Helping a Young Child Connect Fact Family Addition and Subtraction using Tools

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Abstract:

In order to help children become effective at addition and subtraction, it is important to provide them with an opportunity to investigate and discover the interconnectedness of the two operations. Fact families are one method teachers use to try and help children develop and understand how the operations relate to one another. This paper documents a strategy that was used with a seven year old boy to help him connect addition to subtraction. The strategy incorporated flash card tools to help him create logical problems to discover the mathematical relationship of fact families. With just a few trials, the child was able to create and explain problems that demonstrated the interconnectedness of fact families through addition and subtraction. The model was successful in helping the child advance his understanding. Additionally, it can be extended to more complex addition and subtraction problems as well as multiplication and division fact families.

Introduction

According to the National Council of Teachers of Mathematics (NCTM, 2000), pre-K-2 children need to understand the addition and subtraction of whole numbers including the linkage of the operations. More specifically the expectation states, "understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations" (p. 78). Because addition and subtraction are interrelated, children can use a known addition (or subtraction) fact to find an unknown subtraction (or addition) fact (Baroody, 1990). According to Clements and Sarama (2004), number and operations is one of the most important areas in mathematics education for young children. And Baroody (2004) asserts that numbers and operations constitute an important role in young children's daily lives and activities. Furthermore, he states: "understanding their applications is a basic survival skill in our highly technological and information-dependent society and, thus a key basis of mathematical literacy, which is now as important as language literacy" (p. 173)

In relation to early number development, there are six growth points in addition and subtraction (Clark et al., 2001) and children progress through these levels. The growth points: "(1) count all (2) count-on (3) count-back/count-down-to/count-up-from (4) basic strategies (5) derived strategies, and (6) extending and applying addition and subtraction using basic, derived and intuitive strategies" (pp. 3-4). Some of the more advanced strategies include derived strategies; the derived strategies contain fact families (Clarke & Cheeseman, 2000).

Schools sometimes teach the interconnectedness of the two operations with fact families (Cobb, 1987). Fact families are four number facts that are connected through two opposite operations such as addition/subtraction or multiplication/division (Peterson, 1990). In a traditional setting, the teacher will lecture and explain fact family problems, providing examples such as 7+4=11, 4+7=11, 11-7=4, 11-4=7. Then, he/she will ask the children to repeat the process with other fact family numbers (Peterson, 1990) with the hope that children will become more efficient with their calculations. Perhaps the teacher will also point out the pattern, stating that the two small numbers have to be added together to make the bigger number and that the bigger number has to be the first number in the subtraction problem. However, children can fail to make the connection, simply going through the motions of creating addition and subtraction problems with the numbers and procedure given without developing the relationship (Baroody, 1990).

Fact families are one of the many strategies that can be investigated by children to help them make advances in number and operations (Clarke & Cheeseman, 2000; Cobb, 1987; NCTM, 2000). The idea is that the fact families allow children to think about part-whole relationships while also helping them realize that subtraction and addition are opposites of one another (Cobb, 1987; Sun & Zhang, 2001; Zhou & Peverly, 2005). In addition, fact families guide children helping them see how an addition fact can be used to find a subtraction fact or vice versa (Sun & Zhang, 2001). Fact families can be taught in first grade (Zhou & Peverly, 2005) and the concept is at or above the average for most children in this grade (Phillips, Leonard, Horton, Wright & Stafford, 2003) while Burns, VanDerHeyden and Jiban (2006) describe single digit fact families as a skill for second grade.

Fact families take time to develop and are somewhat complex for children (Bryant, Bryant, Gersten, Scammacca & Chavez, 2008; Phillips et al. 2003). Children can fail to make the connection, just stating mathematical problems using the four numbers provided. In addition, they may make up problems using all four numbers that simply do not make mathematical sense. The interconnectedness of addition and subtract is difficult for children to understand. In a study by Baroody (1990), first graders who were given extensive training to help them understand the relationship of the two operations yielded disappointing results. Very few children were able to understand the relationship. Noticing the relation between combinations (whole – part 1 = part 2 and whole – part 2 = part 1) may be a key element (p. 170); this concept ties into fact families.

This paper describes the case of a seven year old boy who was taught fact families using a traditional, lecture-based approach. After two years of exposure, he failed to make any connection whatsoever of interrelating fact families with addition and subtraction. Realizing that tools had to be used, simple hand-made cards were created to successfully guide the child though the development of addition/subtraction fact families. This paper describes the model that was used while also extending the model to more complex mathematical ideas. It is the hope that this paper provides a fact family model to help teachers guide children by interconnecting addition and subtraction using the tools described.

The Case

An individual case was examined with a child who failed to learn fact families after numerous attempts over a two year period (grades 1 and 2 (age 6 and 7)). The child was attending a public school in California with the following demographics: 67% Caucasian, 25% Latino/a, 3% African American and 3% Asian. Eighteen percent of the population was deemed economically disadvantaged by the state. Three percent of the school population was English language learners.

The child was taught using the traditional (non-reform based) approach of using fact families to connect addition to subtraction, as previously described. Using this approach, the child was supplied with multiple triangles printed on a worksheet, and each triangle had three numbers in it that could potentially make up an addition/subtraction fact family, see Figure 1. With lecturing and modeling done by the teacher, he was still unable to come up with problems that made sense. Even though he was corrected and the problems were explicitly explained numerous times, the child clearly failed to make the connection. Sometimes, his problems would make mathematical sense (2+5=7) and other times the problems lacked awareness. He would write problems such as 5-2=7, or 7+5=2; he would use the numbers in the triangle and they were in the right form (addition and subtraction problems), but the location of the numbers was unreasonable. When asked why he made a problem such as 5-2=7, his response was that all the numbers had to be used in the making of the fact family problems. If he was pressed that the problem did not make sense, he would say that he had to use all the numbers and that is the only way he could do it using all the numbers. It was hit or miss, just a randomization of placing numbers in the correct blanks (___ + ___ = ___ or ___-___=___) as interpreted by the child from the teachers' explanations.

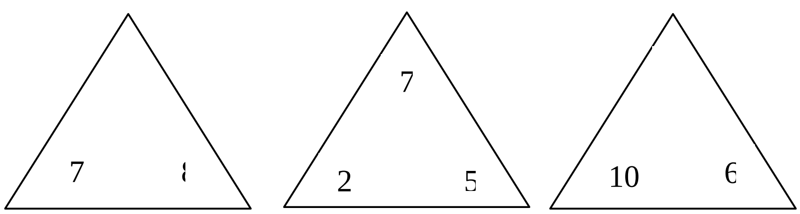

Figure 1. This is a figure of a fact family triangle supplied as a worksheet to the child in a traditional setting.

The Model

It was decided that the child could not learn the fact families without the use of tools to guide his development. He simply could not see the connection and needed to manipulate tools to understand the interconnectedness of addition and subtraction. Through brainstorming, individual tiles as tools with the fact families printed on them were settled on as they could be manipulated and moved to help the child visualize the mathematical problem. Standard, blank, small 3 ½ inch by 5 inch index cards were used. These cards were cut into smaller rectangles, see Figure 2. On each card, a number, operation or equal sign was written. As an alternative, plastic flat squares (inch flats) could have also been used. On each flat, a number or symbol could be written on a sticker or with a dry erase marker. For each fact family, the child was supplied with three rectangles, each with a single number from the fact family. Additionally, the child was supplied with an addition sign, a subtraction sign and an equal sign. The child was also provided with adequate think time; time to develop, build and reflect on the numbers and their relationship to the operations. He was supplied with paper and pencil to keep track of his equations.

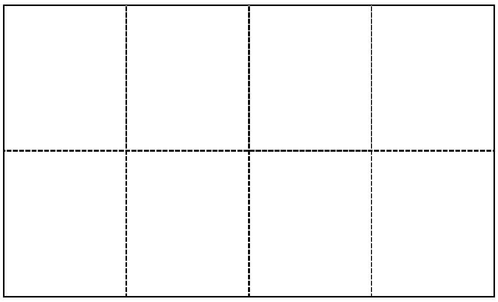

Figure 2. This is a sample showing how to cut the index cards.

The Results

With these six cards (7, 11, 4, +, -, =), the child was asked to make four different mathematics problems that made mathematical sense (see Figure 3). It took awhile; the first attempt at creating the problems took the child about 10 minutes. He created duplicate problems and was guided to find other problems that had not already been discovered. When the child created problems that did not make mathematical sense (7-4=11), the answer was covered up and he was asked to compute the first half of the equation (7-4). When he said the answer was 3, the 11 was then uncovered. He was asked, does what you arranged make sense? Why not? What can you do to fix the problem so that it makes sense? Once the mathematical errors were pointed out, he would rearrange the tiles again to try to find four different problems.

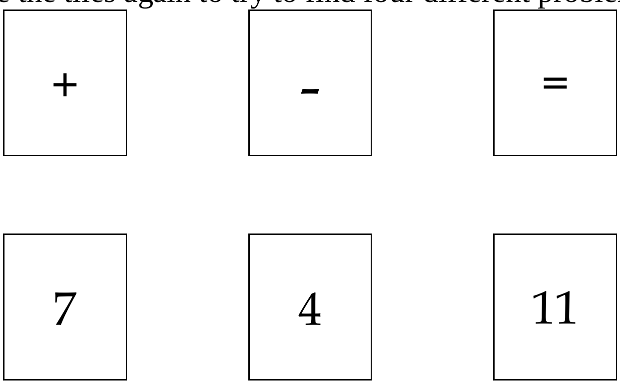

Figure 3. This is a sample showing how the index cards were created.

With each new set of fact family cards, he became quicker and quicker. After he was able to do this with three different sets of fact families and operation cards, he was asked to explain the pattern to making the four problems, as finding patterns may be a key to leading to fact mastery (Baroody, 1990). With this prompting, he was able to explain how the numbers connect to the creation of the problems. He recognized that it was necessary to add the smaller numbers to make an addition problem equal the larger number. As well, he was able to explain that the larger number had to be the first number in the subtraction problem because you are taking amounts away, making a smaller number. He was also able to explain how the operations connected to each other in relation to the fact family. All four numbers made four distinct problems that made sense and connected the two operations.

With the use of flashcard tools and adequate think time, the child was able to grasp the interconnectedness of addition and subtraction after just a few trials with different fact family sets. In addition, he was able to explain, rationalize and justify the problems he created. The knowledge extended beyond the use of the cards as well. The child was able to create the problems without the use of the cards using only fact family numbers after the conceptual understanding was developed. He was able to transfer his knowledge to the original fact family triangles his teachers had started with.

Extending the Model

Linking cubes (such as Omnifix cubes) can also be used to help the child visualize the numbers, see Figure 4. This may be more complex for a child than manipulating number cards, helping them to develop a richer understanding of the interconnectedness of the operations.

Emphasis in textbooks in regards to number and operations is frequently dominated by the use of small numbers (Ashcraft & Christy, 1995). Children are asked to compute small numbers more frequently than they are asked to compute larger numbers, which may lead to the difficulties they have in working with larger numbers (Ashcraft & Christy, 1995). To address this, fact family cards can be extended to more complicated numbers. This model would allow children to extend the idea of fact families to more complex instances in the later elementary grades. For example, children can be supplied with fractional fact families (1 ½ 2 ¾, 4 ¼) decimal fact families (0.15, 1.23, 1.08) and numbers with place values in the tens (28, 11, 17) or hundreds (344, 154, 498).

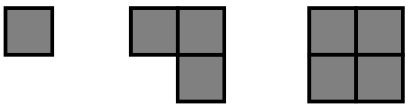

Figure 4. Linking cubes can also be used to help the child visualize the fact families (1, 3, 4).

This index card model can be extended to connect multiplication and division as well. Using blank index cards again, create a set of numbers (3, 4, and 12), the operation cards (multiply (x) and divide (÷)) and the equal sign (=). Have the child come up with four different math problems using the cards you gave him/her (3 x 4 = 12; 4 x 3 = 12; 12 ÷ 4 = 3; 12 ÷ 3 = 4). This investigation can be used to help students discover how the numbers can create distinct problems that relate to one another through multiplication and division. It can also guide them to use a known fact (multiplication problem for example) to find an unknown fact (division problem).

Conclusion

In order for some children to connect addition and subtraction, tools are a necessary building block to facilitate understanding. The child in this case could not make the connection without the tools; the tools were a vital component to advance his understanding. In addition, the think time was essential. Think time provides children with the opportunity to develop strategies and advance ideas (Phillips et al., 2003). With these two components (tools and time), he was able to do in 30 minutes what several hours of classroom time could not do. He was able to explain how numbers relate to one another with respect to addition and subtraction fact families.

This case highlights the importance of teachers taking the extra time to create opportunities for children to discover mathematical ideas with the use of tools. Bryant et al. (2008) pointed out that teaching strategies to children who need intervention is important. In addition, they theorize that concrete, visual representations of number concepts will assist in students' development of mathematical ideas. Specifically, this particular child needed the tools to visualize the connectedness of the numbers in relation to addition and subtraction. The tools made it much easier for him to understand and demonstrate the relationship.

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