Innovations in Educational Research and Teaching of Experimental Calculus

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Abstract

For several decades, there have been a varying number of books on Calculus following the classic line of mathematical thought, where Mathematics is taught for everybody by means of rigorous definitions, theorems, and carefully detailed and extensive demonstrations. For mathematical education into the XXI Century the students need to achieve ability in handling of present mathematical tools and concepts from the beginning of their courses. These needs can be achieved today by means of a paradigmatic change in the focus of mathematics teaching: to learn to develop ideas and to experiment and test those ideas in such way that students can verify their own inferences. In this paper we report an educational research in teaching and learning functions models according to a new paradigm in hands-on experimental mathematics, with applications in the real world, i.e. sciences and engineering by using Computer Algebra Systems. The study of functions is presented, focused into the framing of Exploratory Learning Systems, where students learn by means of the action and their participation in it. It is designed for teachers working together with students in a computer laboratory like hands-on workshops-type activities on other sciences. In this way students have a more “alive”, “realistic” and “accessible” touch in Calculus.

Introduction

For several decades, there have been a varying number of books on Calculus following the classic line of mathematical thought. Without minimizing the quality of the books, and their authors' authority, this line of thought has nonetheless had serious difficulties in being understood and learned by most students of natural sciences, engineering, economic sciences and so on [(1)].

The Mathematical Education into the XXI Century

Students need to achieve ability in handling of present mathematical tools and concepts from the beginning of their courses without getting lost in the labyrinth of demonstrations and tests. These needs can be achieved today by means of a paradigmatic change in the focus of mathematics teaching, particularly of Calculus. Students should assimilate the methodologies of experimentation, simulation, and graphical interpretation in problem solving. For several years now, the interactive graphing and calculation facilities in “Computer Algebra Systems” (CAS) such as “Maple®”, “Mathematica®” “MATLAB®”, and online software releases like “GeoGebra”, have been available. The use of such computer assistance in Mathematics is equivalent to the use of telescopes in astronomy or of microscopes in biology. These tools do not explain the facts, but show new possibilities. This mathematical approach facilitated the discovery of new celestial bodies, and of the effects of mass, gravity and acceleration in space-time. The experimental mathematical approach is maturing and promises extraordinary beneficial effects [(2, 3)].

New focus vision concerning mathematical education at the laboratory

In order to prepare students for the *Mathematical Education into the XXI Century*, an educational research group on experimental mathematics has been created at the Argentine National Technological University. Since many years ago, a complete set of presentations to international congresses in Latin America have been performed [(4-9)]. Based on this new paradigm, we have elaborated a ***collection on topics of Calculus.*** In particular, in VOLUME 1, a study of functions is presented, focused into the framing of Exploratory Learning Systems. We have selected a few of the varied number of tools available in the market for use in mathematical applications.

A group of model functions is selected: lineal, quadratic, exponential, logarithmic, harmonic, and periodic non harmonic. The structure of each description is common:

1. Experimental Study of the model (Assisted by “Mathematica”)
2. Complementary Experimental Activities
3. Experimental Study of the model (Assisted by “GeoGebra”)
4. Integrating problems
5. Numerical and graphical applications

A short series of activities are described in the following paragraph.

Results

A complete book on Functions has been designed and is ready for publication. Some activities contained in this book are shown below, derived from specific stated problems.

Linear model. Representation with Mathematica

Activity 1. Relationship between Fahrenheit and Celsius scales

The water freezing point is 0° Celsius scale, and 32° Fahrenheit scale. The water boiling point is 100° Celsius scale, and 212° Fahrenheit scale.

Which are the increments ΔC and ΔF between the water bowling point and freezing point in both scales?

Which is the relationship between both increments $\frac{\Delta F}{\Delta C}$ *?*

Find the relationships between both scales ºC y ºF, being ºC the independent variable. Draw a graphics of this relation. Complete the following Table.

ºF	**32**	**42**	**150**	*¿?*	*¿?*	*¿?*
ºC	*¿?*	*¿?*	*¿?*	**24**	**37**	**-5**

Table 1. To be completed after experimentation.

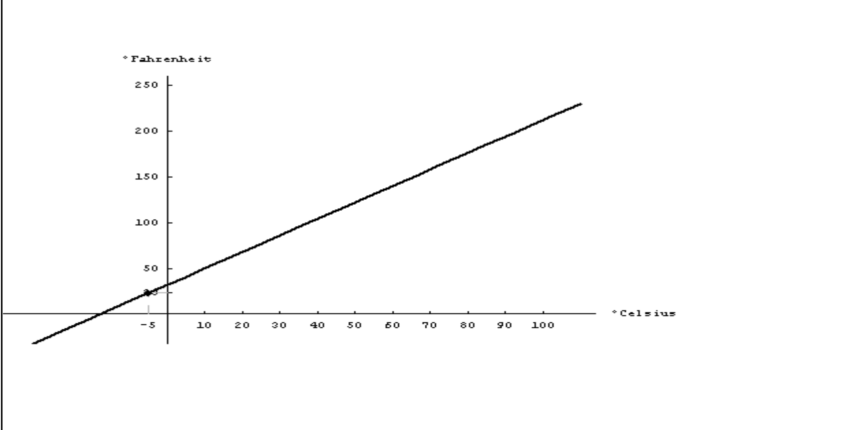


Figure 1. Experimental relationship between Fahrenheit and Celsius scales.

F (º*C*) = 1,8 º*C* + 32

Quadratic model. Representations with GeoGebra and Mathematica

Activity 2. Representation of the "Security Curve"

Find the equation for the "security curve". For a fixed initial projectile velocity, by changing the angle with respect to the horizontal line, the projectile attains different altitudes. These are "limited" by the "security curve".

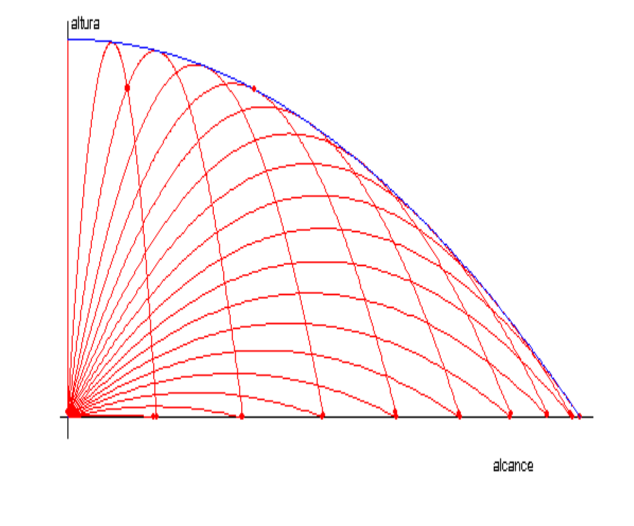


Figure 2. "Security curve" representation.

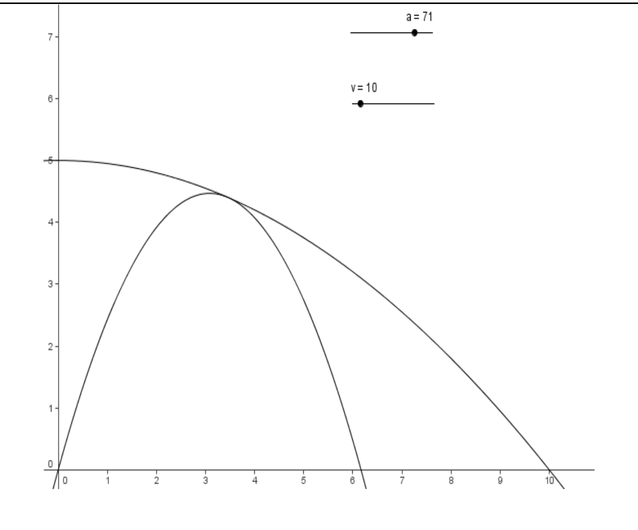
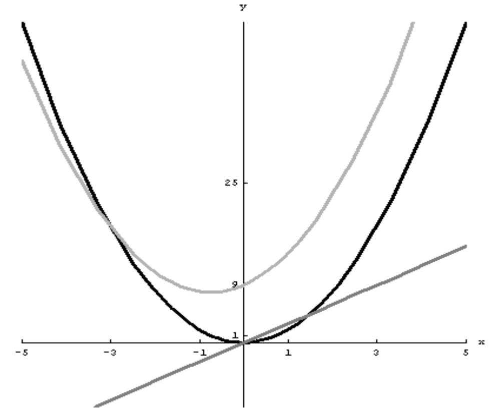


Figure 3. Alpha is the projectile angle with respect to the horizontal line. As alpha varies, the security curve is drawn.

Activity 3. Let are three functions $f_1(x) = 2x^2$, $f_2(x) = 3x$, $f_3(x) = 9$. *Find the sum of them and represent them in a "Mathematica" graph. Let be the function* $f(x) = mx^2 + bx + c$. *Experiment with different values of* ***m***, ***b*** *and* ***c,*** *draw the graphics and write the surmises.*



m = 2, b= 3, c=9

Figure 4. Graphics with Mathematica

Exponential model. Representations with Mathematica

Activity 4. Voltage decay of a capacitor through a resistance as a function of time

Let are a capacitor and a resistor connected in series and a power supply with a certain voltage difference V. As one measures V (t) as function of time one observes an exponential - type decay

$$V(t) = V_0 \cdot e^{-t/\iota}, \qquad (4.1)$$

Where ι = R·C. *Apply the definition of period T and find the relation between it and the time constant ι*

$$T = \ln 2 \cdot \iota = \ln 2 \cdot R \cdot C = 0{,}693 \cdot R \cdot C \qquad (4.2)$$

Represent the function (4.1) for V_0 = 12 V and R = 2000 Ω y C = 5 μF for 5 periods T of time.

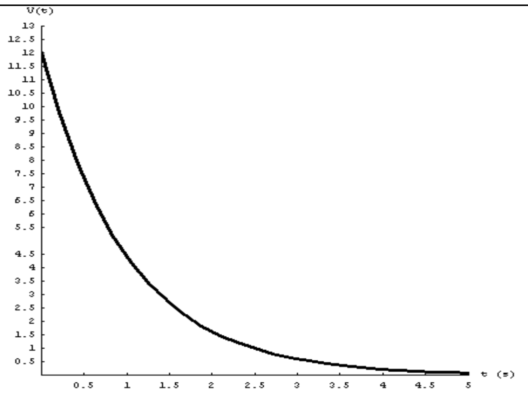


Figure 5. Voltage decay as function of time

Harmonic motion. Representations with Geogebra and Mathematica

Activity 5. Coupling of harmonic motions with GeoGebra. Analyze the two harmonic motions coupling represented by the following conditions. Projection on the x- axis of two vectors of different amplitudes that rotate counterclockwise with the same frequency with a phase difference α = π/3 (Fig. 6).

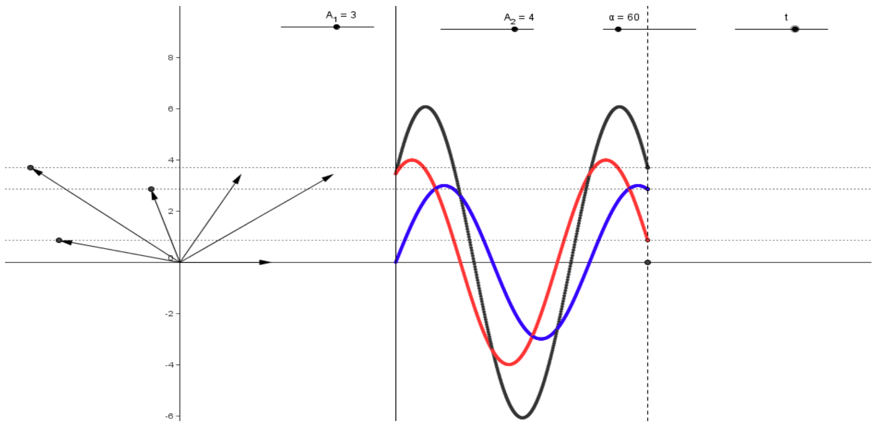


Figure 6. Projection on horizontal axis of two vectors which differ in magnitude and phase π/3, with same frequency. They are coupled to the sum vector, which rotates at the same frequency. The projections on the horizontal axis of the three vectors are represented as function of time.

Activity 6. Mathematica representations

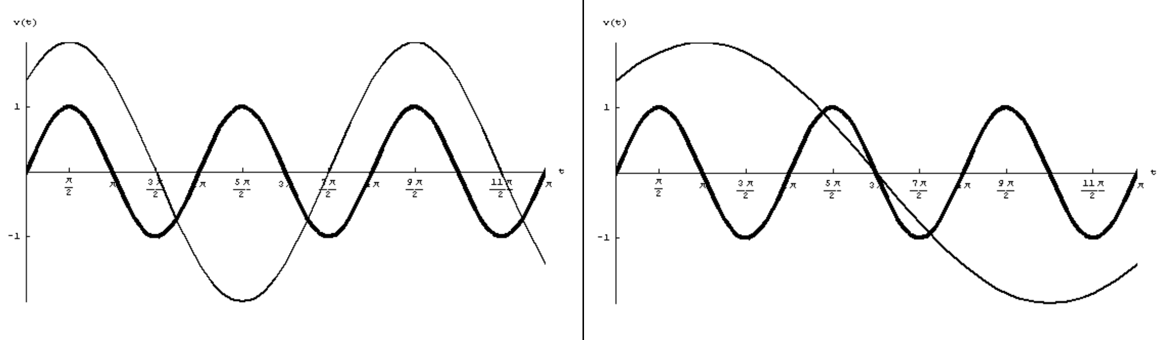
A person is at a Park Wheel. The projection on the y – axis of its position is represented by a function of the type y(t) = sin(t).

Find the time elapsed between two successive identical positions. Define the function period T.

Let are the functions $y(t) = 2.\sin(0.5t + \pi/4)$ *and* $y = y(t) = 2.\sin(0.25t - \pi/4)$.

Which is the period for each function? How are they related with respect to the former function $y(t) = \sin(t)$?.

Find a general harmonic function comprising amplitude A, period T, the relation between ω *and T and initial phase* α. *Find its graphics with specific values of each variable.*



$y(t) = \sin(t)$; $y(t) = 2\cdot\sin(0.5t + \pi/4)$ | $y(t) = \sin(t)$; $y(t) = 2\cdot\sin(0.25t - \pi/4)$

Figure 7. Graphics.

Periodic non harmonic function. Representations with Geogebra

Activity 8. "Tangentoid"

A laser beam rotates vertically and impinges a wall. An angle made by the laser beam and a horizontal line is defined as alpha .Changing the angle, segments on the wall with different "heights" are obtained. If one represents these segments as a function of alpha, one gets the "tangentoid" (Fig. 8).

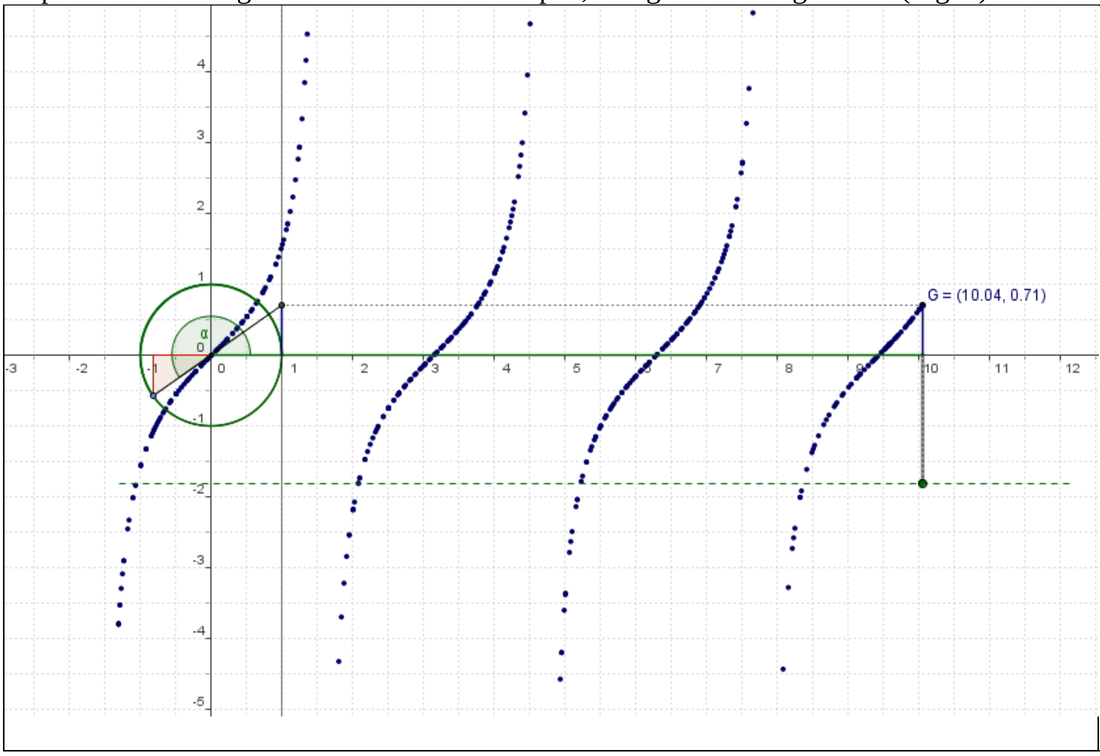


Figure 8. "Tangentoid" Graphics of the projections obtained by changing the α angle.

Transfer of the model to different environments

One of the main concerns the research group has is to disseminate the results in such a way that these can be grasped by teachers, students, graduate schools, and teachers´ careers.

A great effort in this direction has been done organizing local seminars, workshops, and conferences. On the other hand, typical "hands-on" workshops for mathematics teachers at the Technological University have been organized in the last years and for secondary school teachers as well. Many conferences and workshops in Latin American Countries have been arranged [4-9].

Special publications in Spanish, named "Experimental Mathematics Notebooks" have been distributed among several secondary school teachers.

Conclusions

We report some results of an educational research in teaching and learning mathematics carried out for several years, according to a new paradigm in hands-on experimental mathematics, with applications in the real world, i.e. sciences and engineering by using Computer Algebra Systems. A complete book on Function models has been developed according to the above mentioned paradigm. The experimentation with university level professors and students, as well as with secondary school mathematics teachers has been carried out in the last years. A feedback from those experiences has been incorporated by our research group, and we can state that this "new" paradigm has been assimilated and welcome from both, teachers and students.

In this way, a contribution to the actions on mathematical education into the XXI Century is presented highlighting the importance that students need to achieve ability in handling of present mathematical tools. We claim that with this contribution students have a more "alive", "realistic" and "accessible" touch in Calculus.

Nevertheless, a real reconversion of mathematical teaching is still very distant. Many books of the kind we present here have to be written and disseminated in a bigger scale. The fundamental action must be concentrated in teacher's professional careers. As an example, we highlight the thorough efforts done by U.S. National Science Foundation and the U.S. National Academy of Sciences on STEM (Science, Technology, Engineering and Mathematics) [10].

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