Computing and Informatics, Vol. 34, 2015, 1038–1064

NEW GRAPHICAL MODEL FOR COMPUTING OPTIMISTIC DECISIONS IN POSSIBILITY THEORY FRAMEWORK

Ismahane Zeddigha

CRIL-CNRS, UMR 8188, Université d'Artois
Faculté des sciences Jean Perrin
Rue Jean Souvraz, Lens, France
&
Recherche en Informatique Intelligente et Mathematiques Appliquées
Université des Sciences et de la Technologie Houari Boumediene
Algiers, Algeria
e-mail: izeddigha@gmail.com

Salem Benferhat

CRIL-CNRS, UMR 8188, Université d'Artois
Faculté des sciences Jean Perrin
Rue Jean Souvraz, Lens, France
e-mail: benferhat@cril.univ-artois.fr

Faiza Khellaf

Recherche en Informatique Intelligente et Mathematiques Appliquées
Université des Sciences et de la Technologie Houari Boumediene
Algiers, Algeria
e-mail: fkhellaf@usthb.dz

Abstract. This paper first proposes a new graphical model for decision making under uncertainty based on min-based possibilistic networks. A decision problem under uncertainty is described by means of two distinct min-based possibilistic

networks: the first one expresses agent's knowledge while the second one encodes agent's preferences representing a qualitative utility. We then propose an efficient algorithm for computing optimistic optimal decisions using our new model for representing possibilistic decision making under uncertainty. We show that the computation of optimal decisions comes down to compute a normalization degree of the junction tree associated with the graph resulting from the fusion of agent's beliefs and preferences. This paper also proposes an alternative way for computing optimal optimistic decisions. The idea is to transform the two possibilistic networks into two equivalent possibilistic logic knowledge bases, one representing agent's knowledge and the other represents agent's preferences. We show that computing an optimal optimistic decision comes down to compute the inconsistency degree of the union of the two possibilistic bases augmented with a given decision.

Keywords: Decision theory, optimistic criteria, possibilistic networks, possibilistic logic

1 INTRODUCTION

Decision making under uncertainty [19, 1] plays an important role in artificial intelligence (AI) [13]. Several decision making tools [33, 10] have been developed to assist decision makers in their tasks: simulation techniques, dynamic programming [34], logical decision models [18] and graphical decision models [36, 24].

This paper focuses on graphical decision models which provide efficient decision tools and a compact representation of decision problems under uncertainty. Most of decision graphical models are based on Influence Diagrams (IDs) [27, 36] for representing decision maker's beliefs and preferences. In many applications, it is easier to express uncertainty in a qualitative way by ranking different states of the world. Similarly, it is easier to provide a preference relation between different consequences. In these situations, possibility theory [21] is an appropriate framework for representing uncertain knowledge and preferences. A qualitative possibilistic decision model [2] allows a gradual expression of both agent's preferences and knowledge. Few works exist on decision making using possibilistic networks. In [24], the authors proposed a possibilistic counterpart of standard IDs. Uncertainty is expressed by possibility degrees and preferences are considered as satisfaction degrees. In [18], the authors proposed a compact representation of a qualitative decision problem based on possibilistic logic [20, 35]. This logical-based approach [18, 5] allows to express agent's knowledge and preferences by means of valued logical formulas.

This paper proposes a new model for representing decision making under uncertainty based on the use of min-based possibilistic networks. When agent's knowledge and preferences are expressed in a qualitative way, we suggest representing them by two distinct min-based possibilistic networks. The first one encodes a joint possibility distribution representing available knowledge and the second one encodes the qualitative utility. The qualitative possibilistic decision process will be viewed as

a data fusion of these two particular possibility distributions (or two min-based possibilistic networks). Using our new model, we present a unified way for computing optimal optimistic decisions using inference process based on the junction tree associated with the fusion of agent's beliefs and preference-based networks. We show that computing optimal decisions comes down to computing a normalization degree of this junction tree. The second part of this paper explores an alternative way for computing optimal optimistic decisions. This alternative method is useful when it is impossible to construct the junction tree associated with agent's knowledge and preferences. The idea is to transform the two possibilistic networks into two possibilistic logical bases. We show that computing optimal optimistic decisions comes down to computing the inconsistency degree of the two posibilistic knowledge bases representing the fusion of agent's beliefs and agent's preferences.

This paper is an extended and a revised version of the conference paper [9].

The rest of this paper is organized as follows: next section briefly recalls basic concepts of possibility theory, possibilistic logic and min-based possibilistic networks. Section 3 describes the proposed model for encoding decision problems based on min-based possibilistic networks. Section 4 describes how the propagation process in a junction tree can be efficiently adapted for computing optimal optimistic decisions. In Section 5, we deal with an alternative way for computing optimistic decisions based on equivalent transformations between possibilistic knowledge bases and possibilistic networks. Section 6 gives related works and Section 7 concludes the paper. All proofs of propositions of this paper are provided in the Appendix.

2 BACKGROUNDS

This section gives a brief refresher on the possibility theory [21, 28].

Let $\mathcal{V} = \{X_1, \ldots, X_N\}$ be a set of variables. We denote by $\mathbb{D}_{X_i} = \{x_{i1}, \ldots, x_{in}\}$ the domain associated with the variable X_i. x_{ij} denotes the jth instance of X_i. The universe of discourse is denoted by $\Omega = \times_{X_i \in \mathcal{V}} \mathbb{D}_{X_i}$, which is the cartesian product of all variables' domain in $\mathcal{V}$. Each element $\omega \in \Omega$ is called an interpretation which represents a possible state of Ω. It is denoted by $\omega = (x_{1i}, \ldots, x_{Nj})$. ϕ, ψ, ... represent subsets of Ω.

One of basic elements in the possibility theory is the notion of a possibility distribution π which corresponds to mapping from Ω to the scale $[0, 1]$. $\pi(\omega) = 1$ means that ω is completely possible and $\pi(\omega) = 0$ means that it is impossible for ω to represent the real world. A possibility distribution π is said to be $\alpha-$normalized, if its normalization degree, denoted by $h(\pi)$, is equal to α, namely:

$$h(\pi) = \max_{\omega \in \Omega} \pi(\omega) = \alpha. \tag{1}$$

If $\alpha = 1$, then π is said to be normalized. Given a distribution π two dual measures are defined: *possibility measure* $\Pi(\phi) = \max_{\omega \in \Omega}\{\pi(\omega) : \omega \in \phi\}$ and *necessity measure* $N(\phi) = 1 - \Pi(\overline{\phi})$. The first one evaluates to what extent ϕ is consistent with the knowledge encoded by π. The second one evaluates at which level ϕ is

implied by the knowledge represented by π. Possibilistic conditioning [15] consists in revising of the initial knowledge, encoded by π, by the arrival of a new certain information $\phi \subseteq \Omega$. In this paper, we only focus on min-based conditioning defined by:

$$\pi(\omega \mid \phi) = \begin{cases} 1 & \text{If } \pi(\omega) = \Pi(\phi) \text{ and } \omega \in \phi \\ \pi(\omega) & \text{If } \pi(\omega) < \Pi(\phi) \text{ and } \omega \in \phi \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

2.1 Possibilistic Logic

A possibilistic knowledge base is a finite set of weighted formulas [20]: $\Sigma = \{(\phi_i, \alpha_i), i = 1..n\}$, where ϕ_i is a propositional formula and $\alpha_i \in]0, 1]$ represents the certainty level of ϕ_i. Each piece of information (ϕ_i, α_i) of a possibilistic knowledge base can be viewed as a constraint that restricts possibility degrees associated with interpretations. If an interpretation ω satisfies ϕ_i then its possibility degree is equal to 1 (ω is completely compatible with the belief ϕ_i), otherwise it is equal $1 - \alpha_i$ (the more ϕ_i is certain, the less ω is possible). Hence, the possibility distribution associated with one weighted formula (ϕ_i, α_i) is:

$$\forall \omega \in \Omega,\ \pi_{(\phi_i,\alpha_i)}(\omega) = \begin{cases} 1 - \alpha_i & \text{If } \omega \notin \phi_i \\ 1 & \text{otherwise.} \end{cases} \tag{3}$$

More generally, the possibility distribution associated with Σ is the result of combining possibility distributions associated with each formula (ϕ_i, α_i) of Σ:

$$\forall \omega \in \Omega,\ \pi_\Sigma(\omega) = \min \left\{ \pi_{(\phi_i,\alpha_i)}(\omega), (\phi_i, \alpha_i) \in \Sigma \right\}. \tag{4}$$

Let Σ_1 and Σ_2 be two possibilistic bases and π_1, π_2 be their associated possibility distributions. The syntactic counterpart Σ_{min} of the fusion of two possibility distributions using min operator, defined by $\pi_{min}(\omega) = \min(\pi_1(\omega), \pi_2(\omega))$, is [11, 30]:

$$\Sigma_{min} = \Sigma_1 \cup \Sigma_2. \tag{5}$$

Now, we briefly review logical-based models for modelling possibilistic decision problems. A qualitative decision problem [25] is modeled by a finite set of possible states of the world $\mathcal{X}$, a finite set of consequences $\mathcal{C}$, a set of decisions D, such that each decision $d_i : \mathcal{X} \to \mathcal{C}$ associates for each possible state a consequence. Preferences among consequences are encoded by a utility function $\mu : \mathcal{C} \to \mathcal{U}$, where $\mathcal{U}$ is an ordinal scale. The uncertainty on $\mathcal{X}$ is expressed by a normalized distribution π mapping a set of state variables values into $[0, 1]$. Agent's preferences are represented by means of another distribution μ mapping a set of consequences into $[0, 1]$. A decision is represented by a function d from X to $\mathcal{C}$. The consequence $d(x) \in \mathcal{C}$ associated with a decision d on the state x can be evaluated by combining possibility degrees $\pi(x)$ and utilities $\mu(d(x))$ for all possible states. In [18], the authors proposed a compact representation of a qualitative decision problem by means of

valued logical formulas. Besides, in [8, 7], two algorithms for computing optimal decisions using syntactic possibilistic fusion have been proposed.

2.2 Min-Based Possibilistic Networks

A min-based network $\Pi G_{min} = (G, \pi)$ [5] over a set $\mathcal{V}$ is characterized by:

1. A graphical component: which is represented by a directed acyclic graph (DAG), where nodes correspond to variables and arcs represent dependence relations between variables.
2. Numerical components: they quantify different links in the DAG using local possibility distributions for each node X in the context of its parents, denoted by $Par(X)$. More precisely, uncertainty is represented by the conditional possibility degree $\pi(x \mid u_X)$ for each instance $x \in \mathbb{D}_X$ and for any instance $u_X \in \mathbb{D}_{Par(X)}$, such that $\max_{x \in \Omega} \pi(x \mid u_X) = 1$, for any u_X.

The set of a priori and conditional possibility degrees induces a unique joint possibility distribution π_{min} defined by:

$$\pi_{min}(X_1, \ldots, X_N) = \min_{i=1..N} \pi(X_i \mid U_i). \tag{6}$$

Computing posteriori distributions is known to be a hard problem except for singly connected graphs which ensure the propagation in polynomial time [23]. One of well-known propagation algorithm is the so-called junction tree algorithm [4, 16]. More recent works are based on compilation process [16, 3] of parameters.

Let $\Pi G_{min} = (G, \pi)$ and $\Pi G'_{min} = (G', \pi')$ be two min-based possibilistic networks. The result of merging ΠG_{min} and $\Pi G'_{min}$ is the min-based network $\Pi G_{\oplus} = (G_{\oplus}, \pi_{\oplus})$ [12] $\pi_{G_{\oplus}}$ defined by:

$$\forall \omega \in \Omega, \pi_{G_{\oplus}}(\omega) = \min\left(\pi_G(\omega), \pi'_{G'}(\omega)\right). \tag{7}$$

In [12], the authors proposed two classes for merging possibilistic networks:

1. Fusion of same-structure networks: namely when $G = G'$. In this case:
   - The resulting network $\Pi G_{\oplus}$ keeps the same structure: $G_{\oplus} = G' = G$,
   - For each variable X, $\pi_{\oplus}(X \mid U_X) = \min(\pi(X \mid U_X), \pi'(X \mid U_X))$.
2. Fusion of networks with different structures: when the two networks have different structures and their union is acyclic, ΠG_{min} and $\Pi G'_{min}$ are expanded to a same structure (their union) by adding variables or arcs as follows:
   (a) Adding variables: The extension of $\Pi G_{min} = (G, \pi)$ by adding a new variable $X \notin \mathcal{V}$ provides a new min-based possibilistic network $\Pi G_X = (G_X, \pi_X)$ which induces a joint possibility distribution π_{G_X}, such that:
   - $G_X = G \cup \{X\}$,

- the additional node X will represent the total ignorance: $\forall x \in \mathbb{D}_X$, $\pi_X(x) = 1$,
- the remaining variables preserve the same possibility distributions, $\forall Y \in \mathcal{V}$, $Y \neq X$, $\pi_X(Y \mid U_Y) = \pi(Y \mid U_Y)$.

(b) Adding arcs (links): The extension of $\Pi G_{min} = (G, \pi)$ by adding a link from X to Y ($X \notin Par(Y)$) provides a new min-based possibilistic network $\Pi G_L = (G_L, \pi_L)$ which induces a joint possibility distribution π_{G_L}, such that:

- $\forall y \in \mathbb{D}_Y$, $x \in \mathbb{D}_X$, $u_Y \in \mathbb{D}_{Par(Y)}$ $\pi_L(y \mid u_Y x) = \pi(y \mid u_Y)$;
- $\forall Z, Z \neq Y$, $\forall z \in \mathbb{D}_Z$, $u_Z \in \mathbb{D}_{Par(Z)}$, $\pi_L(z \mid u_Z) = \pi(z \mid u_Z)$.

Since, both initial min-based possibilistic networks have now the same structure, then the fusion of same-structure networks is applied. For more details on the fusion of possibilistic networks see [12].

3 NEW GRAPHICAL MODEL FOR POSSIBILISTIC DECISION PROBLEMS: PROBLEM DESCRIPTION

Our starting point is a possibility distribution π and a utility function μ which represent respectively an uncertainty on possible states of the world and agent's preferences.

We propose to compactly encode these two possibility distributions (uncertainty and utility) using two distinct min-based possibilistic networks: one representing agent's beliefs and the second representing the qualitative utility. The first min-based possibilistic network, denoted by $\Pi K_{min} = (G_K, \pi)$, represents agent's knowledge and induces a unique possibility distribution $\pi_K = \pi$ using Equation (6). The second min-based possibilistic network, denoted by $\Pi P_{min} = (G_P, \mu)$, defines agent's preferences and induces a unique qualitative utility $\mu_P = \mu$ using also Equation (6). As in logical-based approach [18], the graphical components G_K and G_p of the two min-based possibilistic networks ΠK_{min} and ΠP_{min} are defined on two types of variables: decision variables denoted by $\mathcal{D} = \{d_1, \ldots, d_p\}$ and state variables denoted by $\mathcal{X} = \{X_1, \ldots, X_n\}$. Making a decision comes down to choosing a decision d which maximises the following qualitative utility given by [25]:

Optimistic utility: $u^*(d) = \max_{\omega \in \Omega} \min(\pi_d(\omega), \mu(\omega))$.

Example 1. Let us consider a simple decision problem to illustrate our proposed model.

1. Agent's knowledge $\Pi K_{min} = (G_K, \pi)$: its graphical component G_K is given by Figure 1. It contains four possible state variables $\{X_1, X_2, X_3, X_4\}$ and one decision variable $\{D\}$. Initial conditional distributions of ΠK_{min} are given by Tables 1 and 2. We assume that all variables are binary. Using the min-based chain rule (Equation (6)), we obtain the joint distribution given in Table 3.

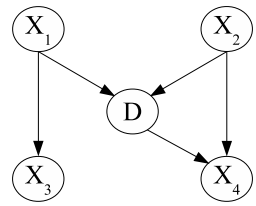

Figure 1. Min-based possibilistic network representing available knowledge

X_1	$\pi(X_1)$	X_2	$\pi(X_2)$	X_3	X_1	$\pi(X_3 \mid X_1)$	X_3	X_1	$\pi(X_3 \mid X_1)$
x_1	.5	x_2	.8	x_3	x_1	.5	$\neg x_3$	x_1	1
$\neg x_1$	1	$\neg x_2$	1	x_3	$\neg x_1$	.6	$\neg x_3$	$\neg x_1$	1

Table 1. Initial possibility distributions ΠK_{min} on X_1, X_2 and X_3 given X_1

2. Agent's preferences: they are expressed by another min-based possibilistic network $\Pi P_{min} = (G_P, \mu)$, where its DAG is given by Figure 2. Initial conditional possibility distributions associated with ΠP_{min} are given in Table 4.

   Using the min-based chain rule (Equation (6)), we obtain the joint qualitative utility given in Table 5.

4 ON THE COMPUTATION OF OPTIMISTIC DECISIONS USING MIN-BASED FUSION OF POSSIBILISTIC NETWORKS

Given the graphical model for representing decision making under uncertainty, we propose in this section an algorithm for dealing with decision evaluations. We recall that each set of decision d induces a possibility distribution π_{K_d} as follows [25]:

$$\pi_{K_d}(\omega) = \min(\pi_K(\omega), \pi_d(\omega)), \tag{8}$$

where

$$\pi_d(\omega) = \begin{cases} 1 & \text{If } \omega \models d \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

and $\omega \models d$ means that the value of D in ω is equal to d.

D	X_1	X_2	$\pi(D\|X_1X_2)$	D	X_1	X_2	$\pi(D\|X_1X_2)$	X_4	D	X_2	$\pi(X_4\|DX_2)$	X_4	D	X_2	$\pi(X_4\|DX_2)$
d	x_1	x_2	.1	$\neg d$	x_1	x_2	1	x_4	d	x_2	.1	$\neg x_4$	d	x_2	1
d	x_1	$\neg x_2$	.6	$\neg d$	x_1	$\neg x_2$	1	x_4	d	$\neg x_2$	.9	$\neg x_4$	d	$\neg x_2$	1
d	$\neg x_1$	x_2	.7	$\neg d$	$\neg x_1$	x_2	1	x_4	$\neg d$	x_2	.2	$\neg x_4$	$\neg d$	x_2	1
d	$\neg x_1$	$\neg x_2$	.4	$\neg d$	$\neg x_1$	$\neg x_2$	1	x_4	$\neg d$	$\neg x_2$	1	$\neg x_4$	$\neg d$	$\neg x_2$	.2

Table 2. Initial possibility distributions ΠK_{min} on D given X_1X_2 and X_4 given DX_2

X_1	X_2	X_3	X_4	D	π_K	X_1	X_2	X_3	X_4	D	π_K
x_1	x_2	x_3	x_4	d	.1	$\neg x_1$	x_2	x_3	x_4	d	.1
x_1	x_2	x_3	x_4	$\neg d$	.2	$\neg x_1$	x_2	x_3	x_4	$\neg d$	.2
x_1	x_2	x_3	$\neg x_4$	d	.1	$\neg x_1$	x_2	x_3	$\neg x_4$	d	.6
x_1	x_2	x_3	$\neg x_4$	$\neg d$	.5	$\neg x_1$	x_2	x_3	$\neg x_4$	$\neg d$	.6
x_1	x_2	$\neg x_3$	x_4	d	.1	$\neg x_1$	x_2	$\neg x_3$	x_4	d	.1
x_1	x_2	$\neg x_3$	x_4	$\neg d$	.2	$\neg x_1$	x_2	$\neg x_3$	x_4	$\neg d$	.2
x_1	x_2	$\neg x_3$	$\neg x_4$	d	.1	$\neg x_1$	x_2	$\neg x_3$	$\neg x_4$	d	.7
x_1	x_2	$\neg x_3$	$\neg x_4$	$\neg d$	.5	$\neg x_1$	x_2	$\neg x_3$	$\neg x_4$	$\neg d$	.8
x_1	$\neg x_2$	x_3	x_4	d	.5	$\neg x_1$	$\neg x_2$	x_3	x_4	d	.4
x_1	$\neg x_2$	x_3	x_4	$\neg d$	.2	$\neg x_1$	$\neg x_2$	x_3	x_4	$\neg d$	.6
x_1	$\neg x_2$	x_3	$\neg x_4$	d	.5	$\neg x_1$	$\neg x_2$	x_3	$\neg x_4$	d	.4
x_1	$\neg x_2$	x_3	$\neg x_4$	$\neg d$	.2	$\neg x_1$	$\neg x_2$	x_3	$\neg x_4$	$\neg d$	.2
x_1	$\neg x_2$	$\neg x_3$	x_4	d	.5	$\neg x_1$	$\neg x_2$	$\neg x_3$	x_4	d	.4
x_1	$\neg x_2$	$\neg x_3$	x_4	$\neg d$	.5	$\neg x_1$	$\neg x_2$	$\neg x_3$	x_4	$\neg d$	1
x_1	$\neg x_2$	$\neg x_3$	$\neg x_4$	d	.5	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg x_4$	d	.4
x_1	$\neg x_2$	$\neg x_3$	$\neg x_4$	$\neg d$	.2	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg x_4$	$\neg d$	.2

Table 3. The joint possibility distribution ΠK_{min} on X_1, X_2, X_3, X_4, D

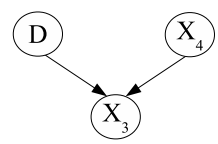

Figure 2. A min-based possibilistic network representing agent's preferences

X_3	D	X_4	$\mu(X_3\|DX_4)$	X_3	D	X_4	$\mu(X_3\|DX_4)$	D	$\mu(D)$	X_4	$\mu(X_4)$
x_3	d	x_4	1	$\neg x_3$	d	x_4	.7	d	.1	x_4	1
x_3	d	$\neg x_4$	.6	$\neg x_3$	d	$\neg x_4$	1	$\neg d$	1	$\neg x_4$	.3
x_3	$\neg d$	x_4	.8	$\neg x_3$	$\neg d$	x_4	1				
x_3	$\neg d$	$\neg x_4$	.2	$\neg x_3$	$\neg d$	$\neg x_4$	1				

Table 4. Initial possibility distributions ΠP_{min} on X_3 given DX_4, D and X_4

X_3	D	X_4	μ_P	X_3	D	X_4	μ_P
x_3	d	x_4	.1	$\neg x_3$	d	x_4	.1
x_3	d	$\neg x_4$	.1	$\neg x_3$	d	$\neg x_4$	.1
x_3	$\neg d$	x_4	.8	$\neg x_3$	$\neg d$	x_4	1
x_3	$\neg d$	$\neg x_4$	.2	$\neg x_3$	$\neg d$	$\neg x_4$	.3

Table 5. The joint qualitative utility ΠP_{min} on DX_3X_4

4.1 Optimistic Decisions as a Fusion Process

An optimal optimistic decision d is defined by the assignment of variables of D that maximizes the expression:

$$u^*(d) = \max_{\omega \in \Omega} \min(\pi_{K_d}(\omega), \mu_P(\omega)). \tag{10}$$

Using Equation (8), the optimistic utility decision $u^*(d)$ becomes:

$$u^*(d) = \max_{\omega \in \Omega} \min(\min(\pi_K(\omega), \mu_P(\omega)), \pi_d(\omega)). \tag{11}$$

Merging two min-based possibilistic networks (Equation (7)), Equation (11) comes down to:

$$u^*(d) = \max_{\omega \in \Omega} \min(\pi_{G_\oplus}(\omega), \pi_d(\omega)). \tag{12}$$

where $\pi_{G_\oplus}(\omega) = \min(\pi_K(\omega), \mu_P(\omega))$. Besides, in Section 2 we recalled how to compute the syntactic counterpart of $\min(\pi, \pi')$ described in [12]. Let $\Pi G_\oplus = (G_\oplus, \pi_\oplus)$ be the syntactic counterpart of $\min(\pi_K(\omega), \mu_P(\omega))$, where its structure $G_\oplus$ is built depending whether the initial graphs are identical or not. The resulted min-based possibilistic network $\Pi G_\oplus$ induces the unique possibility distribution $\pi_{G_\oplus}$.

Example 2. Let us consider the two DAGs (G_K and G_P) given in Example 1 (Figure 1 and Figure 2). They have different structures but their union is free of cycles. The result of merging ΠK_{min} and ΠP_{min} is the min-based possibilistic network $\Pi G_\oplus = (G_\oplus, \pi_\oplus)$, where $G_\oplus$ is given in Figure 3. $G_\oplus$ is simply the union of the two graphs of Figure 1 and Figure 2. The resulted min-based possibilistic network $\Pi G_\oplus$ induces a unique possibility distribution $\pi_{G_\oplus}$ using Equation (6).

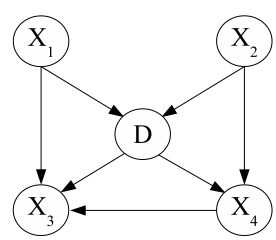

Figure 3. The DAG $G_\oplus$

Initial conditional possibility distributions associated with the merged network $\Pi G_\oplus$ are given by Tables 6 and 7, which are obtained using the minimum of local possibility distributions ΠK_{min} and ΠP_{min}.

Using min-based chain rule (Equation (6)), we obtain the joint possibility distribution given in Table 8.

D	X_1	X_2	$\pi_\oplus(D\|X_1X_2)$	D	X_1	X_2	$\pi_\oplus(D\|X_1X_2)$	X_4	D	X_2	$\pi_\oplus(X_4\|DX_2)$	X_4	D	X_2	$\pi_\oplus(X_4\|DX_2)$
d	x_1	x_2	.1	$\neg d$	x_1	x_2	1	x_4	d	x_2	.1	$\neg x_4$	d	x_2	.3
d	x_1	$\neg x_2$	.1	$\neg d$	x_1	$\neg x_2$	1	x_4	d	$\neg x_2$	.9	$\neg x_4$	d	$\neg x_2$	.3
d	$\neg x_1$	x_2	.1	$\neg d$	$\neg x_1$	x_2	1	$\neg x_4$	$\neg d$	x_2	.2	$\neg x_4$	$\neg d$	x_2	.3
d	$\neg x_1$	$\neg x_2$	.1	$\neg d$	$\neg x_1$	$\neg x_2$	1	$\neg x_4$	$\neg d$	$\neg x_2$	1	$\neg x_4$	$\neg d$	$\neg x_2$	.2

Table 6. Initial possibility distributions $\Pi G_\oplus$ on D given X_1X_2 and X_4 given DX_2

X_3	X_1	D	X_4	$\pi_\oplus(X_3\|X_1DX_4)$	X_3	X_1	D	X_4	$\pi_\oplus(X_3\|X_1DX_4)$	X_1	$\pi_\oplus(X_1)$	X_2	$\pi_\oplus(X_2)$
x_3	x_1	d	x_4	.5	$\neg x_3$	x_1	d	x_4	.7	x_1	.5	x_2	.8
x_3	x_1	d	$\neg x_4$	.5	$\neg x_3$	x_1	d	$\neg x_4$	1	$\neg x_1$	1	$\neg x_2$	1
x_3	x_1	$\neg d$	x_4	.5	$\neg x_3$	x_1	$\neg d$	x_4	1				
x_3	x_1	$\neg d$	$\neg x_4$	.2	$\neg x_3$	x_1	$\neg d$	$\neg x_4$	1				
x_3	$\neg x_1$	d	x_4	.6	$\neg x_3$	$\neg x_1$	d	x_4	.7				
x_3	$\neg x_1$	d	$\neg x_4$	.6	$\neg x_3$	$\neg x_1$	d	$\neg x_4$	1				
x_3	$\neg x_1$	$\neg d$	x_4	.6	$\neg x_3$	$\neg x_1$	$\neg d$	x_4	1				
x_3	$\neg x_1$	$\neg d$	$\neg x_4$	.2	$\neg x_3$	$\neg x_1$	$\neg d$	$\neg x_4$	1				

Table 7. Initial possibility distributions $\Pi G_\oplus$ on X_3 given X_1DX_4, X_1 and X_2

We can check that the joint possibility distribution $\pi_{G_\oplus}$ induced by $\Pi G_\oplus$ is equal to the minimum of the joint possibility distributions π_K and μ_P induced by ΠK_{min} and ΠP_{min}, respectively.

X_1	X_2	X_3	X_4	D	$\pi_{G_\oplus}$	X_1	X_2	X_3	X_4	D	$\pi_{G_\oplus}$
x_1	x_2	x_3	x_4	d	.1	$\neg x_1$	x_2	x_3	x_4	d	.1
x_1	x_2	x_3	x_4	$\neg d$	.2	$\neg x_1$	x_2	x_3	x_4	$\neg d$	.2
x_1	x_2	x_3	$\neg x_4$	d	.1	$\neg x_1$	x_2	x_3	$\neg x_4$	d	.1
x_1	x_2	x_3	$\neg x_4$	$\neg d$	.2	$\neg x_1$	x_2	x_3	$\neg x_4$	$\neg d$	.2
x_1	x_2	$\neg x_3$	x_4	d	.1	$\neg x_1$	x_2	$\neg x_3$	x_4	d	.1
x_1	x_2	$\neg x_3$	x_4	$\neg d$	.2	$\neg x_1$	x_2	$\neg x_3$	x_4	$\neg d$	.2
x_1	x_2	$\neg x_3$	$\neg x_4$	d	.1	$\neg x_1$	x_2	$\neg x_3$	$\neg x_4$	d	.1
x_1	x_2	$\neg x_3$	$\neg x_4$	$\neg d$	.3	$\neg x_1$	x_2	$\neg x_3$	$\neg x_4$	$\neg d$	.3
x_1	$\neg x_2$	x_3	x_4	d	.1	$\neg x_1$	$\neg x_2$	x_3	x_4	d	.1
x_1	$\neg x_2$	x_3	x_4	$\neg d$	.5	$\neg x_1$	$\neg x_2$	x_3	x_4	$\neg d$	.6
x_1	$\neg x_2$	x_3	$\neg x_4$	d	.1	$\neg x_1$	$\neg x_2$	x_3	$\neg x_4$	d	.1
x_1	$\neg x_2$	x_3	$\neg x_4$	$\neg d$	.2	$\neg x_1$	$\neg x_2$	x_3	$\neg x_4$	$\neg d$	.2
x_1	$\neg x_2$	$\neg x_3$	x_4	d	.1	$\neg x_1$	$\neg x_2$	$\neg x_3$	x_4	d	.1
x_1	$\neg x_2$	$\neg x_3$	x_4	$\neg d$	.5	$\neg x_1$	$\neg x_2$	$\neg x_3$	x_4	$\neg d$	1
x_1	$\neg x_2$	$\neg x_3$	$\neg x_4$	d	.1	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg x_4$	d	.1
x_1	$\neg x_2$	$\neg x_3$	$\neg x_4$	$\neg d$	.2	$\neg x_1$	$\neg x_2$	$\neg x_3$	$\neg x_4$	$\neg d$	.2

Table 8. The joint possibility distributions $\Pi G_\oplus$ on X_1, X_2, X_3, X_4, D

4.2 Computing Optimal Optimistic Decisions Using Junction Trees

In this section, we present the propagation process based on junction tree to compute optimal decisions. The computation of $\min(\pi_{G_\oplus}(\omega), \pi_d(\omega))$ is performed using junction tree algorithm on $\Pi G_\oplus$ with an additional step in the initialization process. The main difference with standard possibilistic propagation algorithms is that our junction tree is parameterized by some decision d. The parameter d makes a difference between a propagation algorithm from a possibilistic network representing only agent's knowledge and the one representing knowledge and preferences. Note that the construction of junction tree is only done once. However, the propagation and the initialization (which are both polynomial) are repeated for each possible decision. The following provides main steps of our algorithm:

1. Building junction tree $\mathcal{JT}$: Min-based propagation algorithms begin by transforming the initial graph $G_\oplus$ into a secondary structure corresponding to a junction tree by eliminating existing loops. This is done in three steps [16]:

   - Moralization of the initial graph $G_\oplus$: It consists of creating an undirected graph from the initial one by adding links between the parents of each variable, and replacing directed arcs by undirected ones.
   - Triangulation of the moral graph: It allows to identify sets of variables that can be gathered as clusters or cliques denoted by C_i.
   - Construction of a junction tree $\mathcal{JT}$: A junction tree is built by connecting the clusters, representing cliques of the triangulated graph, identified in the previous step. Once adjacent clusters have been identified, between each pair of clusters C_i and C_j, a separator S_{ij} containing their common variables, is inserted.

2. Initialization for a given decision $\boldsymbol{d}$: The idea is to transform initial conditional distributions into local joint distributions attached to clusters and separators taking into account the decision d. A potential $\pi^t_{C_i}$ (resp. $\pi^t_{S_{ij}}$) is assigned to each cluster C_i (resp. separator S_{ij}) of the junction tree $\mathcal{JT}$, where t is relative to the propagation step. The initialization step is done as follows:

   (a) For each cluster C_i (resp. S_{ij}), $\pi^I_{C_i} \leftarrow 1$ (resp. $\pi^I_{S_{ij}} \leftarrow 1$).

   (b) For each variable X_i, select a cluster C_i containing $\{X_i\} \cup Par(X_i)$,

   $$\pi^I_{C_i} \leftarrow \min(\pi^I_{C_i}, \pi_\oplus(X_i \mid U_i)).$$

   (c) Encode the evidence $D = d$ as a likelihood $\Lambda_D(d)$:

   $$\Lambda_D(d) = \begin{cases} 1 & \text{If } D \text{ is instantiated as } d \\ 0 & \text{If } D \text{ is instantiated by a value} \\ & \text{different from } d \end{cases} \tag{13}$$

(d) Identify a cluster C_i containing D, and update C_i as follows:

$$\pi^I_{C_i} \leftarrow \min(\pi^I_{C_i}, \Lambda_D). \tag{14}$$

Note, that Equations (13) and (14) do not appear in standard initialization of junction trees. It is proper to the possibilistic decision problem. These two steps are added for computing optimal optimistic possibilistic decisions. By entering the fact $D = d$, the junction tree $\mathcal{JT}$ encodes $\pi_{\mathcal{JT}}(\omega) = \min(\pi_{G_\oplus}(\omega)\ , \pi_d(\omega))$, where $\pi_{\mathcal{JT}}$ can be redefined from $\mathcal{JT}$ as follows:

$$\pi_{\mathcal{JT}} = \min_{i=1..m} \pi_{C_i}, \tag{15}$$

where m is the number of clusters in $\mathcal{JT}$. Then the qualitative utility associated with a decision d is summarized by the following proposition:

Proposition 1. Let $\Pi K_{min} = (G_K, \pi)$ and $\Pi P_{min} = (G_P, \mu)$ be two min-based networks representing agent's beliefs and preferences. Let $\Pi G_\oplus = (G_\oplus, \pi_\oplus)$ be the result of merging ΠK_{min} and ΠP_{min} using the min operator. Let $\mathcal{JT}$ be the junction tree associated with $\Pi G_\oplus$ presented above. Then,

$$u^*(d) = h(\pi_{\mathcal{JT}}) = \max_{\omega \in \Omega} \pi_{\mathcal{JT}}(\omega), \tag{16}$$

where $u^*(d)$ is given in Equation (12).

Hence, after the initialization step, the junction tree encodes the possibilistic optimistic decision. Algorithm 1 summarizes the initialization step which is performed by a call to the function $\boldsymbol{Init(\mathcal{JT}, d)}$. This function has two parameters: the junction tree $\mathcal{JT}$ having m clusters and issued from $\Pi G_\oplus$, and a decision d which will parameterize $\mathcal{JT}$.

3. Global propagation: the global propagation is performed in order to make it globally consistent, namely: $\max_{C_i \setminus S_{ij}} \pi^t_{C_i} = \pi^t_{S_{ij}} = \max_{C_j \setminus S_{ij}} \pi^t_{C_j}$. When a cluster C_i sends its potential to one of its adjacent cluster C_j, then the potential of C_j and their separator S_{ij} are updated as follows:

(a) Update the potential of S_{ij} : $\pi^{t+1}_{S_{ij}} \leftarrow \max_{C_i \setminus S_{ij}} \pi^t_{C_i}$.

(b) Update the potential of C_j : $\pi^{t+1}_{C_j} \leftarrow \min\left(\pi^t_{C_j}, \pi^{t+1}_{S_{ij}}\right)$.

Steps (a)–(b) are repeated until no modifications appear in the potentials of clusters. Once stability is reached, the computation of the qualitative utility relative to a decision d can be achieved.

Proposition 2. Let $\Pi K_{min} = (G_K, \pi)$ and $\Pi P_{min} = (G_P, \mu)$ be the min-based networks representing agent's beliefs and preferences. Let $\Pi G_\oplus = (G_\oplus, \pi_\oplus)$ be the result of merging ΠK_{min} and ΠP_{min} using the min operator. Let $\mathcal{JT}$ be the

Algorithm 1: Init(JT,d)

```
Data: 𝒥𝒯, a Junction Tree,
      d, a decision instance,
begin
    /*m, the number of clusters in 𝒥𝒯 */
    for i = 1..m do
        π^I_{C_i} ← 1,
    for j = 1..(m − 1) do
        π^I_{S_j} ← 1,
    n, the number of variables in 𝒥𝒯,
    for i = 1..n do
        Select a cluster C_{j∈{1,...,m}} containing X_i ∪ Par(X_i),
        π^I_{C_i} ← min(π^I_{C_i}, π_⊕(X_i | U_i)),
    for i = 1..m do
        if D ∈ C_i then
            if D = ¬d then
                π^I_{C_i} ← 0
```

junction tree associated with $\Pi G_{\oplus}$ generated using the above global propagation procedure. Then, the computation of optimistic decisions amounts to compute a normalization degree of $\mathcal{JT}$:

$$u^*(d) = h(\pi_{\mathcal{JT}}) = \max_{C_i} \pi_{C_i}. \tag{17}$$

The optimal optimistic decisions are those maximizing the qualitative utility. The computation of these optimal optimistic decisions is obtained using Algorithm 2.

In Algorithm 2, the function $\boldsymbol{Fusion(\Pi K_{min}, \Pi P_{min}, \Pi G_{\oplus})}$ defines the fusion step of the two initial min-based networks ΠK_{min} and ΠP_{min}. The result of the fusion step is the min-based network $\Pi G_{\oplus}(G_{\oplus}, \pi_{\oplus})$. The construction of the junction tree $\mathcal{JT}$ associated with the resulted fusion min-based network $\Pi G_{\oplus}$ is ensured by calling the function $\boldsymbol{Junction-Tree(\Pi G_{\oplus}, \mathcal{JT})}$. In addition, the function $\boldsymbol{Init(\mathcal{JT}, d)}$ corresponds to the initialization step defined by Algorithm 1. Similarly, the function $\boldsymbol{Prog(JT)}$ corresponds to the global propagation and returns a normalization degree relative to $\mathcal{JT}$. As it was already stated, the construction of the junction tree is only done once but the initialization and the propagation steps are repeated for each decision. More precisely, for each decision $d_i \in \{d_1, \ldots, d_p\}$, a call to the initialization and propagation functions occurred. The initialization function $\boldsymbol{Init(\mathcal{JT}, d)}$ allows the parametrization of the junction tree by the decision d_i. As for the propagation function $\boldsymbol{Prog(JT)}$, it allows the computation of a normalization degree associated to the parameterized junction tree. Finally, the

Algorithm 2: Graph-based computation of optimal optimistic decisions

Data: $\Pi K_{min} = (G_K, \pi)$, a knowledge possibilistic network,
$\Pi P_{min} = (G_P, \mu)$, a preferences possibilistic network,
$\mathcal{D} = \{D_1, \ldots, D_p\}$, set of decisions.
Result: *Decisions*, u^*.
begin
  $Fusion(\Pi K_{min}, \Pi P_{min}, \Pi G_{\oplus})$, /* $\Pi G_{\oplus}$ is the fusion of ΠK_{min} and ΠP_{min} */,
  $Junction - Tree(\Pi G_{\oplus}, \mathcal{JT})$, /* $\mathcal{JT}$ the junction tree issued from $\Pi G_{\oplus}$*/,
  $i \leftarrow 1$,
  $Norm \leftarrow 0$, /*normalisation degree*/,
  $u^* \leftarrow 0$, /*the optimistic utility*/,
  $Decisions \leftarrow \emptyset$, */optimal optimistic decisions*/,
  for $i = 1..p$ **do**
    $Init(\mathcal{JT}, d_i)$,/*Initialization step*/
    $Norm \leftarrow Prog(\mathcal{JT})$, /*global propagation*/
    if $Norm > u^*$ **then**
      $Decisions \leftarrow \{d_i\}$,
      $u^* \leftarrow Norm$,
    else
      if $Norm = u^*$ **then**
        $Decisions \leftarrow Decisions \cup \{d_i\}$

algorithm returns optimal decisions, those that maximize the normalization degree relative to the junction tree.

A good feature of our approach is that the decision process has basically the same complexity as the reasoning process. Hence, there is no important extra computational cost added by the presence of decision variables. Indeed, the extra cost due to the presence of decision variables corresponds to the computational complexity of the fusion process. Once the two graphical models (knowledge-based and preference-based networks) are fused, the computational complexity of the variant of the junction tree presented in the paper is the same as the one of standard junction tree. Besides, the computational complexity of the fusion process is smaller than the one of the inference of the junction tree, in particular when the union of the two graphs is free of cycles. In this case, the complexity of the fusion process is linear with respect to the number of variables and parameters of the two graphs.

Example 3. Let us continue Example 2. We need to compute the optimal optimistic decision $D = \{d, \neg d\}$. As indicated in Algorithm 2, we first begin by constructing the junction tree (see Figure 4) associated with the graph $G_{\oplus}$ (Figure 3) representing the fusion of ΠK_{min} and ΠP_{min}. The resulting junction tree contains two clusters $C_1 = \{X_1, X_2, X_4, D\}$ and $C_2 = \{X_1, X_3, X_4, D\}$ and their separator $S_{12} = \{X_1, X_4, D\}$.

Figure 4. The junction tree associated with $G_{\oplus}$ of Example 2

Then for each decision value $D = \{d, \neg d\}$, we need to run the initialization and the propagation algorithm in order to compute the normalization degree associated with the junction tree.

Step 1: $D = d$, in this case, the fact $D = d$ is encoded as follows:

$$\Lambda_D(d) = \begin{cases} 1 & \text{If } D \text{ is instantiated as } d \\ 0 & \text{If } D \text{ is instantiated as } \neg d. \end{cases}$$

Conditional possibility distributions will be transformed into local joint distributions attached with clusters and separators as follows: $\pi_{C_1} = \min(1, \pi_{\oplus}(X_1), \pi_{\oplus}(X_2), \pi_{\oplus}(X_4 \mid DX_2))$ and $\pi_{C_2} = \min(1, \pi_{\oplus}(X_3 \mid DX_1X_4), \Lambda_D)$. Using the initialization procedure, potentials of C_1 and C_2 are given by Table 9.

X_1	X_2	X_4	D	π_{C_1}	X_1	X_2	X_4	D	π_{C_1}	X_1	X_4	D	X_3	π_{C_2}	X_1	X_4	D	X_3	π_{C_2}
x_1	x_2	x_4	d	.1	$\neg x_1$	x_2	x_4	d	.1	x_1	x_4	d	x_3	.5	$\neg x_1$	x_4	d	x_3	.6
x_1	x_2	x_4	$\neg d$	.2	$\neg x_1$	x_2	x_4	$\neg d$	.2	x_1	x_4	d	$\neg x_3$	.7	$\neg x_1$	x_4	d	$\neg x_3$	.7
x_1	x_2	$\neg x_4$	d	.1	$\neg x_1$	x_2	$\neg x_4$	d	.1	x_1	x_4	$\neg d$	x_3	0	$\neg x_1$	x_4	$\neg d$	x_3	0
x_1	x_2	$\neg x_4$	$\neg d$	.3	$\neg x_1$	x_2	$\neg x_4$	$\neg d$	.3	x_1	x_4	$\neg d$	$\neg x_3$	0	$\neg x_1$	x_4	$\neg d$	$\neg x_3$	0
x_1	$\neg x_2$	x_4	d	.1	$\neg x_1$	$\neg x_2$	x_4	d	.1	x_1	$\neg x_4$	d	x_3	.5	$\neg x_1$	$\neg x_4$	d	x_3	.6
x_1	$\neg x_2$	x_4	$\neg d$	.5	$\neg x_1$	$\neg x_2$	x_4	$\neg d$	1	x_1	$\neg x_4$	d	$\neg x_3$	1	$\neg x_1$	$\neg x_4$	d	$\neg x_3$	1
x_1	$\neg x_2$	$\neg x_4$	d	.1	$\neg x_1$	$\neg x_2$	$\neg x_4$	d	.1	x_1	$\neg x_4$	$\neg d$	x_3	0	$\neg x_1$	$\neg x_4$	$\neg d$	x_3	0
x_1	$\neg x_2$	$\neg x_4$	$\neg d$	.2	$\neg x_1$	$\neg x_2$	$\neg x_4$	$\neg d$	.2	x_1	$\neg x_4$	$\neg d$	$\neg x_3$	0	$\neg x_1$	$\neg x_4$	$\neg d$	$\neg x_3$	0

Table 9. Potential assigned to C_1 and C_2

Lastly, the global propagation allows to compute the normalization degree of the junction tree which corresponds to the normalization degree of any cluster. We obtain: $u^*(d) = \max_{C_1} \pi_{C_1} = \max_{C_2} \pi_{C_2} = .1$.

Step 2: $D = \neg d$, we repeat the same procedure described in the previous step, with $\Lambda_D(\neg d) = 1$ If D is instantiated as $\neg d$ and 0 If D is instantiated as d. In the same way, potentials of C_1 and C_2, given $D = \neg d$ are: $\pi_{C_1} = \min(1, \pi_{\oplus}(X_1), \pi_{\oplus}(X_2), \pi_{\oplus}(X_4 \mid DX_2))$ and $\pi_{C_2} = \min(1, \pi_{\oplus}(X_3 \mid DX_1X_4), \Lambda_D)$. The corresponding results are reported in Table 10.

X_1	X_2	X_4	D	π_{C_1}	X_1	X_2	X_4	D	π_{C_1}	X_1	X_4	D	X_3	π_{C_2}	X_1	X_4	D	X_3	π_{C_2}
x_1	x_2	x_4	d	.1	$\neg x_1$	x_2	x_4	d	.1	x_1	x_4	d	x_3	0	$\neg x_1$	x_4	d	x_3	0
x_1	x_2	x_4	$\neg d$	.2	$\neg x_1$	x_2	x_4	$\neg d$	.2	x_1	x_4	d	$\neg x_3$	0	$\neg x_1$	x_4	d	$\neg x_3$	0
x_1	x_2	$\neg x_4$	d	.1	$\neg x_1$	x_2	$\neg x_4$	d	.1	x_1	x_4	$\neg d$	x_3	.5	$\neg x_1$	x_4	$\neg d$	x_3	.6
x_1	x_2	$\neg x_4$	$\neg d$	.3	$\neg x_1$	x_2	$\neg x_4$	$\neg d$	.3	x_1	x_4	$\neg d$	$\neg x_3$	1	$\neg x_1$	x_4	$\neg d$	$\neg x_3$	1
x_1	$\neg x_2$	x_4	d	.1	$\neg x_1$	$\neg x_2$	x_4	d	.1	x_1	$\neg x_4$	d	x_3	0	$\neg x_1$	$\neg x_4$	d	x_3	0
x_1	$\neg x_2$	x_4	$\neg d$	.5	$\neg x_1$	$\neg x_2$	x_4	$\neg d$	1	x_1	$\neg x_4$	d	$\neg x_3$	0	$\neg x_1$	$\neg x_4$	d	$\neg x_3$	0
x_1	$\neg x_2$	$\neg x_4$	d	.1	$\neg x_1$	$\neg x_2$	$\neg x_4$	d	.1	x_1	$\neg x_4$	$\neg d$	x_3	.2	$\neg x_1$	$\neg x_4$	$\neg d$	x_3	.2
x_1	$\neg x_2$	$\neg x_4$	$\neg d$	.2	$\neg x_1$	$\neg x_2$	$\neg x_4$	$\neg d$	.2	x_1	$\neg x_4$	$\neg d$	$\neg x_3$	1	$\neg x_1$	$\neg x_4$	$\neg d$	$\neg x_3$	1

Table 10. Potential assigned to C_1 and C_2

From the global propagation, we get: $u^*(\neg d) = \max_{C_1} \pi_{C_1} = \max_{C_2} \pi_{C_2} = 1$. Finally, the optimal decision $D^* = \neg d$ with the maximal qualitative utility is equal to 1. This result is exactly the same as the one obtained in Example 2.

5 LOGICAL-BASED MODEL FOR COMPUTING OPTIMAL OPTIMISTIC DECISIONS

The logical counterpart of our new graphical model offers an alternative way to compute optimal decisions. It is particularly suitable when it is impossible to apply the junction tree due to space memory. Namely, when the size of cliques is large, then the logical-based encoding is more appropriate.

The first step consists of transforming the two initial min-based networks into two logical possibilistic knowledge bases. We explore the algorithm proposed in [6] for transforming the graphical model to a logical one. The min-based network $\Pi K_{min} = (G_K, \pi)$ can be represented by a set of triples $\Pi K_{min} = \{(x_i, u_i, \alpha_i) : \alpha_i = \pi(x_i \mid u_i) \neq 1$ is an element of the graph$\}$, where $x_i \in \mathbb{D}_{X_i}$ and u_i is an element of the cartesian product of the domains $\mathbb{D}_j$ of the variables $X_j \in Par(X_i)$. Then the possibilistic knowledge base associated with ΠK_{min} is [6]:

$$\Sigma_{K_{min}} = \{(\neg x_i \vee \neg u_i, 1 - \alpha_i) : (x_i, u_i, \alpha_i) \in \Pi K_{min}\}. \tag{18}$$

The possibilistic base $\Sigma_{K_{min}}$ induces the same joint distribution π_K induced by the min-based possibilistic network $\Pi K_{min} = (G_K, \pi)$ using Equation (4). Similarly, the min-based network ΠP_{min} encoding agent's preferences can be represented by a set of triples $\Pi P_{min} = \{(y_j, u_j, \beta_j) : \beta_j = \pi(y_j \mid u_j) \neq 1$ is an element of the graph$\}$, where $y_j \in \mathbb{D}_{Y_j}$ and u_j is an element of the cartesian product of the domains $\mathbb{D}_l$ of the variables $Y_l \in Par(Y_j)$. The min-based network $\Pi P_{min} = (G_P, \mu)$ is transformed into a logical possibilistic base as follows:

$$\Sigma_{P_{min}} = \{(\neg y_j \vee \neg u_j, 1 - \beta_j) : (y_j, u_j, \beta_j) \in \Pi P_{min}\}. \tag{19}$$

The preference possibilistic logic base $\Sigma_{P_{min}}$ induces again the same joint possibility distribution μ_P generated by $\Pi P_{min} = (G_P, \mu)$ using Equation (4).

Example 4. Let us consider the decision problem of Example 1, where the decision problem will be encoded in a logical model: the min-based network ΠK_{min} encoding agent's knowledge is transformed into a logical possibilistic base Σ_K using Equation (18):

$$\begin{aligned}\Sigma_{K_{min}} = \ & \{(\neg d \vee \neg x_1 \vee \neg x_2, .9), (\neg x_4 \vee \neg d \vee \neg x_2, .9), (\neg x_4 \vee d \vee \neg x_2, .8),\\ & (x_4 \vee d \vee x_2, .8), (\neg d \vee x_1 \vee x_2, .6), (\neg x_1, .5), (\neg x_3 \vee \neg x_1, .5), (\neg x_3 \vee x_1, .4),\\ & (\neg d \vee \neg x_1 \vee x_2, .4), (\neg d \vee x_1 \vee \neg x_2, .3), (\neg x_2, .2), (\neg x_4 \vee \neg d \vee x_2, .1)\}.\end{aligned}$$

Similarly, using Equation (19), we have:

$$\begin{aligned}\Sigma_{P_{min}} = \ & \{(\neg d, .9), (x_4, .7), (x_3 \vee \neg d \vee \neg x_4, .3), (\neg x_3 \vee \neg d \vee x_4, .4),\\ & (\neg x_3 \vee d \vee \neg x_4, .2), (\neg x_3 \vee d \vee x_4, .8), (x_3 \vee \neg d \vee \neg x_4, .3)\}.\end{aligned}$$

Once the first step is achieved, the logical-based approach can then be applied on the two logical bases $\Sigma_{K_{min}}$ and $\Sigma_{P_{min}}$ associated with the initial networks ΠK_{min} and ΠP_{min} respectively. We recall that the optimistic utility decision is given by:

$$u^*(d) = \max_{\omega \in \Omega} \min(\min(\pi_K(\omega), \mu_P(\omega)), \pi_d(\omega)). \tag{20}$$

The syntactic counterpart of $\min(\pi_K(\omega), \mu_P(\omega))$ is the possibilistic base $\Sigma_\oplus = \Sigma_{K_{min}} \cup \Sigma_{P_{min}}$. So Equation (20) becomes:

$$u^*(d) = \max_{\omega \in \Omega} \min(\pi_{\Sigma_\oplus}(\omega), \pi_d(\omega)). \tag{21}$$

Using Equation (4), the optimistic utility decision $u^*(d)$ comes to:

$$u^*(d) = \max_{\omega \in \Omega} \pi_{\Sigma_{d_\oplus}}(\omega), \tag{22}$$

where $\Sigma_{d_\oplus} = \Sigma_{K_{min}} \cup \Sigma_{P_{min}} \cup \{(d, 1)\}$. The syntactic counterpart of Equation (22) is:

$$u^*(d) = 1 - Inc(\Sigma_{d_\oplus}), \tag{23}$$

where $Inc(\Sigma_{d_\oplus})$ is the inconsistency degree of $\Sigma_{K_{min}} \cup \Sigma_{P_{min}} \cup \{(d, 1)\}$. Computing optimal optimistic decisions using Equation (23) comes down to computing an inconsistency degree of the possibilistic base $\Sigma_{d_\oplus}$ issued from the fusion phase of the knowledge base $\Sigma_{K_{min}}$, the preference possibilistic logic base $\Sigma_{P_{min}}$ and a decision $d_i \in \{d_1, \ldots, d_p\}$. Optimizing the decision utility means to take a decision $d_i \in \{d_1, \ldots, d_p\}$ which has the minimal inconsistency degree with $\Sigma_{K_{min}} \cup \Sigma_{P_{min}}$. The computation of optimal optimistic decisions is summarized in Algorithm 3–4 which is a slight adaptation of the one proposed in [8]. Our approach basically adds a test that stops the algorithm when the inconsistency degree cannot be improved.

The computation of inconsistency degree is performed by a call to the function ***Incons*** which has three parameters: a stratified base Σ, an integer Inc representing the current inconsistency degree and a boolean variable *Bool*.

Algorithm 3: Logical-based computation of optimal optimistic decisions

Data: $\Pi K_{min} = (G_K, \pi)$, a knowledge possibilistic network,
$\Pi P_{min} = (G_P, \mu)$, a preferences possibilistic network,
$\mathcal{D} = \{D_1, \ldots, D_p\}$, set of decisions.
Result: *Decisions*, u^*.
begin
  $\Sigma_{K_{min}} = \{(\neg x_i \vee \neg u_i, 1 - \alpha_i) : (x_i, u_i, \alpha_i) \in \Pi K_{min}\}$, a possibilistic knowledge base
  $\Sigma_{P_{min}} = \{(\neg y_j \vee \neg u_j, 1 - \beta_j) : (y_j, u_j, \beta_j) \in \Pi P_{min}\}$, a preferences possibilistic network
  $min \leftarrow 1$,
  $Inc \leftarrow 1$, inconsistency degree,
  $u^* \leftarrow 0$, the optimistic utility,
  $Decisions \leftarrow \emptyset$, optimal optimistic decisions,
  for $i = 1..p$ **do**
    $\Sigma_{d_\oplus} = \Sigma_{K_{min}} \cup \Sigma_{P_{min}} \cup \{(d_i, 1)\}$,
    Call Algorithm 4 $Incons(\Sigma_{d_\oplus}, Inc, Bool)$, to compute inconsistency degree of $\Sigma_{d_\oplus}$
    if $bool = true$ **then**
      if $Inc < min$ **then**
        $min \leftarrow Inc$, $Decisions \leftarrow \{d_i\}$, $u^* \leftarrow 1 - Inc$,
      else
        if $Inc = min$ **then**
          $Decisions \leftarrow Decisions \cup \{d_i\}$

The function $\boldsymbol{Incons(\Sigma \cup \{(\neg\phi, 1)\}, Inc, Bool)}$ (defined in Algorithm 4) is an adaptation of a dichotomous algorithm proposed in [29] to compute inconsistency degree of a possibilistic base. Indeed, when the inconsistency degree of $\Sigma_K \cup \Sigma_P \cup \{(d_{i\in[1,p]}, 1)\}$ is greater than the inconsistency degree of $\Sigma_{K_{min}} \cup \Sigma_{P_{min}} \cup \{(d_{j\in[1,i-1]}, 1)\}$, the algorithm stops. This is ensured by the use of the boolean variable *Bool*.

Example 5. Let us continue Example 4, where agent's knowledge and preferences are now transformed into the two possibilistic logic-based bases $\Sigma_{K_{min}}$ and $\Sigma_{P_{min}}$. The set of decisions is: $D = \{d, \neg d\}$. Let us apply the logical approach detailed in Algorithm 3. For each decision $d_i \in D$, an inconsistency degree is computed of the possibilistic logic base $\Sigma_\oplus = \Sigma_{K_{min}} \cup \Sigma_{P_{min}}$ when the decision $\{(d_i, 1)\}$ is added. The Algorithm applies as follows:

Step 1: $D = d$, the function $Incons(\Sigma_\oplus \cup \{(d, 1)\}, Inc, Bool)$ returns $Inc = .9$ and $Bool = true$. In this case, $Inc <$ min (initially min $= 1$), so $Decisions \leftarrow \{d\}$ and min $\leftarrow .9$. The optimistic utility decision is then, $u^*(d) = 1 - Inc = .1$.

Algorithm 4: $Incons(\Sigma \cup \{(\neg\phi, 1)\}, Inc, Bool)$

```
Data: Σ, a stratified base
      {(¬φ, 1)}, a weighted formula,
      Inc, inconsistency degree,
Result: Bool, a boolean
begin
    Σ, stratified base,
    φ, weighted formula,
    n, number of strate in Σ,
    l ← 0, initially pointed on the last strate of Σ,
    u ← n, initially pointed on the first strate of Σ,
    Bool ← true, while (l < u) and (Bool = true) do
        r ← [(l + u)/2], pointer uses for dichotomy,
        if (Σ*_{≥α_r} = {φ_i/α_i ≥ α_r}) ∧ ¬φ consistent then
            u ← r − 1, check inconsistency in the most big base,
        else
            if Inc ≥ α_r then
                l ← r, check inconsistency in the base delimited by u and l,
            else
            Bool ← false,
    if u = 0 then
        Inc = 0
    else
        if Bool = true then
            Inc = α_r,
```

Step 2: $D = \neg d$, the function $Incons(\Sigma_{\oplus} \cup \{(\neg d, 1)\}, Inc, Bool)$ returns $Inc = 0$ and $Bool = true$. In this case, $Inc <$ min, so $Decisions \leftarrow \{\neg d\}$ and $min \leftarrow 0$. The optimistic utility decision is then, $u^*(\neg d) = 1 - Inc = 1$. Finally, as in Example 3, we conclude that the optimal optimistic decision $D^* = \neg d$ with the maximal qualitative utility equal to 1.

In the logical-based approach, the complexity of the transformation from a min-based possibilistic network to a qualitative possibilistic base is linear. The complexity of the function ***Incons*** (for computing inconsistency degrees) requires $\log_2 n$ satisfiability checks [29], where n is the number of strate or degrees present in the possibilistic logic base. This is the same complexity as the one of standard possibilistic logic.

6 RELATED WORKS

Several graphical approaches for dealing with decision problems under uncertainty have been proposed in the literature. In probability theory, among these models,

we can mention decision trees [31, 14], influence diagrams [27], valuation based systems [32], etc. Despite its popularity, a decision tree is not appropriate in the huge decision problems. An alternative to a decision tree is the notion of influence diagrams which have been proposed as a compact representation of a decision making problem under uncertainty. In a qualitative setting, possibility theory allows an agent to express uncertainty by ranking different states of the world. Both agent's knowledge and preferences can be expressed in a possibilistic qualitative way. However, few works exist on decision making using the possibility theory. For multi-stage (sequential or non sequential) decision making, the possibilistic counterparts of the standard graphical models have been proposed: possibilistic decision trees proposed in [24], possibilistic influence diagrams [24, 26, 22], etc. Uncertainty and preference relations are expressed on the same structure by using ordinal data. Like the probabilistic influence diagrams, the possibilistic influence diagrams contain three types of nodes: chance, decision and utility nodes. Uncertainty is described by means of possibility distributions on chance nodes and preferences are expressed as satisfaction degrees on utility nodes. To compute optimal decisions, either the possibilistic influence diagram is transformed into a secondary structure (into a possibilistic decision tree [24] or into a qualitative possibilistic network [26]) or the initial structure is directly used but this method requires additional computations to update possibility distribution tables [24]. In one-stage decision making, a logical-based representation has been proposed in [18]. In this model, agent's knowledge and preferences are encoded separately by two possibilistic bases. A method for computing optimal decisions based on ATMS (assumption-based truth maintenance system) [17] has been proposed in [18]. The proposed solution cannot deal with the important number of variables. An alternative approach exploiting the syntactic counterpart of possibilistic data fusion techniques has been proposed in [7, 8].

Our new graphical model for possibilistic decision making offers several advantages over existing ones.

The first advantage is the separation of knowledge and preferences which does not appear in the existing graphical models including possibilistic influence diagrams. This is an advantage because in practice a decision problem contains two distinct components: the uncertain distribution and the utility function. Besides, the separation of knowledge and preferences is in full accordance with the semantics of decision making in the possibility theory. Indeed, the semantics definitions of a possibilistic decision accepts as inputs two possibility distributions: one representing knowledge and the other representing preferences [18]. Hence, we provided a compact representations of these two possibility distributions and a graphical model for computing optimistic optimal possibilistic decisions. According to the semantic definition, the proposed model is based on the use of two distinct min-based possibilistic networks: one representing agent's knowledge and the second one encodes his preferences. This separation makes the model more intuitive since it naturally reflects the semantics of a possibilistic decision problem.

The second important feature of our approach is that we enriched the expressive power of the graphical-based representation language, by adding decision variables, without increasing its computational complexity. As we already pointed out in the paper, our decision process (that included both state and decision variables) has basically a same complexity as the one of the reasoning process (that only accepts state variables). This is true for both graphical-based and logical-based approaches. For instance, in the logical-based model, the extra computational cost, due to the addition of decision variables, corresponds to the cost of the equivalent translation of min-based networks into possibilistic logic bases. This translation is done in linear time with respect to the number of parameters (possibility degrees) present in min-based networks.

A third advantage of our approach is that it benefits from the simplicity of using graphs to elicit preferences and knowledge. It also benefits from intensive works on inference algorithms developed in graphical models such as junction tree algorithms or network-based computation algorithms. Indeed, any new heuristic for building compact junction trees or any new compiler for possibilistic networks can be easily reused in our model for integrating decision variables.

Lastly, the fourth important advantage of our approach is the use of a unique kind of nodes to represent both state variables and decision variables. This is not the case with other decision models, such as PIDs [24], where different kinds of nodes are used: chance, decision and utility nodes. Hence, our approach offers a simple format for representing both knowledge and preferences.

7 CONCLUSIONS

This paper first proposed a graphical model for representing possibilistic decision making under uncertainty in a compact way using qualitative possibilistic networks. We proposed an encoding of agent's beliefs and preferences by means of two distinct min-based possibilistic networks. The first min-based possibilistic network encodes a joint possibility distribution representing available knowledge and the second one encodes the qualitative utility. Then we proposed a new approach for computing optimal optimistic decisions that takes advantages of existing inference algorithms for qualitative possibilistic networks. Our approach first merges possibilistic networks associated with uncertain knowledge and possibilistic networks associated with agent's preferences. Then we showed that computing optimistic decisions comes down to computing a normalization degree of the junction tree associated to the result graph of merging agent's beliefs and preference-based networks. This allows an efficient computation of optimal decisions. In the second part of this paper, we proposed an alternative algorithm for computing optimistic decisions when the problem is expressed by means of two distinct min-based possibilistic networks. We showed that the logical counterpart is equivalent to the junction tree associated to the result graph of merging agent's beliefs and preference-based networks. The logical-based network is particularly suitable when it is difficult to built a junction tree.

As future works, we plan to extend our graphical model for the representation of decision problems to deal with more complex problems involving sequential decisions. We also plan to show how to encode a possibilistic influence diagram [26] into our new model based on possibilistic networks. The idea consists in decomposing the graph into two possibilistic networks, one encoding a possibility distribution representing agent's beliefs, and the second one encodes the qualitative utility. Another future work is to use the proposed graphical model to deal with the pessimistic decisions for possibilistic decision problems.

A APPENDIX

Proof. [Proof of Proposition 1] We need to prove that using the initialization procedure given in Algorithm 1, the qualitative utility associated with a decision d given in Equation (12) can be rewritten as follows: $u^*(d) = h(\pi_{\mathcal{JT}}) = \max_{\omega\in\Omega} \pi_{\mathcal{JT}}(\omega)$. We proceed in two steps

Step 1 (standard junction tree): By definition, using Equation (15) the joint possibility distribution associated with $\mathcal{JT}$ is expressed by: $\pi_{\mathcal{JT}}(X_1, \ldots, X_n) = \min_{j=1..m} \pi_{C_j}$. From steps 1 and 2 of the initialization procedure, we get: $\pi_{\mathcal{JT}}(X_1, \ldots, X_n) = \min(\min_{i=1..n} \pi_{\oplus}(X_i \mid U_i), 1) = \min_{i=1..n} \pi_{\oplus}(X_i \mid U_i) = \pi_{G_{\oplus}}(X_1, \ldots, X_n)$.

Step 2 (parameterized junction tree): By applying Equations (13) and (14), the joint possibility associated with $\mathcal{JT}$ comes down to compute: $\pi_{\mathcal{JT}}(X_1, \ldots, X_n) = \min(\pi_{G_{\oplus}}(X_1, \ldots, X_n), \Lambda_D) = \min(\pi_{G_{\oplus}}(X_1, \ldots, X_n), \pi_d(X_1, \ldots, X_n))$ where:

$$\pi_d(X_1, \ldots, X_n) = \begin{cases} 1 & \text{If } X_1, \ldots, X_n \models d \\ 0 & \text{otherwise.} \end{cases}$$

This result can be replaced in Equation (12), then we obtain: $u^*(d) = \max_{\omega\in\Omega} \pi_{\mathcal{JT}}(\omega)$. □

Proof. [Proof of Proposition 2] Our aim is to prove that the optimistic utility corresponds to the normalization degree of any cluster once the consistency in the junction tree $\mathcal{JT}$ is reached. Proposition 1 indicates that computing optimistic utility comes down to computing a normalization degree of the junction tree $\mathcal{JT}$. Then, it is enough to prove that optimistic utility corresponds to a normalization degree of some cluster amounts to prove that a normalization degree of the junction tree $\mathcal{JT}$ corresponds to a normalization degree of some cluster once the propagation algorithm is achieved (consistency reached). Formally, this amounts to prove the following equality: $\max_{\omega\in\Omega} \pi_{\mathcal{JT}} = \max_{C_i} \pi_{C_i}$. This equality is satisfied in the case of a junction tree with a unique cluster. Indeed, in this case we have: $\pi_{\mathcal{JT}} = \pi_{C_i}$. Assume that this property is true in the case of a junction tree with m clusters, and let us show that it is also true with $m+1$ clusters. Let $\mathcal{JT}$ be a junction tree with $m+1$ clusters defined on a set of variables V. Let $\mathcal{JT}' = \mathcal{JT} \backslash C_{m+1}$ defined on m clusters and V' be the universe relative to $\mathcal{JT}'$. The cluster C_{m+1} must be

a leaf of $\mathcal{JT}$, connected to the cluster C_m via the separator $S_{(m+1)m}$, otherwise C_{m+1} is connected to another cluster in $\mathcal{JT}'$ namely $\{C_{m+1} \backslash S_{(m+1)m}\} \cap V' \neq \emptyset$ which contradicts the hypothesis $\mathcal{JT}' = \mathcal{JT} \backslash C_{m+1}$. Let $L = C_{m+1} \backslash S_{(m+1)m}$, then $V' = V \backslash L$. From the induction hypothesis, we obtain: $\max_{V \backslash L} \pi_{\mathcal{JT}'} = \max_{C_i} \pi_{C_i}$.

By definition, using Equation (15) we obtain: $\pi_{\mathcal{JT}} = \min(\pi_{\mathcal{JT}'}, \pi_{C_{m+1}})$. Then, $\max_{\omega \in \Omega} \pi_{\mathcal{JT}} = \max_{\omega \in \Omega} \min(\pi_{\mathcal{JT}'}, \pi_{C_{m+1}}) = \min(\max_{V \backslash L} \pi_{\mathcal{JT}'}, \max_{C_{m+1}} \pi_{C_{m+1}})$. Using the induction hypothesis, we get: $\max_{\omega \in \Omega} \pi_{\mathcal{JT}} = \min(\max_{C_i} \pi_{C_i}, \max_{C_{m+1}} \pi_{C_{m+1}})$.

In the case, where C_i corresponds to the cluster C_m (the cluster adjacent to C_{m+1}) and since the junction tree $\mathcal{JT}$ is consistent, then: $\max_{C_m} \pi_{C_m} = \max_{C_{m+1}} \pi_{C_{m+1}}$. So, the normalization degree associated with the junction tree $\mathcal{JT}$ corresponds to the normalization degree of any cluster $C_i \in \{C_1, \ldots, C_{m+1}\}$: $\max_{\omega \in \Omega} \pi_{\mathcal{JT}} = \max_{C_i} \pi_{C_i}$. Hence, the optimistic utility corresponds to the normalization degree of any cluster C_i: $u^*(d) = \max_{C_i} \pi_{C_i}$. $\square$

REFERENCES

[1] Achs, A.: Computed Answer from Uncertain Knowledge: A Model for Handling Uncertain Information. Computing and Informatics, Vol. 26, 2007, No. 1, pp. 63–76.

[2] Anzilli, L.: A Possibilistic Approach to Investment Decision Making. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, Vol. 21, 2013, No. 2, pp. 201–221.

[3] Ayachi, R.—Ben Amor, N.—Benferhat, S.: A Comparative Study of Compilation-Based Inference Methods for Min-Based Possibilistic Networks. Symbolic and Quantitative Approaches to Reasoning with Uncertainty – 12th European Conference, Springer, LNCS, Vol. 7958, 2013, pp. 25–36.

[4] Ben Amor, N.—Benferhat, S.—Mellouli, K.: Anytime Propagation Algorithm for Min-Based Possibilistic Graphs. Soft Computing, A Fusion of Foundations, Methodologies and Applications, Springer-Verlag, Vol. 8, 2003, No. 2, pp. 150–161.

[5] Benferhat, S.: Graphical and Logical-Based Representations of Uncertain Information in a Possibility Theory Framework. Scalable Uncertainty Management – 4th International Conference, Toulouse, France, Springer, LNAI, Vol. 6379, 2010, pp. 3–6.

[6] Benferhat, S.—Dubois, D.—Garcia, L.—Prade, H.: On the Transformation Between Possibilistic Logic Bases and Possibilistic Causal Networks. International Journal of Approximate Reasoning, Vol. 29, 2002, No. 2, pp. 135–173.

[7] Benferhat, S.—Khellaf, F.—Mokhtari, A.—Zeddigha, I.: On the Use of Syntactic Possibilistic Fusion for Computing Pessimistic Qualitative Decisions. Third International Conference on Software and Data Technologies (ICSOFT 2008), Porto, Portugal, 2008, pp. 356–359.

[8] Benferhat, S.—Khellaf, F.—Mokhtari, A.—Zeddigha, I.: Using Syntactic Possibilistic Fusion for Modeling Optimal Optimistic Qualitative Decision. The Joint International Fuzzy Systems Association World Congress and European Society of Fuzzy Logic and Technology Conference, Lisbon, Portugal, 2009, pp. 1712–1716.

[9] Benferhat, S.—Khellaf, F.—Zeddigha, I.: A Possibilistic Graphical Model for Handling Decision Problems Under Uncertainty. The 8th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), Milano, Italy, 2013, pp. 736–743.

[10] Bélanger, M.—Martel, J. M.: Explanations for a Decision Support System Based on MCDA. Computing and Informatics, Vol. 25, 2006, No. 2-3, pp. 195–221.

[11] Benferhat, S.—Sossai, C.: Merging Uncertain Knowledge Bases in a Possibilistic Logic Framework. The Fourteenth Conference on Uncertainty in Artificial Intelligence (UAI'98), Madison, USA, 1998, pp. 8–15.

[12] Benferhat, S.—Titouna, F.: Min-Based Fusion of Possibilistic Networks. Fourth Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), 2005, pp. 553–558.

[13] Brézillon, P.: Context in Artificial Intelligence: I. A Survey of the Literature. Computers and Artificial Intelligence, Vol. 18, 1999, No. 4, pp. 321–340.

[14] Cernak, M.: A Comparison of Decision Tree Classifiers for Automatic Diagnosis of Speech Recognition Errors. Computing and Informatics, Vol. 29, 2010, No. 3, pp. 489–501.

[15] Coletti, G.—Petturiti, D.—Vantaggi, B.: Independence in Possibility Theory under Different Triangular Norms. Symbolic and Quantitative Approaches to Reasoning with Uncertainty – 12th European Conference, Utrecht, The Netherlands, 2013, Springer, LNAI, Vol. 7958, 2013, pp. 133–144.

[16] Darwiche, A.: Modeling and Reasoning with Bayesian Networks. Cambridge University Press, 1st edition, New York, USA, 2009.

[17] De Kleer, J.: Problem Solving with the ATMS. Artificial Intelligence, Vol. 28, 1986, No. 2, pp. 197–224.

[18] Dubois, D.—Le Berre, D.—Prade, H.—Sabbadin, R.: Using Possibilistic Logic for Modeling Qualitative Decision: ATMS-Based Algorithms. Fundamenta Informaticae, Vol. 37, 1999, No. 1–2, pp. 1–30.

[19] Dubois, D.—Fargier, H.—Perny, P.: Qualitative Decision Theory with Preference Relations and Comparative Uncertainty: An Axiomatic Approach. Artificial Intelligence, Vol. 148, 2003, No. 1-2, pp. 219–260.

[20] Dubois, D.—Lang, J.—Prade, H.: Possibilistic Logic. In Handbook of Logic in Artificial Intelligence and Logic Programming, Oxford University Press, 1994, pp. 439–513.

[21] Dubois, D.—Prade, H.: Possibility Theory and Formal Concept Analysis: Characterizing Independent Sub-Contexts. Fuzzy Sets and Systems, Vol. 196, 2012, pp. 4–16.

[22] Fargier, H.—Ben Amor, N.—Guezguez, W.: On the Complexity of Decision Making in Possibilistic Decision Trees. The Twenty-Seventh Conference on Uncertainty in Artificial Intelligence, Barcelona, Spain, 2011, pp. 203–210.

[23] Fonck, P.: Propagating Uncertainty in Directed Acyclic Graphs. Information Processing and Management of Uncertainty – 4th International Conference, Mallorca, Spain, 1992, pp. 17–20.

[24] Garcia, L.—Sabbadin, R.: Complexity Results and Algorithms for Possibilistic Influence Diagrams. Artificial Intelligence, Vol. 172, 2008, No. 8-9, pp. 1018–1044.

[25] Godo, L.—Zapico, A.: On the Possibilistic-Based Decision Model: Characterization of Preference Relations under Partial Inconsistency. Applied Intelligence, Vol. 14, 2001, No. 3, pp. 319–333.

[26] Guezguez, W.—Ben Amor, N.—Mellouli, K.: Qualitative Possibilistic Influence Diagrams Based on Qualitative Possibilistic Utilities. European Journal of Operational Research, Vol. 195, 2009, No. 1, pp. 223–238.

[27] Howard, R. A.—Matheson, J. E.: Influence Diagrams. In Readings on the Principles and Applications of Decision Analysis, Strategic Decisions Group, 1984, pp. 721–762.

[28] Kramosil, I.: Partial Convergence and Continuity of Lattice-Valued Possibilistic Measures. Computing and Informatics, Vol. 27, 2008, No. 3, pp. 297–313.

[29] Lang, J.: Possibilistic Logic: Complexity and Algorithms. Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 5, 2000, pp. 179–220.

[30] Qi, G.—Liu, W.—Bell, D. A.: A Comparison of Merging Operators in Possibilistic Logic. Knowledge Science, Engineering and Management – 4th International Conference, 2010, LNCS, Vol. 6291, 2010, pp. 39–50.

[31] Raiffa, H.: Decision Analysis – Introductory Lectures on Choices under Uncertainty. Addison Wesley, Reading, MA, 1968.

[32] Shenoy, P. P.: Consistency in Valuation-Based Systems. INFORMS Journal on Computing, Vol. 6, 1994, No. 3, pp. 281–291.

[33] Smirnov, A.—Pashkin, M.—Chilov, N.—Levashova, T.: KSNet-Approach to Knowledge Fusion from Distributed Sources. Computing and Informatics, Vol. 22, 2003, No. 2, pp. 105–142.

[34] Sniedovich, M.: Dynamic Programming: Foundations and Principles, 2nd Edition. Pure and Applied Mathematics, CRC Press, Hoboken, 2010.

[35] Thanh, D. V.: A Relationship Between the Probabilistic and the Possibilistic Logic. Computers and Artificial Intelligence, Vol. 17, 1998, No. 1, pp. 51–68.

[36] Zhang, N. L.: Probabilistic Inference in Influence Diagrams. Computational Intelligence, 1998, pp. 514–522.

Ismahane Zeddigha is a Ph.D. student in computer science at University of Sciences and Technology Houari Boumediene (Laboratoir de Recherche en Informatique Intelligente et Mathematiques Appliquées) and at University of Artois (Centre de Recherche en Informatique de Lens, CNRS UMR-8188). She received her Master degree in artificial intelligence and advanced data bases from University of Science and Technology Houari Boumediene, Algiers, Algeria, in 2006, and her Master 2 degree in computer science from University of Paris 11, France, in 2008. Her current research interests include knowledge representation and decision-making under uncertainty.

Salem Benferhat is Full Professor at University of Artois, France since February 2003. He is doing research at CRIL (Centre de Recherche en Informatique de Lens), CNRS UMR 8188. From October 1994 to January 2003, he was a CNRS researcher at IRIT (Insitut de Recherche en Informatique de Toulouse), Toulouse, France. He received his Ph.D. degree in 1994 and his "Habilitation à Diriger des Recherches" in 2000 from University Paul Sabatier, Toulouse, France. His current research interests include knowledge representation, uncertainty handling, data fusion, possibilistic networks, causality, non-classical logics, description logics, decision under uncertainty, plausible and nonmonotonic reasoning with applications to computer security. He is author or co-author of a large number of research papers (more than 50 international journal papers and more than 150 international conference papers) on uncertainty modelling, knowledge representations and applications. In particular, he published in all major conferences and journals on artificial intelligence. He supervised or co-supervised 20 Ph.D. thesis and currently is supervising four Ph.D. thesis. He was involved in several European and national projects. He was or is in program committee of all major international conferences on artificial intelligence such as IJCAI, AAAI, ECAI, KR, JELIA, ECSQARU, UAI, etc. He also co-organized several national and international conferences such as SUM2011 (Fifth International Conference on Scalable Uncertainty Management), ECSQARU '01 (Sixth European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty), NMR'02 (Ninth International Workshop on Non-Monotonic Reasoning NMR 2002) or LFA '08 (Rencontres Francophones sur la Logique Floue et ses Applications). He is also a co-editor in chief of The Information-Interaction-Intelligence (I3) Journal. He is a member of editorial board of Fundamenta Informaticae journal, Annals of Mathematics and Artificial Intelligence AMAI, and he was a guest editor of 10 journal special issues in different artificial intelligence journals or uncertainty management journals (Artificial Intelligence AIJ, Annals of Mathematics and Artificial Intelligence AMAI, International Journal of Approximate Reasoning, etc).

Faiza Khellaf is Lecturer at University of Science and Technology Houari Boumediene, Algiers, Algeria, since 2002. She is affiliated as a researcher to the laboratory RIIMA (Recherche en Informatique Intelligente, Mathématiques et leurs Aplications). She received her Ph.D. degree in the context of a joint supervision between the University of Science and Technology Houari Boumediene and the University of Artois in 2007. She received her "Habilitation à Diriger des Recherches" degree from the University of Science and Technology Houari Boumediene, Algiers, Algeria, in 2010. Her current research interests include knowledge representation, uncertainty handling, possibilistic decision under uncertainty, multi-agents possibilistic logic. She took part in several national projects. She is the author or the co-author of 5 international journal papers and 20 international conference papers on representation and reasoning with uncertain knowledge. She is co-supervising three Ph.D.

thesis. She was a program committee member of several national and international conferences on artificial intelligence such as International Symposium on Programming and Systems ISPS 2013, Rencontres Francophones sur la Logique Floue et ses Applications LFA 2012, International Conference on Machine and Web Intelligence, ICMWI 2010, The Florida Artificial Intelligence Research Society, FLAIRS 2005.