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Mathematics meets physics

A contribution to their interaction in the 19th and the first half of the 20th century

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The Interrelation between Mathematics and Physics at the Universities Jena, Halle-Wittenberg and Leipzig – a Comparison

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1 Introduction to the project

The study of the interrelationship between mathematics and physics at the neighbouring universities of Halle-Wittenberg, Jena and Leipzig in the centre of Germany has been the topic of a project of the Saxon Academy of Sciences in Leipzig since 2000. It focuses on how these interrelations developed at each of these universities during the 19th and the first half of the 20th century, and on how this fits in with more general ideas, firstly on the formation of theoretical physics, secondly on the changes in mathematical physics and, thirdly, regarding the mathematization of physics.[1] Due to their different status in Germany's university system and very different regional conditions – Jena and Leipzig universities were the only ones in Thuringia[2] and Saxony respectively, whereas Halle-Wittenberg was one of many universities in Prussia (after the redistribution of area following the Congress of Vienna in 1815) – the selection of the universities ensured that a wealth of impact factors and reactions could be studied. The results, presented in numerous publications[3], have confirmed these expectations. The large amount of detailed evidence obtained verifies the great variation in the development of the interrelation between mathematics and physics.

1.1 The focal points of the study

In analysing the interrelation, the following four focal points were studied for each university and for each of the two disciplines, i. e. mathematics and physics: the changes in personnel, the research carried out by the university teachers, the extent of their lecturing and finally their

[1] Cf. [Jungnickel/McCormmach 1986] for an overview on the formation of theoretical physics in Germany.

[2] Until the formation of Thuringia in 1920, Thuringia consisted of several dukedoms. However, not all of the Thuringian dukedoms were involved in running the University of Jena. From 1826 until 1920, the supporting dukedoms were Sachsen-Weimar-Eisenach, Sachsen-Gotha, Sachsen-Altenburg and Sachsen-Meiningen. When using the term "Thuringian dukedoms" in the following text we always mean these four dukedoms.

[3] Schlote 2004; Schlote 2008, Schlote/Schneider 2009a, Schlote/Schneider 2009b; Schlote/Schneider 2011. A planed second publication on the University of Jena containing an analysis of the interrelations between mathematics and physics during the period from 1900 to 1945 could not be realised during the course of the project.

activities in local scholarly societies. Depending on regional peculiarities, activities in neighbouring disciplines like astronomy, geophysics and physical chemistry were also included in the investigation – but only for a shorter period of time.

The first focal point deals with the appointment of university teachers for mathematics and physics as well as institutional changes. Since the state was ultimately responsible for appointing university posts, we can see to what extent it supported the faculties' or the universities' efforts to obtain certain scientists. At the same time we can see whether the university's various bodies or the state recognized the general tendencies in scientific development and acted accordingly or whether and to what extent other interests – varying from regional to subjective – influenced the development of the discipline. Our research focuses in particular on the following questions: How was mathematics used to understand physical connections? What kind of theorizing was chosen in physics? In how far did these interdisciplinary problems give new impulses to mathematics and physics? An analysis of the courses of lectures which were offered reveals how the process of interaction found its expression in the teaching of the two fields. This also includes the question for which parts of physics was a theoretical foundation offered in the teaching, and when and how a complete lecture course in theoretical respectively mathematical physics extending over several terms was developed. This kind of analysis turned out to be rather difficult since the exact content of the lecture is not usually known. Finally, the study of the local scholarly societies reveals to what extent the scientists could use regional structures of communication to shape the interrelation between the two disciplines.

In evaluating the teaching and the research of the individual scientists we were faced with a fundamental problem which we were unable to solve satisfactorily, namely that of a reliable characterization of mathematical and theoretical physics at a given time (and place/country). Physicists and mathematicians have characterized these concepts very differently at different times. It was not without good reason that Ludwig Boltzmann stated in 1895 that determining the concept of theoretical physics was "not without difficulty".[4] C. Neumann pointed out that "the

[4] Boltzmann 1925, p. 94

most excellent works of theoretical physics have a definite *constructive* character in that a small number of simple and explicitly stated premises are at the basis of the investigations from which the phenomena of the field in question are constructed with mathematical consequence, or at least are attempted to be constructed."[5] To him, the most critical step in this procedure was the choice of appropriate simple premises on which the theory should be based. Moreover, according to Carl Neumann, papers of descriptive character appeared ever so often, which should be seen as "preparatory efforts" with respect to the theories first mentioned.[6] Neumann himself realized this constructive approach in particular in his contributions to electrodynamics by using methods from the field of potential theory. Albert Einstein, too, regarded the constructive theories as the most important part of theoretical physics, but put the "Prinzip-Theorien" on par. The starting point of the "Prinzip-Theorien" were "empirically found general properties of the processes of nature (principles) from which mathematically formulated criterions follow, which the individual process or rather their theoretical concepts have to satisfy".[7] In particular he considered the relativity theory such a "Prinzip-Theorie". In 1923, Eduard Study, a mathematician and expert in invariant theory, methodologically differentiated between three essential parts of physics: (pure) mathematics with a deductive method, experimental physics with an incomplete inductive method and an area that borders on the two parts ("Grenzgebiet") using idealization as its method. This last part "extends into [the other two parts] and relates them to each other". Study continued: "Together with mathematical theory this border area ["Grenzgebiet"] is usually (and appropriately so)

[5] «die am meisten hervorragenden Werke der theoretischen Physik, [. . .] einen entschiedenen *constructiven* Charakter besitzen, indem der jedesmaligen Betrachtung eine geringe Anzahl einfacher und deutlich ausgesprochener Prämissen zu Grunde liegt, von denen aus die Erscheinungen des betreffenden Gebietes mit mathematischer Consequenz construirt, oder wenigstens zu construiren versucht werden.» Neumann 1896, p. III

[6] «Die *descriptiven* Werke hingegen dürften anzusehen sein als vorbereitende Bemühungen [. . .]» Neumann 1896, p. IV

[7] «Ausgangspunkt und Basis bilden nicht hypothetische Konstruktionselemente sondern empirisch gefundene allgemeine Eigenschaften der Naturvorgänge (Principe), aus denen dann mathematisch formulierte Kriterien folgen, denen die einzelnen Vorgänge bezw. deren theoretische Bilder zu genügen haben.» Einstein 1919, p. 206

summed up as theoretical physics”[8] Thus, according to Study’s point of view, mathematical physics was subsumed under theoretical physics. In 1938, Werner Heisenberg was of the opinion that from the start no difference could be made between experimental and theoretical physics with respect to their aims.[9]

Compared with these different attempts to characterize theoretical physics, little was undertaken to give a more precise meaning to the concept of mathematical physics. For some physicists, the function of mathematical physics was mainly to aid physics. For example Max Wien expressed this as follows in 1915: “Mathematical physics consists [...] in the development of the mathematical aids necessary for the enhancement of theoretical physics.”[10] Some mathematicians emphasized the mutual influence the two fields had on each other. For example, Leon Lichtenstein said in his inaugural lecture at Leipzig in 1923: “The work of the mathematician is [...] of fundamental importance to the physicist. On the other hand, new fields open up for the mathematician from the stimulation that physics provides.”[11] One possible reason why such few attempts were made to define mathematical physics was probably because at that time mathematical physics was already well established as a part of mathematics.

1.2 The institutionalization of mathematical and theoretical physics at the universities of Central Germany

Before describing some of the characteristic features of the development at the universities of Jena, Halle-Wittenberg and Leipzig we would like

[8] «Dazwischen schaltet sich ein Grenzgebiet ein, das in beide [Bestandteile] übergreift und sie zueinander in Beziehung setzt. Dieses Grenzgebiet wird üblicher- (und zweckmäßiger-)weise mit der mathematischen Theorie als *theoretische Physik* zusammengefaßt.» Study 1923, p. 24 f.

[9] Heisenberg 1938, p. 61

[10] «Die mathematische Physik besteht [...] in der Ausbildung der für die Weiterbildung der theoretischen Physik erforderlichen mathematischen Hilfsmittel.» Wien 1915, p. 242

[11] «Die Arbeit des Mathematikers ist [...] für den Physiker von grundlegender Bedeutung. Auf der anderen Seite erwachsen wiederum dem Mathematiker aus den Anregungen, die ihm die Physik bietet, reiche Aufgaben.» Lichtenstein 1923, p. 148

to give a brief outline of the institutionalization of mathematical and theoretical physics at these universities:

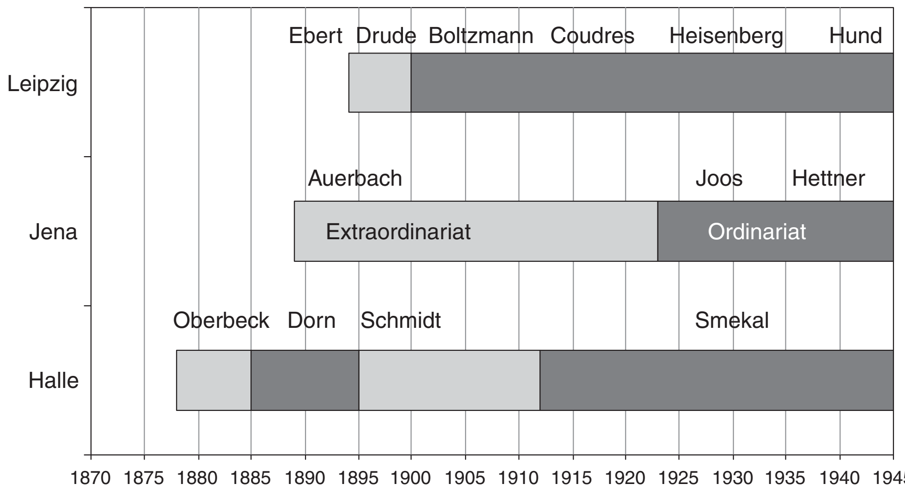

Figure 1
Institutionalization of theoretical physics

The diagram alone makes it clear that there were significant differences in the establishing of theoretical physics at the three universities. The creation of an associate professorship (Extraordinariat) of theoretical physics in Leipzig took place considerably later than at other German universities, in particular later than at the other two universities studied here. But on the other hand the physicists in Leipzig quickly managed to create a permanent chair.

Let us now turn to the development of mathematical physics at the three universities. Unlike theoretical physics, its development did not usually manifest itself in the creation of a professorship. It was only in Leipzig University that a professorship for mathematical physics was created – however only temporarily (and also in different departments). Karl von der Mühll held this associate professorship (Extraordinariat) in the maths department from 1872 to 1889. When a division of mathematical physics was set up within the theoretical-physical institute, George Jaffé was appointed associate professor (Extraordinarius) for mathematical physics in 1924. He was succeeded by Gregor Wentzel in 1926 and later, in 1929, by Friedrich Hund. It might seem surprising that there were hardly any professorships especially designated to mathematical physics, but most professorships were designated to larger fields like analysis, higher analysis, geometry or pure mathematics or the professorship was not specified despite the fact that it was indeed dedicated to mathematical physics. As soon as mathematics was represented by two professorships, the teaching and research areas, too, were split up among the lecturers. This can clearly be seen in the reports concerning the faculty's proposals for professorships (Denominationsberichte). However, this did not result in any specification of the professorships in question.

2 Jena – between philosophy and technology

After this rough outline, we will continue with the development at the University of Jena. One of the special features of the representation of mathematics and physics at the University of Jena is that, until the beginning of the 1880s, both disciplines were represented by only one joint chair. The main reason for this were financial restrictions. As the only university of the small and financially relatively weak dukedoms of Thuringia, Jena had only a small budget at its disposal. Although the proportion of the dukedoms' total budget allocated to the university was higher than that of other German states running a university, in particular higher than in Prussia, the University of Jena had far less resources than the universities in Halle, Bonn, Marburg, Gießen and Freiburg. Thus it was not so easy for Jena to create, for example, a chair for a new science discipline. In view of the great advances made in science during the 19th century, in particular in the natural sciences, and of the limited financial means at its disposal, the university was faced with serious problems, as the 1854 general report of the university's curator Moritz Seebeck shows.[12]

The limited financial resources was the main reason why in 1802, after the death of the physics professor (Ordinarius) Laurenz Johann Daniel Succow, the faculty of philosophy proposed the conversion of his chair to one of finance (Kameralistik) and a combination of the chair

[12] Thüringisches Staatsarchiv Altenburg, Geheimes Ministerium Nr. 1522, unpaginated, Generalbericht des Kurators vom 7. März 1854

of physics with that of mathematics, thereby securing the representation of economics and finance in a chair. This request was supported by the university's senate and curator, and approved by the Thuringian states. Physics was then assigned to Johann Heinrich Voigt who was professor (Ordinarius) of mathematics. This combination of mathematics and physics in one chair was not unusual during the first half of the 19th century, since in the tradition of the 18th century, the treatment of a lot of physical questions was seen as a part of applied mathematics.

2.1 The philosophical foundation of the interrelationship

However, one special feature of the development in Jena was the additional combination of the two disciplines with philosophy which occurred when Jakob Friedrich Fries succeeded Voigt to the chair in 1824. Fries had received his habilitation at Jena with a work on philosophy in 1801. In 1805 he was assigned the chair (Ordinariat) of philosophy and mathematics at the University of Heidelberg, where, in 1813, he was awarded the chair (Ordinariat) of physics. After an unsuccessful call to Berlin University Fries returned to the chair (Ordinariat) of metaphysics and logic at Jena in 1816. Three years later, as a result of the passing of the Karlsbad Decrees[13], the Grand Duke Carl August of Weimar was forced to suspend him from teaching (although he continued to pay his salary[14]). Fries' appointment to the chair of mathematics and physics allowed him an active return to university life in 1824. At that time the faculty did not discuss the question of a separation of the joint chair of mathematics and physics.

Fries dealt intensively with epistemological questions concerning the gaining of knowledge in the natural sciences and the role of mathematics in the formation of scientific theories. In a critical way he took up ideas developed in Immanuel Kant's monograph *Metaphysische Anfangsgründe der Naturwissenschaft*. Mathematics was for him one of the fundamental pillars of the natural sciences (Naturlehren). It imposed on all of them "with necessity the principal laws of motion and the basic forces through

[13] Cf. [Eggeling 1878] and [Gäbe 1971] for a biography of Fries. Through a malicious interpretation of some private letters found on arrested students Fries was drawn into the investigations concerning the killing of August Koetzebue.

[14] Kreiser 2001, p. 70

which everything is caused as well as the highest forms of all processes under which the material substances interact".[15] This mathematical knowledge of nature "should give us the laws of possible hypotheses about the nature of bodies, should determine which prerequisites are admissible, which are to be regarded as the most simple of all and which mathematical consequences each such single hypotheses involves."[16]

With respect mainly to physics Fries noted: There is "no theory at all, no explanation of phenomena by general laws that is not, at least indirectly, determined by mathematical knowledge".[17] According to him, throughout all parts of the natural sciences, it is mathematics that tells us how to construct the tools, only by means of which exact observation can succeed. Mathematics follows observation with precise measurement and calculation, and aims at predicting the course of phenomena by general laws on the basis of observation by means of calculation.[18]

Taking this general philosophical viewpoint as his starting point Fries developed a systematic and coherent overview of mathematics. In particular, his reflections on the probability theory deserve mentioning.[19] In his monographs, Fries gave the interrelation between mathematics and physics[20] a careful philosophical foundation, an analysis which turned out to be forward-looking. In great detail, Fries described the interplay of exact observations including precise measurements, physical abstractions and mathematical methods in the theoretical permeation of individual problems and fields. The focal points were in accordance

[15] «Reine Mathematik schreibt allen Naturlehren mit Nothwendigkeit die obersten Gesetze der Bewegung und der Grundkräfte, durch welche alles bewirkt wird, so wie die obersten Formen aller Processe, unter denen die körperlichen Stoffe in Wechselwirkung kommen, vor.» Fries 1822, p. 32

[16] «Sie [die ganze mathematische Naturphilosophie] soll uns die Gesetze möglicher Hypothesen über die Natur der Körper angeben; bestimmen, welche Voraussetzungen zulässig seyen, welche als die einfachsten von allen anzusehen und welche mathematisch bestimmbaren Folgen jede einzelne solche Hypothese mit sich führe.» Fries 1822, p. 30

[17] «[...] die Mathematik selbst ist die Grundfeste für die ganze Physik, denn es gibt überhaupt keine Theorie, keine Erklärung der Erscheinungen aus allgemeinen Gesetzen, welche nicht wenigstens mittelbar durch die mathematische Erkenntniß bestimmt würde.» Fries 1826, p. 25

[18] Fries 1826, p. 25

[19] Fries 1842

[20] Cf. [Schlote/Schneider 2011], section 5.1.2 for a more detailed account.

with the very programme which came to fruition in the founding of the seminar on mathematical physics at Königsberg and which led to the founding of the successful and influential school of mathematical physics.[21] In his publications Fries integrated the latest physical results, as his references to Ørsted's discovery of the deflection of a magnetic needle by a current-carrying conductor and to Fechner's *Lehrbuch der Experimentalphysik* show. However, he failed to illustrate his theories with detailed concrete examples. This might be seen as a short-coming, but Fries' exposition is quite complete in itself. The lecturers (Privatdozenten) and scientists Carl Temler, Ernst Mirbt, Ernst Friedrich Apelt, Gustav Suckow and Ernst Erhard Schmid taught alongside Fries, some of whom were his students and well acquainted with his ideas. They, too, chose not to illustrate or apply Fries' theory by working out a concrete example.

In their search for Fries' successor in 1843, the faculty adhered to the combination of mathematics and physics in one chair and, with Karl Snell's appointment to the chair, the connection with philosophy continued.[22] However, Snell failed to give any sustainable new insight with regard to this relation with philosophy. And, as at that time the combination of mathematics and physics in one chair no longer met the current requirements with regard to teaching duties and to an appropriate representation of both disciplines in research, the development of both disciplines at Jena fell behind the general level in Germany in those decades.

2.2 Theorizing with regard to technology

In the second half of the 19th century, Jena became a centre for the construction of optical instruments. Industry and the university cooperated very closely in terms of personnel, money and research. Carl Zeiss, the founder of the optical industry, and Ernst Abbe, a lecturer at Jena University, were the initiators of this close cooperation. They were later joined by Otto Schott together with his glass works, which had been co-founded by Abbe and Zeiss, Abbe became the co-director at Zeiss'

[21] The history of the foundation of the seminar on mathematical physics at Königsberg and the formation of the Königsberg school was analysed by K. Olesko in great detail. Olesko 1991

[22] Cf. [Schlote/Schneider 2011], section 3.4 for details on Snell's appointment.

and founded the Carl-Zeiss-Stiftung, a foundation trust with social and scientific ambitions.[23]

Such trusts (Stiftungen) were essential for the development of research and teaching at Jena University. The Reichenbach-Stiftung enabled the separation of the joint chair of mathematics and physics in 1879 and 1882 respectively. The Carl-Zeiss-Stiftung enabled the creation of an associate professorship (Extraordinariat) for theoretical physics in 1889. Both foundations supported improvements in the facilities for mathematical and physical research and for teaching in Jena, e. g. they paid for a new building for the physical institute, equipped it with instruments and purchased books and models etc. for the mathematical seminar.

Towards the end of the 19th century, the research carried out by the physicists at Jena was increasingly geared towards questions that were of interest to the local optical industry and (increasingly) also towards their technical implementation.[24]

It was the university lecturer (Privatdozent) Ernst Abbe who initiated this development. In 1864, Abbe was recruited as a scientific collaborator by Zeiss. Abbe realized that an improvement of microscopes could be achieved by a profound mathematical-theoretical analysis of the system of lenses and of the opening angle of the objectives. Joseph Fraunhofer, Joseph Petzval and Carl August von Steinheil had already thought along these lines. But it was Abbe who realized that the wave nature of light should be taken into account because of the tiny size of the objects studied under the microscope. He thus established a refraction theory for microscopic images. Abbe determined the so-called sinus-condition and the resolution of objectives. He studied the consequences of the central and non-central illumination of objects and the use of immersion substances. He also derived a formula for the smallest distance possible for distinguishing objects for a given microscope. In Abbe's research constructive-technical, experimental and theoretical aspects blended together. The laws of dioptrics enhanced by physical ideas were the mathematical-theoretical basis for his investigations.

[23] Cf. [Schomerus 1955] for the foundation of the Carl-Zeiss-Stiftung. See also [Stolz/Wittig 1993] for some aspects of Abbe's ans Zeiss' work.

[24] Cf. [Schlote/Schneider 2011], section 5.2 for a detailed exposition as well as for the bibliographic references concerning the results mentioned here

The experimental physicist Leonhard Sohncke, appointed in 1882, as well as his successor Adolph Winkelmann, appointed four years later, took up research concerning optics and also the properties of glass. Sohncke published papers on the theory of Newtonian rings, on diffraction patterns created by wedge-shaped particles as well as papers on the rotation of the polarization-plane of natural light under the influence of electromagnetic forces. Winkelmann studied the properties of different kinds of glass, which he got from Schott's glass factory, with respect to refraction, elasticity, resistance to tension and pressure as well as thermal resistance. Moreover, Winkelmann and Straubel explored the diffraction of X-rays by metal-prisms. The studies of Sohncke and Winkelmann were mainly of experimental character. Felix Auerbach, associate professor (Extraordinarius) of theoretical physics, who was appointed in 1889, also supported – at least indirectly – research in optics with his theory of hardness. In cooperation with Schott's glass works he determined the different degrees of hardness of various kinds of glass. Furthermore, he constructed measuring instruments.

While Sohncke, Winkelmann and Auerbach were also engaged in other research topics, Rudolf Straubel who had received most of his training in Jena concentrated exclusively on the construction of instruments. He habilitated in Jena in 1893, was appointed associate professor (Extraordinarius) five years later and, another five years later, became a member of the Zeiss management. In Abbe's tradition he tried to improve instruments by giving a more exact mathematical version of the theory. But he also engaged in the determination of number and modulus of the elasticity of different kinds of glass.

While Jena physicists were concerned with questions of optics and the technical implementation or utilization of their results, there was no such activity going on among the Jena mathematicians at that time. What can be found in the papers of Frege, Piltz and Thomae, are remarks in which they point to the possibility of utilizing their results in physics – however vaguely and generally they are expressed.[25] Otherwise – apart from one exception – topics of mathematical physics did not play any role in their research. The exception is an article by Frege published in 1891 in which Frege demonstrated how he would like to apply the principles

[25] Cf. [Schlote/Schneider 2011], section 5.3 for a treatment of the mathematical contributions.

of the *Begriffsschrift* to physics.[26] Discussing the wording of the law of inertia he gave a thorough criticism of the definition of a so called inertial system introduced by Ludwig Lange five years ago. However, Frege could not solve the problem either, he also made a mistake and anyway it seems that the physicists took no notice of Frege's article. Neither a remark nor an article could be found that referred to Frege's contribution. Hence one should doubt that Frege's ideas and contributions to a logical exact foundation of arithmetic had an impact on the theory-forming process done by physicists.

Thus, with the exception of Fries' philosophical foundation of the interrelation between mathematics and physics there was no significant interaction between the two disciplines in Jena during the first half of the 19th century. The turn towards theoretical physics in Jena took place in close relation with technological questions right from the outset.

3 Halle – 'normal' science

There are two features that are characteristic of the interaction between mathematics and physics at the University of Halle-Wittenberg during 1890 and the end of the Second World War. On the one hand, one notices that mathematicians showed little interest in engaging in questions related to physics, on the other hand, the scientists appointed to a chair of theoretical physics undertook experimental investigations. With the exception of Gustav Mie, the physicists and mathematicians at Halle did not contribute to the latest physical theories, i. e. relativity theory and quantum mechanics, in their research. The focus of their research lay in other fields. Their methods of research were in a certain sense traditional. Therefore, it seems appropriate to characterize the research done at Halle by using the term 'normal' science.

The characteristic features mentioned will be explored in the following two sections, starting with the close connection between theory and experiment in the research of the theoretical physicists at Halle.

[26] Frege 1891

3.1 The close connection between experiment and theory

The close connection between experiment and theory can well be seen in the unusual way that professors in Halle were appointed.[27] In this period theoretical physics was represented at Halle by physicists who included applied and technical physics in their field of research. In 1895 the Hallensian lecturer (Privatdozent) Karl E. Schmidt succeeded his colleague Ernst Dorn who took over the Hallensian chair (Ordinariat) of experimental physics from Hermann Knoblauch. At the same time, the chair (Ordinariat) of theoretical physics was downgraded to an associate professorship (Extraordinariat). This did not become a professorship again until 1912, and then only one to last for Schmidt's lifetime (persönliches Ordinariat). After Schmidt's retirement in 1927, Adolf Smekal was appointed Schmidt's successor and also director of the newly established department of theoretical physics. Against the opposition of the experimental physicist Gerhard Hoffmann, Smekal managed in the end to completely separate the departments of experimental and theoretical physics. Both Schmidt and Smekal allowed a lot of scope for experimentation.

When Schmidt was appointed, he had already done research in optics, on animal tissue as an electric conductor and on photography. During the following years he widened his fields of interest to include investigations on high frequency physics and wireless telegraphy. To this purpose at the beginning of the 20th century, he set up two experimental laboratories with his own money in Halle and allowed his PhD-students to carry out their research there. When he closed down these laboratories in 1910, he initiated the establishment of a university laboratory of theoretical physics, equipping it with instruments again at his own expense. Schmidt's experimental investigations were so extensive that at times his chair was referred to as one of "theoretical and applied physics".[28]

When the experimental physicist Dorn died in 1916, Schmidt would have liked to become his successor. The faculty rejected this out of finan-

[27] Cf. [Schlote/Schneider 2009b], chapter 4 for a detailed description of the appointments made at the Physics' department at Halle.

[28] «theoretische und angewandte Physik» Geheimes Staatsarchiv Preußischer Kulturbesitz Berlin, Rep. 76, Va, Sekt. 8, Tit. IV, Nr. 48, Vol. VI, p. 383

cial-institutional considerations, because then the chair (persönliches Ordinariat) would have been transformed into an associate professorship (Extraordinariat) again. The faculty pursued a different strategy. They wanted a professor (Ordinarius) of experimental physics who was also versed in theoretical physics. And they found the very person: Gustav Mie. So the situation in Halle was indeed quite paradoxical: the physicist appointed for theoretical physics devoted most of his research to experimental investigations closely connected with technology, whereas the experimentalist was expected to cover theoretical physics, too. In a certain sense there was no change in this situation under Mie's successors – Gustav Hertz, Hoffmann and Wilhelm Kast – and Schmidt's successor – Smekal. The faculty's attempt to create a chair of technical physics, as in Göttingen and Jena, thereby solving the structural problem, repeatedly failed because of opposition from the Prussian ministry. Schmidt devoted himself increasingly to questions of application and in the 1920s studied ways of how to preserve fodder.[29]

In 1927 the faculty looked for a theoretical physicist who was also acquainted with technology as Schmidt's successor. Smekal was appointed professor (Ordinarius) of theoretical physics and director of the institute of theoretical physics. Smekal had already pointed out in the negotiations concerning his appointment that apart from theoretical physics he was also doing experimental research. Relations between Smekal and the professor (Ordinarius) for experimental physics Hoffmann and later Kast were often strained – partly for this very reason.

One of the consequences of this policy on appointments was that little research was done which dealt exclusively with theory-formation in physics.[30] The research of the theoretical and experimental physicists had both a theoretical and experimental component. Most of the time, the physicists were concerned with the elaboration of existing theories. For example, in 1898 Schmidt proved the deflection of cathode rays under the influence of electric forces, more precisely under electromagnetic waves. This result was based on many experiments. He thus discovered a "new" form of deflection besides those known by magnetic and electrostatic fields. He embedded his result in a wider context, saying e. g.: "Each

[29] Cf. [Schlote/Schneider 2009b], sections 7.2 – 7.4 with respect to Schmidt's research.

[30] An analysis of the research done at the physics department is given in [Schlote/Schneider 2009b], chapter 7.

change of state in a field caused by magnetic or electric forces changes the path of cathode rays passing through the field."[31] In addition to that, he determined a fundamental law of deflection.

Schmidt's successor Smekal also combined a theoretical with an experimental investigation of a specific problem. He developed new approaches particularly in the field of fracture mechanics and thus made an important contribution to solid-state physics. In the 1930s, among other things, Smekal analysed fractures, in particular the formation of fractures, and provided a theoretical description. He introduced the concept of molecular tensile strength (molekulare Zerreißfestigkeit) and derived an improved formula for tensile strength (Zugfestigkeit). He also explored the dependencies of fractures on various kinds of properties, like the state of the surface or temperature. Extensive series of experiments on crystals and glass-like substances facilitated and accompanied his research on fracture theory. In the broader context of the elucidation of the properties of materials, let us also mention his explanation of so-called structure-sensitive properties of solid materials by the introduction of "loose building blocks" (Lockerbausteine) as deviations from the ideal structure of the crystal. Among other things, this helped to explain ion conduction. Finally, it should be mentioned that Halle's experimental physicists also developed new theoretical concepts. For example, Hoffmann contributed to an explanation of cosmic radiation by introducing a concept known today as "Hoffmann'sche Stöße", which was developed and tested by experiments conducted over a long period of time in Halle.

We believe that the research in physics done at Halle University was, to a certain extent, typical of the time and of small and middle-sized German universities. This kind of research can be characterized by a close connection between theory and experiment as well as by a lot of contributions to the consolidation of existing theories and by a few outstanding papers published every now and again. This is the reason why we characterize this research as 'normal' or 'ordinary' science.

[31] «Jede durch magnetische oder elektrische Kräfte hervorgerufene Zustandsänderung in einem Felde ändert den Gang der das Feld durchlaufenden Kathodenstrahlen.» Schmidt 1896, p. 174

3.2 Mathematical research without any contact to physics or application

As to the interaction between mathematics and physics, the mathematical formulation of relations did play a certain role in the research of the physicists, but not an outstanding one. However, physical problems were hardly of any importance to Hallensian mathematicians from the mid 1920s onwards. With the appointments of Heinrich Jung in 1920, Helmut Hasse in 1925 and Hasse's successor Heinrich Brandt in 1930, algebra and number theory became the new focal points of research at Halle. Along with them came the methods of "modern" algebra to Halle. Jung pursued the arithmetization of algebraic geometry. Hasse published his famous "Klassenkörper-Bericht" in 1926. Brandt introduced the so-called "Brandt's gruppoid" with a system of axioms. He also developed an arithmetic of algebras. In the early 1930s, he invited Emmy Noether to lecture in Halle.[32]

With the exception of Heinrich Behmann who mainly did research in logic, the research carried out by Hallensian private lecturers (Privatdozenten) and associate professors (Extraordinarien) went in the same direction. Reinhold Baer studied different systems of axioms of topology. Erhard Tornier introduced a system of axioms for probability calculus. Their research did not bear any relation to applications nor to questions of mathematical physics.

However, the appointment of Heinrich Grell as a private lecturer in 1934 seemed to change the situation. The lecture he gave on the occasion of his "Umhabilitierung" from Jena to Halle was entitled *The significance of hyper-complex systems for quantum mechanics*.[33] Algebras first emerged in quantum mechanics in Wolfgang Pauli's paper on spin in 1926 and in Paul Dirac's paper on the relativistic wave equation of the electron. In 1934 Pascual Jordan, John von Neumann and Eugene Wigner studied them systematically in the context of an improved mathematization of the quantum mechanical formalism. Grell's lecture on the mathematical foundations of quantum mechanics that he announced for the summer

[32] Cf. [Schlote/Schneider 2009b], sections 3.3 and 3.4 concerning their appointments and sections 6.5 and 6.1 concerning their research.

[33] «Die Bedeutung von hyperkomplexen Systemen für die Quantenmechanik» He gave the talk in February 1935. Universitätsarchiv Halle, PA 6887

term of 1935 had to be cancelled, because Grell was arrested on the charge of homosexuality in April. In 1936, he was replaced by William Threlfall. Thus, the mathematicians' only attempt at that time to engage in modern physical theories from a mathematical perspective came to an abrupt end.

This brief outline of mathematics at Halle in the 20s, 30s and 40s of the last century does not mean that there was no research in applied mathematics and mathematical physics at Halle before that time. In this respect we would like to mention Albert Wangerin's and Ernst Neumann's research in potential theory, Friedrich Pfeiffer's and Richard Grammel's papers on aero- and hydrodynamics and Gustav Doetsch's pioneering contributions to Laplace-transformations.[34] However, for various reasons, they did not have a lasting influence on the research in Halle. The mathematicians tried to ensure the representation of applied mathematics in teaching. From the beginning of the 20th century onwards they tried to create an associate professorship (Extraordinariat) of applied mathematics, but did not succeed until 1939. With the appointment of Behmann who had been teaching applied mathematics in Halle since 1925, the chair was occupied by a logician who hardly did any research in applied mathematics.

Halle's concentration of mathematical research on algebra and number theory, i. e. a specialization on areas of pure mathematics, was not planned but was simply a result of the lecturers that were appointed. It helped to establish and consolidate "modern" algebra.

4 Leipzig – at the forefront of research

Let us now turn to the development at the University of Leipzig. At the period in question, the University of Leipzig was one of the universities in Germany with the highest numbers of students, even taking first place until the end of the 1870s, then to be overtaken by the universities of Berlin and Munich by the end of the century. One of the outstanding features of the development of mathematics and physics in Leipzig

[34] Cf. [Schlote/Schneider 2009b], sections 6.2 and 6.4 as well as [Schlote/Schneider 2009a], section 7.5.

during the 19th and 20th century was the almost uninterrupted tradition of mathematical physics, with such famous names as August Ferdinand Möbius, Carl Neumann, Karl von der Mühll, Gustav Herglotz, Leon Lichtenstein and Herbert Beckert.

4.1 The predominance of mathematical physics during the 19th century

After there had been the chance, for a short time and as a result of the work of Wilhelm Weber and Gustav Theodor Fechner in the 1840s, of Leipzig taking up a leading position on par with the seminar on mathematical physics at Königsberg, the representation of mathematical physics was surprisingly strengthened in 1868 by a generational change in appointments to the chairs of mathematics.[35] When Moritz Wilhelm Drobisch changed to philosophy and Möbius died, their positions went to Wilhelm Scheibner and Carl Neumann respectively.[36] The latter's appointment was surprising in that he only came third on a par with Richard Baltzer in the faculty's report to the ministry, and thus was by no means the candidate preferred by the faculty of philosophy. Alfred Clebsch, the candidate placed first, did not consider coming to Leipzig because he had just been appointed to Göttingen. The Saxonian ministry at Dresden then appointed the 36-years old Carl Neumann as professor (Ordinarius) passing over Hermann Hankel who was second on the list.

Neumann stemmed from the Königsberg school of mathematical physics and had already produced remarkable contributions to mathematical physics.[37] He had also started to develop his own ideas about the relation between mathematics and physics which he made more precise, modified and put into practice in Leipzig. To him the aim of mathematical physics was to provide a strictly logical structure of physical theory which was as simple as possible and rested on a few fundamental ideas which could not be explained any further.[38]

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[35] Cf. [Schlote 2004] for an overview.

[36] Cf. [Schlote 2004], section 4.4.2 concerning the appointment of Neumann.

[37] Cf. [Schlote 2005] as well as [Schlote 2004], sections 6.8 and 11.8 for details on Neumann's contributions to potential theory and mathematical physics.

[38] Neumann 1865, Neumann 1870

Since two scientists, the private lecturer (Privatdozent) of mathematical physics von der Mühll and Adolph Mayer, who were also interested in questions concerning physics, worked alongside Neumann at Leipzig, mathematical physics became a focal point of research and teaching at Leipzig in the following decades. Neumann shaped this process by his strongly mathematically oriented view on the treatment of physical questions. He repeatedly demonstrated this view with respect to the foundation of electrodynamics. By doing this in a form which was not very appropriate for physicists he, indirectly, increased at the same time the awareness of the distinction between mathematical and theoretical physics.

Another outcome of the Leipzig emphasis on mathematical physics was the fact that lecture courses on theoretical physics were mainly given by mathematicians and that the physicists hardly contributed to teaching them. Neither did the Leipzig physicist participate in the rise of theoretical physics that took place during these decades. In 1894, Leipzig was one of the last universities in Germany to establish an associate professorship (Extraordinariat) of theoretical physics. The physicists at Leipzig, however, caught up with the institutionalization and representation of theoretical physics rather quickly, without specializing in any particular field.

4.2 A centre of quantum mechanics under Heisenberg

At the end of the 1920s, a historical coincidence, namely the need to appoint two professors (Ordinariate) of physics and an associate professor (Extraordinarius) in mathematical physics within one and a half years, made it possible to create a centre of theoretical physics at Leipzig.[39] In spring 1926, the advocates of the latest trends in theoretical physics succeeded in obtaining the associate professorship (Extraordinariat) of mathematical physics that had been established at the physical institute in 1923. The position was to be given to a younger representative of theoretical physics, to someone who wanted "to engage actively in the elaboration of the modern theory of quanta

[39] Cf. [Schlote 2008], sections 3.2, 3.3 and 7.2 for a more detailed analysis of the situation at the Leipzig Physical Institute and the course of events which led to a new profil of research.

and relativity".[40] After Werner Heisenberg and Wolfgang Pauli refused to take the position, it was given to Gregor Wentzel. Shortly afterwards, the two scientists who held the chairs (Ordinariate) in physics at Leipzig died: Theodor Des Coudres, who held the chair of theoretical physics, in October 1926 and Otto Wiener, who held that of experimental physics, in January 1927. Thus, the path to a new orientation of research in physics was opened that previously had been blocked by Wiener in particular. The chair of experimental physics was given to Peter Debye. Werner Heisenberg was appointed professor (Ordinarius) of theoretical physics. Both took up their positions at the beginning of the winter term 1927/28.

Within a few months the situation at Leipzig had changed completely. The directors of the physical and the theoretical-physical institute were two young outstanding scientists. The programme of research was given a new profile and was oriented towards modern theoretical physics, in particular towards quantum mechanics and its application to atomic and nuclear physics as well as physics of solid bodies.[41] Heisenberg and Debye attracted a lot of students and young physicists and successfully established research teams. Leipzig soon became one of the leading centres of theoretical research in physics and many colleagues from other universities sent their students to Leipzig to study and work with Heisenberg's team. From summer 1928 onwards, Debye organized the well-known Leipzig lecture weeks («Leipziger Vortragswochen») that were a kind of summer school. Lots of physicists met there to exchange ideas and Debye invited leading specialists like Paul Dirac and Enrico Fermi as lecturers.[42]

Between the autumn of 1927 and the summer term of 1930 the number of physics students in Leipzig tripled. In the 1930s, the

[40] «Bei ihrer Wahl hat sich die Fakultät von dem Wunsche leiten lassen, daß der jüngere Vertreter der theoretischen Physik an unserer Universität an dem Ausbau der modernen Theorie der Quanten und Relativität aktiv beteiligt sein möchte, [...]» Universitätsarchiv Leipzig, PA 1051, p. 6

[41] Cf. [Schlote 2008], sections 9.1 – 9.4. for research at the Physical Institute. For a detailed description of the bloom of physics in Leipzig and in particular Heisenberg's leading role cf. [Cassidy 1992], chapters 13 – 16; [Kleint/Wiemers 1993] and [Kleint/Rechenberg/Wiemers 2005].

[42] The talks were edited by Hans Falkenhagen and Debye as conference's proceedings in the series «Leipziger Vorträge».

number of PhD-theses in physics rose significantly and at times made up for more than a third of the PhDs of the mathematical-scientific (mathematisch-naturwissenschaftliche) department of the philosophical faculty. When Wentzel left for the ETH Zürich in October 1928, it had little effect on this development. The appointment of Friedrich Hund as his successor in summer 1929 was an additional boost to the Leipzig institute as a centre of atomic physics because of his research related to molecular structure and also because of his close personal relation to Heisenberg. Another positive factor was the appointment of the mathematician Bartel Leendert van der Waerden as successor to Otto Hölder in 1931. One of his reasons for accepting the call to Leipzig was the prospect of working with Heisenberg and his team. Van der Waerden was not only an excellent algebraist and geometer, but he was also interested in modern quantum theory.[43] He participated in Heisenberg's and Hund's seminars and contributed to the mathematical elaboration on quantum mechanical questions.

This blooming of theoretical physics in Leipzig lasted until the mid 1930s. By then the Leipzig institute had become one of the leading centres, if not the leading centre of theoretical physics in Germany. One of the reasons for this was that the scientists responsible for the rise, i. e. Heisenberg, Debye and Hund, were quick to identify new lines of research and integrated them into their research. They also succeeded in continuing the direction of their research despite opposition from representatives of technical physics. From the mid 1930s onwards, the damage inflicted by the dictatorship of the Nazis was getting more serious and a couple of years later, in the late 1930s, very little was left of Heisenberg's school.

To round off the picture let us note that Leon Lichtenstein, too, was very successful in mathematical physics at the same time as the Leipzig physicists' research bloomed. However, with the hostilities against Lichtenstein during the first months of the Nazi's regime and his sudden death in August 1933, this development came to an abrupt end.[44]

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[43] Cf. [Schneider 2011] for van der Waerden's early contributions to physics.
[44] Cf. [Schlote 2008], sections 6.5 and 9.6 as well as [Beckert 1981].

5 Concluding remarks

The above account of the three universities shows a remarkable diversity in the shaping of the interaction between mathematics and physics. This variety is the result of many factors. It depends not only on the position of a university within Germany's system of universities, but also on its traditions, on local and regional contexts and, last but not least, on the individual scientist. These factors blend together in very different ways leading to different, but sometimes also similar results.

Leipzig stands out because of its contributions to research in mathematical physics and later also to "modern" theories of physics. For longer periods of time it was one of the centres of these research fields. In Jena, the immense, not only financial support of the university by local industry, in particular by Abbe and the Carl-Zeiss-Stiftung, should be emphasized. The close connection between physical research in Jena, which was geared towards technological application, and the interests and needs of the two enterprises Zeiss and Schott – that could have been motivated by this financial arrangement – was unique in Germany at that time. In contrast, the development of the interaction in Halle, a small provincial university, proceeded without any heights. Mathematical and theoretical physics was not specially supported there, no traditions developed in these fields and only a few outstanding papers emerged. Theoretical physics or rather the research done by scientists appointed as theoretical physicists at Halle was not purely of theoretical character, but intimately linked with experiments and technical aspects. We have used the term 'normal' science to describe this, because we believe that this kind of development could probably be found at many small and middle-sized universities in Germany at that time and is thus quite typical.

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