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VISUALIZATION OF AIRCRAFT LONGITUDINAL-AXIS MOTION

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Abstract. In this paper, the use of continuous mathematical models of an aircraft in an aircraft simulator is described. The models are of lower degree and less time-consuming for calculation. Computer implementation of the models capable to work faster and more accurately and efficiently is also described. The suggested approach allows to achieve the required precision at accelerated simulation speed using the continuous mathematical models of an aircraft. Frequency of the computation of continuous mathematical models of an aircraft is higher, reaching up to 200 times per second. The main focus of the paper is designing continuous mathematical models of an aircraft, their simulation and visualization in aircraft simulators. Current mathematical models of a control of objects motion are based on aircraft aerodynamics. In our approach, these models have impact on the quality and completeness of simulation process and are crucial for computer modeling and visualization of equations of the continuous mathematical models of an aircraft. In the paper, creating continuous mathematical models of an aircraft and the way of visualization of simulation results are described in detail. The main aim of computer simulation of continuous mathematical models of an aircraft is pilots training. Aircraft simulator plays a key role in the process of pilots training; it enables pilots to control the aircraft and its equipment. Standard computer with a graphic processing unit for the visualization results from continuous mathematical models of an aircraft can be utilized.

Keywords: Analytical design, mathematical model, modelling, simulation, 3-dimensional virtual reality

Mathematics Subject Classification 2010: 00A67, 34A30, 65L80, 15A03, 15A24, 34K06, 65L03

1 INTRODUCTION

A large part of science deals with designing mathematical models of aircraft, their simulation and visualization. Each of these parts (mathematical models of aircraft, simulation and visualization) has developed separately. Our attempt is to unite the three parts in flight simulator. Analytical design contains a number of mathematical approaches based on mathematical models description. Many types of outdated aircraft employ mainly control systems for control in a longitudinal direction. Due to this fact, designing aircrafts with desired stability and controllability properties gains ever more attention.

Extending the interval limiting the flight values, picking up the speed, maximum height and other parameters have created conditions under which current and next piloted objects need to have very precise, accurate semi-automated and automated control systems. All flight stages are automated; sequence of operations is designed and comprises sequence of programmes [17]. Control of algorithm parameters of these systems needs designing their mathematical models, by means of which users can learn about their properties for using them.

The paper consists of the following parts: The Introduction offers essential characteristics of aircraft, aircraft control, flight limits, modelling of these characteristics, etc. Section 2 discusses a complex design of mathematical models written in differential equations; these models are employed in an analytical process of the aircraft design. The system of equations representing the flight conditions and coefficient calculations is discussed in Section 3. The results obtained in the process of visualization prove the suitability of the methods for practice; they are given in Section 4.

Our attention is focused on mathematical models design, input analysis, and creating mathematical model of control, computer implementation and visualization. The paper is aimed at connecting the process of simulation and visualization of mathematical models of aircraft motion running in real time.

1.1 Mathematical Models of Aircraft

Simulators provide users with real flight conditions and training and learning experience. Most software developers have relied on the advances in hardware to increase the speed of applications of mathematical models under the hood. Properties of piloted aircrafts, as objects to be controlled automatically, are complex. This complexity is due to unstable characteristics, aero elasticity impact, fuel density and some other parameters. The latest facts on theory of control implemented by means of computer technology, employing mathematical and combined (mathematical and physical) modelling are needed to define them. In the process of pilots training simulators are employed. Model designing substantially consists of structural analysis of aircrafts, board control system and also frequency characteristics. Simulators belong to the group of combined simulation abbreviated as Hardware in the Loop (HL). This type of simulation is important as in the loop with a mathematical model there is a part of real equipment connected to a simulation surrounding.

In simulators the real equipment includes an aircraft pilot's cockpit, loading mechanism, real control lever etc. While intervening pilots control the object − an aircraft; mathematical models interact with real equipment that responds to the pilot's interventions and in line with it they change mathematical models inputs; data on the control panel are observed by the pilot. In analytical designing of a mathematical model the criterion for control is functional of coordinates x_i, $i = 1, \dots, n$ of the object and necessary control values u_j, $j = 1, \dots, n$. Applying the analytical design methods is based on differential equations of the type [9]:

$$\dot{x}_i + f_i(x_1, \dots, x_n, t) = \sum_{j=1}^{n} \varphi_{ij}(x_1, \dots, x_n, t) u_j, (i = 1, 2, \dots, n), \tag{1}$$

defining the functional,

$$I = V_E[x_1(t_2), \dots, x_n(t_2)] + \int_{t_1}^{t_2} Q(x_1, \dots, x_n, t) + 1/q \int_{t_1}^{t_2} \sum_{j=1}^{n} \left(\frac{u_j}{k_j}\right)^q \mathrm{d}t, \tag{2}$$

where f_i is the function describing the object, φ_{ij} is the object characteristics. The searched minimum of the functional (2), V_E is the given function of the phase coordinates, Q is the function of changing phase coordinates of the object, k_j represents the function of given influencing values in the final time $t = t_2$, $q > 1$ is a defined number where z_q is an even function z. Control algorithm parameters are defined by items $u_j(x_1, \dots, x_n, t)$ and minimize the functional (2) while controlling is optimal. Finding optimal control parameters results in solving non-linear equations with variables of time. In this case, the expression $(1/p + 1/q) = 1$ holds for parameters.

Today there is no solution to non-linear objects of the quadratic equation of optimal control parameters. For linear objects with quadratic functional (V_E, Q quadratic forms, $p = q = 2$) the solution aims at integration of ordinary differential equations of Riccati type. The functional of the object given by (1) can be written as follows:

$$\begin{aligned} I &= V_E[x_1(t_2), \dots, x_n(t_2)] + \int_{t_1}^{t_2} Q(x_1, \dots, x_n, t) + \frac{1}{q} \int_{t_1}^{t_2} \sum_{j=1}^{n} \left(\frac{u_j}{k_j}\right)^q \mathrm{d}t \\ &\quad + \frac{1}{p} \int_{t_1}^{t_2} \sum_{j=1}^{m} \left(k_j \sum_{k=1}^{n} \varphi_{jk} \frac{\partial V}{\partial x_k}\right)^p \mathrm{d}t, \end{aligned} \tag{3}$$

where V represents the solution of the linear control in variables of time

$$\frac{\partial V}{\partial t} - \sum_{i=1}^{n} f_i \frac{\partial V}{\partial x_k} = -Q, \tag{4}$$

where the limiting condition is represented by $V_{t=t_2} = V_E$. The optimal control minimizes the functional (3)–(4) and referring to [9]:

$$u_j = -k_j^p \left(\sum_{k=1}^{n} \varphi_{jk} \frac{\partial V}{\partial x_k} \right)^{p-1} . \tag{5}$$

Linearization of Equation (4) for the function V makes the task of finding the equation root simpler and enables to show possible solving options that are identical with those of solving linear ones and also of some general groups of large non-linear objects. In this functional there are only V_E and Q functions. The last item below the functional integral (2) as a function of phase coordinates is defined by means of solving Equation (4). Finding the coefficients of the given minimized functional is considered to be an attempt to determine the link between the coefficient functionals and corresponding characteristic values, e.g. roots of an equation characteristic for a closed circuit.

Mathematical model is formed to define a given group of properties of real object of control. In this case the same flying device, e.g. the object of control, can be defined by means of different mathematical models depending on the purpose of searching and the flight phase [2]. Furthermore, the mathematical models may depend on conditions like intervals of coordinates changes, i.e. what interval parameters are needed for frequencies of changes and actuating signals under the analyzed conditions. This is the reason why simple or simplified mathematical models defining the control object are used.

1.2 Physical Basis of Mathematical Models

One of the physical basis of mathematical models is mass and its inertia defined by Newton. According to Newton's Law III, to every action there is always an equal opposite reaction, or the mutual action of two bodies upon each other is always directed to opposite parts. Another important term used in physical basis of mathematical models is *moment of momentum* in a system of mass points that describes rotary inertia of the system in motion about an axis [3]. The moment of momentum of a collection of particles is the sum of all moments of momentum of all mass points within a system upon the origin point. These well-known laws are utilized in the construction of mathematical models. Many tasks of controlled and uncontrolled flights can be done on the basis of linear motion model.

The basic system of equations has the form [1]:

$$\dot{x}_1 + f_1(x_1, \ldots, x_n, u_1, \ldots, u_m, \xi_1, \ldots, \xi_\gamma) = 0, (i = 1, 2, \ldots, p) \tag{6}$$

where $x_1, \ldots, x_n$ are object coordinates, $u_1, \ldots, u_m$ are elements of control, $\xi_1, \ldots, \xi_\gamma$ are failure functions. In a vector form, Equation (6) has the form:

$$\dot{\mathbf{x}} + \mathbf{f}(\mathbf{x}, \mathbf{u}, \boldsymbol{\xi}) = \mathbf{0},$$

where $\mathbf{x}$, $\mathbf{u}$, $\boldsymbol{\xi}$ are vectors and $\mathbf{f}$ is a vector function. Initial conditions are

$$x^0 + x^0(t), u^0 + u^0(t), \tag{7}$$

where $\boldsymbol{\xi} = \mathbf{0}$. Forced motion can have the form of [7]:

$$x = x^0 + \Delta(x), u = u^0 + \Delta(u), \tag{8}$$

satisfying Equation (7) and $\boldsymbol{\xi} \neq \mathbf{0}$. In this case:

$$\Delta\dot{x} + f(x^0 + \Delta(x), u^0 + \Delta(u), \xi) - f(x^0, u^0, 0) = 0. \tag{9}$$

Then the final equation has the form:

$$\Delta\dot{x} + \left(\frac{\partial f}{\partial x}\right)_0 \Delta(x) + \left(\frac{\partial f}{\partial u}\right)_0 \Delta(u) + \left(\frac{\partial f}{\partial \xi}\right)_0 \Delta(\xi) = 0. \tag{10}$$

Simulation of linear mathematical models is simpler than that of nonlinear mathematical models, stochastic mathematical models or models defined by partial differential equations, etc. Simulation and synchronization of mathematical models on computers is difficult and complicated from the point of view of computer power. Simplification of mathematical models impacts software designers, enables to create simpler interface and generate sufficiently accurate results. Many tasks of controlled and uncontrolled flights can be done on the basis of linear motion model. Precision is the difference between results of transient response of the nonlinear mathematical models if compared to linear mathematical models and represents only several per cent. The steady-states in linear and non-linear models are represented only by a small percentage, few tenth of per cent. From the point of view of human precision this percentage can be neglected.

In a linear model of longitudinal motion, the following items seem to be the weakest couples for flight height: $a_x^H \Delta H$, $a_y^H \Delta H$, $a_{mz}^H \Delta H$. The equation of longitudinal motion model has the form [9]:

$$\begin{aligned}
\Delta\dot{V} + a_x^V \Delta V + a_x^\alpha \Delta\alpha + a_x^\theta \Delta\theta &= a_x^{\delta_M} \Delta\delta_M \\
\Delta\theta + a_y^V \Delta V + a_y^\alpha \Delta\alpha + a_y^\theta \Delta\theta &= a_y^{\delta_V} \Delta\delta_V \\
\Delta\dot{\omega}_z + a_{mz}^V \Delta V + a_{mz}^\alpha \Delta\alpha + a_{mz}^{\dot{\alpha}} \Delta\dot{\theta} + a_{mz}^{\omega z} \Delta\omega_z &= a_{mz}^{\delta_V} \Delta\delta_V
\end{aligned}$$

$$\begin{aligned}
\Delta\dot{\upsilon} &= \Delta\omega_z, \\
\Delta\upsilon &= \Delta\theta + \Delta\alpha
\end{aligned} \tag{11}$$

In a stationary case under initial values of reference Laplace's transformation [1, 17], the equation system (11) has the form:

$$\begin{aligned}
\left(s+a_x^V\right)\Delta V(s)+a_x^\alpha\Delta\alpha(s)+a_x^\theta\Delta\theta(s) &= a_x^{\delta_M}\Delta\delta_M(s)\\
a_y^V\Delta V(s)+a_y^\alpha\Delta\alpha(s)+(s+a_y^\theta)\Delta\theta(s) &= a_y^{\delta_V}\Delta\delta_V(s)\\
a_{mz}^V\Delta V(s)+(a_{mz}^\alpha+a_{mz}^{\dot\alpha}(s))\Delta\alpha+(s^2+a_{mz}^{\omega z}(s))\Delta\varpi_z &= a_{mz}^{\delta_V}\Delta\delta_V(s)\\
\Delta\alpha(s)+\Delta\theta(s)-\Delta\upsilon(s) &= 0.
\end{aligned} \tag{12}$$

The characteristic equation corresponding with Equations (11) and (12) is of the fourth order:

$$\Delta(s)=\begin{vmatrix}
\left(s+a_x^V\right) & a_x^\alpha & a_x^\theta & 0\\
a_y^V & a_y^\alpha & \left(s+a_y^\theta\right) & 0\\
a_{mz}^V & \left(a_{mz}^{\dot\alpha}(s)+a_{mz}^\alpha\right) & 0 & \left(a_{mz}^{\dot\alpha}(s)+a_{mz}^\alpha\right)\\
0 & 1 & 1 & -1
\end{vmatrix}. \tag{13}$$

We do not need to express the ratio of roots of the characteristic equation, because this ratio does not represent stability of the systems. The system stability is defined by the magnitude and phase by vector in the s-plane, drawn from zeros and poles, special real part of the roots (Root-locus technique). Stability of linear systems is defined by compliance with the algebraic or frequency stability criterions. The stability characteristics of a linear time-invariant system is determined from the system's characteristic equation. It will be better to modify this part in the following way.

When the equation equals zero, then we get two pairs of complex compound roots. Large roots (negative real parts) depend on coefficients a_3, a_2 defining the oscillation of motion in longitudinal direction. The second pair – small roots – are defined mainly by a_1, a_0 values. Coefficients a_3, a_2, a_1, and a_0 are denoted by the following formulas:

$$\begin{aligned}
a_3 &= a_{mz}^{\omega z}-a_{mz}^{\dot\alpha}+a_x^V+a_y^\theta-a_y^\alpha,\\
a_2 &= a_{mz}^\alpha-\left(a_y^\theta*a_{mz}^{\omega z}\right)+\left(a_x^V*a_{mz}^{\omega z}\right)+\left(a_y^V*a_{mz}^{\omega z}\right)\\
&\quad+\left(a_x^V*a_y^\theta\right)+\left(a_y^V*a_x^\alpha\right)+\left(a_y^V*a_x^\theta\right)-\left(a_x^V*a_y^\alpha\right),\\
a_1 &= a_{mz}^\alpha*\left[\left(a_x^V*a_y^\alpha\right)+\left(a_y^V*a_y^\theta\right)-\left(a_x^V*a_y^\theta\right)-\left(a_y^V*a_x^\alpha\right)\right]+\left(a_x^V*a_y^\theta*a_{mz}^{\alpha'}\right)\\
&\quad-\left(a_x^\alpha*a_{mz}^V\right)-\left(a_y^V*a_x^\theta*a_{mz}^{\alpha'}\right)+\left(a_x^V*a_{mz}^\alpha\right)+\left(a_y^\theta*a_{mz}^\alpha\right),\\
a_0 &= \left(a_y^\alpha*a_x^\theta*a_{mz}^V\right)+\left(a_x^V*a_y^\alpha*a_{mz}^\alpha\right)-\left(a_x^\alpha*a_y^\theta*a_{mz}^V\right)-\left(a_y^V*a_x^\theta*a_{mz}^\alpha\right).
\end{aligned} \tag{14}$$

Strong couple coefficients are represented by a_y^α, a_{mz}^α, $a_{mz}^{\omega z}$. These coefficients define periodic vibrations of a short period and define longitudinal motion. The other couple of roots defines periodic vibrations of long periods and defines longitudinal aircraft motion.

2 MATHEMATICAL MODEL OF CONTROL

Complexity of mathematical models of objects of control and of the tasks defined by them needs the systems design automation. In design, the general mathematical models defined by systems of differential equations are difficult to employ in practice. These models are employed in the analytical process of aircraft designing. Therefore linear models of objects are of importance, as the parameters of equations define the selected properties of objects. Linear models are used in the process of analysing general processes which are determined by practical requirements and algorithms of control algorithm parameters [5].

2.1 Mathematical Model in Longitudinal Motion

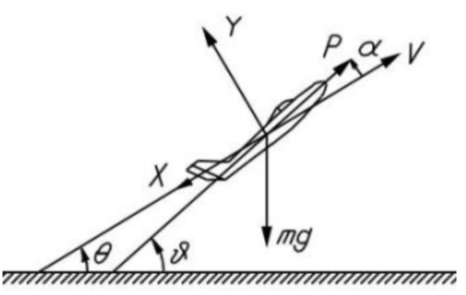

Figure 1. The relation between force components and angles of an aircraft

Equations of longitudinal aircraft motion comprise: an equation of forces projection into tangent and normal to the trajectory and an equation of moments directed towards cross axis in an aircraft motion; they have the form:

$$\begin{aligned} m\dot{V} &= P\cos\alpha - X - mg\sin\theta, \\ mV\dot{\theta} &= Y + P\sin\alpha - mg\cos\theta, \\ I_z\dot{\omega}_z &= M_{z1}, \end{aligned} \tag{15}$$

where $\theta = \upsilon - \alpha$ is the angle of trajectory, P is thrust of engines, X is aircraft drag, Y is lift (Figure 1). In standard conditions of atmosphere:

$$
\begin{aligned}
P &= P(V, H, \delta_M),\\
X &= X(V, \alpha, H),\\
Y &= Y(V, \alpha, H),\\
M_{z1} &= M_{z1}(\alpha, \omega_z, \dot{\alpha}, V, H, \delta_V).
\end{aligned} \tag{16}
$$

The dependence of engines thrust P on the position of the throttle lever δ_M in the equation of projections forces into normal towards the trajectory is neglected due to its little influence. To get the linear model a standard methodology is employed. We define equilibrium of general motion:

$$V = V_0, \quad H = H_0, \quad \delta_V = \delta_V^0, \quad \delta_M = \delta_M^0, \quad \alpha = \alpha_0, \quad \theta = \theta_0. \tag{17}$$

In line with Equation (12) we get [9]:

$$
\begin{aligned}
\Delta\dot{V} + a_x^V \Delta V + a_x^\alpha \Delta\alpha + a_x^\theta \Delta\theta + a_x^H \Delta H &= a_x^{\delta_M} \Delta\delta_M\\
\Delta\theta + a_y^V \Delta V + a_y^\alpha \Delta\alpha + a_y^\theta \Delta\theta + a_y^H \Delta H &= a_y^{\delta_V} \Delta\delta_V\\
\Delta\dot{\omega}_z + a_{mz}^V \Delta V + a_{mz}^\alpha \Delta\alpha + a_{mz}^{\dot{\alpha}} \Delta\dot{\theta} + a_{mz}^{\omega z} \Delta\omega_z + a_{mz}^H \Delta H &= a_{mz}^{\delta_V} \Delta\delta_V\\
\Delta\dot{H} - \sin(\theta^0)\Delta V - \cos(\theta^0) V \Delta V &= 0\\
\Delta\dot{\upsilon} &= \Delta\omega_z\\
\Delta\upsilon &= \Delta\theta + \Delta\alpha.
\end{aligned} \tag{18}
$$

Four transfer functions define the outcome values of the model: ΔV is the speed change, $\Delta\alpha$ is the angle of attack displacement, $\Delta\theta$ is the angle of trajectory displacement, $\Delta\upsilon$ is the angle of pitch displacement. In them, there are two inputs – control values: $\Delta\delta_M$ stands for throttle lever displacement of the engine, $\Delta\delta_V$ is the elevator displacement and they have the form [9]:

$$
\begin{aligned}
\Delta V(s) &= -G_{\delta_M/V}(s)\Delta\delta_M(s) - G_{\delta_V/V}(s)\Delta\delta_V(s)\\
\Delta\alpha(s) &= -G_{\delta_M/\alpha}(s)\Delta\delta_M(s) - G_{\delta_V/\alpha}(s)\Delta\delta_V(s)\\
\Delta\theta(s) &= -G_{\delta_M/\theta}(s)\Delta\delta_M(s) - G_{\delta_V/\theta}(s)\Delta\delta_V(s)\\
\Delta\upsilon(s) &= -G_{\delta_M/\upsilon}(s)\Delta\delta_M(s) - G_{\delta_V/\upsilon}(s)\Delta\delta_V(s)
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
G_{\delta_M/V}(s) &= a_x^{\delta_M}\tfrac{\Delta_{11}}{\Delta},\\
G_{\delta_M/\alpha}(s) &= a_x^{\delta_M}\tfrac{\Delta_{12}}{\Delta},\\
G_{\delta_M/\theta}(s) &= a_x^{\delta_M}\tfrac{\Delta_{13}}{\Delta},\\
G_{\delta_M/\upsilon}(s) &= a_x^{\delta_M}\tfrac{\Delta_{14}}{\Delta},\\
G_{\delta_M/V}(s) &= a_y^{\delta_V}\tfrac{\Delta_{21}}{\Delta} - a_{mz}^{\delta_V}\tfrac{\Delta_{31}}{\Delta},\\
G_{\delta_M/\alpha}(s) &= a_y^{\delta_V}\tfrac{\Delta_{22}}{\Delta} - a_{mz}^{\delta_V}\tfrac{\Delta_{32}}{\Delta},
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
G_{\delta_V/\theta}(s) &= a_y^{\delta_V}\tfrac{\Delta_{23}}{\Delta} - a_{mz}^{\delta_V}\tfrac{\Delta_{33}}{\Delta},\\
G_{\delta_M/\upsilon}(s) &= a_y^{\delta_V}\tfrac{\Delta_{24}}{\Delta} - a_{mz}^{\delta_V}\tfrac{\Delta_{34}}{\Delta},
\end{aligned}
\tag{21}
$$

where Δ_{ij} is the sub-determinant of the i^{th} column and the j^{th} row. In our examples the second equation of the system (19) is employed to describe the change of the attack angle $\Delta\alpha$ and also the equations describing displacement of the attack angle depending on the displacement of throttle engine lever $G_{\delta_M/\alpha}(s)$ or the displacement of the elevator $G_{\delta_V/\alpha}(s)$ are employed.

2.2 Calculation of Parameters of Mathematical Models of Aircraft in Defined Surrounding

The given real data define a hypothetical ultrasound aircraft for which aerodynamic coefficients for its properties are calculated. The considered flight of the aircraft is steady and without any random interferences (wind, storm, or other outer interferences). Based on mathematical models some other decisions on control [13] are done. Coefficients c_i and e_j represent aerodynamic parameters.

These parameters have numeric values in every flight phase; some parameters are important in the "take off" phase, other parameters are important in the "landing" phase, other parameters are important in the "climb" phase, etc. To illustrate this some parameters dominant in certain flight phases are stable and non-variable. This is the step for linearization of the defined mathematical models. Coefficient c_1 presents the dependence of the aircraft speed on the coordinate x, coefficient c_2 presents the dependence of the aircraft angle of attack on the coordinate x, coefficient c_3 presents the dependence of the aircraft trajectory angle on the coordinate x etc. [12].

To calculate the coefficients $c_1 = a_x^V$, $c_1 = a_x^\alpha$, $c_3 = a_x^\theta$, $c_4 = a_y^V, \ldots, e_1 = a_x^{\delta_M}$, $e_2 = a_y^{\delta_V}$, ... in the flight phases the following data are given:

- mass $m = 16\,000\,\mathrm{kg}$;
- thrust of engines $P = 35\,000\,\mathrm{N}$;
- velocity $M = 0.6$; $V_0 = a * M = 330 * 0.6 = 198\,\mathrm{m.s^{-1}}$;
- flight height $H = 900\,\mathrm{m}$;

- wings area $S = 34.5\,\mathrm{m}^2$;
- wingspan $l = 14.3\,\mathrm{m}$;
- standard mean chord b_A of the wing $= 2.1\,\mathrm{m}$;
- air density ρ (at flight height $H = 900\,\mathrm{m}$) $= 1.2\,\mathrm{kg.m}^{-3}$;
- moment of inertia around the z axis $J_{ZZ} \approx 1.6 \cdot 10^4\,\mathrm{N}$;
- in horizontal balanced flight deviation moment J_{XZ} and also deviation moment J_{YZ} and J_{XY} are not taken into consideration;
- longitudinal aerodynamic moment dependence angle speed ω_z around axis z $m_z^{\omega_z} = -0.021\,\mathrm{s}^{-1}$;
- longitudinal aerodynamic moment dependence angle of attack α around axis z $m_z^{\alpha} = -0.054\,\mathrm{s}^{-1}$;
- longitudinal aerodynamic moment dependence control stick of elevator around axis z $m_z^{\delta_V} = -0.0038\,\mathrm{s}^{-1}$;
- aerodynamic drag coefficient $c_x = 0.032$.

Coefficients define the aircraft position and are calculated in line with the following formula:

$$c_1 = -\frac{m_z^{\omega_z}}{J_{zz}}\frac{\rho V}{2} S b_A^2 \approx 0.105\,[\mathrm{s}^{-1}]. \tag{22}$$

The coefficient c_2 can be calculated as follows:

$$c_2 = -\frac{m_z^{\alpha}}{J_{zz}}\frac{\rho V^2}{2} S b_A \approx 0.202\,[\mathrm{s}^{-2}], \tag{23}$$

and also the coefficient c_3 is calculated:

$$c_3 = -\frac{m_z^{\delta_V}}{J_{zz}}\frac{\rho V^2}{2} S b_A \approx 0.301\,[\mathrm{s}^{-2}]. \tag{24}$$

The coefficient c_4:

$$c_4 = \frac{c_\alpha^y + c_x}{m}\frac{\rho V}{2} S \approx 0.005\,[\mathrm{s}^{-1}]. \tag{25}$$

The coefficient c_5 is calculated using the formula:

$$c_5 = -\frac{m_z^{\dot{\alpha}}}{J_{zz}}\frac{\rho V}{2} S b_A \approx 0.0112\,[\mathrm{s}^{-1}]. \tag{26}$$

According to similar dependencies other coefficients $c_7 - c_9$ can be calculated. The coefficient e_1 is calculated by means of the formula:

$$e_1 = -\frac{\rho S V}{m} c_x \left\{1 + \frac{c_y^M M}{2c_x} - \frac{P^V}{\rho V S c_x}\right\} \approx -0.097\,[\mathrm{s}^{-1}]. \tag{27}$$

The coefficient e_2 is calculated according to the formula:

$$e_2 = -\frac{57.3\rho S}{m} c_y \left\{ 1 + \frac{c_y^M M}{2c_y} \right\} \approx 0.014\,[\mathrm{s}^{-1}]. \tag{28}$$

The coefficient e_3 can be calculated in line with the following equation:

$$e_3 = -\frac{57.3}{J_{zz}} \left[\frac{m_z^M}{a} + 2\frac{(c_x + c_{YRP}\sin(\theta_0) y_R}{V b_A} \right] \frac{\rho V^2}{2} S b_A - P_{y_R}^V \approx -0.015. \tag{29}$$

The parameters calculated by means of the above Equations (22)–(29) are as follows: $c_1 \approx 0.105$; $c_2 \approx 0.202$; $c_3 \approx 0.301$; $c_4 \approx 0.005$; $c_5 \approx 0.0112$; $c_6 \approx 3.500$; $c_7 \approx 0.171$; $c_8 \approx -0.314$; $c_9 \approx 0.077$; $e_1 \approx -0.097$; $e_2 \approx 0.014$; $e_3 \approx -0.015$. Other given values are: $T \approx 0.1$; $k_{SM} \approx 0.1$. In other flight phases similar calculations are done.

2.3 Controlling Input

Impact of control stick and throttle displacement on the aircraft longitudinal motion (see Figure 2) are discussed below.

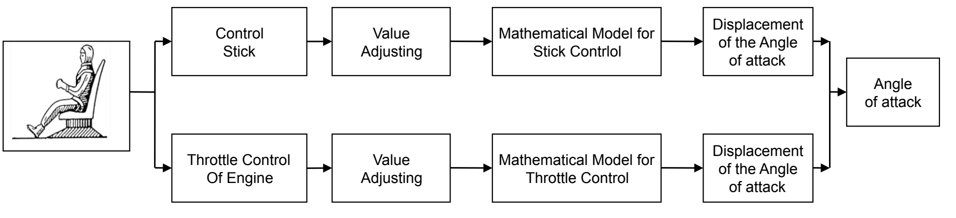

Figure 2. Mathematical model control of the aircraft angle of attack

The control value is represented by displacement of the control stick and displacement of the throttle. The values are adjusted according to the required ones and are forwarded to the input of the mathematical model of the aircraft attack angle control. Based on the realized analysis the mathematical model simulates displacement of the aircraft angle of attack $\Delta\alpha$ depending on control stick and throttle displacement.

For the angle of attack displacement next equation is valid. The equation defines the change in fuel supply or displacement of the angle of the aircraft elevator:

$$\Delta\alpha(s) = -G_{\delta_M/\alpha}(s)\Delta\delta_M(s) - G_{\delta_V/\alpha}(s)\Delta\delta_V(s), \tag{30}$$

where $G_{\delta_M/\alpha}(s)$ stands for mathematical model – transfer function for fuel supply, $\Delta\delta_M(s)$ is the input function for fuel supply in Laplace transformation, $G_{\delta_V/\alpha}(s)$ is mathematical model – transfer function for the aircraft elevator, $\Delta\delta_V(s)$ stands for the input function of the aircraft elevator in Laplace transformation.

When we induce the computed specifications from Section 2.2 for aerodynamic derivations into (12) we get the characteristic equation, that is used as numerator in the mathematical model. According to (19) we compute Δ_{12} for the transfer function, that means omitting the 1st column and the 2nd row. Making this change we define the fuel supply expression, in radians [9]:

$$G_{\delta_M/\alpha}(s) = 5\frac{0.002s^2 - 0.252s - 0.1}{s^4 + 1.134s^3 + 62.798s^2 + 28.659s + 4.093}. \tag{31}$$

From control theory the different ways of the transfer function notation of the system in computer are known. To simulate the values of the mathematical models the transfer function in the form of polynomial is used. The numerator and the denominator are polynomials of variable s. For pole-zero of transfer function we induce the computed specifications from Section 2.2 for aerodynamic derivations; they are in the form

$$G_{\delta_M/\alpha}(s) = -5\frac{(s + 0.3956)(s - 126.3956)}{[(s + 0.2292)^2 + 0.1142^2]\,[(s + 0.3378)^2 + 7.8936^2]}.$$

According to (19) we compute Δ_{22} or Δ_{32} for a transfer function, that means omitting the 2nd column and the 2nd row or omitting the 3rd column and the 2nd row. After this arrangement we define the expression for the aircraft elevator mathematical model, in radians [9]:

$$\begin{aligned} &G_{\delta_V/\alpha}(s) \\ &\quad= \frac{(-0.11(-s^3 + 0.866s^2 + 0.012s - 2.453)) - (0.42(-s^2 - 0.414s - 0.025))}{s^4 + 1.134s^3 + 62.798s^2 + 28.659s + 4.093}. \end{aligned}$$

$$G_{\delta_V/\alpha}(s) = -5\frac{(s + 2.7007)\,[(s + 0.1156)^2 + 0.9646^2]}{[(s + 0.2292)^2 + 0.1142^2]\,[(s + 0.3378)^2 + 7.8936^2]}. \tag{32}$$

In the aircraft mathematical model that expresses the dependence on the fuel supply – numerator of Equation (31) has zeros: -0.3956 and 126.3956. The poles of this mathematical model can be seen in the denominator in (31) and they express: 1st pair of complex conjugated poles $-0.2292 \pm j0.1142$ and the 2nd pair of complex conjugated poles $-0.3378 \pm j7.8936$. Comparing zeros and poles results in steady state, that is defined by gain in the equation and it is 0.0244. This value must be multiplied by the coefficient $a_x^{\delta_M} = -5$, see systems of Equations (20) and (21); the steady state value is 0.122. The time response to the unit-step function of input of transfer function means fuel supply to the aircraft (see Figure 3).

In the aircraft mathematical model that expresses the dependence on the fuel supply – numerator of Equation (32) has zeros: -2.7007 and the pair of complex conjugated zeros $-0.1156 \pm j0.9646$. The poles of this mathematical model can be seen in the denominator in Equation (32); namely 1st pair of complex conjugated poles $-0.2292 \pm j0.1142$ and 2nd pair of complex conjugated poles $0.3378 \pm j7.8936$. Comparing zeros and poles results in the steady state, that is defined by gain in

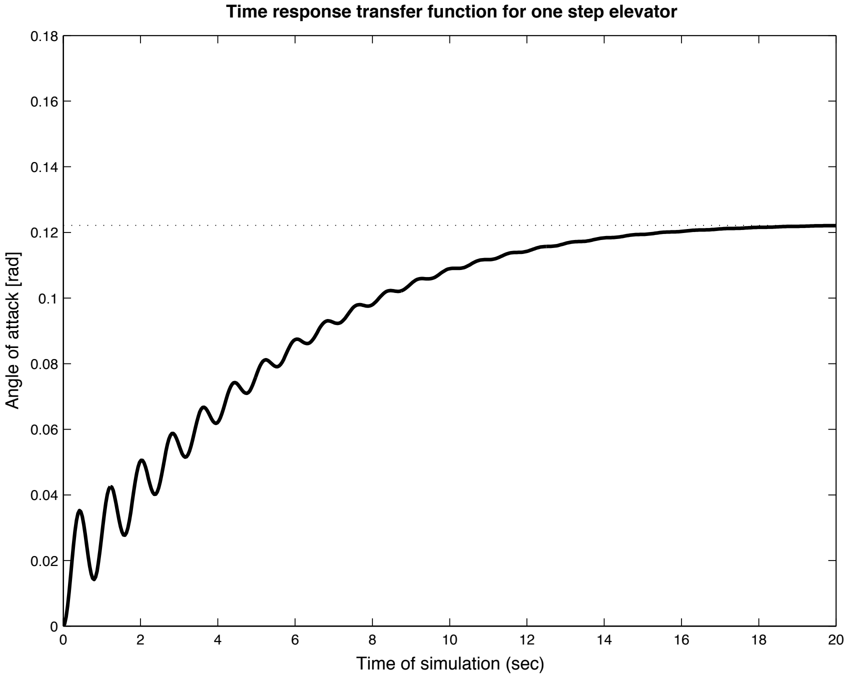

Figure 3. Time response of the transfer function for unit-step function of elevator of the aircraft angle of attack

the equation and it is 0.0685. This value must be multiplied by the coefficient $a_y^{\delta_V} = -0.11$ or $a_{mz}^{\delta_V} = -0.42$, see Equations (20) and (21); the steady state value is 0.0685. The time response to the unit-step function of input of transfer function means angle of elevator to the aircraft (see Figure 4).

The oscillations are caused by two pairs of complex conjugated poles of the characteristic equation, in accordance with Equation (13). Aircrafts are designed to have these properties (oscillations) and due to a control circuit, tailplanes, automated devices, etc. the aircraft loses the above mentioned properties.

3 THE MATHEMATICAL MODEL SIMULATION IN COMPUTER SYSTEMS

The system of equations can be represented with reference to initial or limiting restrictions in the given flight phase, for which the aircraft motion is calculated [14]. Current development in computer technology with the available calculation output meets the requirements for outputs to simulate mathematical models of control (see Figure 3). The development is aided by object-oriented programming [11].

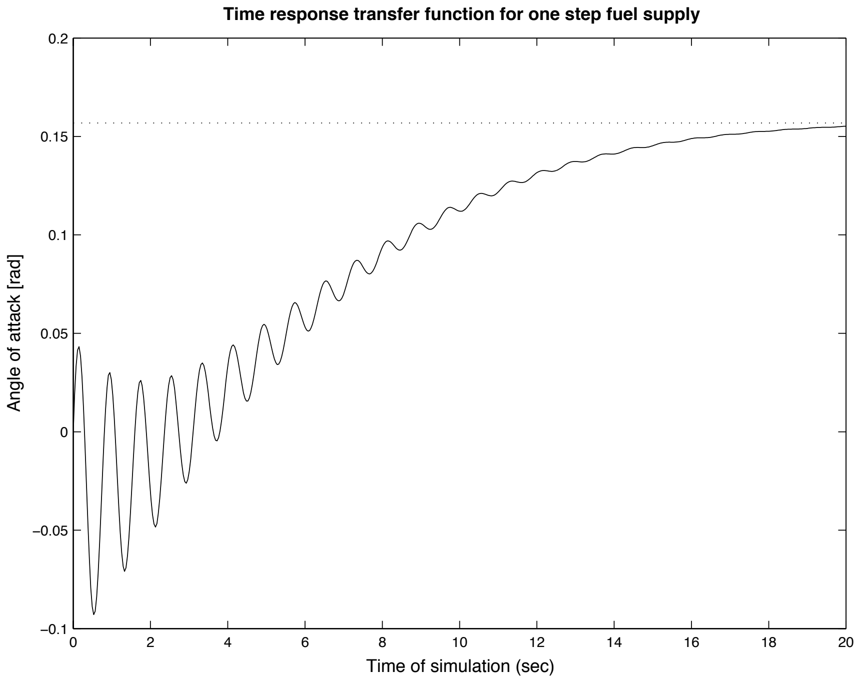

Figure 4. Time response of the transfer function for unit-step function of fuel supply of the aircraft angle of attack

3.1 Simulation

Simulation attempts to get the information on properties of a real system by means of an experiment, the so-called simulation model [18]. Computer simulation is employed usefully as enlarging or replacing model systems for which a simple form of analytical solution is impossible.

Simulation is employed in the process of examining dynamic properties of simulators where in the simulation programme time factor in the form of simulation time is employed explicitly and the conditions of the model are changed. There are two different methods of dealing with simulated time that are reflected in the employed programming technology [17]. The principle of this programme is partially analogical with the activity of a camera as the conditions of the system in equidistant moments are registered. This is the main reason why the method of variable time step has been employed lately. If mathematical models of control simulated in simulators are supposed to train pilots to react properly and in time, data in real time must be available. A standard time step is employed in these situations.

The key factors indicating the quality of simulators are represented by precise elements of mathematical models of control and modularity of designed applications on computer architecture. Some mathematical models can run independently on separate computers to increase the computation performance of the system [4, 6].

Mathematical models of control are programmed in C++ programming language. The basic class of languages *CMmlBase* is derived from the language class *CGssBaseObj*; the given class *CMmlBase* is a standard class and includes standard methods and data on the aircraft. In the constructor of this class initiation data are loaded by means of calling the method *MmlInit*. The basic class *CMmmModel* is based on the class *CGssBaseObj*. The class *CMmmModel* is a standard class and includes virtual methods and data on the engine. In the constructor of the class, initial data on parameters of aircraft turbocompressor engine of the set *Mmmdata* are loaded by means of calling the method of *Motorinic*.

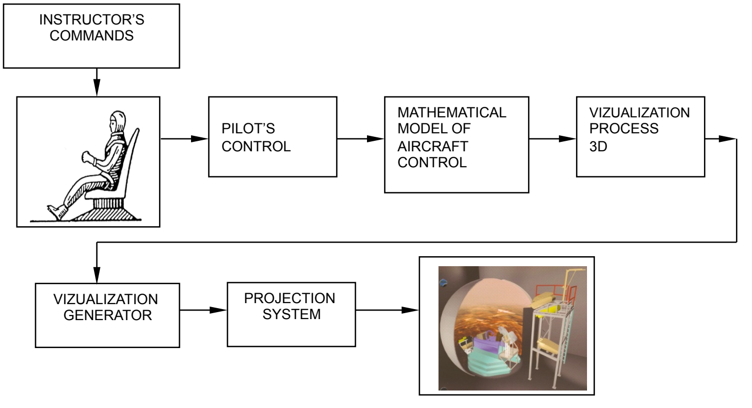

Figure 5. Principle of pilots activity and its visualization

To enable these programme codes to start in the computer in a sequence, mathematical models of control must be transformed at the level of the resource code. The needed mathematical models of control such as impact of the control stick on the speed in the longitudinal direction, impact of the control stick on the angle of attack of the aircraft, the impact of engine throttle lever on the speed in the longitudinal direction must be ranked in a stable logic sequence. The sequence is constant and in this way the resource code is compiled and calculations of created mathematical models are done in this sequence.

3.2 Angle of Attack Displacement Depending on the Control Stick Position

According to Equation (30) the first member represents transfer function (aircraft mathematical model) of displacement angle of attack with dependence on the fuel supply with "−" sign. In pole-zero format of transfer function we induce the following form for transfer function of the computed attack angle of displacement from the fuel supply in radians [9]:

$$G_{\delta_{M/\alpha}}(s) = 5\frac{0.002s^2 - 0.252s - 0.1}{s^4 + 1.134s^3 + 62.798s^2 + 28.659s + 4.093}. \tag{33}$$

If displacement of the attack angle is taken into account in Equation (35) this is conditioned by the speed of change in fuel supply (unit speed). In Laplace Transformation $\Delta\delta_M(s) = A/s^2$, where $A \in (0;1)$, it is necessary to calculate the value of $G_{\delta_{M/\alpha}}(s)$, which represents the mathematical model. If the input function is more complicated, then it is necessary to modify the mathematical model of control and to carry out the process of simulation accordingly.

3.3 Displacement of Angle of Attack Depending on the Throttle Position

According to Equation (30), the second member represents transfer function (aircraft mathematical model) of displacement angle of attack with dependence on the angle of elevator with "-" sign. In pole-zero format of transfer function we define the following expression for transfer function of the computed displacement angle of attack from the position of elevator in radians [9]:

$$G_{\delta_{V/\alpha}}(s) = \frac{-0.11(-s^3 + 0.87s^2 + 0.012s - 2.45) - 0.42(-s^2 - 0.414s - 0.025)}{s^4 + 1.134s^3 + 62.798s^2 + 28.659s + 4.093}. \tag{34}$$

If we consider displacing the angle of attack in Equation (30) depending on the speed of displacement of the elevator (unit speed). In Laplace Transformation $\delta_V(s) = A/s^2$, where $A \in (0;1)$, so it is necessary to calculate $G_{\delta_{V/\alpha}}(s)$, which is the mathematical model. If the input function is more complicated, then the mathematical model of control must be modified accordingly and then simulation must be carried out. When the elevator position is assigned as shown in Figure 6, then it is necessary to analyze the displacement as follows.

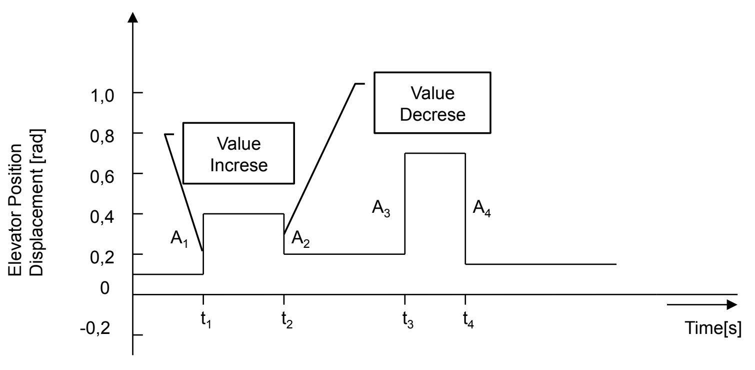

Figure 6. Elevator position in mathematical model of control

In time t_1 positive displacement of the elevator position is done by the value A_1. In time t_2 the elevator is displaced by the value A_2 in a negative way. Then in time t_3 the positive elevator displacement is done by the value A_3 and in time t_4 the elevator is displaced by the value A_4 in a negative way. According to (30) we have the position change of the elevator ($\delta_V(s)$); the positive value $A_1 = 0.3$ is taken into account in time t_1 on the input of the mathematical model of control; it is simulated and has the form:

$$\Delta\alpha(s) = -G_{\delta_V/\alpha}(s)\Delta\delta_V(s) = -G_{\delta_V/\alpha}(s)\frac{A_1}{s} = -G_{\delta_V/\alpha}(s)\frac{0.3}{s}. \tag{35}$$

In Equations (36), (37) and (38) only the shape input function for aircraft elevator position is modified. In line with Equation (30) negative value $A_2 = -0.2$ is considered from time t_2, mathematical model of control is simulated and has the following form:

$$\Delta\alpha(s) = -G_{\delta_V/\alpha}(s)\Delta\delta_V(s) = -G_{\delta_V/\alpha}(s)\frac{A_2}{s} = -G_{\delta_V/\alpha}(s)\frac{-0.2}{s}. \tag{36}$$

During time t_3 positive elevator displacement is done by $A_3 = 0.5$; since time t_3 simulated mathematical model of control has the form:

$$\Delta\alpha(s) = -G_{\delta_V/\alpha}(s)\Delta\delta_V(s) = -G_{\delta_V/\alpha}(s)\frac{A_3}{s} = -G_{\delta_V/\alpha}(s)\frac{0.5}{s}. \tag{37}$$

Then in t_4 negative elevator displacement is done by $A_4 = -0.6$; since time t_4 mathematical model of control is simulated and has the form:

$$\Delta\alpha(s) = -G_{\delta_V/\alpha}(s)\Delta\delta_V(s) = -G_{\delta_V/\alpha}(s)\frac{A_4}{s} = -G_{\delta_V/\alpha}(s)\frac{-0.6}{s}. \tag{38}$$

4 VISUALIZATION OF MATHEMATICAL MODEL OF CONTROL

The process of visualization is conditioned by the type of computer technology and graphic cards, monitors and application software employed. The computer image is artificial and is based on data stored in the memory and external memory device. These data can be gained from a real surrounding or are interpreted mostly by imaginative data and in this way they create artificial surrounding. In case of 3-dimensional data projection, an artificial 3-dimensional scene is created. In this setting it is important to state *what* is presented and *how* it is presented.

4.1 3-dimensional System and Field of View

The MMI (Man Machine Interface) represents a virtual level of information exchange between the operator and the machine; aircraft [15] plays a key role in simulator systems. In the process of designing information exchange a very important task must be fulfilled as the simulator is speedy, punctual, effective and not adjustable in its activity. On the other hand the operator is slow, prone to making mistakes and non-effective from the point of view of performance.

To simulate optical, sound, motion and touch equipments controlled by computer in interaction between the instructor and the artificial surrounding, it is important to create a surrounding that is identical with real aircraft surrounding. In this process methods of modelling and simulation along with the virtual reality (VR) systems are employed [16]. To generate virtual flight surroundings in a 3-dimensional scene, it is important to develop the necessary skills needed to run the training systems.

Visualization needs creating 3-dimensional models of objects employed in simulators. In this process the object is represented by space (airports, buildings, mountains) where an aircraft with its moving parts and all the equipment moves. Another task to be fulfilled is the motion control of 3-dimensional objects in the scene formed by the data of mathematical models of motion control of 6-degree freedom. The objects are actuated by the throughput data of a mathematical model.

Figure 7. Hemispherical Projection System: 3D simulator visualization [VRM, 2000]

4.2 Stationary Aircraft Cockpit

Being dived means being completely surrounded by something and this condition of being dived in space affects human's perception, thus enabling people to create and update the model of surroundings in pilot's perception. To create the condition of being dived into virtual reality, we need to be surrounded by elements stimulating the imagination of flight control in pilot's mind. That means that while turning our head left, things placed on the left must be seen by us (Figure 7). Moving forwards we approach objects placed in front of us, so they must change their position; this is the basic feature of pilot's perceptions [16].

Pilot – operator is placed in the focus of the hemispherical space, where objects in 3-dimensional visualization are projected. The image of objects on hemispherical screen is created by two projectors placed one above the other, thus generating one channel (see Figure 8). These three channels create a surrounding for visualization in aircraft-pilot simulator [10].

Supplying fuel by the pilot results in positive speed increase along the longitudinal axis; the aircraft flies faster, at higher speed. It results in momentum displacement around the transverse axis causing attack angle displacement of the aircraft and trajectory. The visualization system responds by changing the scene and the aircraft position in a 3-dimensional space, enabling the pilot to feel the change of the position in space.

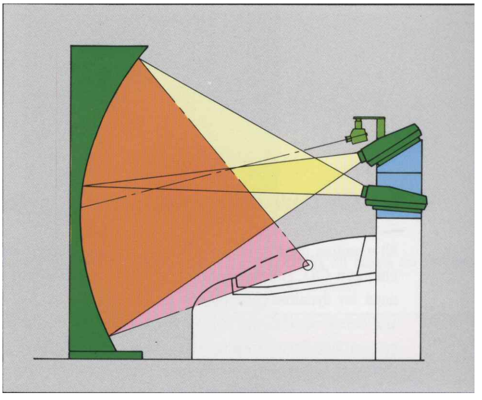

Figure 8. Principle of projecting of one channel projection system in simulator

Lever control displacement by the pilot results in height control displacement, attack angle displacement, change of the trajectory slope and aircraft coordinates in the system. The cockpit position does not change, the image generated by VR changes on the projection screen. The pilot perceives this real condition as a relative aircraft motion in space.

4.3 Changing Scene in the Visualization System and Calculation of Coordinates

In virtual 3-dimensional surrounding that is generated artificially according to Figure 7, the motion of an aircraft is relative in this surrounding. The pilot's cockpit does not move; it is stationary. The surrounding, the background projected on the hemispherical screen changes, thus generating the pilot's perception that he/she moves, "flies" in 3-dimensional space (Figure 9). This perception is created and achieved as the result of object position transformation via the converse matrix (39) into virtual surrounding of the image generator [15]. In the figure, according to the mathematical model throughout attack angle α is displaced by the value $\Delta\alpha$. The desired effect of the visual image can be achieved when the virtual 3-dimensional scene is displaced in the generated window at the horizontal level in the $-\Delta\alpha$ angle, that means displacement in the reverse direction. The aircraft position defined by simulating mathematical model of control comprising 6 degrees of freedom is given by the following variables: x, y, z, h, p, r. The aircraft position defined by the coordinates system of the visualization device has 6 degrees of freedom and is given by the variables $x_v, y_v, z_v, \alpha_v, \beta_v, \gamma_v$. The input values of different types are loaded into the model and the results are controlled in the point of "output". Steady-state validation of the mathematical models as the basic level of the validation process, in which values are compared, is discussed first.

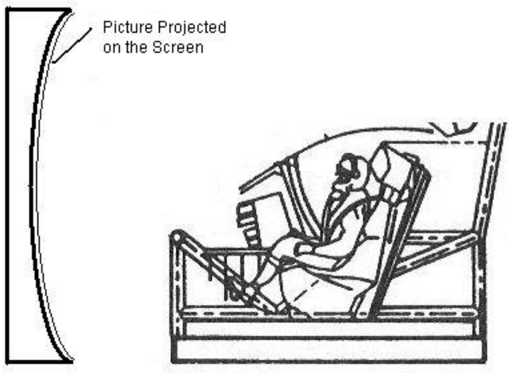

Figure 9. Attack angle displacement visualisation in simulator

In continuous systems for the input of values various kinds of aperiodic functions such as steps, ramps, or various periodic functions such as sine waves or square waves are discussed first. We often make distinction between the transient and steady-state characteristics of the system behaviour. The acceptor is the slot in the experimental frame where conditions limiting the observation of behaviour, such as steady-state versus transient, can be specified. The steady-state characteristics of system behaviour are controlled by licensed pilots or by computing numerical values by Newton's motion laws. The transient characteristics of system behaviour are controlled visually on the pilot's devices and by changing 3D scene position.

The transducer processes the output values, where the postprocessing may range from none at all to very different intervals when only certain features of interest are extracted. In steady-state conditions, the final values reached may be of our interest. The aircraft position image in 3-dimensional scene of VR needs the following form:

$$\begin{pmatrix} x_v \\ y_v \\ z_v \\ \alpha_v \\ \beta_v \\ \gamma_v \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ h \\ p \\ r \end{pmatrix} \tag{39}$$

Graphic results are presented and are compatible with the given mathematical form and mathematical model simulation.

5 CONCLUSIONS

Calculations and their results of visualization are considered to be a form of distributed calculations and are oriented towards a number of applied devices and throughput visualisation systems, etc. [8]. The paper presents methods of creating a mathematical model of control: model designing, linear model, model implementation and visualization. Current computer technology enables implementing advantages of multi-core processors while simulating mathematical models and graphic cards. The process of simulation is carried out on single-processor architecture with its known limits and the options of parallel computing in the future are considered.

Calculation results in the mathematical model of control prove to be correct, effective and are visualized precisely via VR. Simulation results conform to the requirements defined for simulation of the real time mathematical model. The aim of the paper is visualization of results of the mathematical model of control in vertical plane. Most publications address design of mathematical models and their simulation by means of computers and graphic presentation of results.

The results obtained in simulation of the mathematical models using real time visualization in 3D scene environment created in a virtual reality are presented. By means of changed input values of the mathematical models, different results are obtained. Based on the results of diverse analyses of stability, transient states, steady-states, precise simulation can be achieved. In this way, series of measurements can be made.

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