Modelling the Transition from Secondary to Tertiary Mathematics Education: Teacher and Lecturer Perspectives

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Abstract The transition from school to tertiary study of mathematics is rightly coming under increasing scrutiny in research. This paper employs Tall's model of the three worlds of mathematical thinking to examine key variables in teaching and learning as they relate to this transition. One key variable in the transition is clearly the teacher/lecturer and we consider the perspectives of both teachers and lecturers on teaching related matters relevant to upper secondary and first year tertiary calculus students. While this paper deals with a small part of the data from the project, which aims to model the transition, the results provide evidence of similarities and differences in the thinking of teachers and lecturers about the transition process. They also show that each group lacks a clear understanding of the issues involved in the transition from the other's perspective, and there is a great need for improved communication between the two sectors.

Introduction

Concerns about decreasing numbers of students opting to study mathematics at university and beyond (e.g. the ICMI Pipeline Project) have encouraged research interest in the transition from school to university. A widespread decrease in levels of mathematical competence, including a lack of essential technical facility, a marked decline in analytical powers, and changed perceptions of what mathematics is, especially with regard to the place of precision and proof, have been noted in reports (LMS, 1995; Smith, 2004), with these difficulties even extend to 'high-attaining' students. Research on the transition period from school to university education in mathematics confirms the mathematical under-preparedness of students entering university (Hourigan & O'Donoghue, 2007; Kajander & Lovric, 2005; Luk, 2005), and the impact this has on students' success in university mathematics (Anthony, 2000; D'Souza & Wood, 2003). Moreover, the problem of a possible widening gap between school and university has been described by studies in a number of different countries around the world (e.g. Brandell, Hemmi & Thunberg, 2008; Engelbrecht & Harding, 2008).

In the research described here we have been using a developing theory by Tall (2004, 2008), which suggests that mathematical thinking exists in three *worlds*, the embodied, symbolic and formal, to examine the possibility of qualitatively different approaches to thinking about mathematics at school and tertiary levels. The embodied world is where we make use of physical attributes of concepts, combined with our sensual experiences to build mental conceptions. The symbolic world is where the symbolic representations of concepts are acted upon, or manipulated, where it is possible to switch from processes to *do* mathematics, to concepts to *think* about mathematics. The formal world is where properties of objects are formalised as axioms, and learning comprises the building and proving of theorems by logical deduction from these axioms. There is some evidence that one specific problem in mathematical thinking relates to an emphasis in school mathematics on symbolic world procedural understanding of algebraic material (Novotna & Hoch, 2008). Tertiary mathematics courses are usually trying to build formal world thinking based on a deductive, axiomatic approach; so if students are primarily symbolic thinkers, then tensions and difficulties will naturally arise. One outcome is that many students who are exposed to a formal deductive approach in mathematics for the first time on entry to university experience a significant amount of cognitive conflict in their first year (Tall, 1997).

Method

This study is part of a much larger research project entitled 'Analysing the Transition from Secondary to Tertiary Education in Mathematics' involving teachers, lecturers and

students, that employs questionnaires, interviews and teaching observations. A questionnaire on the transition was sent to all 350 secondary schools and 31 tertiary Institutions (Polytechnics, Universities, Wanangas and Institutes of Technology) in New Zealand to be completed by all teachers who teach calculus in Years 12 or 13 (age 17-18 years) and by all the calculus lecturers.

The questionnaire was posted, complete with a stamped addressed return envelope and teachers and lecturers were given three weeks to answer. After this a follow-up copy was sent by email to remind the teachers and lecturers to reply. Using this approach we received a total of 178 teacher and 26 lecturer responses, and some of these were later interviewed. There are no figures available on the total number of calculus teachers and lecturers in the schools/institutions, which vary in size from fewer than 30 students (small country school, Polytechnics, Wanangas and Institutes of Technology) to 3000 (inner city schools and Universities), but we estimate the response rate at about 30% of the calculus teaching school population, and a little less of the tertiary one. In this paper we present and compare the teachers' and lecturers' responses to two questions from the questionnaire, along with some interview comments, in the light of Tall's (2004, 2008) model of thinking. The questions (22 and 23) asked "Do you think that there are any differences between Year 13 and first year tertiary calculus teaching in any of the following areas? If so please describe them." and "Do you think students have any problems on moving from school calculus to tertiary calculus." Of the 178 teachers and 26 lecturers who responded to the survey, only 154 teachers and 23 lecturers gave personal demographic details. Table 1 highlights some of the demographic differences.

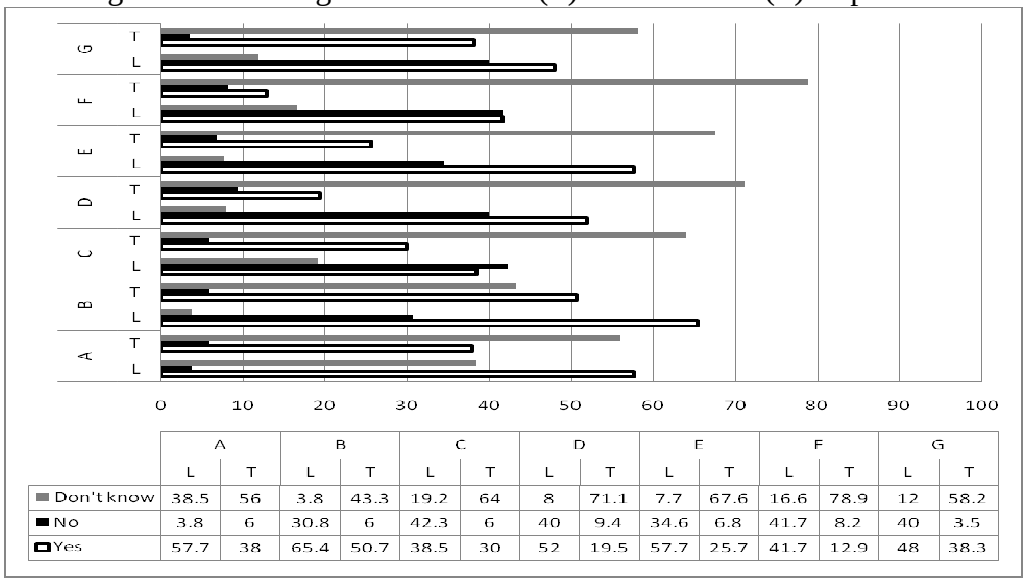
Table 1: A Comparison of the Personal Demographic Details

	Male	Female	Predominant age group(s)	English first language	Taught >11 years
Teachers (*N*=154)	82 (52%)	79 (48%)	41-50 (35%) 51-60 (29%)	90%	55%
Lecturers (*N*=23)	19 (83%)	4 (17%)	51-60 (44%) 41-50 and >61 (21.7%)	78.3%	17%

Results

In question 22 of the survey, teachers and lecturers were asked if they thought that there were differences in the following areas: A-assessment, B-teaching style, C-teaching resources, D-teaching emphasis, E-technology use, F-teacher preparedness, and G-students' experiences, and if so why. These areas were considered to be possible key variables in the transition and in order to be able to begin the process of constructing a model of transition we wanted to identify what variables are important, the relative level of their importance, and the relationships between them. Figure 1 (below) shows a comparison of percentages of the lecturers' and teachers' responses.

Figure 1: Percentages of lecturers' (L) and teachers' (T) responses.



	A		B		C		D		E		F		G	
	L	T	L	T	L	T	L	T	L	T	L	T	L	T
Don't know	38.5	56	3.8	43.3	19.2	64	8	71.1	7.7	67.6	16.6	78.9	12	58.2
No	3.8	6	30.8	6	42.3	6	40	9.4	34.6	6.8	41.7	8.2	40	3.5
Yes	57.7	38	65.4	50.7	38.5	30	52	19.5	57.7	25.7	41.7	12.9	48	38.3

While more than 30% of the teachers perceived differences in assessment, teaching style, teaching resources and student experiences, the most common response was to answer, "don't know" whether there are any differences. This could be of concern when considering the transition from school to tertiary study since it implies a lack of knowledge of the tertiary situation. Some teachers alluded to this as a possible reason in their interviews, with one saying.

> I think that we don't.. we haven't got a lot of uniformity amongst schools in presenting to students what to expect at university, and I don't think the universities do that brilliant a job in feeding back to schools what they want…I do believe that, where schools are trying to find out what's required at university. (T018)

Comparatively speaking, a majority of the lecturers perceived differences in assessment, teaching style, technology and student experiences. However, for the assessment area 38.5% of the lecturers responded that they 'did not know', possibly implying a lack of knowledge of the school assessment system. This may be because many lecturers have not taught in schools, and even those who have may not have taught to the recently introduced National Certificate in Educational Achievement assessment system. Whatever the case, they seemed to have some knowledge of other areas such as technology use, teaching style and teaching emphasis, since the percentage responding "did not know" was relatively lower for these.

The following analysis will be an attempt to match Tall's (2004, 2008) notion of mathematical thinking with these four variables: assessment, teaching style, teaching emphasis and teacher preparedness and support.

Assessment

For the assessment area the lecturers' comments on differences presented only a vague perspective of school assessment in terms of types of assessment and how they are graded:

Assignments differ from NCEA internal assessments (L3)

NCEA does not require a student to get 'more than half' correct to pass (achieve) (L10)

The teachers who commented about differences in assessment between school and tertiary level made observations such as, "A lot more assessment" (4, 6.1%), although it is not clear whether they felt that school or tertiary had more. References were also made to the differences in assessment styles, such as "Standards-based versus norm-referenced" (4, 6.1%) and "Presumably universities are not using the type of marking used in NCEA [national] exams." (2, 3.0%). In their interviews, teachers talked at length about the NCEA assessment and the attitudes of students and themselves in dealing with this summative assessment. A theme of tailoring work to assessment at a specific, often lower, level was prevalent.

I think that the internal assessments...because you know what you're going to be assessing them and because of time constraints, you can teach the content that's in the assessment. I'm afraid that that's the sort of thing that has crept in. (T156)

Let me think of an example, let us go back to my expectations with the majority of the class, if I'm aiming at achieved or merit I might skip out the excellence part work at the end. (T134)

This suggests that teachers who tend to teach to the assessment may promote procedural, symbolic world mathematical thinking to achieve student passes.

Teaching Style

The prevalent perception of differences in teaching styles was agreement that the level of interaction between lecturers and students at tertiary level is not sufficient (41 teachers, 64.1% and 10 lecturers, 80%). The lecturers' comments included "primarily lecture format less interactive than school" (L2) and "[our] teaching style [is] more formal less individualistic" (L3). Of course this is partly due to large class size, and this was evident in these kinds of comments "My class is 420 students! Determines style." (L8) and "Lecture style is all one way for large 200+ classes" (L13). Some areas where the lecturers' comments were consistent with the teachers' comments included: "Tertiary students are taking more responsibility for their own learning. Teaching style is more teacher-centred" and "less personal interaction with students" (41, 64.1%).

Teaching Emphasis

It is significant that 71.1% of the teachers answered that they did not know of any

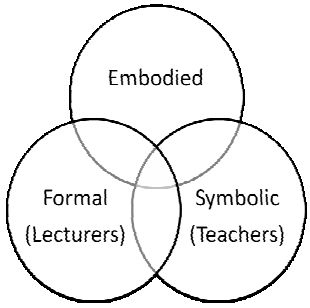
differences in teaching emphasis. Those who commented mostly felt there was greater depth to the understanding (2, 11.1%), an emphasis on the theory, and a more formal approach (2, 11.1%) at tertiary level than at Year 13. Some felt there were "Different approaches to certain sections, inclusion/exclusion of topics at school" (2, 11.1%), and "more on pure mathematics (and) less on applications." On the other hand, 52% of the lecturers also reflected the teachers' view that the lecturers "focus on understanding concepts rather than learning techniques" and have an emphasis on "applications in particular areas, example engineering, science". Question 10 of the survey considered the level of importance (1=Not important to 5=Very important) that lecturers attribute to various factors when teaching calculus. In particular, 92% of the lecturers valued applications in calculus teaching, but only 44% of them valued procedural learning. These results are consistent with Tall's (2004, 2008) model, which promotes the notion that procedural learning is more common and valued in schools, while formal thinking tends to be promoted, and valued, at tertiary level, whereas procedural work is less valued, and hence less common.

Teacher Preparedness and Support for Teachers

The results here showed that lecturers would be more prepared to teach in a formal way that engenders Tall's (2004, 2008) notion of mathematical thinking than the teachers who were over-burdened with administrative and classroom matters and possibly received lesser support. In the teacher interviews they discussed the predominant issue of being burdened with administrative work and classroom management. "If you're tired and you're wrapped off your feet because you're doing your reports and ninety thousand other things.., you don't prepare." (T156). The lecturers also echoed understanding of the teachers' frustration, "High school teachers are generally very under-prepared for their classes compared with tertiary teachers. They are often discouraged by the impossible situations which they face in the classroom." (L10). The teachers also stated that the lecturers get "possibly more support/preparedness at university and perhaps time." (2, 13.3%); the "University has more access to support for resource preparation." and there are "More colleagues and departmental discussion at university. "

By comparing the lecturers' and teachers' responses for the four variables, it reinforces the idea that lecturers tend to promote formal world of mathematical thinking while teachers may focus on developing symbolic world thinking in their teaching. Hence, based on the inter-relationships of variables, Figure 2 shows how they may embrace Tall's (2004, 2008) notions of mathematical thinking.

Figure 2: How lecturers and teachers fit in Tall's model.



Transition

Following question 22, the results from question 23 reinforce the cognitive conflict (Tall, 1997) faced by students as the teaching/learning paradigm shifts from symbolic to formal thinking during the school-tertiary transition. The results also show that most lecturers (60%) and some teachers (25.3%) tend to agree that students faced problems during their transition period. Based on Tall's (1997) notion of cognitive conflict, it would appear that the under-prepared first year students face problems in their learning, whereas the more prepared ones coped well during the transition. These statements show the teachers' perception of how the transition would be made easier or harder,"If calculus is well taught at school, the first year of university calculus can be 'too easy." and "Only if it were properly taught at school first year university mathematics is sometimes easier than L3 maths and there is little challenge for the top students in first year...".

Other reasons cited for under-prepared students were low achievement in school. The most common teachers' suggestion was that "students should aim higher to get merit or excellence as the tertiary education assumes they have a sufficient knowledge of Yr 13 calculus." It appears that these teachers observe students simply aiming to 'pass' rather than understand at a deeper level. Another possible problem faced by the students is the low lecturer-student interaction. Nearly 9% of the teachers believed that the amount and quality of interaction between lecturers and students was a problem, mentioning the importance of 'one-to-one contact and help'.

In summary, this paper uses Tall's (2004, 2008) notion of the three mathematical worlds of thinking in comparing teacher and lecturer perspectives on calculus teaching and learning. Both groups perceive differences between Year 13 and first year tertiary calculus teaching, including: there is a more formal approach to the teaching at tertiary level; secondary teachers interact more with their students; secondary teachers spend a large amount of time on administration at the expense of lesson preparation; and there is more procedural teaching, especially to the NCEA assessment, at school. There was a great deal of ignorance expressed about school and tertiary calculus teaching, notably by the teachers, and to a lesser extent, the lecturers. Clearly there are important roles for secondary teachers and tertiary lecturers to play in helping students with their transition. They can help to ease the cognitive conflict (Tall, 1997) faced by the students and be more aware of changes, including the shift in mathematical thinking, during the school-tertiary transition.

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