An innovative model for developing critical thinking skills through mathematical education

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Abstract

In a challenging and constantly changing world, students are required to develop advanced thinking skills such as critical systematic thinking, decision making and problem solving. This challenge requires developing critical thinking abilities which are essential in unfamiliar situations. A central component in current reforms in mathematics and science studies worldwide is the transition from the traditional dominant instruction which focuses on algorithmic cognitive skills towards higher order cognitive skills. The transition includes, a component of scientific inquiry, learning science from the student's personal, environmental and social contexts and the integration of critical thinking. The planning and implementation of learning strategies that encourage first order thinking among students is not a simple task. In an attempt to put the importance of this transition in mathematical education to a test, we propose a new method for mathematical instruction based on the infusion approach put forward by Swartz in 1992. In fact, the model is derived from two additional theories., that of Ennis (1989) and of Libermann and Tversky (2001). Union of the two latter is suggested by the infusion theory. The model consists of a learning unit (30h hours) that focuses primarily on statistics every day life situations, and implemented in an interactive and supportive environment. It was applied to mathematically gifted youth of the Kidumatica project at Ben Gurion University. Among the instructed subjects were bidimensional charts, Bayes law and conditional probability; Critical thinking skills such as raising questions, seeking for alternatives and doubting were evaluated. We used Cornell tests (Ennis 1985) to confirm that our students developed critical thinking skills.

Introduction

In his monograph "Smart Schools," (Perkins, 1992) the author focuses on how to change our teaching to enable children to learn more meaningful information. One of his suggestions is that we should have a thoughtful school, which means: teachers should teach by using a language of thinking. In this paper, we are focusing on the language of critical thinking. We begin by first explaining the term critical thinking (CT). Actually critical thinking is not a new concept; we can find it in ancient times. Socrates, as reported by Plato, used to the streets of Athens asking people all kinds of philosophical questions about the purpose of life, morality, justice, etc apparently for the purpose of stimulating a form of critical thinking. These questions and answers were collected and published in the “Socratic dialologues” (eg The Meno, and The Protagoras). In the field of education, it is generally agreed that CT capabilities are crucial to one’s success in the modern world, where making rational decisions is increasingly becoming an important part of everyday life. Students must learn to test reliability, raise doubts, and investigate situations and alternatives, both in school and in everyday life. There have been many attempts to define it, but we would like to focus on three of them. Mecpeck defines critical thinking as “skills and dispositions to appropriately use reflective skepticism” (Mecpeck, 1981). Lipman claims that critical thinking is “thinking which enables judgment, is based on criteria, corrects itself, and is context sensitive” (Lipman, 1991). The third definition is the one we have based our research on. Ennis (1962) defines it as “a correct evaluation of statements". Twenty-three years later Ennis broadened his definition to include a mental element, so now the improved definition is “reasonable reflective thinking focused on deciding what to believe or do” (Ennis, 1985). Our research is based on three key elements: a CT taxonomy that includes CT skills (Ennis, 1987); the learning unit "probability in daily life" (Liberman & Tversky 2002); and the infusion approach of integrating subject matter with thinking skills (Swartz, 1992). Ennis’ Taxonomy (Ennis, 1987), the learning unit "probability in the daily life" (Liberman & Tversky 2002) and the infusion approach (Swartz,

1992) collectively provide a firm basis for our thesis. In light of his definition, Ennis developed a CT taxonomy that relates to skills that include intellectual aspect as well as behavioural aspect. In addition to skills, Ennis's (1987) taxonomy also includes dispositions and abilities. Ennis claims that CT is a reflective and practical activity aiming for a moderate action or belief. There are five key concepts and characteristics defining CT: practical, reflective, moderate, belief and action. In our novel learning unit, which is a part of the formal syllabus of the Ministry of Education, the student is required to analyse problems, raise questions and think critically about the data and the information. The purpose of the learning unit is not to be satisfied with a numerical answer but to examine the data and its validity in order to. In cases where there is no single numerical answer, the students are required to know what questions to ask and how to analyse the problem qualitatively, not only quantitatively. Along with being provided with statistical instruments, students are redirected to their intuitive mechanisms to help them estimate probabilities in daily life. Simultaneously, students examine the logical premises of these intuitions, along with misjudgments of their application. A combination of the two theories was put forward by the infusion approach. There are two main approaches to fostering CT: the general skills approach which is characterized by designing special courses for instructing CT skills, and the infusion approach which is characterized by providing these skills through teaching the set learning material. According to Swartz, the Infusion approach aims for specific instruction of special CT skills during the course of different subjects. According to this approach there is a need to reprocess the set material in order to combine it with thinking skills. In this report, we will show how we combined the mathematical content of "probability in daily life" with CT skills from Ennis' taxonomy, restructured the curriculum, tested different learning units and evaluated the subjects' CT skills.

Methodology

Our methodological challenge is to investigate the development of the language of critical thinking" through critical thinking skills incorporated into a structured mathematics lesson, based on the proposed models.

Setting, Population, and Data**:** Fifty-five children between the ages of fifteen and sixteen participated in an extra curriculum program aimed at enhancing the critical thinking skills of students from different cultural backgrounds and socio-economical levels. An instructional experiment was conducted in which probability lessons were combined with CT skills. The study consisted of fifteen 90 minutes lessons, spread out over the course of an academic year, in which the teacher was also one of the researchers.

Data sources were students' products, pre and post questionnaires, personal interviews and class transcriptions. Students' products (papers, homework, exams etc.) were collected:. Personal interviews were conducted randomly. Five students were interviewed at the end of each lesson and one week after. The personal interviews were conducted in order to identify any change in the students' attitudes throughout the academic year.

All lessons were video-recorded and transcribed. In addition, the teacher kept a journal (log) on every lesson. Data was processed by means of qualitative methods intended to follow the students' patterns of thinking and interpretation with regards to the material taught in different contexts. As already mentioned, the probability unit combines CT skills with the mathematical content of "probability in daily life". This new probability unit included questions taken from daily life situations, newspapers and surveys, and combined CT skills. Each of the fifteen lessons that comprised the probability unit had a fixed structure: a generic (general) question written on the blackboard; the student's reference to the question and a discussion of the question using probability and statistical instruments and; an open discussion of the question that included practicing the CT skills. The mathematical topics taught during the fifteen lessons were: Introduction to set theory, probability rules, building a 3D table, conditional probability and Bayes theorem, statistical connection and causal connection, Simpson's paradox, and judgment by representative. The following CT skills were incorporated in all fifteen lessons: A clear search for

an hypothesis or question, the evaluation of reliable sources, identifying variables, "thinking out of the box," and a search for alternatives (Aizikovitsh & Amit, 2008). Each lesson followed the same four part structure.

1. Given text :In the beginning of the lesson the teacher presented a short article or text.
2. Open class discussion in small groups: Discussion in small groups about the article and the question.

- Initial suggestions for the resolution of the question
- No intervention by the teacher

3. Further discussion directed by the teacher: Open class discussion. During the discussion the teacher asked the students different questions to foster the students' thinking skills and curiosity and to encourage them to ask their own questions.

- Various suggestions from students in class.
- Interaction between groups of students.
- Reaching a consensus across the whole class (or just across the group).

4. Critical thinking skills and mathematical knowledge (teaching)

The teacher referred to the questions raised by the students and encouraged CT, while instilling new mathematical knowledge: the identification of and finding a causal connection by a third factor and finding a statistical connection between C, and A and B, Simpson's paradox and Bayes Theorem.

Case study- The Aspirin Case

Below, I have provided a detailed description of one lesson called the Aspirin Case. Following the description, I outline the analysis of the lesson using the following techniques: referring to information sources, raising questions, identifying variables, and suggesting alternatives and inferences. The lesson topic was conditional probability. The CT skills practiced in the lesson were evaluating source reliability, identifying variables, and suggesting alternatives and inference.

1. A Given Text

Your brother woke up in the middle of the night, crying and complaining he has a stomachache. Your parents are not at home and you don't know what to do. You gave your brother aspirin, but an hour later he woke up again, suffering from bad nausea and vomiting. The doctor that takes care of your brother regularly is out of town and you consider whether to take your brother to the hospital, which is far from your home. You read from a book about children's diseases and find out that there are children that suffer from a deficiency in a certain type of enzyme and as a result, 25% of them develop a bad reaction to aspirin, which could lead to paralysis or even death. Thus, giving aspirin to these children is forbidden. On the other hand, the general percentage of cases in which bad reactions such as these occur after taking aspirin is 75%. 3% of children lack this enzyme.

(Taken from "probability thinking" p. 30+slight changes made by researcher)

2. Open Class Discussion in Small Groups

Discussion in small groups about the generic question:

Should you take your brother to the emergency room? What should you do?

Can aspirin consumption be lethal?

3. Critical Thinking Skills and Mathematical Knowledge (Teaching)

This phase of the lesson focused on encouraging critical thinking and instilling new mathematical knowledge (Bayes formula) statistical connections by referring to students' questions and further discussion.

A teacher-led discussion focused on methods of analysis using such Critical Thinking skills as: Source identification: Medicine book; Source reliability: High; Variable identification: A – enzyme deficiency, D – adverse reaction to aspirin; Mathematical Knowledge: Data: $P(D/A)=0.25$ $P(D)=0.75$ $P(A)=0.03$, To prove: $P(A/D)=?$

Using Bayes formula (or a two dimensional matrix) the result is:

Lesson Conclusion is that only 1% of the children without the enzyme develop an adverse reaction to aspirin, thus there is no need to go to the hospital.

Even so, is it worth taking the risk? What do you think? (question to the class).

Discussion

Research analysis according to critical thinking skills in this case study through the model of the infusion approach, students practice their CT while acquiring technical probability skills. In this lesson, the following five skills are exercised: raising questions – asking question about the article and probing on the main question about the connection between aspirin and death; referring to information sources and evaluating the source's reliability - the text took from Medicine book; the students skepticism, and identification of variables – students identified the enzyme deficiency and adverse reaction to aspirin. Following these skills, another skill, searching for alternatives, was presented. In class the teacher and the students spoke about suggesting alternatives, not taking things for granted, but examining what had been said and suggesting other explanations. Hence, the skills that were practiced in the described lesson were: raising questions, evaluating the source's reliability, identifying variables, and suggesting alternatives and inference. In order to understand and monitor the students' attitudes toward CT as manifested by the skills specified above, interviews were conducted with five students after the aforementioned lesson. In these interviews, the students acknowledged the importance of CT. Moreover, students were aware of the infusion of instructional strategies that advance CT skills. Results of our study should be considered with some caution, as this case study presents one lesson which was designed in a fixed pattern – a generic question, a discussion of the question, the practice of statistical connection, introduction to causal connection and experiencing the use of CT skills such as: raising questions, evaluating the source's reliability, identifying variables, and suggesting alternatives and inferences. On the basis of the interviews conducted and questionnaires that were qualitatively analyzed, it is not known, at this stage, whether these skills had been acquired. Skill acquisition will be evaluated at a later phase in this study, using quantitative measures – the Cornell Critical Thinking Scale and the CCTDI (Facion, 1992) scale. This case study provides encouraging evidence of the effectiveness of this approach and further investigation in this direction is needed. The small scale research described here constitutes a small step in the direction of developing additional learning units within the traditional curriculum. Current research is exploring additional means of CT evaluation, including: the Cornell CT scale (Ennis, 1987), questionnaires employing various approaches, and a comprehensive test composed for future research. The general educational implications of this research suggest that we can and should lever the intellectual development of the student beyond the technical content of the course, by creating learning environments that foster CT, and which will, in turn, encourage the student to investigate the issue at hand, evaluate the information and react to it as a critical thinker. It is important to note that, in addition to the skills mentioned above, in the course of this lesson the students also gained intellectual skills such as conceptual thinking and developed a class culture (climate) that fostered CT. Students practiced critical thinking by studying probability. In this lesson, the following skills were demonstrably practiced: referring to information sources, encouraging open-mindedness and mental flexibility (all questions), a change in attitude and searching for alternatives. A very important intellectual skill is the fostering of cognitive determination – to be able to express one's attitude and present an opinion that is supported by facts . In this lesson, students could be seen to be searching for the truth, they were open-minded and self-confident. In other words, they practiced critical thinking skills. A new language was being created: the language of critical thinking.

References

Aizikovitsh, E. & Amit, M. (2008) Developing critical thinking in probability lesson. In O. Figueras, J. Cortina, S. Alatorre, T. Rojano & A. Sepulveda (Eds.), Proceedings of the 32th Annual Conference of the International Group for the Psychology of Mathematics Education (vol.2 pp. 9-13). Mexico: PME.

Ennis, R.H. (1987). A taxonomy of critical thinking: Dispositions and abilities. In J. B. Baron, and R. J. Sterngerg (Eds.), Teaching for Thinking (pp. 9-26). New York: Freeman

Facione PA & Facion NC (1992) The California Critical Thinking Disposition Inventory.
Millbrae:The California academic Press.

Liberman, V. & Tversky, A. (2002). Probability Thinking in Daily Life. Tel-Aviv, Israel: The Open University.

Perkins, D. (1992). Smart Schools: Better Thinking and Learning for Every Child. New York: The Free Press.

Swartz, R. (1992). Critical thinking, the curriculum, and the problem of transfer. In D. Perkins, J. Bishop, & J. Lochhead (Eds.), Thinking: The Second International Conference (pp. 261-284). Hillsdale, NJ: Erlbaum.