Toward Calculus via Real-time Measurements

Tine Golež, MSc
Physics teacher
St. Stanislav's Institution, Ljubljana, Slovenia
tine.golez@guest.arnes.si

Abstract

Several years of my experiences in the use of real-time experiments are now upgraded in order to enhance also the teaching of mathematics. The motion sensor device enables us to get real time *x(t)* and *v(t)* graphs of a moving object or person. We can productively use these graphs to introduce differentiation on visual level as well as to show the integration procedure. The students are fully involved in the teaching as they are invited to walk in front of the sensor. This approach motivates them by the realistic aspects of mathematical structures. The method could help to fulfill the credo of teaching: comprehension before computation. The steps of such an approach are explained and discussed in further detail below.

Introduction

Computers are more and more involved in the teaching. Physics teachers widely take advantage of them. It is important not to use them only for various kinds of physics simulations and applets but also as a part of measuring system. There is no doubt - the experiments should be the essential part of the teaching of physics. Virtual realty can not and may not replace them.

However, according to my experiences the use of computer has not reached its full potential in the teaching of mathematics. What could make mathematics more realistic, alive and accessible than real-time experiments? The answer to this difficult question is a major challenge for the mathematics teacher. Therefore I strongly suggest and prove by praxis that the physics teacher should support his colleagues – math teachers – in introducing some real-time measurement approach in order to enhance the teaching of mathematics. One of the most versatile equipment is motion sensor device.

Real-time measurements

An example *par excellence* of novel measurement techniques that can be used for this purpose is the so-called motion sensor, a device which uses ultrasonic pulse technology to measure the object's position. An ultrasonic transducer generates 40 kHz sound pulses and the device measures the time it takes for each pulse to travel out, bounce off a target and travel back to the sensor. The travel time of the ultrasound pulse is proportional to the distance. By connecting the sensor to a PC, it enables us to measure the position of a moving body; such as student walking back and forth in a straight line in front of the sensor, or a ball falling under the sensor. The computer plots real-time graphs of such linear motion.

The emitter can emit ultrasonic pulses up to 100 times per second, which is sufficient for a fast moving object. Some additional features like using two devices simultaneously to plot the path of a an object moving in a plane or following two objects during collision events increase the teaching value of this equipment. The necessary software was developed independently all over the world by firms and individuals. In the case of Slovenia [1, 2] its development was supported by the state and therefore it is available at no charge for all schools. The apparatus itself is in the price range of 120 EUR.

It is my opinion the "Homo Sedens" – a term which almost perfectly corresponds to today's student. Students spend the majority of the day in the classroom, sitting and listening to the lectures. Teaching is no longer a two-dimensional (blackboard) activity as it is still commonly practiced. For the best learning environment the teacher must find creative ways to engage the whole student. I have been successful using the motion sensor device during my lessons, which allows students to stand up and walk during learning. As a student is walking in front of the motion sensor he is active in creating changes and these changes (x(t)) are simultaneously displayed and seen on the screen. If he stops for several moments he/she can observe that the independent variable (time in this case) run actually independently – the position just doesn’t change (Fig. 1).

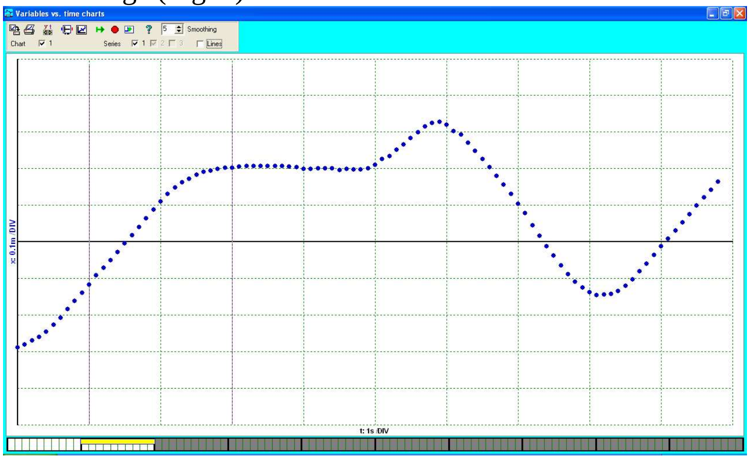

Fig. 1. Linear motion of a walking student; graph x(t) obtained by motion sensor connected to a PC.

Graph analyses-precursor of calculus (derivation and integration)

At the age of 15 or 16, which corresponds to the second year of high school in Slovenia, students are able to analyze a graph. As a result, they observe and examine the two graphs (*x* vs. time and *v* vs. time) for an object with variable velocity. They soon determine that the slope of the curve *x(t)* is exactly the velocity at that instant. Looking at the position graph, *x(t)*, they are able to sketch the velocity graph, *v(t)*. It should be noted that this is a qualitative task (Fig. 2).

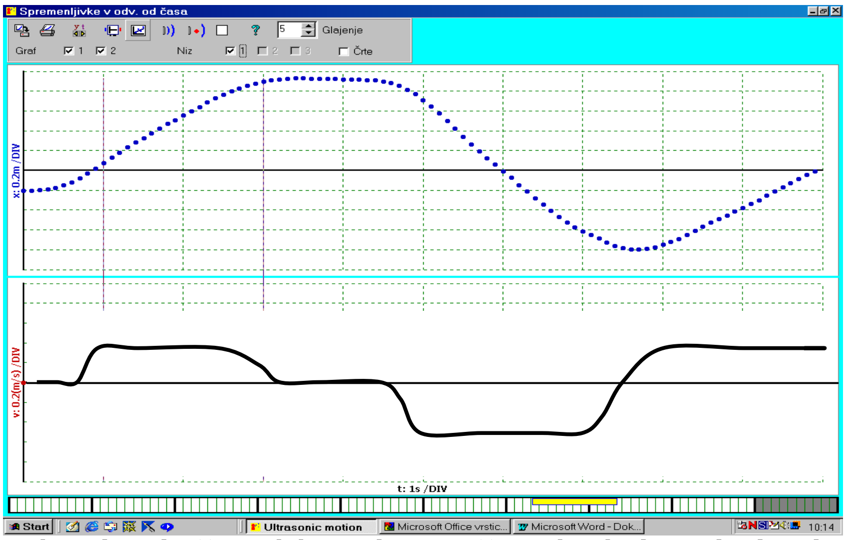

Fig. 2. A student has plotted *v(t)* graph by analyzing *x(t)* graph. The latter displayed *v(t)* graph computed by the measurement system confirmed the student’s prediction of *v(t)* graph.

The quantitative task is to draw a tangent and calculate its slope in order to determine the instantaneous velocity [3]. The result is checked by the graph *v(t)*, plotted by computer. However, taking into account the geometric approach one can not expect that all the students will calculate exactly the same coefficient (slope). I am convinced these activities are the right moment to tell them that this process will be called differentiation and that it will be fully explained in mathematical terms two years later during their mathematical lessons (Fig. 3).

As the reverse process is also possible, we will use the opportunity to deal with it, too. Let us now focus on the velocity vs. time graph. We choose a time interval and observe the area under *v(t)* graph during the chosen time interval, Δt. Let us first consider the case of non-

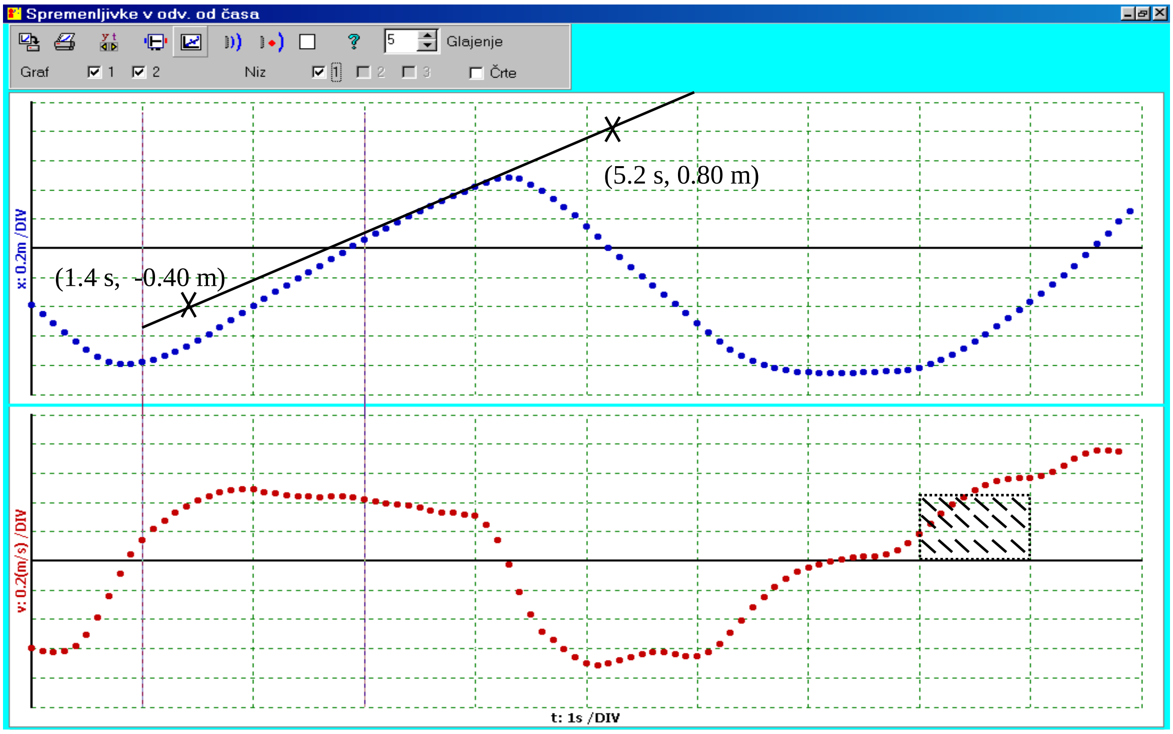

Figure 3: The slope of the curve *x(t)* is defined as the slope of the tangent at that point. The calculated slope at t = 4,0 s equals the instantaneous velocity at that instant ($\Delta x/\Delta t$ = 0.32 m/s). Graph *v(t)* confirms the result. The reverse process is shown on *v(t)* graph. The displacement during the time interval is determined by calculating the area (in our case Δx = 1,0 s·0,45 m/s = 0,45 m)

uniform motion, Fig. 3. The graph is not a straight line. We estimate the average velocity during the time interval and plot a horizontal line; therefore, we obtain a rectangle which has approximately the same area as the area under *v* vs. *t* graph during that time interval. The area of the rectangle is calculated and is equal to the displacement (during the same time interval). We can check our result using *x(t)* graph: $\Delta x = x_2 - x_1$. If the curve *v(t)* is under the time axis (negative velocity), the "area" is negative; it corresponds to a backwards displacement. Again, a physics teacher has an unique opportunity to explain to his students the importance of such a procedure, and one day they will call it integration.

But there is another quantity defined in kinematics. It is the acceleration. This excellent equipment enables us also to measure the most known acceleration, the acceleration of gravity. In addition, we can use very ordinary ball to measure it, no special physics equipment is needed.

The acceleration is defined as the rate of change in velocity. Hence we simply drop an ordinary volleyball or basketball from rest under the motion sensor, Fig. 4. It is right to assume the air resistance is negligible. The system displays both graphs (*x(t)* and *v(t)*). As the ball rebounds several times from the floor, a part of this motion is like an object thrown straight up. During one bounce it continues to move upwards for a certain time *t* and then drops back to the floor. Graph *x(t)* clearly shows the two time intervals, upwards and downwards, are the same. At the same height the instantaneous speed is the same. The velocities are opposite, as we can see from graph *v(t)*. And finally, it is possible to measure the acceleration – by calculating the slope of the graph *v(t)*. It is the same all the time, regardless of the ball moving upwards or downwards … with the exception of a short time interval when the ball hits the floor. What about graph *a(t)* for this experiment? The brightest students can draw it, so I expect the readers of this article will find this task as a minor challenge.

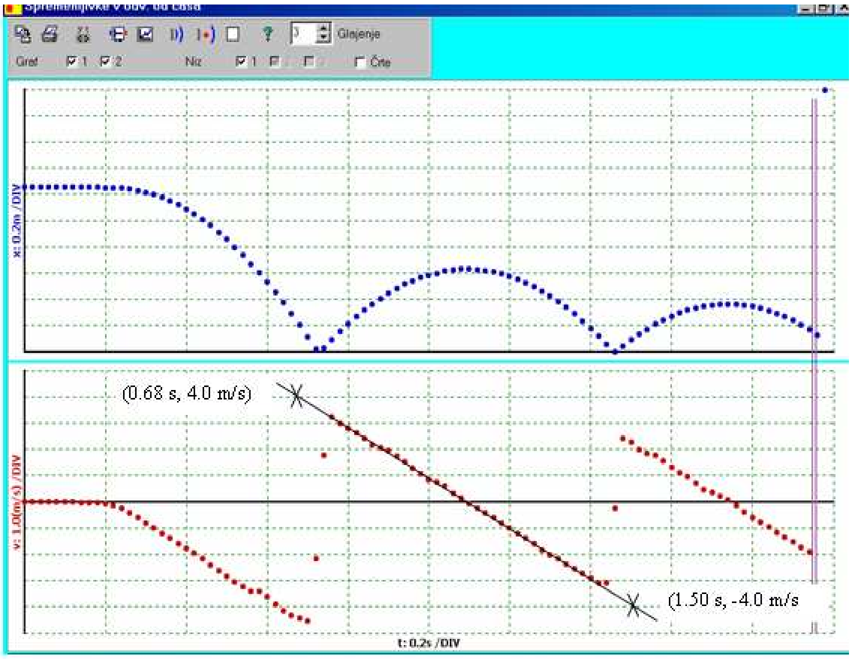

Figure 4. An ordinary ball was held under the motion sensor and dropped from rest. It bounced twice from the floor during the measurement. Graphs show that the instantaneous speed (= the magnitude of the instantaneous velocity) at points of equal elevation in the path is the same regardless of the ball moving upwards or downwards during the bounce (e. g., compare t = 0.80 s and t = 1.40 s). In addition, during one bounce the ball slows from the initial upward velocity to zero velocity. At the highest point it changes its direction of motion. Certainly, it experiences the same acceleration the way downwards. The acceleration, which is the rate of change of velocity, is constant. Therefore this part of the v(t) graph is linear. The slope of the line equals the acceleration. As calculated for this case:

$$a = \frac{\Delta v}{\Delta t} = \frac{-4.0\,ms^{-1} - 4.0\,ms^{-1}}{1.50\,s - 0.68\,s} = -9.76\,ms^{-2} \approx -9.8\,ms^{-2}$$

The software can also plot a tangent to the curves *x(t)* and *v(t)* and display its coefficient (slope). However, according to the expressed *credo* we use this option only after the students have mastered the same procedure themselves.

Once these basic experiments have been performed, it is possible to play with other options. The program allows us to select the place of each graph. In addition, it is possible to hide the legend. Therefore, students can check their understanding by “hidden graphs”. For example, we measured a pendulum motion. The three graphs were displayed without legends and not in the usual order (*x(t)*, *v(t)*, *a(t)*), Fig. 5. One can assume the first graph is *x(t)* and then examine its slope looking for the *v(t)* graph.

If one of the two graphs could be *v(t)*, the task is not finished yet. Now it is time to prove that the last graph corresponds to the slope of the second one. If not, one would simply assume the graph in the middle could be *x(t)* and repeat a similar procedure until the two consecutive graphs are “in slope relationship with the previous”.

Transfer from physics to mathematics

Mathematics is a compulsory subject for all students in the last year of gymnasium in Slovenia, while physics is already an elective subject. All students must pass a state prepared exam called Matura. Mathematics is one of three compulsory subjects, which gives students additional motivation to fully engage during the lessons.

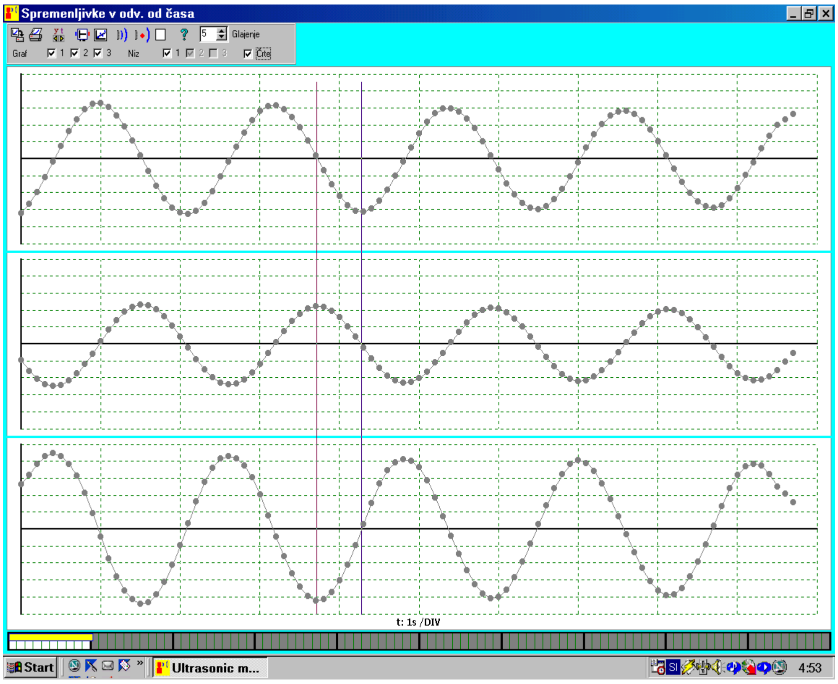

Figure 5: Motion sensor analyzed pendulum motion. The graphs *x(t)*, *v(t)* and *a(t)* are not displayed in this order. Students must find out the legend of each graph by investigating their slope and interrelationship between the graphs.

My colleagues, mathematics teachers at my school, are fully aware of the activities we are carrying out during the physics lessons. Consequently they are not at all reluctantly when I offer a combined lesson. I bring motion sensor at their lessons and students repeats some visual level calculus activities. Certainly some things can be done at higher level.
The bouncing ball experiment (Fig. 4) is now revisited using the advanced mathematical tool, calculus. First we find out the equation of parabola which corresponds to the first rebound. This equation is x(t). As the students are now capable of using derivative techniques they find the derivative of the parabola equation. Next derivative, dv/dt corresponds to the acceleration. They find out that the acceleration is constant and its value is exactly the acceleration of gravity [4].
Examining the same experiment as two years earlier but at higher level with a new mathematical tool is a superb longitudinal (regarding time) and transversal (regarding two subjects: mathematics and physics) approach. Such an approach will help to create less parceled knowledge. The abundances of connections put more logic and excitement in the learned topics and therefore helping to make these subjects more "alive", more "realistic" and more "accessible".

Conclusion The international ScienceMath Comenius project is the framework which strongly supports the cooperation between mathematics and other subjects. The project offers me both financial support as well as some directions to fulfill the ideas of combined lessons. I am convinced that such cooperation is a logical step. We must admit that during the developing of calculus (Newton, Leibnitz) there was much more connection between these two sciences. The curriculum of both subjects will soon bring more demand toward connecting both subjects via combined lessons.

References

[1] KOCIJANČIČ, Slavko: GIREP Conference Physich Beyond 2000, Barcelona, 2000
[2] KOCIJANČIČ, Slavko, GOLEŽ, Tine: Ultrazvočni slednik, Fizika v šoli 1(2000)
[3] Golež, Tine: The use of motion sensor in the teaching of mathematics; Proceedings of FISER, N. Cyprus 2009
[4] Rugelj, Marina, GOLEŽ, Tine: Bouncing ball – a mathematisation for second year high school and Matura students; Proceedings of FISER, N. Cyprus 2009