

Plenary Address: Language and Mathematics, A Model for Mathematics in the 21st Century

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“Human language and thought are crucially shaped by the properties of our bodies and the structure of our physical and social environment. Language and thought are not best studied as formal mathematics and logic, but as adaptations that enable creatures like us to thrive in a wide range of situations” (Feldman, 2006, p. 7).

Language and Mathematics: A Complex Symbiotic for Learning

In order to know how to use this language correctly requires an integrated knowledge of multiple facets of communicative competence and mathematical knowledge. Vosniadou and Vamvakoussi (2006) suggest that if knowledge is viewed as a process instead of a product that the emphasis of teaching shifts from one focused on subject-matter content to thinking and learning skills. They further assert that mathematics is not only a process in which one participates but the knowledge products of complex social interaction. It can be argued then that there exists a complex interplay between language and mathematics, both as processes and products. For Vygotsky (1978), “thought is not merely expressed in words; it comes into existence through them” (p. 218). Learning is dependent upon this relationship between thinking and language. To improve our conceptions of learning requires exploring the complex questions about the mediation between thought, language, and mathematics.

Walshaw and Anthony (2008) argue that student outcomes are dependent on an array of cultural scripts and imperatives that are part of a pedagogical activity system. They challenge that classroom discourse, the means of constructing mathematical knowledge, will expand only when there is a viable cohesion between all the elements and interrelated contingencies. The model presented here is a response to that challenge: an effort to describe this cohesion between all the elements and interrelated contingencies. I concur that this understanding is essential if we are to develop the capacities to affect this multidimensional system.

Explicating a Model

Language and competence in mathematics are not separable. The model that is presented in this paper [See Figure 1] is intended to be an emerging work to stimulate discussion and to provide opportunities for dialogue about the complex nature of this relationship. The work of Erik DeCort has been substantive in moving this discussion forward. De Cort (2007) eloquently articulates five components that are necessary for developing competence in mathematics:

- (1) A well-organized and flexibly accessible domain-specific knowledge base involving the facts, symbols, algorithms, concepts, and rules that constitute the contents of mathematics as a subject-matter field.
- (2) Heuristic methods, i.e., search strategies for problem analysis and transformation (e.g., decomposing a problem into subgoals, making a graphic representation of a problem) which do not guarantee, but significantly increase the probability of finding the correct solution.
- (3) Meta-knowledge, which involves knowledge about one’s cognitive functioning (metacognitive knowledge; e.g., knowing that one’s cognitive potential can be developed through learning and effort), on the one hand, and knowledge about one’s motivation and emotions (metavolitional knowledge; e.g., becoming aware of one’s fear of failure when confronted with a complex mathematical task or problem), on the other hand.
- (4) Positive mathematics-related beliefs, which include the implicitly and explicitly held subjective conceptions about mathematics education, about the self as a learner of mathematics, and about the social context of the mathematics classroom.
- (5) Self-regulatory skills, which embrace skills relating to the self-regulation of one’s cognitive processes (metacognitive skills or cognitive self-regulation; e.g., planning and monitoring one’s problem-solving processes), on the one hand, and skills for regulating one’s volitional processes/activities (metavolitional skills or volitional self-regulation; e.g., keeping up one’s attention and motivation to solve a given problem), on the other hand. (p. 20-21).

These five components describe prominent features that elucidate the link between mathematical competence, language, and thought. In the model offered in this paper, the importance of these components is evident. The proposed model is intended to identify the multifaceted interaction of complex features that impact thinking in the communication circuit or loop with the goal of demonstrating the dependency of multiple components in creating an effective multidimensional communicative process.

External factors. In the communication loop, the receiver is influenced by both external and internal factors. While internal factors represent those facets that are most complex and abstract, the role of external factors in this process should not be downplayed. The features and nature of the communication provides the first interaction between the message and the receiver. In written communication, features of text comprise a linguistic register and include phonological, lexical, grammatical, and sociolinguistic elements (Scarcella, 2003). The dense conceptual level of mathematical texts and the technical register present problems for the reader (Pugalee, 2007). These difficulties are further compounded by a lack of metalinguistic awareness that supports the reader as he reflects on the structural and functional features of the text as decisions are made about how to communicate information and manipulate units of language (MacGregor & Price, 1999; Pugalee, 2007). Metacognition or self-monitoring and application of learning strategies assists the reader in comprehending the text. Oral language also must pass through similar filters. Oral language must be considered within the broader framework of the content features and the quality of the interaction (Bussi, 1998).

Internal Factors. As previously presented, DeCort (2007) provides a rich discussion of salient features of the cognitive elements that are necessary in any model for which mathematical competence is the core. Also of interest is the sociocultural dimension which represents a complex interplay between the physical world and how those in discourse communities construct meaning through language. The sociocultural dimension involves norms, values, beliefs, attitudes and practices of language within cultural settings which includes the learning environment. Mathematics might be thought of as a “cultural activity that involves inventing, using and improving symbols” (Cobb, 2000, p.20). In the sociocultural sense, mathematical discourse comprises perceiving and doing in addition to speaking and writing (Sfard, 2000; Dorfler, 2000). Communication in this symbol rich environment of mathematics is challenging both in terms of executing effective communication but also in terms of interpreting information and constructing knowledge within this environment.

“There is also the question of whether symbolism can be used as a tool for cutting through the relevant noise during the abstraction process or whether it can only be used to formulate what has already been abstracted. If the latter, then every learning situation will have a ceiling determined by (1) the amount of noise generated, and (2) the amount of noise the learner is able to cut through” (Dienes, 1963, pp. 160-161).

Ernest (1997) argues that the physical world is only describable through linguistic means (categories) and that such description is the result of interpretation. Language and the construction of mathematical meaning are social phenomena mediated by environmental and individual factors.

The cognitive processes that are required to give students access to mathematics cannot be separated from the linguistic aspects of the information received and how that communication interacts with multiple variables as a result of cognition. Higher order thinking, strategic competence, and metalinguistic awareness (Scarcella, 2003) along with metaknowledge, metacognition, heuristic and procedural knowledge, and conceptual understanding (DeCort, 2007; Scarcella, 2003) are included in the cognitive register. The student’s strategic competence will impact their ability to formulate, represent, and do mathematics. Their capacity for adaptive reasoning will influence the quality of the product or output as they reflect, explain, and justify their thinking. The student’s procedural and conceptual understandings further arbitrate the degree to which they are capable of producing meaningful and acceptable results of their thinking. All of this is mediated by language – and produces a cognitive load that some students are ill equipped to handle.

Some Concluding Thoughts

The progress of students depends on the advancement of our thinking about the relationship between language and the learning and teaching of mathematics. Anna Sfard (2001) posits that communication is the heart of mathematics education and should be viewed “not as a mere aid to thinking, but as almost tantamount to the thinking itself” (p. 13). Consider the model presented in this paper. How can it inform our thinking about the complexities of our practice? What is the nature of student’s failure and success in mathematics? Is it possible to have mathematical learning with understanding void of a thoughtful and deliberate consideration of the role of language and communication?

The model offered in this paper is an attempt to represent complex and multifaceted processes that affect mathematics teaching and learning. Models are but an attempt to simplify complex activities so that they can be better conceptualized. Understanding communication and language is essential in understanding mathematics learning. Consider the model; discuss it; critique it. Accept the challenge to inform our practice as mathematics educators through the consideration of how our work is both informed and constructed by language.

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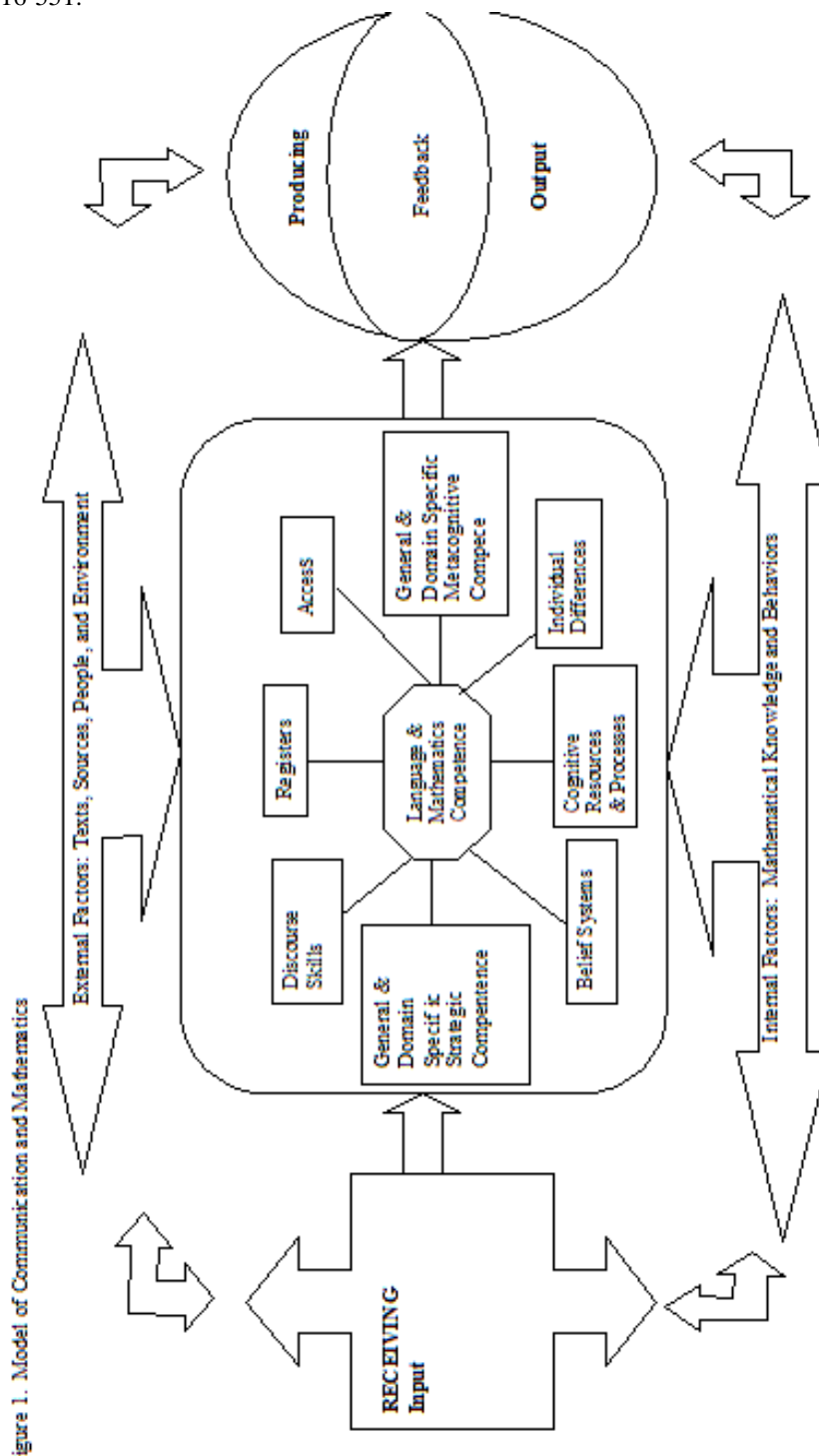


figure 1. Model of Communication and Mathematics