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Mathematics meets physics

A contribution to their interaction in the 19th and the first half of the 20th century

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Mathematical Foundations and physical Visions: Pascual Jordan and the Field Theory Program

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1 Introduction

The work of Pascual Jordan (1902 – 1980) offers rich material for a study of the complex interactions between mathematics and physics in the twentieth century, and especially for its possibly most eventful period, the years 1925 – 1927 when modern quantum mechanics and quantum field theory were established. Jordan was truly a scion of the unique closeness if not amalgamation of physics and mathematics characteristic for Göttingen in the days of Felix Klein and David Hilbert. Within two years of his arrival there in 1922, he had been a student assistant with the theoretical physicist Max Born revising his article «Dynamik der Kristallgitter» [Born, 1923], with the mathematicians Richard Courant and David Hilbert working on the textbook *Methoden der Mathematischen Physik* [Courant and Hilbert, 1924], and with the experimentalist James Franck coauthoring the review article «Anregungen von Quantensprüngen durch Stöße» [Franck and Jordan, 1926]. The present contribution will discuss the connection of this educational background with Jordan's program and achievements in quantum field theory.

Jordan was the earliest and most ambitious visionary of the quantum field theory program: long before this became commonly accepted in the second half of the twentieth century, he saw in quantum field theory a unified basis for all of modern physics.[1] Jordan's formulation of this goal and his work towards it depended on a rather unique combination of a foundationalist universalism that would befit an Einstein or Planck, and a radical positivism that rejected vehemently the demand for a visualizable and intuitive understanding of physics. While it is not hard to discern these two tendencies in Jordan's work and see the tension between them, it is less obvious to understand how they relate to the balance between mathematics and physics in Jordan's work. Nevertheless, I will claim that there is an intimate connection between the two relationships.

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[1] Jordan's seminal contributions to quantum field theory are described in more detail in [Cini, 1982] and [Darrigol, 1986]. Duncan and Janssen [2008] give a detailed account of Jordan's derivation of Einstein's fluctuation formula for radiation and the role this played in the emergence of quantum field theory.

2 Neither waves nor particles

In his dissertation,[2] Jordan had attempted to find a way to avoid Einstein's conclusion [Einstein, 1917] that the emission of radiation by the Bohr atom had to be directed. Einstein [1925a] quickly showed that Jordan's argument rested on the physically implausible assumption that also the absorption of radiation could not be directed, i. e., that an atom could not absorb a light wave coming in from a specific direction. After this paper and a correspondence about it with Einstein, Jordan accepted Einstein's argument about the irreducibly dual nature of light. However, the lessons he had learned about the statistics of the equilibrium of radiation and matter would have a decisive impact on his further development: When Jordan read Einstein's papers on the Bose statistics of the ideal gas [Einstein, 1924, 1925b], he immediately noticed the impact that the new statistics had on the theory of the interaction between radiation and matter. Jordan used the new statistics, as well as de Broglie's idea of matter waves to which Einstein had referred in order to motivate it, to study the thermodynamical equilibrium of light quanta and the ideal gas. This led him to make a strikingly novel stipulation:

> "The elementary acts of dispersion [between radiation and matter] can be viewed not only as dispersion of light radiation on material corpuscles but also as dispersion of matter radiation on corpuscular light quanta; therefore, the probabilistic law will be symmetric ... [between the densities of radiation and matter]."[3]

Schrödinger had taken Einstein's theory of the ideal gas as evidence that matter and radiation both had to be understood as waves [Schrödinger, 1926b]. Jordan agreed that matter and radiation were of the same nature, but he did not accept that this nature was correctly expressed by a classical wave picture. Instead, he postulated that both matter and radiation should be representable equivalently either as waves or as particles, thus establishing a complete symmetry between the two representations.

In an interview with Thomas Kuhn for the Archives for the History of Quantum Physics (AHQP),[4] Jordan credited the idea of the symmetry

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[2] Published as [Jordan, 1924].

[3] Jordan 1925

[4] Interview of Pascual Jordan with Thomas S. Kuhn, June 18, 1963. AHQP, Transcripts of Oral History Recordings, Microfilm 1419-03, Jordan interview 2, p. 19.

of representations to William Duane's treatment of the scattering of light quanta by a grid [Duane, 1923]. Duane had shown that the interference on a grid, which had always been seen as a paramount wave phenomenon, could also be explained in the light quantum theory if one quantized the periodic structure of the grid. Jordan saw this argument as evidence that the dualism of particle and wave character of light should find its theoretical expression in the possibility to represent the same physical situation equivalently in particle and in wave description. For Jordan, this symmetry of representations was a convincing argument that all previous mechanical pictures had to be insufficient. The symmetry of representations would become the fundamental heuristic principle underlying Jordan's work both in quantum mechanics and quantum field theory during the following years. Jordan claimed in the AHQP interview[5] that already at this point he was hoping that a quantum theory of waves could deliver this symmetrical representation for both matter and radiation. Although there is no direct contemporary evidence, the circumstances described above make this plausible.

In the summer of 1925, Jordan got recruited by Max Born to help in the mathematical elaboration of Werner Heisenberg's idea of *Umdeutung*. Born and Jordan [1925] showed that the matrix calculus was the appropriate mathematical form for Heisenberg's new mechanics. However, Jordan did not limit himself to the formalization of Heisenberg's ideas: the paper contains an application of matrix mechanics to the electromagnetic field. This section did not lead to any concrete empirical predictions, and was largely ignored. But it is an indication of Jordan's program of a quantized field theory, rooted in his earlier insights from gas theory. Also the subsequent *Dreimännerarbeit* [Born et al., 1926] contains a section on the quantization of a field, this time with a much more striking result: the derivation of Einstein's famous and puzzling fluctuation formula for radiation from matrix mechanics applied to a field. As we know from a letter from Heisenberg to Pauli,[6] it was written

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[5] Interview of Pascual Jordan with Thomas S. Kuhn, June 19, 1963. AHQP, Transcripts of Oral History Recordings, M/f 1419-03, Jordan interview 3, p. 9.

[6] Heisenberg to Pauli, October 23, 1925 [Pauli, 1979, p. 252].

by Jordan who later considered it as “almost the most important thing I have contributed to quantum mechanics.”[7]

Einstein had used the thermodynamic entropy of radiation to derive its fluctuation properties: the energy fluctuations in a small volume of a small band of frequencies contained two terms. One could be interpreted as expressing fluctuations due to a varying number of light quanta in the volume, the other as due to the interference of light waves. Their simultaneous presence was a striking illustration of the dual nature of light but also posed the problem to find a theory of light that could account for the presence of both terms. Einstein struggled for the rest of his life to provide such a theory of light. In a study of Einstein’s fluctuation formula, Paul Ehrenfest [1925] had introduced the model of a vibrating elastic string fixed at both ends as the simplest possible situation for the study of wave fluctuations. Each characteristic frequency of its vibration (or wave mode) can be treated as an independent harmonic oscillator. The total energy of each mode (and thus of the string as a whole) is constant. But the energy content of a small number of neighboring wave modes in a small segment of the string fluctuates because of the interference of the neighboring wave modes. Ehrenfest calculated this fluctuation and obtained only the wave fluctuation term, even if the individual wave modes were quantized in the sense of the old quantum theory. In the *Dreimännerarbeit*, Jordan quantized Ehrenfest‘s model using matrix mechanics – harmonic oscillators being one of the few things one could quantize with matrix mechanics in 1925 – and discovered that the non-commutativity of the matrix calculus leads to an additional term for the energy fluctuations: it is exactly the particle fluctuation term. For the first time, Einstein’s fluctuation formula had been derived from an underlying dynamical theory.

Jordan concluded his considerations with the remark:

> “If one considers that the question treated here [the fluctuation of radiation] is rather removed from the problems out of which quantum mechanics arose, one will perceive the result [. . .] as especially encouraging for the further extension of the theory.”[8]

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[7] Jordan to van der Waerden, April 10, 1962, AHQP M/f 1419-006, p. 604. The quotes from Heisenberg and Jordan are given in [Duncan and Janssen, 2008].

[8] Born et al. 1926, p. 615

The full meaning of this remark would have eluded a contemporary reader, but it fits very well with Jordan's later reminiscences that he saw in this derivation the first lead to the quantized field theory he had been looking for. However, even his coauthors Heisenberg and Born were skeptical about the need to quantize the electromagnetic field [Duncan and Janssen, 2008, p. 640 – 642]. One obvious problem was that Jordan's method implied that each of the quantized oscillators representing the radiation field had a zero-point energy, so that the vacuum had an infinite energy density. This led Heisenberg to state that the method is only suitable for the treatment of oscillations of a discrete crystal lattice where such infinities would not occur. Jordan, on the other hand, had an even more ambitious goal: His principle of symmetries of representations implied that also matter should be represented by quantized waves in the same manner. As he claimed in [Jordan, 1927g, p. 480] and in a letter to Schrödinger, his occupation with the quantum theory of the ideal gas had suggested this further application of the theory of quantized waves. Jordan writes in the letter:

> "Then your hydrogen paper [i. e., Schrödinger [1926a]] gave hope that by following up this correspondence also the non-ideal gas could be represented by quantized waves – that therefore a complete theory of light and matter could be derived in which, as an essential ingredient, this wave field itself operates in a quantum, non-classical way."[9]

Jordan saw Schrödinger's wavefunctions as a generalization of the simple plane waves that he had quantized in the *Dreimännerarbeit* and interpreted as the quantum mechanical representation of the Bose-Einstein ideal gas; he was convinced that the quantization of these wavefunctions was the method necessary to apply quantum mechanics to the case of several interacting particles.[10] In the letter to Schrödinger, Jordan gives two reasons why he did not pursue this program immediately: The problem to account for Fermi-Dirac statistics, since it seemed that the wave picture would always lead to Bose-Einstein statistics, and the reservations of his colleagues Heisenberg, Pauli, and Born.

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[9] Jordan to Schrödinger, reply to Schrödinger's letter from July 28, 1927, AHQP M/f 41 Sect. 8-009b, quoted after Darrigol, p. 224.

[10] Interview of Pascual Jordan with Thomas S. Kuhn, June 20, 1963. AHQP, Transcripts of Oral History Recordings, M/f 1419-03, Jordan interview 4, p. 3.

By the summer of 1926, Jordan was thus convinced that the correct treatment of a system of interacting particles was the quantization of their associated matter waves. This approach was fundamentally different both from Schrödinger's and from Heisenberg's and Dirac's ideas about the application of quantum mechanics to the many-particle problem. While Schrödinger was searching for a way to represent the many-body problem as the self-interaction of a continuous charge distribution, Heisenberg and Dirac had constructed symmetrical and antisymmetrical many-particle wavefunctions from single-particle wavefunctions and given phenomenological arguments why they should account for the characteristics of atomic spectra. Dirac showed that symmetrical wavefunctions led to Bose-Einstein statistics and that antisymmetrical wavefunctions explained the Pauli exclusion principle for electrons and therefore should be the basis of a statistics for matter particles. The success of the Heisenberg-Dirac method in the explanation of atomic spectra made Jordan's much more abstract program seem superfluous.

The transformation theory, developed in 1926/27 by Dirac [1927a] and Jordan [1927e, f] independently, was for Jordan further evidence for his principle of symmetry of representations. To Jordan, it showed that there is no preferred ontological basis in which quantum mechanics should be explicated. Jordan's transformation theory did not use the concept of a state at all; rather, what he used for the description of a physical system was the totality of all possible transition amplitudes between the values of physical quantities, the squares of which give the probability of finding the value of one quantity given the value of another quantity. Instead of specifying, e. g., one specific state of a hydrogen atom by a wavefunction, Jordan's transformation theory describes all possible states of the hydrogen atom by the transition amplitudes between a basis diagonalizing the energy matrix and a basis diagonalizing the position matrix of the electron. Jordan now identified "particle" properties with the basis diagonalizing the position matrix and "wave" properties with the basis diagonalizing the momentum matrix conjugate to the position matrix. Since the theory is invariant with regards to the choice of basis, the system can be described equally in particle or wave language. Therefore, neither description of the system (as a particle or as a wave) is fundamental.

This conviction about the symmetry of representations was also the background for Jordan's attack on Schrödinger's physical wave interpretation of wave mechanics [Jordan, 1927d]. Jordan agreed with Schrödinger that light and matter show analogous behavior and should be treated analogously in quantum theory. But he argued that just as classical wave optics fails for the effects that made the light quantum theory necessary, so wave mechanics alone cannot account for the particulate aspects of matter. Otherwise, there would be a disanalogy between the theories of light and matter.

3 The beginning of quantum field theory

The idea of a quantized field only came to the attention of a wider group of physicists through Paul Dirac's "The quantum theory of emission and absorption of radiation."[11] Paradoxically, the notion of quantizing a field appears nowhere in the paper. Dirac started with standard perturbation theory and observed that the expansion of the perturbed state ψ in terms of the eigenstates ψ_r of the unperturbed Hamiltonian H_0

$$\psi = \sum_r a_r \psi_r \tag{1}$$

can be interpreted as describing how a statistical ensemble of noninteracting systems reacts to an external perturbation, since the squared expansion coefficients $|a_r|^2$ can be read as giving the ratio of systems in each eigenstate. Standard perturbation theory gives for a perturbed Hamiltonian $H = H_0 + V$ the following time-dependence of the expansion coefficients:

$$i\hbar \dot{a}_r = \sum_s V_{rs} a_s \tag{2}$$

Dirac now showed that if one treated the a_r as quantum numbers, the same equations can be interpreted as describing an ensemble of systems obeying Bose-Einstein statistics. In this case, $N_r = a_r^\dagger a_r$ gives the number of systems in state r. If one applies this procedure to a system of light quanta interacting with an atom, one can represent the interaction in

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[11] Dirac 1927b

terms of the changes that it causes in the atomic states and in the number of light quanta.

Dirac never tried to relate the a_r directly to field amplitudes. Rather, he connected the two by observing that a given number of light quanta determines through Einstein's $E = h\nu$ the energy density of the corresponding electromagnetic field and thus the field amplitudes acting on the atom. Using this equation, he could connect Einstein's emission and absorption coefficients with the matrix elements of the atomic electron in matrix mechanics – something that Heisenberg had only postulated in the *Umdeutung* paper. However, Dirac explicitly denied that the "wave function of the light quanta" is the same as the electromagnetic field. He also argued that while an ensemble of light quanta can be associated with a light wave, there is no such physical wave associated with an ensemble of matter particles such as electrons. Therefore, he did not see the quantization procedure as an explanation of the quantum nature of radiation. It was to him only an elegant way to take into account the Bose statistics of light quanta. Since electrons do not obey Bose statistics, the procedure is not applicable to them. Dirac maintained particle number conservation for light quanta by introducing a 'sea' of zero-momentum light quanta. This is another piece of evidence that for Dirac the particle concept was primary.[12]

Unlike Jordan's earlier attempt, Dirac's theory was greeted with enthusiasm, since it first derived the link between quantum mechanics and Einstein's theory of absorption and emission, and so offered a quantum-mechanical representation of the interaction of matter and radiation. Today, Dirac's paper is often seen as the seminal work for quantum field theory. This is somewhat ironic, as Dirac explicitly rejected the idea that his method was to be understood as the quantization of the classical field. Jordan thought for the rest of his life that he did not get due credit for his work:

> "It has always saddened me somehow that the attack on the light-quantum problem already contained in our Dreimännerarbeit was rejected by everyone for so long (I vividly remember how Frenkel, despite his very friendly disposition toward me,

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[12] It also shows the problems that interpreting light quanta as particles leads to, foreshadowing the even more problematic notion of a sea of negative-energy electrons that would appear in Dirac's 1928 electron theory.

> regarded the quantization of the electromagnetic field as a mild form of insanity) until Dirac took up the idea from which point onward he was the only one cited in this connection.”[13]

Instigated by Dirac’s success, Jordan quickly returned to the theory of the quantized field. However, what he did was in conflict with Dirac’s ideas and a clear continuation of his earlier program based on the principle of symmetry of representations. Therefore, his first paper [Jordan, 1927g] explicitly rejected Dirac’s assessment that the ideal gas obeying Fermi statistics cannot be represented by a wave field. Jordan observed that in the case of Bose-Einstein statistics, the number operator has arbitrary integer eigenvalues, while in the case of Fermi-Dirac statistics, the number operator can only have eigenvalues 0 or 1. He now constructed an algebra of field operators that yield these eigenvalues for the number operator using Pauli’s spin matrices. This construction was made possible by Jordan’s concept of conjugate variables that was more general than Dirac’s: While Dirac relied on commutation relations of the standard form $pq - qp = -i\hbar$, Jordan’s transformation theory relied on a more general notion of conjugate variables (motivated by the need to represent angle and angular momentum as conjugate variables[14]) and allowed for a generalization of these commutation rules. However, as Darrigol [1986, p. 232] has pointed out, Jordan’s actual calculations were full of mistakes: “Although Jordan knew he was on the right track, his paper was only a sketch, full of misprints and imprecisions. The draft received by Alfred Landé resembles a bad student paper overcorrected by the professor.” What had gotten lost in the imprecisions were the correct phase relations between the creation and annihilation operators. Only in the fall of 1927, Jordan would return to the topic and, with the help of Eugene Wigner, present the correct algebra (now called Jordan-Wigner second quantization) using anticommutation relations [Jordan and Wigner, 1928].

Despite its technical flaws, [Jordan, 1927g] already defines Jordan‘s program: a unified quantum field theory for matter and radiation.

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[13] Jordan to Born, July 3, 1948, AHQP M/f 1419-006, p. 596; quoted after Duncan and Janssen, 2008

[14] Interview of Pascual Jordan with Thomas S. Kuhn, June 19, 1963. AHQP, Transcripts of Oral History Recordings, M/f 1419-03, Jordan interview 3, p. 22 – 23.

Particles and waves are only two different aspects of the same underlying quantum field both in the case of light and in the case of matter:

> "Despite the validity of the Pauli instead of Bose statistics for electrons, the results achieved so far leave hardly a doubt that a quantum-mechanical wave theory of matter can be formulated, in which electrons are represented as quantized waves in ordinary three-dimensional space and that the natural formulation of the quantum theory of the electron will have to be achieved by comprehending light and matter on equal footing as interacting waves in three-dimensional space. The fundamental fact of electron theory, the existence of discrete electrical particles, thus manifests itself as a characteristic quantum phenomenon, namely as equivalent to the fact that matter waves only appear in discrete quantized states."[15]

Jordan pointed out that the antisymmetrical wavefunctions that Heisenberg and Dirac had constructed for many-particle systems were therefore not at all physical waves but simply "a special case of the general probability amplitudes which have to be used as a mathematical tool for the description of the statistical behavior of quantized light and matter waves" [Jordan, 1927g, p. 480]. These quotes show clearly the difference in perspective between Jordan and Dirac: Unlike Dirac, Jordan treated second quantization of the Schrödinger wave function as the quantization of a physical field and saw this procedure as an explanation of the corpuscular character of matter. Unlike Schrödinger, however, Jordan did not attempt to find an objective physical description behind the mathematical formalism. Transformation theory to him still implied that neither the particle nor the wave description were fundamental and therefore neither picture could be used to construct a complete description of objective reality.

Jordan's vision was not yet a full theory. So far, he only could treat free fields nonrelativistically. In the following months, Jordan made quick progress towards a complete theory in a series of three papers with different collaborators. First, he collaborated with Wolfgang Pauli [Jordan and Pauli, 1928], giving relativistically invariant commutation rules for the free field. The second paper was written together with Oskar Klein in Copenhagen [Jordan and Klein, 1927]. Klein had been thinking

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[15] Jordan 1927g, p. 480

about a relativistic quantum theory of interacting particles, based on Dirac's quantized waves. As he wrote to Dirac, he worried about the problem of self-energies arising from the field-theoretical treatment of the Coulomb interaction.[16] When Jordan stayed in Copenhagen in the summer of 1927, they introduced field operators $\phi(r)$ to represent the field strength in a specific spacetime point and solved the problem of self-energies by what is now called normal ordering of these field operators. This allowed for a quantum field theoretical reformulation of the (instantaneous) interaction between particles and demonstrated that quantum field theory can treat the many-particle problem, as Jordan had envisioned already in 1926.

Schrödinger, referring to the programmatic passage from [Jordan, 1927g] cited above, wrote to Jordan in surprise:

> "This is, as far as I understand, also my opinion. So far, I thought that it was decidedly rejected from Göttingen and Copenhagen. Now I am glad to see that prospects are improving that we will come together again."[17]

Born, Heisenberg, and Pauli referred to Jordan's work at the Solvay meeting in October of 1927, as a possible solution to the problems faced when explaining quantum effects based on a wave picture. Also Bohr was impressed and praised the work by Jordan and Klein in [Bohr, 1928]. Dirac, however, was not convinced and, in the discussions at the Solvay meeting (yet never in writing), criticized Jordan's quantization procedure as artificial and *ad hoc*. He also pointed out that there were mistakes in the mathematical derivation of [Jordan, 1927g]. When in 1928 Dirac developed his relativistic theory of the electron [Dirac, 1928], he treated the relativistic wave equation as an analogue of the Schrödinger equation and did not make use of any field-theoretical interpretations.

The first attempt at a full treatment of quantum electrodynamics was given by Heisenberg and Pauli [1929]. But this treatment also showed the problems connected with the quantum field theory program. As Jordan [1929] noted, the infinite self-energy of the electron was not a constant that could be simply ignored as in the case of the free

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[16] See [Darrigol, 1986, p. 234].

[17] Letter from Erwin Schrödinger to Pascual Jordan, 28 July 1927, AHQP, M/f 18, Sect. 7-001.

field. Jordan remarked that this problem was inherited from classical electrodynamics and that therefore it showed the limitations of the procedure of quantizing a classical theory. A new autonomous quantum field theory would have to be found to solve the problem. While various proposals to remove the infinities were made in the following years, it remained unclear how a general theory without inconsistencies could be built up. Possibly even more damaging to the program was the fact that it did not offer empirical predictions going beyond a theory such as Dirac's treating particles with antisymmetrical wave functions.

Only after World War II did the observation of the Lamb shift offer a first empirical confirmation of vacuum fluctuations, leading to a resurgence in interest in the quantum field theory program.[18] Quickly, this led to new renormalization techniques and the successful treatment of perturbation theory with Feynman diagrams. Jordan and Dirac, however, never rejoined the forefront of research in quantum field theory. Jordan's early contributions were mostly forgotten by the time of the postwar renaissance of quantum field theory, even though its modern formulation is closer to Jordan's program than to Dirac's original ideas.

4 Positivism

Schrödinger's hope for a reapproachment between his views and those of Jordan was not shared by the latter. Despite Jordan's polite answer, there was no indication that Jordan was changing his views already expressed in connection with transformation theory, that quantum mechanics did not allow for a reduction to classical models, be they particles or waves. As he would state in 1936 in his programmatic popular account "Physics in the 20th century":

> "The atom as we know it today no longer has the tangible and visualizable properties of the atoms of Democritus. It has been stripped of all sensible qualities and can only be characterized by a system of mathematical equations. The unbridgable opposition of materialistic philosophy and positivistic epistemology stands out especially clearly at this point. With this insight, one of the

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[18] See [Schweber, 1994] for a treatment of the history of quantum electrodynamics after World War II.

> most prominent elements of the materialist world view has been liquidated once and for all. At the same time, the positivistic epistemology has been confirmed and justified decisively."[19]

The fundamental lesson Jordan drew from quantum physics was a confirmation of positivism. The basis for this bold metatheoretical claim[20] was Jordan's conviction that the symmetry of different descriptions established by transformation theory implied that there was no *one* fundamental physical description and that therefore statements about unobservable entities in quantum mechanics (which corresponded to one specific description, i. e. the wave or particle picture) were meaningless.

However, neither Jordan's positivism nor his argument for it from transformation theory harmonize very well with his program for quantum field theory: Jordan's claims about the foundational character of quantum field theory imply a priority of an abstract field concept, with particles as secondary quantum phenomena. This abstract field concept, even though it does not coincide with Schrödinger's more physical concept of a matter field, retains one important characteristic of the classical field: the continuity and classical description of spacetime. No matter what representation is chosen, the states of the theory are defined on this continuum. For that reason, transformation theory does not have the same implications in quantum field theory as it does in quantum mechanics. Even though Jordan is not explicit about how he understands the application of transformation theory to quantum field theory, he seems to assume that particle and wave properties are represented by the two basic quantities of his formalism, the $\phi(r)$, describing the field strength at the position r, and the b_k, describing the amplitude of the excitation with the wavevector k.[21] Although these two quantities are related by a Fourier transform

$$\phi(r) = \sum_k b_k u_k(r) \tag{3}$$

(which resembles the Fourier transform between position and momentum eigenstates in quantum mechanics), this does not mean that $\phi(r)$

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[19] Jordan 1936, pp. 122 – 123

[20] See [Jordan, 1934] for a defense of positivism as a general epistemological principle, and [Darrigol, 1986, pp. 232 – 233], [Cini, 1982] for discussions of Jordan's positivism.

[21] In modern terms, these are the field operator and the annihilation operator, respectively.

can be identified with a particle property (i. e., a particle being in the position r). $\phi(r)$ only specifies the field strength at the position r, not a localization of the field at r. In Jordan's terminology: The matrix $\phi(r)$ is highly degenerate and therefore does not specify a basis that suffices to describe *localized* excitations of the field. Therefore, the Fourier transform is not the formal expression of a symmetry between wave and particle representations, unlike in the case of quantum mechanics. Thus, Jordan's quantum field theory is not symmetrical between wave and particle representations and so does not confirm positivism in the same way that he believed transformation theory did. Rather, one could say that wave and particle picture are represented by Jordan's field theory and Dirac's "many-particle theory" of symmetrized or antisymmetrized wave functions. But these are two distinct theories, which only coincide in certain cases.[22]

More generally, one can observe that Jordan's grand foundationalist visions are at odds with his positivism: According to the 19th century understanding of positivism, physical theory should describe, not explain. But Jordan himself kept invoking the explanatory power of quantum field theory as a justification of its fundamental nature, e. g., in the above quote from [Jordan, 1927g, p. 480]. Despite these tensions, Jordan maintained his positivism by emphasizing the differences between his quantized fields and classical fields. He frequently stressed that the quantum field did not offer hope for picturability in the classical sense. Therefore Jordan could maintain that, although quantum field theory offers a unified foundation of physics, it does not offer a visualizable physical model of the world. All it provides are probability amplitudes connecting possible observations, like in the case of transformation theory. However, this is a much weaker argument than in the case of the explicit argument for the possibility of different representations – it does not exclude the possibility that a *non-classical* but still spatiotemporal field picture could eventually be found as a consistent model for quantum field theory.

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[22] A simple aspect in which they do not coincide is that for Dirac, particle number must be conserved, while for Jordan, this is not necessarily the case.

5 Mathematics and physics

It is somewhat difficult to define the relation between mathematics and physics in Jordan's work. Exactly because of his Göttingen background he does not seem to see the two as distinct research subjects. In the AHQP interviews, he characterizes himself as a «Göttinger» in several places, contrasting his own open-minded attitude towards mathematical formalism to the suspicion if not hostility towards it from other physicists. For example, he relates the well-known story that Pauli accused Born that he would mess up Heisenberg's «Umdeutung» ideas with excessive mathematical formalism. However, and this is the more important observation, Jordan goes beyond what was traditionally seen as the role of a mathematical physicist: the precise elaboration of existent physical theories (say, in analytical mechanics). This difference becomes quite evident in comparison with John von Neumann's work, and his central contribution to quantum physics, the introduction of the Hilbert space formalism. Von Neumann's formalization gave a firm mathematical grounding to transformation theory, avoided Dirac's "improper functions," and allowed for new important concepts and arguments on the basis of the formalized theory, such as projection and density operators, quantum logic, his no-hidden-variable proof, or the formulation of the measurement problem. For all his important contributions, von Neumann's ambition was not to establish a new theory, but to clarify the existing statistical transformation theory. Thus his work is much more easily understood in the traditional sense of mathematical physics.

Jordan's strength, on the other hand, was definitely not the clarification of formal structures. We encountered a striking example above: Jordan needed Wigner's help to formulate the correct commutation relations for fermion fields. Another example is Jordan's half-hearted and confused attempt to present [Jordan, 1927e, f] in axiomatic form. Rather, Jordan's strength laid in his novel and far-reaching ideas about the foundations of quantum physics. In this respect, he was much more in the tradition of the previous generations of theoretical physicists, such as Planck, Einstein, and Schrödinger. Like these, he had the ambition to develop new and fundamental theories encompassing hitherto disjoint phenomena and the talent to find the correct clues in an abundance of experimental data. An indication of his claim to the status of a theoretical physicist are his lucid review papers[23] and his eloquent presentations to general audiences (e. g. [Jordan, 1927a, b, c] in *Die Naturwissenschaften*).

However, there is a fundamental divide between Jordan and Einstein or Schrödinger. What Jordan sees as a proof of positivism from quantum physics is for them a *reductio ad absurdum* of quantum mechanics as a physical theory. Although they disagreed in their specific criticisms and their hopes for a better theory, they agreed in one point: The inability of quantum mechanics to produce unambiguous spatiotemporal models of objective processes disqualified it as a fundamental physical theory.[24] The disagreement about positivism was not merely a philosophical debate disconnected from physical theorizing, it fundamentally affected the definition of theoretical physics itself and its methodological prescriptions.

The central role of (mechanical) models for the foundations of classical theoretical physics is a well-treated subject.[25] I will only touch on one aspect that throws an interesting light on the relation of theoretical physics to mathematics: Elizabeth Garber contrasts the work of Poincaré as a mathematician in electrodynamics with that of Einstein and Lorentz as theoretical physicists. She notes that Poincaré had a different interest in exploring electrodynamics: "Poincaré's net was mathematics and observation, not physical theory."[26] This led Einstein to explore the physical consequences of Lorentz invariance, which Poincaré didn't. Einstein in turn did not see the relevance of Minkowski's geometrical representation of the Poincaré group, until his work on general relativity forced him to deal with it. One can therefore see the distinction between mathematics and theoretical physics in the focus on theoretical models

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[23] E. g. [Jordan, 1928, 1929]

[24] The philosophical principles underlying Schrödinger's critique of quantum mechanics are treated in [Bitbol and Darrigol, 1992], Schrödinger's defense of the need for visualizability of physical theories in [De Regt, 1997]. In the case of Einstein, the existing literature is far too extensive to be cited in detail here. See [Home and Whitaker, 2007] for an overview; I will discuss Einstein's critique of positivism in quantum mechanics in a forthcoming contribution to *The Cambridge Companion to Einstein*, M. Janssen and C. Lehner, eds. Cambridge University Press, Cambridge.

[25] See for example [Lützen, 2005] for the case of Heinrich Hertz, [De Regt, 1999] for the case of Boltzmann, or [Cat, 2001] for the case of Maxwell.

[26] Garber 1999, p. 354

of physical situations and their exploration. While theoretical physicists took them as an expression of fundamental physical principles, mathematicians treated them as secondary illustrations of the fundamental mathematical structure.

This distinction connects the issue of positivism with the demarcation of theoretical physics from mathematics: Pauli, Heisenberg, and Jordan saw matrix mechanics as expressing the impossibility to give a consistent physical picture to quantum mechanical processes and Heisenberg's uncertainty relations as numerical limit to the applicability of such pictures.[27] As we saw, Jordan maintained this position also for the quantized field and was the most explicit in connecting it to a emphatic defense of positivism: Physics is about nothing but a concise mathematical description of the phenomena. Every question going beyond that is a pseudoproblem. The conspicuous absence of the concept of a physical state in Jordan's formalism reflects his conviction that there are no matters of fact beyond the observational data. According to Garber's distinction, his positivism therefore would make him a mathematician rather than a theoretical physicist – or at least it would have done so around the turn of the twentieth century. This verdict would have probably been applauded by Einstein and Schrödinger, who maintained that giving up a fundamental physical picture for quantum theory meant abandoning the core element of physical theorizing. And both, in different ways, kept fighting to regain such a physical picture.

However, as we have seen, this verdict is rather one-sided. It does not do justice to the relevance and foresight of Jordan's vision for quantum field theory. It is not that Jordan was not a theoretical physicist, rather theoretical physics changed radically in the years between 1900 and 1930. But there is something particular about Jordan's quantum field theory that put it in a precarious situation: Not only had Jordan abandoned the theoretical models of old, he also did not have a solid mathematical foundation at the basis of his theory. And this might very well be the reason why the pursuit of his theoretical visions was rather short-lived. When the problems of infinities in the Heisenberg-Pauli theory convinced Jordan that a straightforward quantization of Maxwell-Lorentz electrodynamics was not possible,

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[27] See [Hendry, 1984] for a detailed discussion.

and that “radical new ideas” were necessary, he had no firm ground from which he could have kept trying. It is striking how quickly Jordan abandoned his program after 1929: No direct continuation of his work on the foundations of quantum field theory exists. Rather, in the following years, he turned to biology, to mathematics, and to the right-wing politics that should permanently damage his reputation. Only in the mid-thirties there were some unsuccessful attempts to resuscitate his work on quantum field theory. Unlike Einstein and Schrödinger who kept developing their chosen models, despite success kept evading them, and against the opinion of the mainstream, Jordan had no fundamental structure to fall back on in the face of his problems.

6 Epilogue

It was not just Jordan, but his whole generation that rejected the idea of theoretical physics that Einstein and Schrödinger defended. The triumph of quantum mechanics convinced theoretical physicists ever since that they could do their job without recourse to visualizable models. But unlike in the case of quantum mechanics, where von Neumann’s Hilbert space formalism offered a clear and solid mathematical foundation, quantum field theory up to this day has not been cast into a definite mathematical structure. The algebraic approach has been an attempt in that direction, but has not yet arrived at a point where it successfully reconstructs the theory that physicists use.

In its physical foundations, on the other hand, modern-day quantum field theory equally suffers from lack of clarity and definiteness. Just as in the days of Dirac and Jordan, it sometimes is interpreted as field theory, sometimes as particle theory; its proudest technical achievement, renormalization theory, lacks a physical interpretation or theoretical justification. Nor is there a clear account of the relation of quantum field theory to quantum mechanics, its nonrelativistic limit. In their daily work most physicists use varying visualizations, especially Feynman diagrams, as a substitute for physical models. But they are quite aware that their use is very limited and in the end they just rely on pragmatic rules when the model fails. Just like Jordan, quantum field theorists today still have a grand vision of a unified theoretical framework for all of physics. But (as Einstein might add) just like him they are still suspended in a no-man's land between physics and mathematics.

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