Elementary Teacher Candidates' Understanding of Rational Numbers: An International Perspective

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Abstract

This paper combines data from two different international research studies that used problem posing in analyzing elementary teacher candidates' understanding of rational numbers. In 2007, a mathematics educator from the United States and a mathematician from Northern Ireland collaborated to investigate their respective elementary teacher candidates' understanding of addition and division of fractions. A year later, the same US mathematics educator collaborated with a mathematics educator from South Africa on a similar research project that focused solely on the addition of fractions. The results of both studies show that elementary teacher candidates from the three different continents share similar misconceptions regarding the addition of fractions. The misconceptions that emerged were analyzed and used in designing teaching strategies intended to improve elementary teacher candidates' understanding of rational numbers. The research also suggests that problem posing may improve their understanding of addition of fractions.

Introduction

Research conducted over the past three decades provides evidence of the difficulties that many elementary teacher candidates experience with rational numbers [1], [2], [3]. The international research project described in this paper concurs with past researchers about the importance of identifying the level of mathematical knowledge that should be required of future teachers [4], [5]. Besides acquiring procedural knowledge, elementary teacher candidates must also exhibit a deep conceptual understanding of rational numbers to adequately teach their future students. Identifying understandings as well as analyzing student errors to determine their misunderstandings is critical in preparing candidates for teaching [6], [7]. Additional rational number research also shows that teaching number sense helps learners to understand mathematical symbols by relating them to referents that are meaningful to them. Connecting instructional examples to real life experiences, thus, is extremely important in overcoming difficulties and in developing conceptual understanding [8], [9]. It is also suggested that future teachers should be able to pose a valid problem for their future elementary students that can be solved by using a particular arithmetic operation. By doing so, they will exhibit a deeper understanding and be more likely to help their future students develop their own understanding [10]. In the two studies described in this paper, mathematics educators from Northern Ireland, South Africa, and the United States were involved in similar research projects where the mathematics educator from each country asked their respective elementary teacher candidates to write problems for their future students regarding operations with fractions. The findings of these studies demonstrate the understandings as well as misunderstandings regarding addition of fractions that were exhibited by the elementary teacher candidates from each country.

Fractions Research by United States, Northern Ireland, and South Africa

During the spring of 2007, two mathematics educators from Northern Ireland (NI) and the United States (US) asked their respective elementary teacher candidates to pose problems involving addition (and division) of fractions [11]. Six elementary teacher candidates from NI and 34 US elementary teacher candidates were included in this initial study.

In the spring of 2008, the same US mathematics educator collaborated with a mathematics educator from South Africa on a similar research project that focused solely on the addition of fractions. There were 26 elementary teacher candidates from SA and 18 additional US subjects in this second study.

The research that was used is grounded on previous research that also involved problem posing with the addition and multiplication of fractions [12], [13]. In these two new international studies, elementary teacher candidates were asked to relate the problems that they posed to real life experiences that would be appropriate for their future elementary students.

The following problem was completed by 84 subjects in both studies.

Write a story problem where students in the elementary grades would add ¾ + ½ to complete the problem.

Description of participants

In the three countries involved in the studies, all subjects were current elementary teacher candidates at the universities of their respective researcher. During the 2008 study, the SA and US teacher candidates were simultaneously taking courses that included similar mathematical concepts: whole numbers, fractions, decimals, percents, ratios, and proportions. The NI subjects did not have a similar course that reviewed these concepts in their curriculum. All NI subjects, however, had a concentration in elementary mathematics and were in the senior year of their education program, where the US and SA subjects were both beginning their mathematical studies, and none were concentrating in mathematics. All three countries used the English language in instructing their courses, but the SA subjects were unique in that 73% of their subjects did not use English as their first language (19 out of 26).

Cultural Differences

Mathematics is a universal language, and the procedures for adding fractions are the same, however, in the analysis of the problems, some differences in the cultures of the US and SA emerged in the researchers' conclusions as to whether several of the problems posed were acceptable. Each researcher had first analyzed the problems from their own country, and then analyzed the problems from the other country. When comparing the results, there were several problems in which the researchers initially disagreed. For example, in SA, margarine is usually sold in 250 g, 500 g, and 1 kg blocks, but in the US, margarine may be found in ½ cup sticks. The SA researcher rejected the example below because she was not familiar with "sticks" of margarine, but "sticks" are appropriate for the US culture.

Mrs. Newton is baking cookies. The recipe calls for 1 cup of margarine. Her husband, Mr. Newton, will sometimes open a new stick of margarine before finishing the old one. So Mrs. Newton has 2 used sticks of margarine. One is ½ c long and the other is ¾ c long. Does Mrs. Newton have enough margarine to make her cookies.

The US researcher rejected the problem below regarding loaves of bread, as loaves are not a standard size in the US. The SA researcher explained that loaves of bread are a standard size and can be considered a nonstandard, but informal unit of measurement in SA.

In this story is very poor family, with sick mother, absent father, and 3 children. The mother was worried because there only ½ loaf bread left, but she manage 2 get some bread, it was ¾ of a loaf. So how much bread do they have now

After cultural differences were explained, researchers were in agreement that problems regarding the differences in the cultures should be considered acceptable.

Analysis of the Results from the Three Countries

In the first analysis of all of the problems that were posed, it was immediately noticeable that food was mentioned in a high percentage of the problems. Problems from all countries included categories of pizza, sweets, fruit, and, uniquely from South Africa, loaves of bread.

In a deeper analysis of the results of the addition problems, the researchers found that those who wrote acceptable problems had two basic similarities.

The first similarity in posing acceptable problems was that the teacher candidates from all three countries exhibited number sense that was sufficient to realize that the sum of the two fractions (1/2 + ¾) is greater than one and that two wholes were necessary, and must be included in their problems.

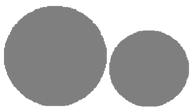
The second similarity in the acceptable problems from SA and US was that the teacher candidates displayed an understanding that referring to two similar unit wholes for both fractions was required. None of the NI subjects made reference to equivalent wholes, but all of the problems written by the NI teacher candidates included two wholes, which showed that all NI subjects exhibit sufficient number sense that the sum of the fractions is greater than one.

Findings from Unacceptable Problems

Difficulties in writing the problem in all three countries were similar in regard to the referent whole. Two themes emerged in analyzing the misconceptions of the unacceptable problems.

First, teacher candidates did not stipulate that the wholes being added are the same size and shape.

Kim has half a cake and Jamie has ¾ of a cake. How much cake is there altogether?



Students did not indicate that half of the left cake and ¾ of the unknown size of the right cake should not be added since they are not the same size. Since a cake is not a standard unit of measurement, the researchers agreed that for the problem to be considered acceptable, the writers should include that the cakes are of a uniform size. This writer, however, indicates an understanding of number sense that the sum of the fractions is greater than one, since two cakes are identified in the problem.

Jack and Jill had just gone trick-or-treating. They realized that if they added their candy together they would have more. Jack's bag was ½ filled and Jill's was ¾ filled. When they combine them together, how many bags of candy will they have?

Here we are unsure if Jack and Jill have the same size bags for their candy, thus adding the candy together may not be appropriate.

The second misconception that emerged in the analysis of the candidates' problems was that SA and US teacher candidates had not acquired sufficient number sense to realize that the sum of the given fractions is greater than one whole.

In Mrs. C's class, ½ of her students got A's on the test, and ¾ of her students got B's. How many students got A's and B's?

My class teacher bought us Pizza, she then gave ½ to boys and ¾ to girls. After we have eaten she told that the other pieces will be shared the following day. How many pieces were there altogether?

Each of these problems could not be modeled in a real life situation since the sum is greater than one in each situation. Acquiring such number sense is necessary to pose a realistic problem.

Findings from Acceptable Problems

Examples of acceptable problems that show understanding of number sense are presented below.

You have 2 pizzas that are exactly the same size. You give one pizza to your friend and you eat one half of your pizza. Your friend eats ¾ of his pizza. How much pizza did you both eat together?

Brian Habanna goes to Debonaires and orders one large Hawaiian pizza and one large chicken pizza. He eats half of his Hawain pizza and his girlfriend eats a quarter of her chicken pizza. How much pizza do they have left altogether?

Both problems also indicate "the same size" in the first problem, and an indication in the second problem that both pizzas are the "large" size from the same vendor, and thus equivalent.

Examples of acceptable problems that show understanding in employing a referent whole are presented below. When students included a referent whole in their problem, many problems included standard units of measurement in the problem so that the sum, which is greater than one, was appropriate for a real life situation.

You are helping your grandma make cupcakes. She tells you to add ½ cup of flour, then add ¾ cup more. How much flour did you put in total.

John ran ½ mile in the race for life on Saturday. Sue ran ¾ mile, how many miles did the run all?

If your mother tells you that you can spend ¾ of an hour at your friends house. Then you need to come home and get ready for school which will take you ½ an hour. In fraction form, how long does it take you to visit your friend and get ready for school?

Standard units of *cups*, *miles,* and *hours* were used in the problems above and easily make use of a familiar standard referent whole. In using these types of units of measurement, however, it was not evident if the students considered if the sum of the two fractions is greater than one.

Acceptable problems were also written that include non-standard units of measurement, as the following example from South Africa.

Dad gave John ½ of a peppermint chocolate and Susan ¾ of a peppermint chocolate too. How much chocolate(s) did dad have in total?

Response to Findings

In the first study none of the problems posed by the Northern Ireland elementary teacher candidates and only 20% of the US problems were categorized as acceptable prior to instruction but all NI candidates showed knowledge of number sense. In the second research study, four (15%) of the South African problems and seven (39%) of the US problems were categorized as acceptable prior to instruction. Of the 84 subjects in the two studies, only 21% were able to write an acceptable problem prior to instruction.

Analyzing student work has been shown to help mathematics educators reflect on their teaching and improve the teaching and learning of mathematics [14], [15]. After analyzing the elementary teacher candidates' problem posing work with addition of fractions, the US researcher shared some of the US sample research problems with her classes. A short discussion ensued concerning whether several of the problems were acceptable. The problems were carefully selected so that problems discussed were not written by any students in that class, but were from another class. Problems were chosen so that each of the identified themes of misconceptions was discussed. The classes also practiced estimating answers regarding the sum of fractions to the nearest whole.

At the end of the rational number unit, a few weeks after the problem posing discussions, when the US students were evaluated on fractions, decimals, and percent, the same problem posing item of the addition of two fractions was included on their unit exam. An analysis of the post test results in the first study shows that there was an increase in understanding such that 82% were able to write acceptable problems after instruction as compared to 20% on the pretest. After identifying the difficulties of the US teacher candidates and addressing them through a different type of instruction than was normally employed, the teacher candidates showed an improvement in their ability to pose a relevant problem.

Conclusions

Groups of elementary teacher candidates from three different countries, representing three different continents, wrote most of their problems about a most basic human need – food. Whether it was a pizza or a loaf of bread, similar understandings and misconceptions emerged from the three countries. The two salient themes that emerged regarding their understanding of addition of fractions and also their misconceptions are number sense regarding the sum of the two fractions being greater than one, and reference to a uniform unit whole. In future discussions, the international examples provided in this research can be used to raise the level of the classroom discussion to a global discussion by presenting problems from different countries. As the identified difficulties are addressed, a deeper understanding of addition of fractions may result. An improvement in the ability to pose problems was shown on the US post evaluations, after the misconceptions were addressed. The similarity in misconceptions held by all groups indicate a possible universal need to address the misconceptions that emerged in this research when teaching addition of fractions. The findings suggest factors to consider when teaching addition of fractions with elementary teacher candidates. This study contributes to the research that supports the use of problem posing to improve conceptual understanding. The findings also alert international researchers that cultural differences should be considered in analyzing qualitative data from different countries.Two questions naturally arise from the results of this study. Does the use of problem posing with elementary teacher candidates deepen their understanding of addition of fractions? Can the use of similar problem posing studies in other areas of mathematics help to clarify other misunderstandings for future teachers?

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