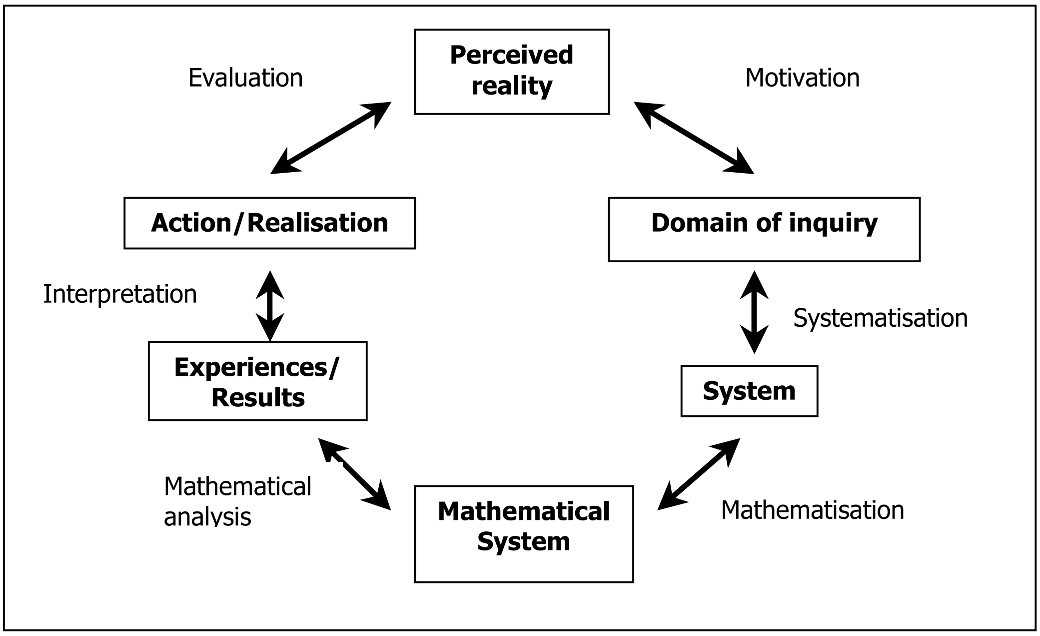


Fig. 3: Modelling Circle Blømhoj & Jensen (2006)

On the basis of the above discussion it can be concluded that the chosen modelling circle from Blømhoj & Jensen is a good basis for developing descriptors for modelling tasks. It will be shown in the following discussion how this modelling circle was simplified into four stages which could possibly be the descriptions for the arrows in Greefrath's model. Further, the model of Blum is very similar to the model from Blømhoj & Jensen. The only thing missing is the presentation of the results, which is also included in the list of descriptors.

## Descriptors for modelling tasks

To make the ideal model of a mathematical modelling process a bit clearer for teachers, it was simplified into four categories:

- Motivation,
- Systematisation and Mathematisation,
- Doing the mathematics and
- Interpretation and Validation.

These resulting topics were then filled with criteria (descriptors), which describe what the learning objectives mean in detail.

| **Learning objectives** | **Descriptors** |
|---|---|
| Motivation | Engagement (personal and societal) |
| | Teaching purpose |
| | Authenticity |
| | Linking existing mathematical knowledge |
| | Challenging |
| Systematisation & Mathematisation | Is data needed? |
| | Abstraction |
| | Assigning variables |
| | Making assumptions |
| | Simplifying |
| | Representation(s) |
| Doing the mathematics | Formalizing and analyzing the math problem |
| | Using data |
| | Approximation and estimation |
| | Use of Information and Communication Technology (ICT) |
| | Use known algorithms |
| | Mathematical common sense |
| | Proof (validation of the math used) |
| | Use of math. representation(s) |
| Interpretation & Validation | Validation of the solution mathematically |
| | Validation of the solution in the 'real world' |
| | Are the results good enough? |
| | Or is another cycle needed? |

Table 1: Results of the first meeting of the research group_1

In addition to that a list of Learning and Teaching styles, especially communication skills has been developed:

| **Learning Objectives** | **Descriptors** |
|---|---|
| Group discussion | Justifying |
| | Discuss and compare different strategies |
| Presenting the results and process | Oral presentation |
| | Written presentation |
| | Posters |
| | Reflection |

Table 2: Results of the first meeting of the research group_2

Although the used modelling circle did not include presenting the results, these learning objectives were also focuses of the discussion to develop a list of descriptors for modelling tasks.

## Comparison of the descriptors with theories about modelling competencies

In literature about mathematical modelling, lists about modelling competencies can be found.

Below you find a table which shows the comparison of the descriptors developed in our project with two concepts about mathematical modelling competencies. This comparison shall show that the descriptors not only include already existing descriptions about what modelling is but also expand these descriptions.

| **DQME II Descriptors** | **Modelling competencies by Blum and Kaiser (in: Maaß, 2000)** | **Competencies by Ikeda, Stephens: What are modelling competencies? (in: Galbraith, Blum, Booker and Huntley, 1998)** |
|---|---|---|
| Engagement (personal and societal) | | |
| Teaching Purpose | | |
| Authenticity | | |
| Linking with existing mathematical knowledge | | |
| Challenging | | |
| Is data needed? | to look for available information and to differentiate between relevant and irrelevant information | |
| Abstraction | to mathematise relevant quantities and their relations | |
| Assigning variables | to recognise quantities that influence the situation, to name them and to identify key variables - to construct relationships between the variables | Were relevant variables correctly identified? (G2) - Did the students identify a principle variable to be analysed? (G4) |
| Making assumptions | to make assumptions for the problem and simplify the situation | Did the students idealise or simplify the conditions and assumptions? (G3) |
| Simplifying | to make assumptions for the problem and simplify the situation - to simplify relevant quantities and their relations if necessary, and to reduce their number and complexity | Did the students idealise or simplify the conditions and assumptions? (G3) |
| Representation(s) | to choose appropriate mathematical notations and to represent situations graphically | |
| Formalising and analysing the mathematics problem | Doing the maths in common: to use heuristic strategies such as division of the problem into part problems, establishing relations to similar or analog problems, rephrasing the problem, viewing the problem in a different form, varying the quantities or the available data, etc. | Did the students identify the key mathematical focus of the problem? (G1) |
| Using data | | |
| Approximation and estimation | | |
| Use of ITC (software and graphics calculator) | | |
| Use of algorithms | | |
| Mathematical common sense | to use mathematical knowledge to solve the problem | |
| Proof (validations of the mathematics used) | | |
| Use of mathematical representations | | |

| | | |
|---|---|---|
| Validation of the solution mathematically | to critically check and reflect on found solutions; to review some parts of the model or again go through the modelling process if solutions do not fit the situation; to reflect on other ways of solving the problem or if the solution can be developed differently; in general, to question the model | Did the student successfully analyse the principal variable and arrive at appropriate mathematical conclusions? (G5) |
| Validation of the solution in the real world | to interpret mathematical results in extra-mathematical contexts; to generalise the solutions that were developed for a special situation | Did the students interpret mathematical conclusions in terms of the situation being modelled? (G6) |
| Are the results good enough? | | |
| Is another cycle needed? | | |
| Justifying | and/or communicate about the solutions | |
| Discuss and compare different strategies, Reflection | to view solutions to a problem by using appropriate mathematical language | |
| Oral presentation, Written presentation, Posters | and/or communicate about the solutions | |

Table 3: Comparison of descriptors and competency concepts

What is very noticeable is that the two competency concepts have nothing comparable to the motivation descriptors of the project list. On the one hand this is obvious because the question whether a task is authentic or not has nothing to do with competencies. On the other hand, it is a competency to choose or find authentic tasks for mathematical modelling. And this is not only a competency a teacher shall have, but also the pupils. So tasks shall also support the development of the competency to find mathematics in the real world.

Another difference to both concepts is that the focus “doing the mathematics” is not included in the competencies of Ikeda and Stephens and only included very generally in the concept of Blum and Kaiser. In my opinion “doing the maths” is a necessary competency for mathematical modelling, but it is also nothing characterising mathematical modelling, because it is also needed for example in problem solving.

Both differences found between the existing concepts and the developed descriptors support that the developed descriptors are good characterisations for the mathematical modelling process.

## Outlook - Checklist for teachers

On the basis of the above discussion there is a good theoretical background to prove the accuracy and usefulness of the named descriptors in the DQME II list. Furthermore it is an expansion of the already existing descriptions of mathematical modelling. The list of descriptors is used in the project for evaluating the developed tasks. This will be part of the oral presentation of this paper.

Another question I want to follow up on in future is: with the help of these descriptors, can a useful checklist be developed for teachers to identify modelling tasks or maybe some kind of “good” modelling tasks? Not every teacher has a research group to ask if the developed, found or modified task is a modelling task and can support modelling competencies of the pupils. They need a tool to check it themselves since they are used to creating modelling tasks.

A checklist has already been created and will be presented to teachers soon. The checklist and the opinions of the teachers will be discussed during the presentation of this paper.

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