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RESEARCH ON TRACKING AND SYNCHRONIZATION OF UNCERTAIN CHAOTIC SYSTEMS

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Abstract. The tracking and synchronization problem of uncertain chaotic system, which is considered to be applied in secure communication in the future by many researchers, is considered in this paper. A double integral sliding mode controller is adopted to cope with the uncertainties of the chaotic system. Adaptive and robust strategies, such as Nussbaum gain method, are used to solve the unmodeled dynamic problem and unknown control direction problem. Meanwhile, the stability of the whole system is guaranteed by constructing of a big Lyapunov function for the whole system. Finally, a four dimension super-chaotic system is used as an example to do the numerical simulation and it testifies the rightness and effectiveness of the proposed method.

Keywords: Synchronization, adaptive, chaos, unknown control direction

1 INTRODUCTION

As a main aspect of nonlinear science, chaos has attracted many researchers of various fields [1, 2, 3]. It also has comprehensive applications in natural science and social science.

Chaos synchronization is an important research direction of chaotic science. It has been researched by many experts since the 1990's [4, 5, 6, 7, 8]. Much progress has also been made in its applications such as secret communication and image manipulation [9]. There are many methods proposed to solve synchronization problem of chaotic systems [9, 10, 11, 12]. In many researches the situation that there only exist static uncertainties between driver system and response system is considered. So the unmodeled dynamics of synchronization between chaotic system with different structure were seldom considered, especially for the situation that there exist static uncertainties, unknown parameters and dynamic uncertainties simultaneously; but it is very possible for the actual system that driven systems have different structure with response systems [13], or parameters may be changed unexpectedly because of the disturbance of environment, or the system model is inevitably inaccurate because of the dynamic uncertainties. The above situations are very possible to happen when synchronization of chaotic systems is used in the application of secure communication. So it is meaningful to study the synchronization of chaotic systems with both static and dynamic uncertainties.

In this paper, four kinds of uncertainties such as unknown parameters, static uncertain functions, unmodeled dynamics and unknown control directions [14, 15, 16, 17, 18] are considered simultaneously for the synchronization of chaotic systems. Adaptive method, robust control and Nussbaum gain control strategy are integrated to handle the above complex uncertainties. Also a Lyapunov function is constructed to guarantee the stability of the whole system with a double integral sliding mode type controller. Finally, numerical simulations are done and the good performance of the controller testifies the effectiveness and rightness of our proposed method. Especially, it is worth pointing out that a novel characteristic of the Nussbaum gain function is firstly defined and used to solve the synchronization problem with unmodelled dynamics.

2 MODEL DESCRIPTION

The following typical uncertain chaotic system with nonlinear functions is considered as a response system:

$$\begin{aligned} \dot{\xi} &= q(x_1, \xi, t) && (1) \\ \dot{x} &= f(x) + \Delta(x, \xi, t) + n(u) && (2) \end{aligned}$$

where $x = [x_1, \ldots, x_n]^T$, $u = [u_1, \ldots, u_n]^T$ are vectors, $n(u)$ are continuous nonlinear input functions. A three dimensional coordinate system is taken as an example, and it can be extended as follows:

$$\begin{aligned} \dot{\xi} &= q(x, \xi, t) && (3) \\ \dot{x}_1 &= f_1(x_1, \ldots, x_4) + \Delta_1(x, \xi, t) + n_1(u) && (4) \\ \dot{x}_2 &= f_2(x_1, \ldots, x_4) + \Delta_2(x, \xi, t) + n_2(u) && (5) \\ \dot{x}_3 &= f_3(x_1, \ldots, x_4) + \Delta_3(x, \xi, t) + n_3(u) && (6) \end{aligned}$$

where $f(x)$ are known functions of the system and $\Delta(x, \xi, t)$ are uncertain nonlinear dynamic functions of the system.

So the objective of tracking problem of chaotic system is to design a control $u = u(x, \hat{\theta})$, $\dot{\hat{\theta}} = g(x, \hat{\theta})$, such that states of the system can track to the desired value. In other word, it satisfies $x \to x^d$, where x^d is the desired value.

Without loss of generality, assume x^d is a constant value; then $\dot{x}_i^d = 0$. Define a new variable $e_i = x_i - x_i^d$; then the error system can be described as

$$\dot{e}_i = f_i(x_1, \ldots, x_4) + \Delta_i(x_1, \ldots, x_4, \xi) + b_i u_i. \tag{7}$$

To make the following illustration and proof easy, the input nonlinearity $n_i(u)$ is neglected in the tracking problem and it will be considered in the synchronization problem. So b_i is a known constant coefficient here. The driven system can be described as

$$\dot{y} = f(y) + \theta f_\theta(y). \tag{8}$$

Taking a three dimensional coordinate system as a example, it can be extended as

$$\begin{aligned} \dot{y}_1 &= f_{y1}(y_1, \ldots, y_4) + \theta_1 f_{\theta 1}(y) && (9)\\ \dot{y}_2 &= f_{y2}(y_1, \ldots, y_4) + \theta_2 f_{\theta 2}(y) && (10)\\ \dot{y}_3 &= f_{y3}(y_1, \ldots, y_4) + \theta_3 f_{\theta 3}(y) && (11) \end{aligned}$$

where θ are unknown parameters, $f_\theta(y)$ are known functions.

So the objective of the synchronization problem is to design a control $u = u(x, \hat{\theta}, \hat{d})$, where $\hat{\theta}' = g_1(x, \hat{\theta}, \hat{d})$ and $\hat{d}' = g_2(x, \hat{\theta}, \hat{d})$ such that the response system can track the driven system, that is to say $y \to x$.

Define a new variable as

$$z_i = \mathrm{y}_i - x_i. \tag{12}$$

Then the error system can be described as

$$\begin{aligned} \dot{z}_i = {} & f_i(x_1, \ldots, x_4) - f_{yi}(y_1, \ldots, y_4) \\ & - \theta_i f_{\theta i}(y) + \Delta_i(x, \xi, t) - n_i(u_i) \end{aligned} \tag{13}$$

where $\Delta_i(\bullet)$ and $q_i(\bullet)$ are unknown continuous Lipschitz functions, the ξ subsystem is the uncertain dynamic part of the above system, and $\Delta_i(\bullet)$ represents the uncertain nonlinearities of the system, which satisfies the following assumption.

3 ASSUMPTIONS

Assumption 1. The ξ subsystem can be viewed that it has a input as state x and there exists an input-to-state practical stability Lyapunov function $V_0(\xi)$. That is to say there exists a smooth positive definite and canonical function $V_0(\xi)$ such that

$$\frac{\partial V_0(\xi)}{\partial \xi} q(x, \xi, t) \leq -\alpha_z(V_0(\xi)) + v_z(|s_i|) + d_z, \forall (x, \xi, t) \in R \times R^{n_0} \times R_+ \tag{14}$$

where $\alpha_z(\bullet)$ and $v_z(\bullet)$ are k_∞ type functions, $s = f(x, y)$ and y are chaotic signals, so they are bounded, and d_z is a nonnegative constant.

Assumption 2. For $1 \le i \le n$, there exists an unknown constant $p_i^* \le d_i$ such that

$$|\Delta_i(X, \xi, t)| \le p_i^* \psi_{i1}(|(x_1, \ldots, x_i)|) + p_i^* \psi_{i2}(|\xi|), \forall (X, \xi, t) \in R^n \times R^{n_0} \times R_+ \quad (15)$$

where d_i is a known constant, $\psi_{i1}(\bullet)$ and $\psi_{i2}(\bullet)$ are known nonnegative smooth functions with $\psi_{i2}(0) = 0$.

Remark 1. Without loss of generality, assume that there exists constant ε_{ci} big enough such that

$$\frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - \alpha_z(V_0(\xi)) < 0. \quad (16)$$

Similarly, there exist parameters big enough ε_{c3i} such that

$$v_z(|s_i|) - \varepsilon_{c3i} s_i^2 < 0. \quad (17)$$

Definition 1. $N(\chi)$ is a Nussbaum-type function, if it has the following characteristics

$$\lim_{s\to\infty} \sup \frac{1}{s} \int_0^s N(x)dx = +\infty \quad (18)$$

$$\lim_{s\to\infty} \inf \frac{1}{s} \int_0^s N(x)dx = -\infty. \quad (19)$$

Meanwhile, it is easy to prove that $N(\chi) + k_d$ also satisfies

$$\lim_{s\to\infty} \sup \frac{1}{s} \int_0^s \{N(x) + k_d\}dx = +\infty \quad (20)$$

$$\lim_{s\to\infty} \inf \frac{1}{s} \int_0^s \{N(x) + k_d\}dx = -\infty. \quad (21)$$

4 TRACKING OF UNCERTAIN CHAOTIC SYSTEM

Considering i^{th} subsystem of the error system about tracking problem, it has

$$\dot{e}_i = f_i(x_1, \ldots, x_4) + \Delta_i(x_1, \ldots, x_4) + b_i u_i. \quad (22)$$

Design the control u_i as follows:

$$u_i = f_{2i}(x)[-f_i(x_1, \ldots, x_4) - \eta(x, z) + f_{zi}(z_i)]. \quad (23)$$

Remember that

$$|z_i \Delta_i(X, \xi, t)| \le p_i^* |z_i| \psi_{i1}(|(x_1, \ldots, x_i)|) + p_i^* |z_i| \psi_{i2}(|\xi|) \quad (24)$$

where $f_{2i}(x) = b_i^{-1}$ and

$$f_{zi}(z_i) = -k_{i1}z_i - k_{i2}\frac{z_i}{|z_i| + \varepsilon_{i1}} - k_{i3}\frac{3}{2}z_{i1}^{1/3}\exp(z_{i1}^{2/3}) - k_{i4}sign(z_{i1}). \tag{25}$$

Then it holds

$$\begin{aligned} z_i\dot{z}_i &= z_i[\Delta_i(x) - \eta(x,z) + f_{zi}(z_i)] \\ &\le z_i f_{zi}(z_i) + p_i^* |z_i| \psi_{i1}(|(x_1,\ldots,x_i)|) + p_i^* |z_i| \psi_{i2}(|\xi|) - z_i\eta(x,z) \\ &= z_i f_{zi}(z_i) + p_i^* |z_i| \psi_{i1}(|(x_1,\ldots,x_i)|) + \varepsilon_{ci}^2 z_i^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - z_i\eta(x,z). \end{aligned} \tag{26}$$

Design the robust control law as

$$\eta(x,z) = \hat{p}_i^* |z_i| \psi_{i1}(|(x_1,\ldots,x_i)|) + \hat{\varepsilon}_{c2i}z_i \tag{27}$$

where $\tilde{p}_i^*$ is defined as

$$\tilde{p}_i^* = p_i^* - \hat{p}_i^*, \tilde{\varepsilon}_{c2i} = \varepsilon_{ci}^2 + \varepsilon_{c3i} - \hat{\varepsilon}_{c2i}. \tag{28}$$

Then it satisfies

$$z_i\dot{z}_i = z_i f_{zi}(z_i) + \tilde{p}_i^* |z_i| \psi_{i1}(|(x_1,\ldots,x_i)|) + \tilde{\varepsilon}_{c2i}z_i^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - \varepsilon_{c3i}z_i^2. \tag{29}$$

Design the adaptive control law as

$$\frac{d\hat{p}_i^*}{\mathrm{d}t} = sign(z_i)\psi_{i1}(|(x_1,\ldots,x_i)|), \frac{d\hat{\varepsilon}_{c2i}}{\mathrm{d}t} = z_i^2. \tag{30}$$

Choose a Lyapunov function as

$$V = \sum_{i=1}^n \frac{1}{2}z_i^2 + \frac{1}{2}(\tilde{\varepsilon}_{2ci})^2 + \frac{1}{2}(\tilde{p}_i^*)^2 + V_0(\xi). \tag{31}$$

Solve its derivative along its trajectory of differential equations; it holds

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n \frac{1}{2}z_i^2 + \frac{1}{2}(\tilde{\varepsilon}_{2ci})^2 + \frac{1}{2}(\tilde{p}_i^*)^2 + V_0(\xi) \\ &\le \sum_{i=1}^n z_i f_{zi}(z_i) + \sum_{i=1}^n \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - \alpha_z(V_0(\xi)) + v_z(|z|) + d_z - \varepsilon_{c3i}z_i^2 \\ &\le \sum_{i=1}^n z_i f_{zi}(z_i) + v_z(|z|) + d_z - \varepsilon_{c3i}z_i^2 \le \sum_{i=1}^n z_i f_{zi}(z_i) + d_z. \end{aligned} \tag{32}$$

Then it is easy to prove that z_i is bounded and it can converge to a small neighborhood of zero with a proper design of $f_{zi}(z_i)$.

Since tracking problem is easy compared with the below synchronization problem situation, the numerical simulation result and details are ignored here.

4.1 Synchronization of Uncertain Chaotic Systems

Consider the subsystem

$$\dot{z}_i = f_{yi}(y_1, \dots, y_4) + \theta_i f_{\theta i}(y) - \Delta_i(x, \xi, t) - f_i(x_1, \dots, x_4) - n_i(u_i). \tag{33}$$

Remark 2. Wanglong Li assumed the nonlinear input function $n_i(u_i)$ is bounded by u_i in paper [13]. It yields positive constants c_{i1} and c_{i2}, such that the following conditions are satisfied.

$$c_{i1} \leq \frac{n_i(u_i)}{u_i} \leq c_{i2}, i = 1, \dots, n. \tag{34}$$

Then they have

$$c_{i1}u_i^2 \leq u_i n_i(u_i) \leq c_{i2}u_i^2 \tag{35}$$

It is still a strict condition for many real systems. In this paper, we further relax the restriction for the nonlinear input of the previous work as following Assumption A3.

Assumption 3. For $1 \leq i \leq n$, there exists an unknown time varying variable $b_i(t)$ such that

$$n_i(u_i) = b_i(t)u_i$$

and assume $b_i(t)$ is bounded. To make it simple, write $b_i(t)$ as b_i; then b_i is an unknown bounded time-varying parameter. Especially, the sign of b_i is unknown.

It is easy to prove that Assumption 3 is more relax than the assumption in [13]. For any $n_i(u_i)$ satisfies $c_{i1} \leq \frac{n_i(u_i)}{u_i} \leq c_{i2}$ in paper [13], b_i can always be chosen as $b_i = \frac{n_i(u_i)}{u_i}$; then $c_{i1} \leq b_i \leq c_{i2}$. b_i is restricted to be positive; but in this paper, b_i can be positive or negative; what is worse, the sign of b_i is changing during a comparatively long time interval. With Assumption 3, the error system can be written as follows:

$$\dot{z}_i = f_{yi}(y_1, \dots, y_4) + \theta_i f_{\theta i}(y) - \Delta_i(x, \xi, t) - f_i(x_1, \dots, x_4) - b_i u_i. \tag{36}$$

Define a double integral sliding mode surface as

$$s_i = z_i + a_{si}\int_0^t z_i \mathrm{d}t + b_{si}\int_0^t\int_0^t z_i \mathrm{d}t\mathrm{d}t. \tag{37}$$

Solve the derivative as

$$\begin{aligned} \dot{s}_i \;&=\; \dot{z}_i + a_{si}z_i + b_{si}\int_0^t z_i \mathrm{d}t = f_{yi}(y_1, \dots, y_4) + \theta_i f_{\theta i}(y) \\ &\quad - f_i(x_1, \dots, x_4) - \Delta_i(x, \xi, t) - b_i u_i + a_{si}z_i + b_{si}\int_0^t z_i \mathrm{d}t \end{aligned} \tag{38}$$

and design the control u_i as

$$\begin{aligned} u_i &= f_{si}(x)u_i^d = f_{si}(x)[-f_{yi}(y) - \hat{\theta}_i f_{\theta i}(y) \\ &\quad + f_i(x) + \eta_i(x, y, z_i, s_i) - a_{si} z_i - b_{si} \int_0^t z_i \mathrm{d}t + f_{sri}(s_i)] \end{aligned} \tag{39}$$

where

$$\begin{aligned} f_{si}(x) &= N(k_i) \\ f_{sri}(s) &= -k_{i1} s_i - k_{i2} \frac{s_i}{|s_i| + \varepsilon_{i1}} - k_{i3} \frac{3}{2} s_{i1}^{1/3} \exp(s_{i1}^{2/3}) - k_{i4} sign(s_{i1}). \end{aligned} \tag{40}$$

Then

$$\begin{aligned} s_i \dot{s}_i &= s_i f_{si}(s_i) + s_i \{\tilde{\theta}_i f_{\theta i}(y) + \eta_i(x, y, z_i, s_i) \\ &\quad - \Delta_i(x, \xi, t)\} + s_i(-b_i N(k_i) u_i^d - u_i^d) \end{aligned} \tag{41}$$

$$|s_i \Delta_i(X, \xi, t)| \leq p_i^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|) + p_i^* |s_i| \psi_{i2}(|\xi|) \tag{42}$$

and it also can be written as

$$\begin{aligned} s_i \dot{s}_i &= s_i f_{si}(s_i) + s_i \{\tilde{\theta}_i f_{\theta i}(y) + \eta_i(x, y, z_i, s_i) \\ &\quad - \Delta_i(x, \xi, t)\} + s_i(-b_i N(k_i) u_i^d - u_i^d) \end{aligned} \tag{43}$$

$$\begin{aligned} s_i \dot{s}_i &\leq s_i f_{si}(s_i) + p_i^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|) \\ &\quad + p_i^* |s_i| \psi_{i2}(|\xi|) + s_i \eta(x, y, z_i, s_i) \\ &\quad + s_i \tilde{\theta}_i f_{\theta i}(y) + s_i(-b_i N(k_i) u_i^d - u_i^d). \end{aligned} \tag{44}$$

It can be arranged as follows:

$$\begin{aligned} s_i \dot{s}_i &\leq s_i f_{zi}(z_i) + p_i^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|) \\ &\quad + \varepsilon_{ci}^2 s_i^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} + s_i \eta(x, y, z_i, s_i) \\ &\quad + s_i \tilde{\theta}_i f_{\theta i}(y) + s_i(-b_i N(k_i) u_i^d - u_i^d). \end{aligned} \tag{45}$$

Design

$$\eta(x, y, z_i, s_i) = -\hat{p}_i^* sign(s_i) \psi_{i1}(|(x_1, \ldots, x_i)|) - \hat{\varepsilon}_{c2i} s_i \tag{46}$$

and define

$$\begin{aligned} \tilde{p}_i^* &= p_i^* - \hat{p}_i^* \\ \tilde{\varepsilon}_{c2i} &= \varepsilon_{ci}^2 + \varepsilon_{c3i} - \hat{\varepsilon}_{c2i} \\ \frac{d\hat{\theta}_i}{\mathrm{d}t} &= s_i f_{\theta i}(y). \end{aligned}$$

Then the following equation holds:

$$s_i \dot{s}_i = s_i f_{si}(s_i) + \tilde{p}_i^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|)$$

$$
\begin{aligned}
&+ \tilde{\varepsilon}_{c2i} s_i^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - \varepsilon_{c3i} s_i^2 \\
&+ s_i(-b_i N(k_i) u_i^d - u_i^d).
\end{aligned} \tag{47}
$$

Define

$$
d\hat{p}_i^*/\mathrm{d}t = |s_i|\, \psi_{i1}(|(x_1, \ldots, x_i)|), d\hat{\varepsilon}_{c2i}/\mathrm{d}t = s_i^2 \tag{48}
$$

and choose a Lyapunov function as

$$
V_i = \frac{1}{2}\left[s_i^2 + (\tilde{\theta}_i)^2\right] + \frac{1}{2}(\tilde{\varepsilon}_{2ci})^2 + \frac{1}{2}(\tilde{p}_i^*)^2 + V_0(\xi) \tag{49}
$$

and the derivative of the Lyapunov function can be written as

$$
\dot{V}_i \leq s_i f_{zi}(s_i) + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - \alpha_z(V_0(\xi)) + v_z(|s_i|) + d_z - \varepsilon_{c3i} s_i^2. \tag{50}
$$

According to the assumption, it is easy to prove that

$$
\dot{V}_i \leq s_i f_{zi}(s_i) + d_z + s_i(-b_i N(k_i)) u_i^d - u_i^d. \tag{51}
$$

With the discussion of Assumption 3, it is necessary to adopt a new kind of control strategy to solve the unknown control direction of b_i. Then use the Nussbaum gain method and design the Nussbaum gain regulation law as

$$
\dot{k}_i = -s_i u_i^d. \tag{52}
$$

Then,

$$
\dot{V}_i \leq d_z + \dot{k}_i(1 + b_i N(k_i)). \tag{53}
$$

With integral computation on both sides of the inequality, we have

$$
V_i(t) - V_i(0) \leq (k(t) - k(0) + b_i \int_{k(0)}^{k(t)} (N(k_i) + d_z)\mathrm{d}k. \tag{54}
$$

Remark 3. Use the apagoge method; assume that $k(t)$ will be unstable in finite time, so when $t \to t_n$, it has $k(t) \to \infty$. With the help of Nussbaum gain function characteristics, it is easy to prove the above inequality is contradicting. So $k(t)$ is bounded in finite time.

Now, it is also easy to prove that s_i is bounded and design $f_{si}(s_i)$ such that s_i can be converged to a small enough interval near zero. Furthermore, because of the design of sliding mode coefficients, it is easy to guarantee that $s_i \to 0$; then it has $z_i \to 0$. So the system is proved to be stable.

5 EXAMPLE AND SIMULATION

Taking the three dimensional coordinate chaotic system as an example to make a numerical simulation, the model can be described as

$$\begin{aligned}
\dot{\xi} &= -5\xi + 3x_1 + 0.2x_2 + 1.4x_3 + 2.7x_1x_3 \\
\dot{x}_1 &= a(x_2 - x_1) + k_{lb}(x_2 \cos x_2 + \xi) + \lambda_1 u_1 \\
\dot{x}_2 &= bx_1 - x_1x_3 - x_2 + k_{lb}[(1 + \sin(x_2x_3))x_2 + 0.7\xi x_2] + \lambda_2 u_2 \\
\dot{x}_3 &= -cx_3 + x_1x_2 + k_{lb}[(2 - \cos(x_1x_2x_3))x_1 + 3.5\xi] + \lambda_3 u_3
\end{aligned}$$

where a, b, c are unknown constants, which are set as $(a, b, c) = (10, 28, 8/3)$, and the uncertain nonlinear function obviously satisfies all assumptions of this paper. The initial state of the system can be chosen as

$$(\xi, x_1, x_2, x_3) = (0, 1, 1, 1).$$

The model of the driven system can be described as a Genesio system

$$\begin{aligned}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= y_3 \\
\dot{y}_3 &= -a_1y_1 - b_1y_2 - c_1y_3 + y_1^2
\end{aligned}$$

where the unknown parameters are chosen as

$$(a_1, b_1, c_1) = (6, 2.92, 1.2)$$

and the initial states are chosen as

$$(y_1, y_2, y_3) = (1, 1, 1).$$

The comparison between free trajectory of driven system and it of response system without control can be seen in Figures 1 and 2. It is obvious that the synchronization between the above two system can not be realized.

Assume that the unknown control direction switches twice at the time of 2.5 s and 4.5 s, respectively. Using the proposed method, the synchronization of chaotic system can be achieved (Figure 3, 4 and 5).

The curve of the error of synchronization is shown in Figures 6, 7 and 8.

The curve of Nussbaum gains can be seen in Figures 9, 10 and 11. They are converged to a new value at the time of 1 s when the input direction switches. According to the figures, a conclusion can be made that synchronization between the driven and response systems can be achieved quickly.

The curve of real control gains is given in Figures 12, 13 and 14.

The figures show that the gain of control can be adapted to the change of input directions such that the chaotic systems with both input unmodeled dynamics and uncertain input can be synchronized.

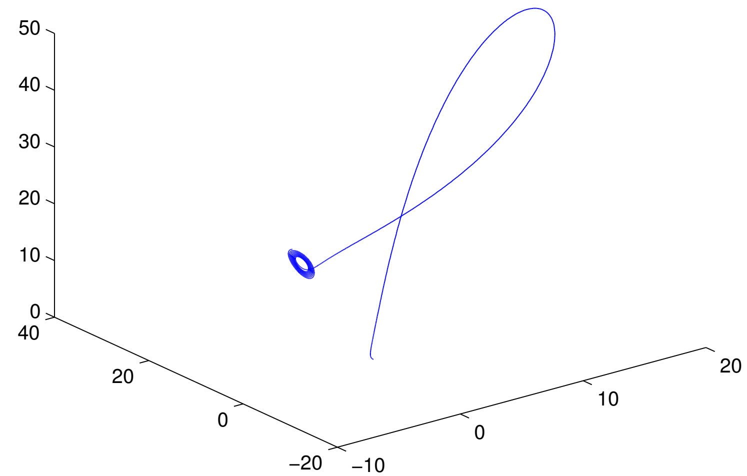

Figure 1. Chaotic behavior of x_1,x_2,x_3

6 CONCLUSIONS

The main contribution of this paper can be summarized as follows. First, to make the synchronization problem easy to be understood, a simple situation of superchaotic system is considered and the tracking problem is investigated. Second,

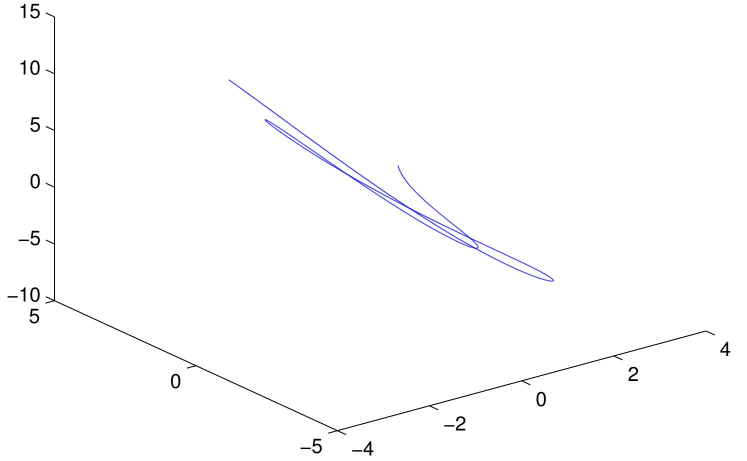

Figure 2. Chaotic behavior of y_1, y_2, y_3

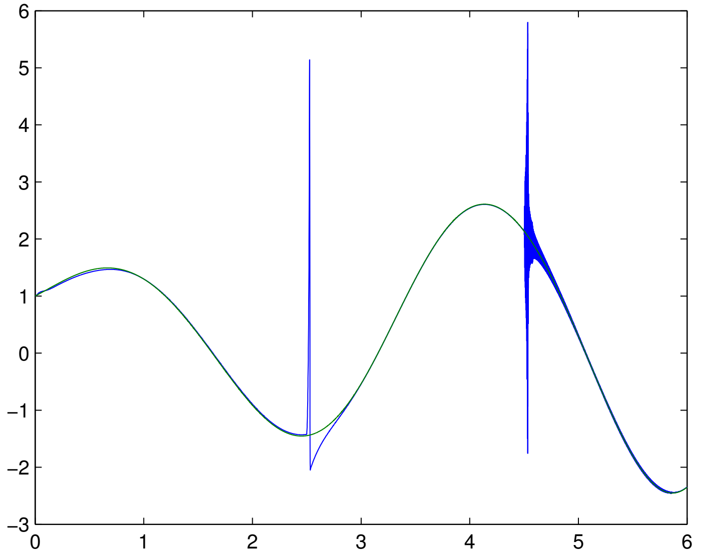

Figure 3. Synchronization of x_1 and y_1

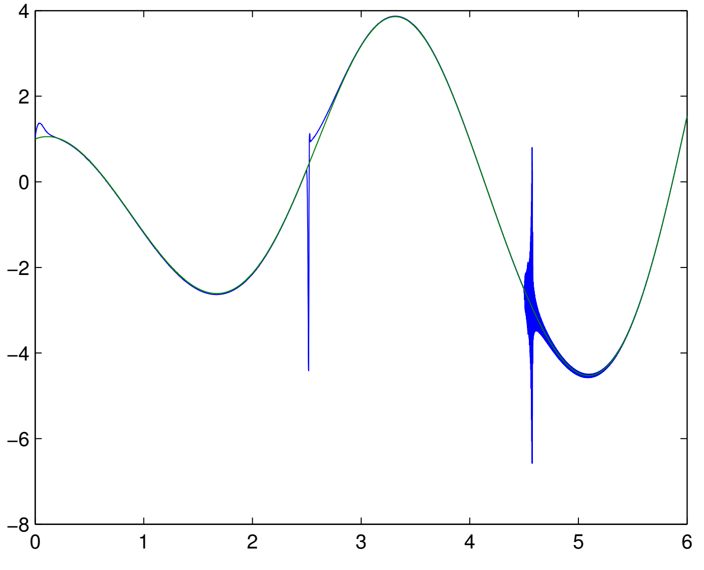

Figure 4. Synchronization of x_2 and y_2

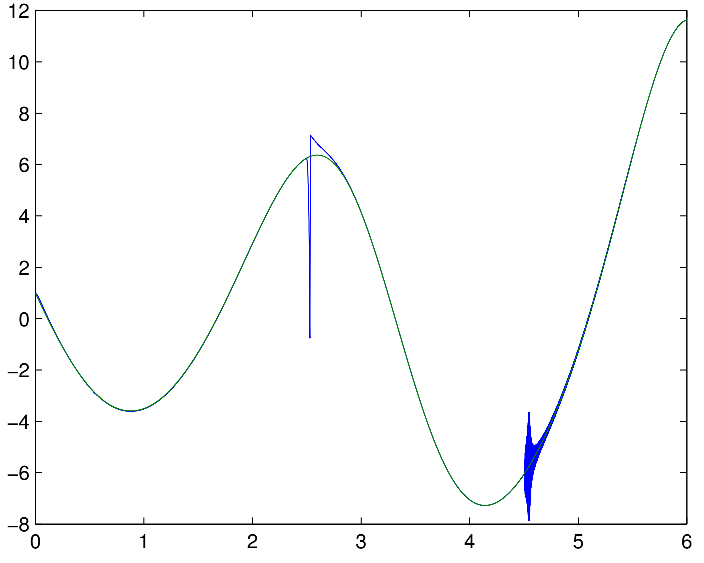

Figure 5. Synchronization of x_3 and y_3

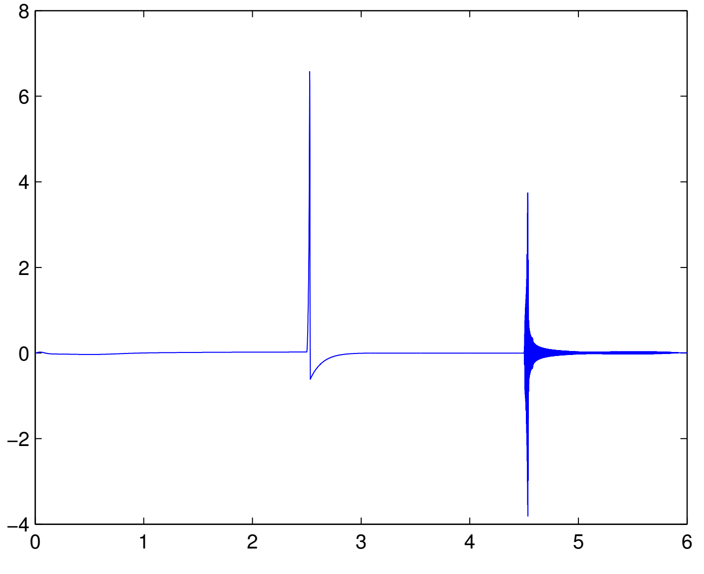

Figure 6. Error of synchronization e_1

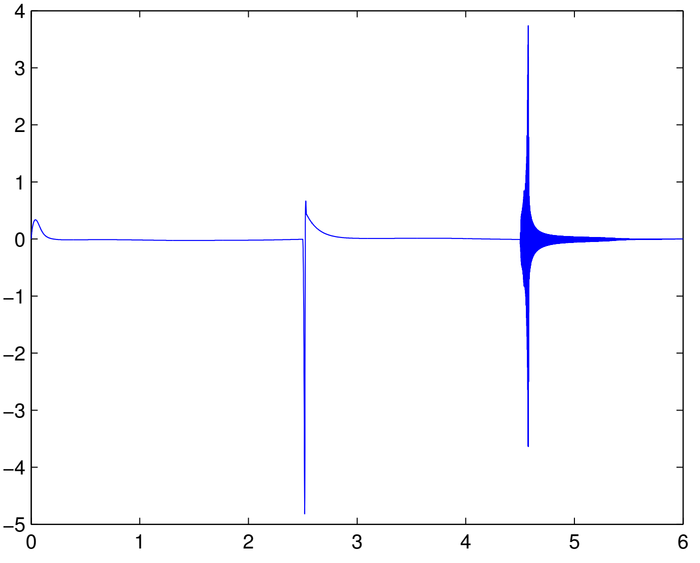

Figure 7. Error of synchronization e_2

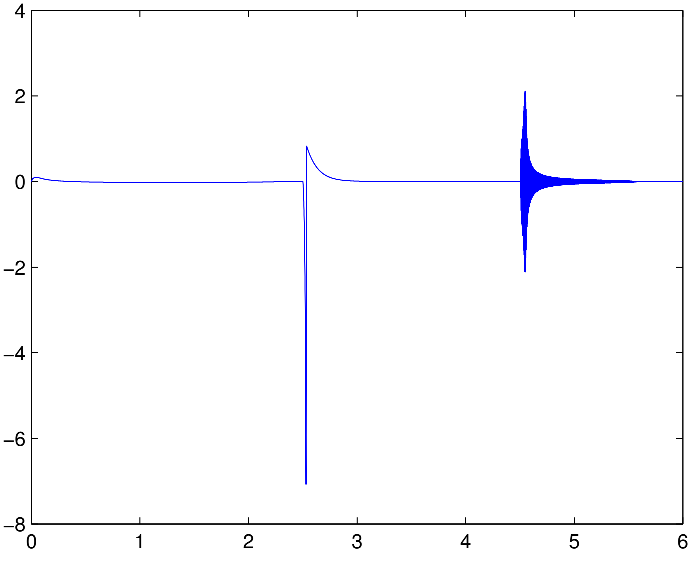

Figure 8. Error of synchronization e_3

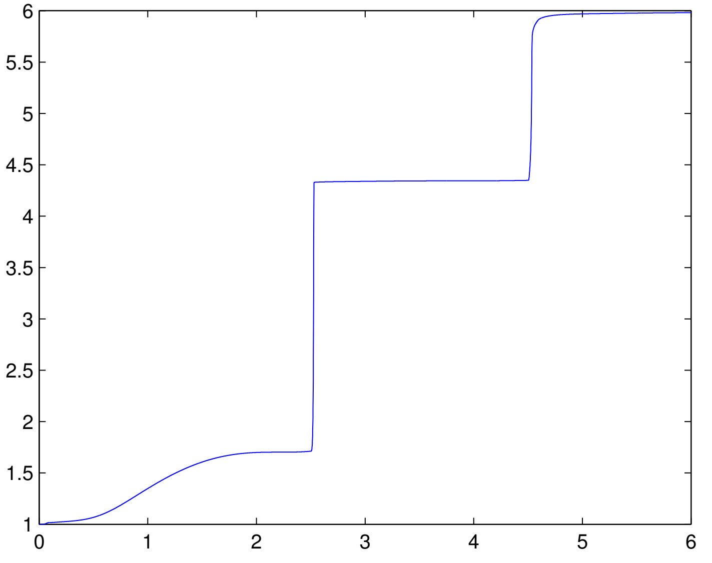

Figure 9. Nussbaum gain of k_3

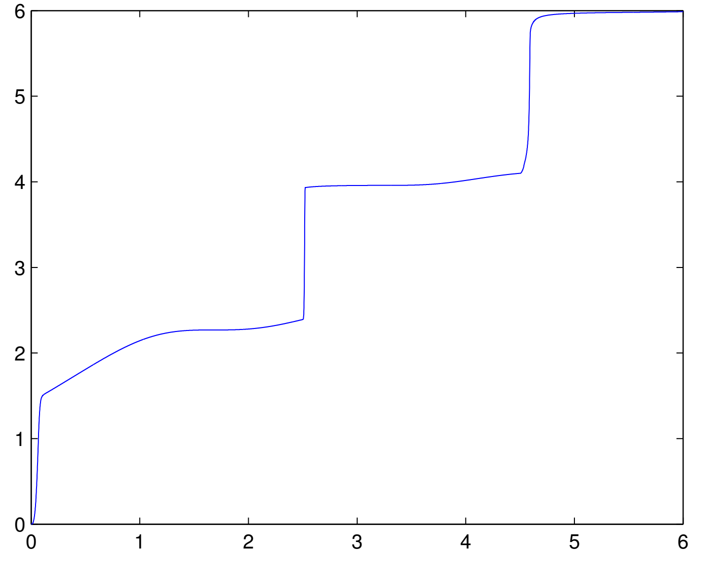

Figure 10. Nussbaum gain of k_2

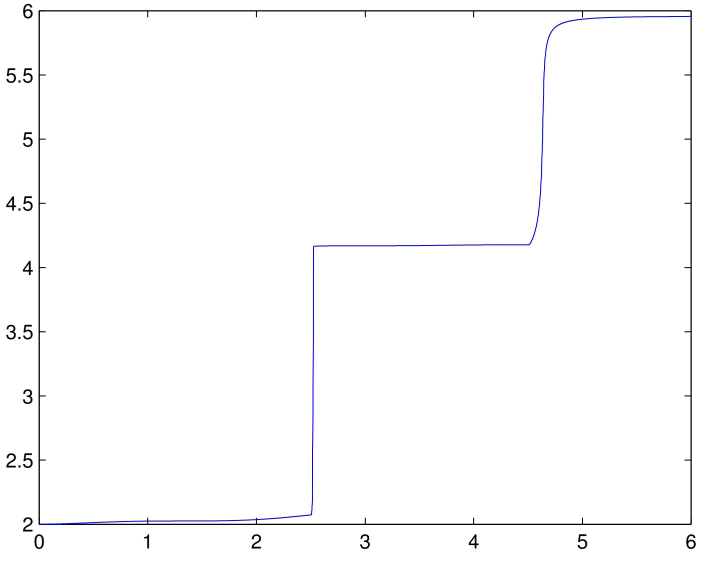

Figure 11. Nussbaum gain of k_3

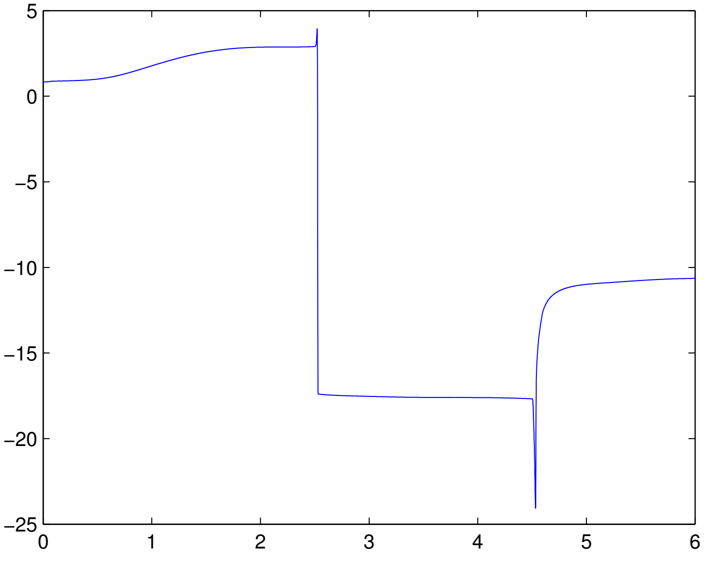

Figure 12. Real control gain of u_1

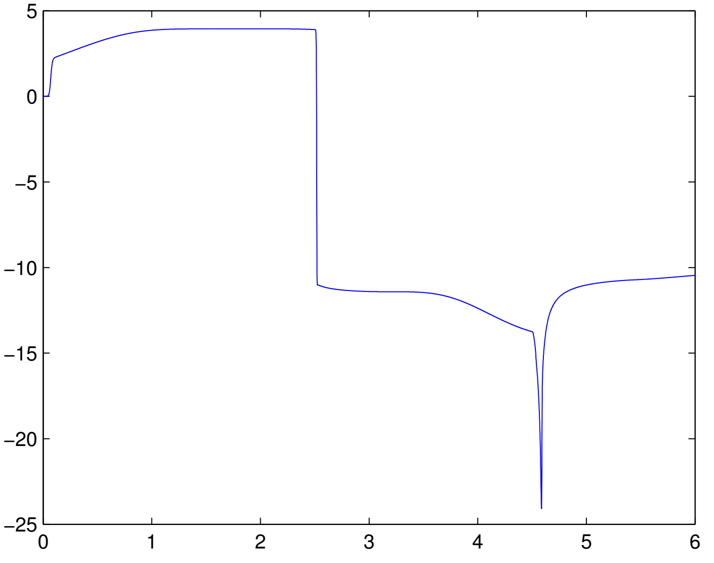

Figure 13. Real control gain of u_2

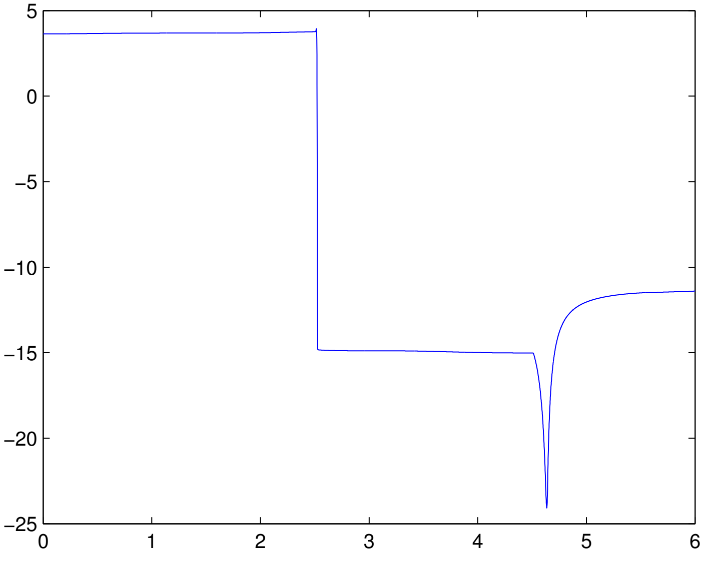

Figure 14. Real control gain of u_3

the synchronization problem is studied and a double integral sliding mode method, robust control, adaptive control strategy and Nussbaum gain method are perfectly integrated to solve complex uncertainties. Third, a novel characteristic of Nussbaum function is proposed and used to cope with dynamic uncertainties in this paper. Also, a numerical simulation is made and good performance is achieved; this testifies the rightness and effectiveness of the proposed method.

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REFERENCES

[1] Lei, J.—Wang, X.–Lei, Y.: How Many Parameters can be Identified by Adaptive Synchronization in Chaotic Systems? Physics Letters A, Vol. 373, 2009, No. 4, pp. 1249–1256.

[2] Lei, J.—Wang, X.–Lei, Y.: A Nussbaum Gain Adaptive Synchronization of a New Hyperchaotic System With Input Uncertainties and Unknown Parameters. Communications in Nonlinear Science and Numerical Simulation, Vol. 14, 2009, No. 4, pp. 3439–3448.

[3] Wang, X.—Lei, J.–Pan, C.: Trigonometric RBF Neural Robust Controller Design for a Class of Nonlinear System with Linear Input Unmodeled Dynamics. Applied Mathematics and Computation, Vol. 185, 2007, No. 3, pp. 989–1002.

[4] Hu, M.—Xu, Z.—Zhang, R.–Hu, A.: Parameters Identification and Adaptive Full State Hybrid Projective Synchronization of Chaotic (Hyper-Chaotic) Systems. Physics Letters A, Vol. 361, 2007, No. 4, pp. 231–237.

[5] Gao, T.—Chen, Z.—Yuan, Z.–Yu, D.: Adaptive Synchronization of a new Hyperchaotic System With Uncertain Parameters. Chaos, Solitons and Fractals, Vol. 33, 2007, No. 4, pp. 922–928.

[6] Elabbasy, E. M.—Agiza, H. N.—El-Dessoky, M. M.: Adaptive Synchronization of a Hyperchaotic System With Uncertain Parameter. Chaos, Solitons and Fractals, Vol. 30, 2006, No. 4, pp. 1133–1142.

[7] Tang, F.Wang, L.: An Adaptive Active Control for the Modified Chua's Circuit. Physics Letters A, 346 (2005), pp. 342–346.

[8] Ge, Z. M.Yang, C. H.: Pragmatical Generalized Synchronization of Chaotic Systems With Uncertain Parameters by Adaptive Control. Physica D, Vol. 231, 2007, No. 4, pp. 87–89.

[9] Gauthier, J. P.—Hammouri, H.—Othman, S: A Simple Observer for Nonlinear Systems, Applications to Bioreactors. IEEE Transactions on Automatic Control, Vol. 37, 1992, No. 4, pp. 875–880.

[10] Khalil, H. K.—Saberi, A: Adaptive Stabilization of a Class of Nonlinear Systems Using High-Gain Feedback. IEEE Transactions on Automatic Control, Vol. 32, 1987, No. 2, pp. 1031–1035.

[11] Lei, H.–Lin, W.: Universal Adaptive Control of Nonlinear Systems With Unknown Growth Rate by Output Feedback. Automatica, Vol. 42, 2006, No. 2, pp. 1783–1789.

[12] Lei, H.–Lin, W.: Adaptive Regulation of Uncertain Nonlinear Systems by Output Feedback: A Universal Control Approach. Systems and Control Letters, Vol. 56, 2007, No. 2, pp. 529–537.

[13] Li, W.-L.–Chang, K.-M.: Robust Synchronization of Drive-Response Chaotic Systems via Adaptive Sliding Mode Control. Chaos, Solitons and Fractals, Vol. 31, 2007, No. 2, pp. 321–324.

[14] Nussbaum, R. D.: Some Remarks on the Conjecture in Parameter Adaptive Control. Systems and Control Letters, Vol. 3, 1983, No. 3, pp. 243–246.

[15] Ye, X. D.—Jiang, J. P.: Adaptive Nonlinear Design without a Priori Knowledge of Control Directions. IEEE Transactions on Automatic Control, Vol. 43, 1998, No. 11, pp. 1617–1621.

[16] Ge, S. S.—Hong, F.—Lee, T. H.: Adaptive Neural Control of Nonlinear Time-Delay System with Unknown Virtual Control Coefficients. IEEE Transactions on Systems, Man, and Cybernetics, Part B – Cybernetics, Vol. 34, 2004, No. 1, pp. 499–516.

[17] Li, Y.Chen, Y. Q.: When Is a Mittag Leffler Function a Nussbaum Function? Automatica, Vol. 45, 2009, No. 1, pp. 1957–1959.

[18] Ge, S. S.—Wang, J.: Robust Adaptive Tracking for Time-Varying Uncertain Nonlinear Systems With Unknown Control Coefficients. IEEE Transactions on Automatic Control, Vol. 48, 2003, No. 8, pp. 1463–1469.

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