Educational Simulator for Teaching of Particle Swarm Optimization in LabVIEW

Perangkat Lunak Edukatif Berbasis LabVIEW sebagai Alat Bantu Pengajaran Particle Swarm Optimization

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Abstract
This paper presents an educational software tool for aid the teaching of Particle Swarm Optimization (PSO) fundamentals with friendly design interface. This software were developed in the platform of LabVIEW (Laboratory Virtual Intrumentation Engineering Workbench). The software's best qualities are users can select many different version of the PSO algorithm, a lot of the benchmarks test functions for optimization and set the parameters that have an influence on the PSO performance. Through visualization of particle distribution in the searching, the simulator is particularly effective in providing users with an intuitive feel for the PSO algorithm.

Keyword *:* Benchmark Function, LabVIEW, Particle Swarm Optimization, PSO Algorithm

Abstrak
Paper ini mempresentasikan perancangan perangkat lunak pendidikan sebagai alat bantu pengajaran materi Particle Swarm Optimization dengan desain antarmuka yang mudah digunakan. Perangkat lunak ini dibuat berbasis LabVIEW (Laboratory Virtual Intrumentation Engineering Workbench). Keunggulan dari perangkat lunak ini adalah pengguna mendapat pilihan banyak versi yang berbeda dari algoritma PSO, terdapat banyak fungsi yang bisa digunakan untuk menguji proses optimasi dan dapat memodifikasi parameter-parameter yang akan mempengaruhi performansi dari PSO. Dengan adanya visualisasi dari pergerakan particle, perangkat lunak ini akan efektif untuk memberikan pemahaman mengenai prinsip algoritma PSO kepada pengguna.

***Kata Kunci** – Fungsi benchmark, LabVIEW, Particle Swarm Optimization, Algoritma PSO*

I. INTRODUCTION

Optimization problems are widely encountered in various fields in technology. Some problems can be very complex due to the actual and practical nature of the objective function or the model constraints. With the fast development of industrial applications, optimization algorithms encounter more and more challenges. Optimization algorithms can be classified into classical optimization based on gradient of objective functions (e.g. Steepest Descent, Conjugate Gradient Algorithm and Newton Algorithm) and heuristic optimization algorithms (e.g. Genetic Algorithms (GA), Simulated Annealing (SA) and Particle Swarm Optimization (PSO)). For some optimization problems, there is no explicit analytical formula, so the gradient information cannot be gained. And for high dimension of problem, the classical algorithms are sometimes not satisfying. Hence, population-bases heuristic optimization algorithms, which not require the derivative information of objective functions and return a set of solutions at each iteration, are more convenient for solving these kinds of problems.

In 1995, Kennedy and Eberhart suggested a PSO based on the analogy of swarm of bird and school of fish (J. Kennedy et al., 1995). In PSO, each individual makes his decision using his own experience together with other individuals experiences. The individual particles are drawn stochastically toward the present velocity of each individual, their own previous best performance and the best previous performance of their neighbours.

This paper would introduce an educational simulator for the PSO algorithm. The purpose of this simulator is to provide the users with useable tool for gaining an intuitive feel for PSO algorithm and mathematical optimization problems. To aid the understanding PSO, the simulator has been developed under the user-friendly graphic user interface (GUI) environment using LabVIEW. In this simulator, the users can select many different version of the PSO algorithm and set parameters related to the performance of PSO and can observe the impact of the parameters to the solution quality. This simulator also displayed the movements of each particle and convergence process of a group. This educational simulator is used in Artificial Intelligent and Intelligent Control course of Electrical Engineering and Computer Engineering Graduate Program at Universitas Komputer Indonesia.

Some researchers have integrated PSO into different kinds of toolboxes. Lee and Park developed Educational Simulator for Particle Swarm Optimization and Economic Dipatch Applications based on MATLAB (W. N. Lee et al., 2011). Coelho and Sierakowski developed A Software tool for teaching particle swarm optimization fundamental based on MATLAB (L.d.S. Coelhu et al., 2008). Qi, Hu and Cournede developed a particle swarm optimization in Scilab (R.Qi et al., 2009). However, most of them are implemented in MATLAB or Scilab. As far as the author concerns, there is no work on development of educational simulator for particle swarm optimization based on LabVIEW. This toolbox can be widely used, not simply as a ‘black box’, but also as a basis to understand the principles of optimization algorithms by making it possible for user to easily read, change or tune algorithms and algorithm parameters.

The paper is arranged as follows. Overview of particle swarm optimization are descibed in Section II. In Section III The features of educational PSO simulator are introduced. Mathematical optimization problems is presented in Section IV. Its performance is analyzed in Section V. Finally, the conclusion is given in Section VI.

II. PARTICLE SWARM OPTIMIZATION

PSO is a kind of heuristic optimization algorithms. It is motivated from simulating certain simplified animal social behaviors such as bird flocking. It is an iterative and population-based method. The particles are descibed by their two instinct properties: position and velocity. The position of each particle represents a point in the parameter space, which a possible solution of the optimization problem, and the velocity is used to change the position.

A. The Original Particle Swarm Optimization Algorithm

In order to optimize an unconstrained d-dimensional objective function $f : \mathbb{R}^d \to \mathbb{R}$, the original PSO algorithm [J. Kennedy, et al. (1995)] is initialized with a population of complete solutions (called particles) $\{p_1, \dots, p_k\} = \mathcal{P}$ are randomly initialized in the solution space. The objective function determines the quality of a particle’s position, that is, the quality of the solution it represents.

Each particle p_i at time step t has a position vector $\vec{x_i}^t$ and a associated velocitiy vector $\vec{v_i}^t$. Every particle “remembers” the position in which it has received the best evaluation of the objective function. This memory is represents by vector $pb\vec{e}st_i$. This vector is updated every time particle p_i finds a better position. At the swarm level, the vector $gb\vec{e}st$ stores the best position any particle has ever visited.

The algorithm iterates updating particles velocity and position until a stopping criterion is met, usually a maximum number of iterations or a sufficiently good solution. The update rules are :

$$\vec{v_i}^{t+1} = \vec{v_i}^t + \varphi_1 \cdot \vec{U}_1(0,1) * (pb\vec{e}st_i - \vec{x_i}^t) + \varphi_2 \cdot \vec{U}_2(0,1) * (gb\vec{e}st - \vec{x_i}^t) \quad (1)$$

$$\vec{x_i}^{t+1} = \vec{x_i}^t + \vec{v_i}^{t+1} \quad (2)$$

Where φ_1 and φ_2 are two constant called the cognitive and social acceleration coefficients repectively, $\vec{U}_1(0,1)$ and $\vec{U}_2(0,1)$ are two d-dimensional uniformly distributed random vector (generated every iteration) in which each component goes from zero to one, and $*$ is an element-by-element vector multiplication operator. The values of φ_1 and φ_2 are parameters of the algorithm.

In the original PSO algorithm a particle has two attractors: its own previous best position and the swarm's global best position. Previous experience with population-based optimization algorithms dictated that a strong bias towards the best solution so far may lead to premature convergence. Therefore, the local version of the PSO algorithm was devised.

The variants we include in this study selected either because they are among the most commonly used in the field or because they look promising.

B. Local Particle Swarm Optimizer

An early variant of the original PSO algorithm was proposed by Eberhart and Kennedy [R. Eberhart, et al. (1995)] in which a particle does not accelerate towards the swarm's global best solution. Instead, it accelerates towards the best solution found within its local *topological* neighborhood. A particle p_i has a topological neighborhood $\mathcal{N}_i \subseteq \mathcal{P}$ ($\mathcal{P} = \{p_1, \dots, p_k\}$ is the set of particles in the swarm) of particles.

In the local PSO algorithm, Equation (2) becomes

$$\vec{v_i}^{t+1} = \vec{v_i}^{t} + \varphi_1 \cdot \vec{U}_1(0,1) * (pb\vec{e}st_i - \vec{x_i}^{t}) + \varphi_2 \cdot \vec{U}_2(0,1) * (lb\vec{e}st_i - \vec{x_i}^{t}) \tag{3}$$

Where vector $lb\vec{e}st_i$ stores the best position in the neighborhood has ever visited. Mohais et al. [A. Mohais, et.al. (2005)] reported that random topologies have the same or even better performance than nonrandom topologies.

C. Canonical Particle Swarm Optimizer

Clerc and Kennedy [M. Clerc, et al. (2002)] introduced a constriction factor into the velocity update rule of the original PSO algorithm. The purpose of this factor is to avoid particles velocities to increase towards infinity and to control the convergence properties of the particles.

This constriction factor is added to Equation (3) giving

$$\vec{v_i}^{t+1} = \mathcal{X} \cdot \left(\vec{v_i}^{t} + \varphi_1 \cdot \vec{U}_1(0,1) * (pb\vec{e}st_i - \vec{x_i}^{t}) + \varphi_2 \cdot \vec{U}_2(0,1) * (lb\vec{e}st_i - \vec{x_i}^{t})\right) \tag{4}$$

with

$$\mathcal{X} = \frac{2 \cdot k}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \tag{5}$$

where $k \in [0,1]$, $\varphi = \varphi_1 + \varphi_2$ and $\varphi > 4$. Usually, k is set to 1 and both φ_1 and φ_2 are set to 2.05, giving as a results $\mathcal{X}$ equal to 0.729.

D. Time-Varying Decreasing Inertia Weight Particle Swarm Optimizer

Shi and Eberhart [Y. Shi, et al. (1999)] introduced the idea of a control factor called *inertia weight* to control the diversification-intensification behavior of the original PSO. The velocity update rule was modified as follows

$$\vec{v_i}^{t+1} = w(t) \cdot \vec{v_i}^{t} + \varphi_1 \cdot \vec{U}_1(0,1) * \left(pb\vec{e}st_i - \vec{x_i}^{t}\right) + \varphi_2 \cdot \vec{U}_2(0,1) * \left(lb\vec{e}st_i - \vec{x_i}^{t}\right) \tag{6}$$

where $w(t)$ is the inertia weight which is usually a time-dependent function. φ_1 and φ_2 are set to 2.

Since we want the algorithm to explore the search space during the first iterations and focus on the promising regions afterwards, $w(t)$ should be a time-decreasing function of time.

The function used to schedule the inertia weight is defined as

$$w(t) = w_{max} - \frac{(w_{max} - w_{min}) \cdot t}{t_{max}} \tag{7}$$

where t_{max} marks the time at which $w(t) = w_{min}$, w_{max} and w_{min} are the maximum and minimum values the inertia weight can take. The most widely used approach, is the one that uses a decreasing inertia weight with a starting value of 0.9 and 0.4 as the final one.

E. Time-Varying Increasing Inertia Weight Particle Swarm Optimizer

Zheng [Y.L. Zheng, et al. (2003)] studied the effects of using a time-increasing inertia weight function showing also that, in some cases, it provides a faster convergence rate.

The function used to schedule the inertia weight is defined as

$$w(t) = w_{min} - \frac{(w_{min} - w_{max}) \cdot t}{t_{max}} \quad (8)$$

Zheng et al., use increasing inertia weight with a starting value of 0.4 and the final value 0.9.

F. Time-Varying Stochastic Inertia Weight Particle Swarm Optimizer

Eberhart and Shi [R. Eberhart, et al. (2001)] proposed another variant in which the inertia weight is randomly selected according to a uniform distribution in the range [0..5, 1.0]. This range was inspired by Clerc and Kennedy's constriction factor because the expected value of the inertia weight in this case in 0.75 ≈ 0.729.

G. Fully Informed Particle Swarm Optimizer

Mendes et al. [R. Mendes, et. Al. (2004)] proposed the fully informed particle swarm, in which a particle uses information provided by all its neighbors in order to update its velocity.

The new velocity update equation becomes

$$\vec{v}_i^{t+1} = \mathcal{X}\left[\vec{v}_i^{t} + \sum_{\mathcal{P}_k \in \mathcal{N}_i} \varphi_k \cdot \mathcal{W}(pb\vec{e}st_k) \cdot \vec{U}_k(0,1) * \left(pb\vec{e}st_k - \vec{x}_i^{t}\right)\right] \quad (9)$$

where $\mathcal{N}_i$ is the neighborhood of particle i, $\mathcal{W}(pb\vec{e}st_k)$ is a weighting function. The goal of $\mathcal{W}(pb\vec{e}st_k)$ is to provide information about the quality of the attractor $pb\vec{e}st_i$. For example, the normalized objective function value of the vector $pb\vec{e}st_i$ could serve well.

H. Self Organizing Hierarchical Particle Swarm Optimizer with Time-varying Acceleration Coefficients

Proposed by Ratnaweera [A. Ratnaweera, et al. (2004)], in HPSOTVAC, if any component of a particle's velocity vector becomes zero, it is reinitialized to a value proportional to the maximum allowable velocity V_{max}. To amplify the local search behaviour of the swarm, HPSOTVAC linearly adapts the value of the acceleration coefficients φ_1 and φ_2. The cognitive coefficient, φ_1, is decreased from 2.5 to 0.5 and the social coefficient, φ_2, is increased from 0.5 to 2.5.

To avoid the problem of setting a proper reinitialization velocity, HPSOTVAC linearly decreases it from V_{max} at the beginning of the run to $0.1 \cdot V_{max}$ at the end. As in the time-decreasing inertia weight variant, a low reinitialization velocity near the end of the run, allows particles to move slowly near the best region they found.

I. Hierarchical Particle Swarm Optimizer

In H-PSO, all particles are arranged in a hierarchy that defines the neighborhood structure. Each particle is neighbored to itslef and its parent in the hierarchy. In this research we used reguler tree like hierarchies. The hierarchy is defined by the *height* h, the *branching degree* d, the maximum number of children of the inner nodes and the total number of nodes m of the corresponding tree. We use only hierarchies in which all inner nodes have the same number of children, only the inner nodes on the deepest level might have a smaller number of children.

The new position of the particles within the hierarchy are determined as follows. For every particle j in a node of the tree, its own best solution is compared to the best solution found by the particles in the child nodes. If the best of these particles is better, then child particles and parents particles swap their place within the hierarchy.

For the update of the velocities in AH-PSO, a particle is influenced by its own so far best position and by the best position of the individual that is directly above in the hierarchy.

Similar as in PSO, after the particles velocities are updated and after the particles have moved in H-PSO, the objective function is evaluated at the new position. If the function value at this position is better than the function value at the personal best position, the personal best position is update.

J. Adaptive Hierarchical Particle Swarm Optimizer

Proposed by Janson and Middendorf [S. Janson, et al. (2005)], In the Adaptive H-PSO (AH-PSO) , the branching degree is gradually decreased during a run of the algorithm by k_{adapt} degrees until a certain minimum degree d_{min} is reached. This process takes place every f_{adapt} number of iterations

K. Estimation of Distribution Particle Swarm Optimizer

Proposed by Iqbal, [M. Iqbal, et al.], the EDPSO borrows some ideas from $ACO_{\mathbb{R}}$. EDPSO works as a canonical PSO but with some modifications : after the execution of the velocity update rule shown in Equation (4), EDPSO selects one Gaussian function. Then, the selected Gaussian function is evaluated to probabilistically move the particle to its new position. If the movement is successful, the algorithm continues as usual, but if the movement is unsuccessful, then the selected Gaussian function is sampled in the same way as $ACO_{\mathbb{R}}$.

III. EDUCATIONAL PSO SIMULATOR

This educational PSO simulator can solve maximization or minimization problems without transforming the formulas of optimization problems. It can show convergence curve in real-time and particle distribution in the searching space. This educational PSO simulator considers different PSO algorithm. The variants of PSO algorithm integrated into this educational PSO simulator are listed in **Table 2**.

Table 2. Variants of PSO algorithm integrated into simulator

Algorithm	Reference
Original PSO	J. Kennedy, et al. (1995)
Local PSO	R. Eberhart, et al. (1995)
Canonical PSO	M. Clerc, et al. (2002)
Decreasing Inertia Weight PSO	Y. Shi, et al. (1999)
Increasing Inertia Weight PSO	Y.L. Zheng, et al. (2003)
Stochastic Inertia Weight PSO	R. Eberhart, et al. (2001)
Fully Informed PSO	R. Mendes, et. Al. (2004)
Self-Organizing Hierarchical PSO with Time-Varying Acceleration Coefficients	A. Ratnaweera, et al. (2004)
Hierarchical PSO	C.-C. Chen (2009)
Adaptive Hierarchical PSO	S. Janson, et al. (2005)
Estimation of Ditribution PSO	M. Iqbal, et al

The educational PSO simulator are arranged in three layers : Functions Selection, 3D Function Display dan Optimization Process. In the Functions Selection Layers, the user can select the mathematical function problem. **Figure 1** show the main view of the Educational PSO Simulator.

At least there are 100 function available which user can select. **Figure 2** show the Function Selection Layers. In the left of the front panel is the list of mathematical function available to select.

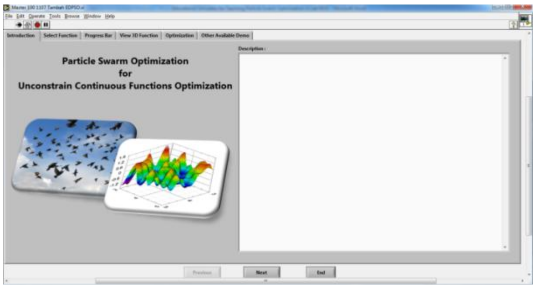


Figure 1. Front Panel of Educational PSO Simulator

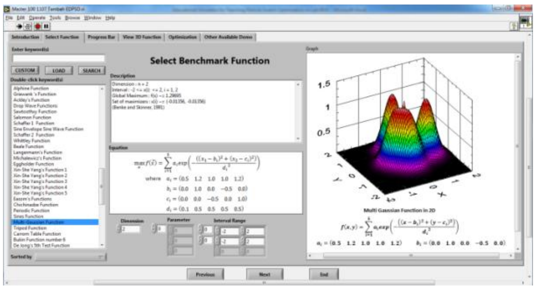


Figure 2. Functions Selection Layers

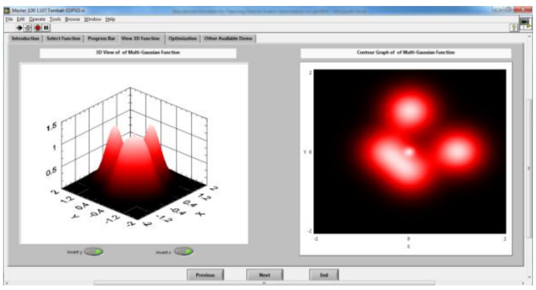


Figure 3. 3D Function Display Layers

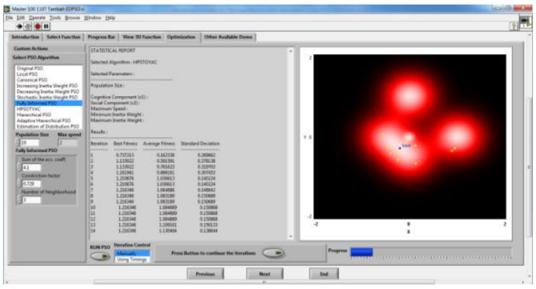


Figure 4. Optimization Process Layers

3D Function Display Layers is used to display the function in 3D view. The user can rotate the function to get better view of the problem. **Figure 3** show the 3D Function Display Layers of the Educational PSO Simulator.

The Optimization Process Layers is used to set the PSO algorithm, to set parameters that have an influence on the PSO performance and to visualization process of each particle. **Figure 4** show the Optimization Process Layers.

IV. MATHEMATICAL OPTIMIZATION PROBLEMS

Several kinds of function benckmark problems are chosen to demonstrate the performace of simulator. For the case study, we choose 5 mathematical examples : (i) Sphere function, (ii) Rosenbrock function, (iii) Ackley's function, (iv) Rastrigin function, (v) Griewank function.

The function and the range of input variables of Sphere Function are as follows :

$$\min_x f(\vec{x}) = \sum_{i=1}^{N} {x_i}^2 \qquad -5.12 \le x_i \le 5.12 \tag{10}$$

The function and the range of input variables of Schaffer Function are as follows :

$$\begin{gathered} \min_x f(\vec{x}) = 0.5 + \frac{a(\vec{x})}{b(\vec{x})} \\ a(\vec{x}) = sin^2\sqrt{{x_1}^2 + {x_2}^2} - 0.5 \\ b(\vec{x}) = \left(1 + 0.001({x_1}^2 + {x_2}^2)\right)^2 \\ -100 \le x_i \le 100 \end{gathered} \tag{11}$$

The function and the range of input variables of Ackley's Function are as follows :

$$\begin{gathered} \min_x f(\vec{x}) = a(\vec{x}) + b(\vec{x}) + 20 + e^1 \\ a(\vec{x}) = -20 \cdot exp\left(-0.2\sqrt{\frac{1}{N}\sum_{i=1}^{N} {x_i}^2}\right) \\ b(\vec{x}) = -exp\left(\frac{1}{N}\sum_{i=1}^{N} \cos(2\pi \cdot x_i)\right) \\ -32.768 \le x_i \le 32.768 \end{gathered} \tag{12}$$

The function and the range of input variables of Rastrigin Function are as follows :

$$\begin{gathered} \min_x f(\vec{x}) = 10N + \sum_{i=1}^{N} ({x_i}^2 - \\ 10 \cos\ (2\pi x_i)) \\ -5.12 \le x_i \le 5.12 \end{gathered} \tag{13}$$

The function and the range of input variables of Shekel's Foxholes Function are as follows :

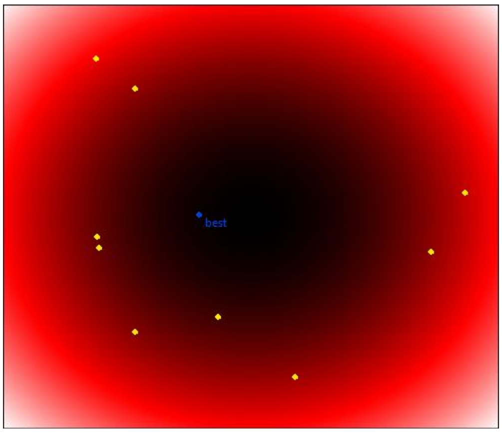
$$\begin{gathered} \min_x f(\vec{x}) = \frac{1}{0.002 + g(\vec{x})} \\ g(\vec{x}) = \sum_{j=1}^{25} \frac{1}{j + (x_1 - a_{1j})^6 + (x_2 - a_{2j})^6} \\ (a_{ij}) = \begin{pmatrix} -32 & -16 & 0 & \dots & 32 \\ -32 & -32 & -32 & \dots & 32 \end{pmatrix} \\ -65.536 \le x_i \le 65.536 \end{gathered} \tag{14}$$

V. EXAMPLES

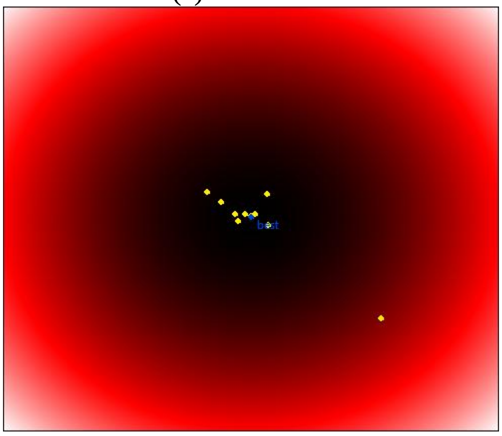
In this section, Simulator is run in Windows 7 platform in the version of LabVIEW 7.

A. Visualization Process

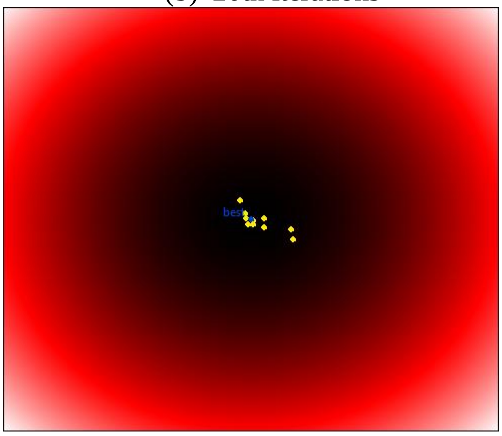
The series picture in **Figure 5** shows a run of the Fully Informed Particle Swarm on the Sphere Function. Specifically it shows the 0th, 20th, 40th, 60th, 80th and 100th iterations of the run. As the particles continue in the run, they reach lower and lower fitness values (signified by an inceased amount of yellow dot) thus minimizing the function. **Figure 5** also displayed the convergence process of a group.



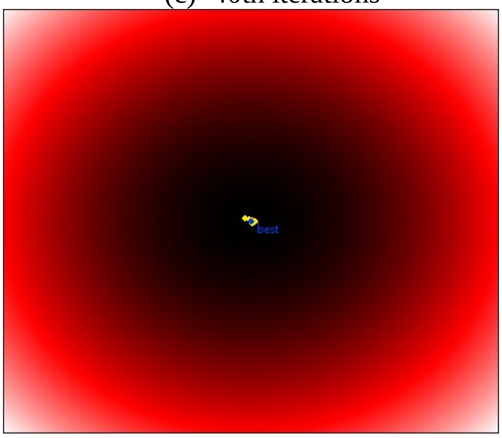
(a) 0th iterations



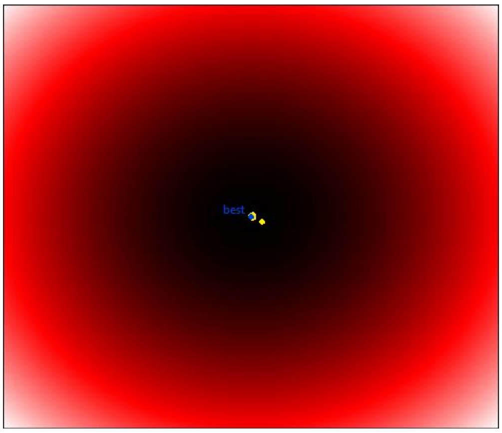
(b) 20th iterations



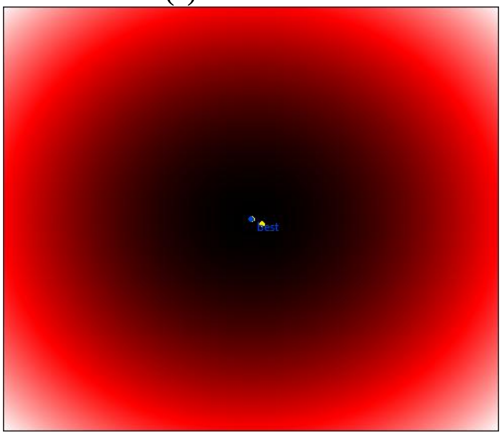
(c) 40th iterations



(d) 60th iterations



(e) 80th iterations



(f) 100th iterations

Figure 5. A run of Simulator on the Sphere FUnction

B. Parameter Settings

We decide to test the particle swarm optimizers that we included in our study without tuning their set of parameters specifically for each test problem. All algorithms were run with the same set of parameters over all test problems. The specific parameter settings were those that are normally used in the literature. **Table 4** lists the algorithms fixed parameter settings that we use in our experiments.

The maximum number of evaluations to find a solution was set to 100. We ran the algorithms 100 times on each problem. Maximum velocity V_{max} is clamped to $\pm X_{max}$ where X_{max} is the maximum of the search range.

Table 4. Algorithm fixed parameter settings

Algorithm	**Settings**
Original PSO	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05
Local PSO	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Number of Neighborhood = 3

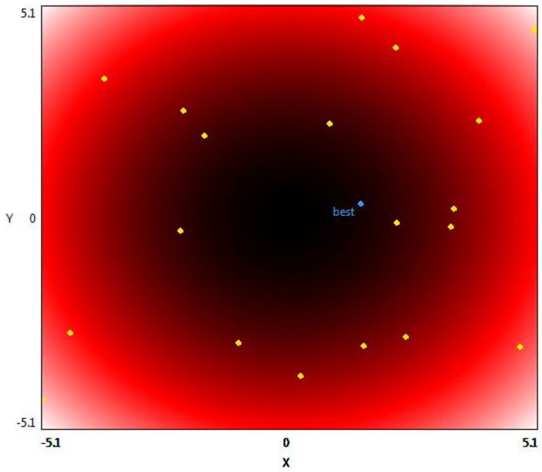
Canonical PSO	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Constriction factor ($\mathcal{X}$) = 0.729 Number of Neighborhood = 3
Decreasing Inertia Weight	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Initial inertia weight = 0.9 Final inertia weight = 0.4 Number of Neighborhood = 3
Increasing Inertia Weight	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Final inertia weight = 0.9 Initial inertia weight = 0.4 Number of Neighborhood = 3
Stochastic Inertia Weight	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Minimum inertia weight = 0.4 Maximum inertia weight = 0.9 Number of Neighborhood = 3
Fully Informed PSO	Sum of the acc. coeff. (φ) = 4.1 Constriction factor ($\mathcal{X}$) = 0.729 Number of Neighborhood = 3
Self Organizing Hierarchical PSO	Initial value of φ_1 = 2.5 Final value of φ_1 = 0.5 Initial value of φ_2 = 0.5 Final value of φ_2 = 0.5 Number of Neighborhood = 3
Hierarchical PSO	Cognitive component (φ_1) = 2 Social component (φ_2) = 2 w = 0.9 r = 0.95 height of the tree (h) = 3 branching degree (d) = 4
Adaptive Hierarchical PSO	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Constriction factor ($\mathcal{X}$) = 0.729 Initial Branching factor = 20 d min = 2 f adapt/m = 10 k adapt = 3
Estimation of Distribution PSO	Cognitive component (φ_1) = 2.05 Social component (φ_2) = 2.05 Constriction factor ($\mathcal{X}$) = 0.729 q = 0.1 epsilone = 0.85

C. Convergence Results

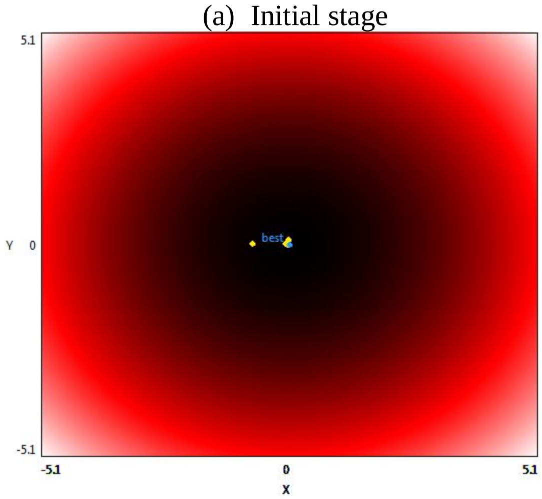
Two input variables (i.e., 2-dimensional space) have been set in order to show the movement of particles on contour. 30 independent trials are conducted to observe the variation during the evoltionary processes and compare the solution quality and convergence characteristics.

To successfully implement the PSO, some parameters must be assigned in advance. The population size NP is set to 20. Since the performance of PSO depends on its parameters such as inertia weight or acceleration coefficients, it is very important to determine the suitable values of parameters.

Initial and final stages of the optimization process for the Sphere function are shown if **Figure 6**.



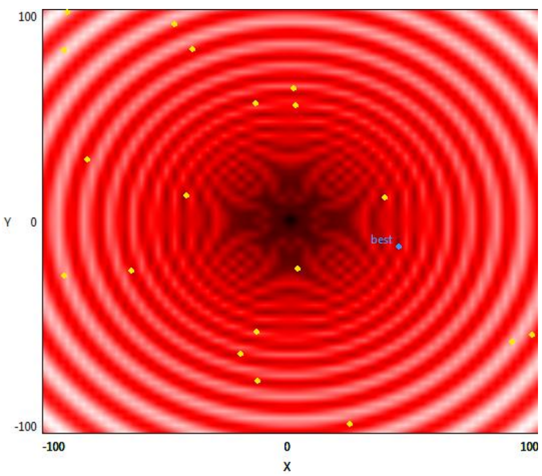
(a) Initial stage



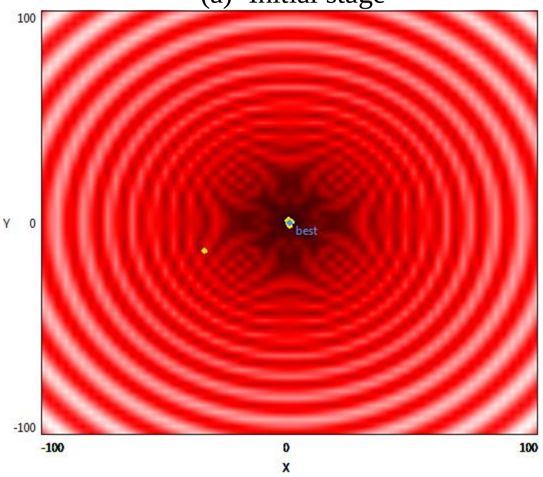
(b) Final stage

Figure 6. Optimization process for the Sphere Function

Initial and final stages of the optimization process for the Schaffer function are shown if **Figure 7**.



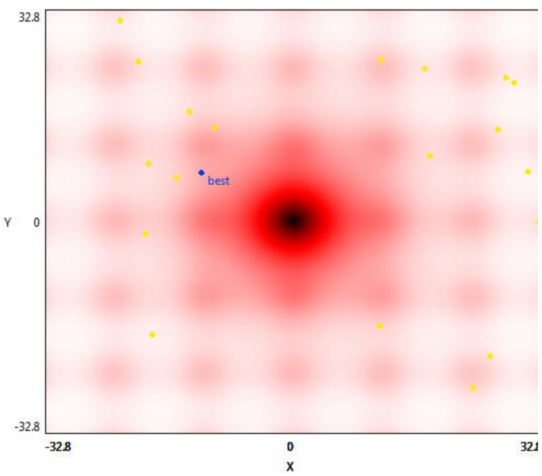
(a) Initial stage



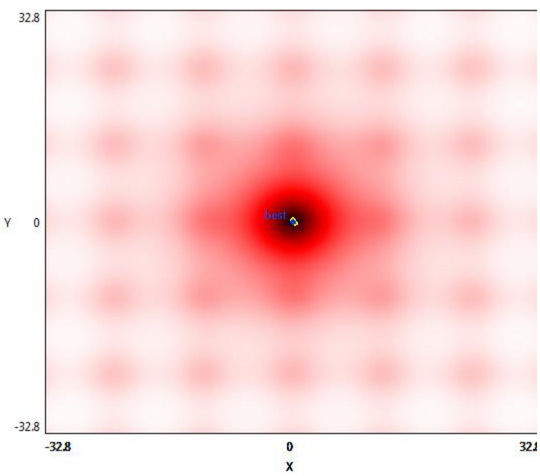
(b) Final stage

Figure 7. Optimization process for the Schaffer Function

Initial and final stages of the optimization process for the Ackley's function are shown if **Figure 8**.



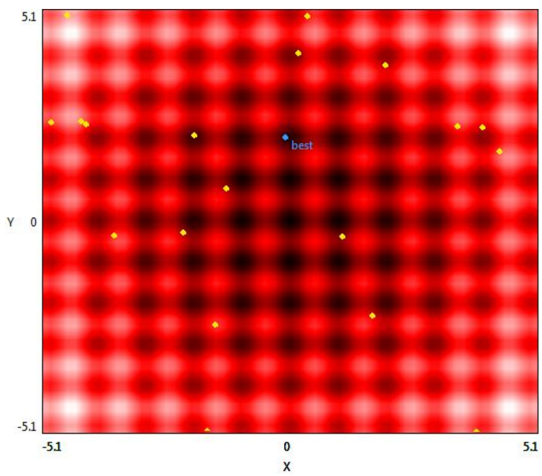
(a) Initial stage



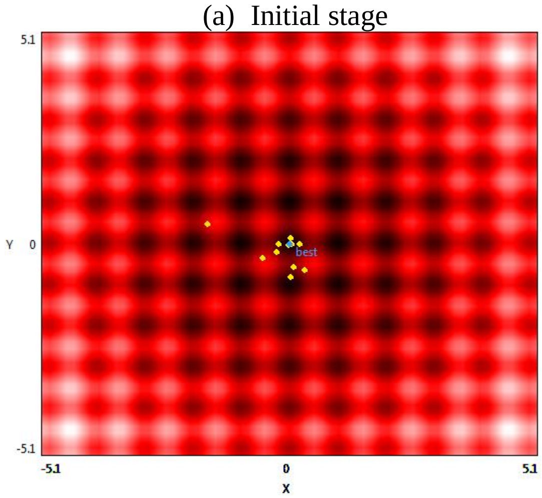
(b) Final stage

Figure 8. Optimization process for the Ackley's Function

Initial and final stages of the optimization process for the Rastrigin function are shown if **Figure 9**.



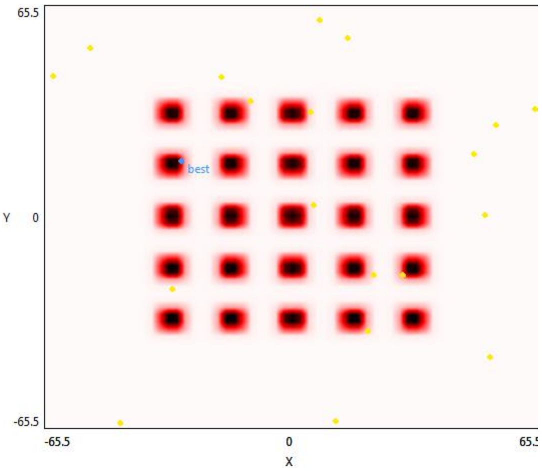
(a) Initial stage



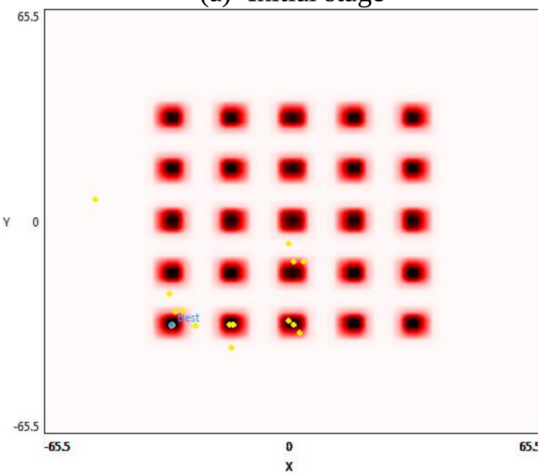
(b) Final stage

Figure 9. Optimization process for the Rastrigin Function

Initial and final stages of the optimization process for the Shekel's Foxholes function are shown if **Figure 10**.



(a) Initial stage



(b) Final stage

Figure 10. Optimization process for the Shekel's Foxholes Function

D. Optimization Results

Table 5 – Table 15 shows the summary result of 10 runs using each PSO algorithm.

Table 5. Summary of result of 10 runs using **Original PSO**

Function	Mean	Min	Max
Sphere	0.001000	0.000000	0.006000
Schaffer	0.382000	0.000000	1.910000
Ackley	0.355000	0.000000	1.965000
Rastrigin	0.000000	0.000000	0.000000
Shekel Foxholes	2.499000	1.001000	3.968000

Table 6. Summary of result of 10 runs using **Local PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.401000	0.000000	1.798000
Ackley	0.337000	0.000000	2.397000
Rastrigin	0.164000	0.000000	1.049000
Shekel Foxholes	2.066000	0.998000	5.914000

Table 7. Summary of result of 10 runs using **Canonical PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.036000	0.000000	0.057000
Ackley	0.001000	0.000000	0.002000
Rastrigin	0.122000	0.000000	0.995000
Shekel Foxholes	5.394000	0.998000	16.441000

Table 8. Summary of result of 10 runs using **Decreasing Inertia Weight PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.122000	0.000000	0.313000
Ackley	0.005000	0.001000	0.013000
Rastrigin	0.119000	0.000000	1.013000
Shekel Foxholes	2.965000	0.998000	10.763000

Table 9. Summary of result of 10 runs using **Increasing Inertia Weight PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.134000	0.055000	0.314000
Ackley	0.004000	0.000000	0.013000
Rastrigin	0.129000	0.000000	0.996000
Shekel Foxholes	1.832000	0.998000	3.975000

Table 10. Summary of result of 10 runs using **Stochastic Inertia Weight PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.348000	0.000000	0.864000
Ackley	0.110000	0.000000	0.693000
Rastrigin	0.108000	0.000000	0.436000
Shekel Foxholes	2.066000	0.998000	4.101000

Table 11. Summary of result of 10 runs using **Fully Informed PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.105000	0.000000	0.298000
Ackley	0.002000	0.000000	0.005000
Rastrigin	0.100000	0.000000	0.996000
Shekel Foxholes	3.271000	0.998000	7.874000

Table 12. Summary of result of 10 runs using **HPSOTYAC**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.062000	0.016000	0.104000
Ackley	0.001000	0.000000	0.004000
Rastrigin	0.199000	0.000000	0.995000
Shekel Foxholes	4.236000	0.998000	12.671000

Table 13. Summary of result of 10 runs using **Hierarchical PSO**

Function	Mean	Min	Max
Sphere	0.000009	0.000000	0.000037
Schaffer	1.308884	0.998004	2.775963
Ackley	0.061050	0.011576	0.138272
Rastrigin	0.081365	0.006938	0.239305
Shekel Foxholes	1.073859	0.998013	1.461168

Table 14. Summary of result of 10 runs using **Adaptive Hierarchical PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.059000	0.015000	0.128000
Ackley	0.001000	0.000000	0.006000
Rastrigin	0.299000	0.000000	0.995000
Shekel Foxholes	3.530000	0.998000	15.504000

Table 15. Summary of result of 10 runs using **Estimation of Distribution PSO**

Function	Mean	Min	Max
Sphere	0.000000	0.000000	0.000000
Schaffer	0.062000	0.006000	0.128000
Ackley	0.000000	0.000000	0.002000
Rastrigin	0.241000	0.000000	0.997000
Shekel Foxholes	1.595000	0.998000	3.968000

VI. CONCLUSION

This paper presents an educational simulator for particle swarm optimization (PSO) and application for solving mathematical test functions. Using this simulator, instructor and students can select the test functions for simulation and set the parameters that have an influence on the PSO performance. Through visualization process of each particle and variation of the value of objective function, the simulator is particularly effective in providing users with an intuitive feel for the PSO algorithm. This simulator is expected to be an useful tool for students.

In the present version of educational simulator of PSO, only unconstrained optimization problems can be solved. The PSO algorithms for constrained optimization problems will be integrated to simultor soon.

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Appendix A. A Collection of Benchmark Optimization Test Problems

No	Name	Source	No. Of Vars.	Upper Bound, Lower Bound	Global Optimum
1.	Ackley Function	Storn & Price (1997)	1 – N	(-32.768, 32.768)	0
2.	Alphine Function	R. Thangaraj et al. (2012)	1 – N	(-10, 10)	0
3.	Aluffi-Pentini Function	Aluffi-Pentiti et al. (1985)	2	(-10,10)	-0.3523
4.	Banana Shape Function		2	(-2, 2)	-25
5.	Beale Function	Ernesto P. Adorio (2005)	2	(-4.5, 4.5)	0
6.	Becker & Lago Function	Price (1997)	2	(-10, 10)	0
7.	Bird Function		2	(-20, 20)	-106.764536
8.	Bohachevsky 1 Function	Bohachevsky et al. (1986)	2	(-50, 50)	0
9.	Bohachevsky 2 Function	Bohachevsky et al. (1986)	2	(-50, 50)	0
10.	Booth Function	Ernesto P. Adorio (2005)	2	(-10, 10)	0
11.	Box and Betts Function	Kaj Madsen (2000)	3	(0.9, 1.2); (9, 11.2); (0.9, 1.2)	0
12.	Branin RCOS Function	Dixon and Szego (1978)	2	(-5, 10); (0, 15)	0.397887357
13.	Bukin Function		2	(-20, 20)	0
14.	Camel Function		2	(-2, 2)	-7.0625
15.	3-Hump Camel Function	Dixon and Szego (1975)	2	(-5, 5)	0
16.	6-Hump Camel Function	Dixon and Szego (1975)	2	(-5, 5)	-1.03162845
17.	Carrom Table Function	Rody P. S. O. (2009)	2	(-10, 10)	-24.1568155
18.	Chichinadze Function	Rody P. S. O. (2009)	2	(-30, 30)	-42.9443870
19.	Colville Function	Ali, M. et. Al (2009)	4	(0, 10)	0
20.	Corana's Parabola Function	Corana (1987)	4	(-1000, 1000)	0
21.	Cosine Mixture Function	Breiman and Culter (1993)	1 – N	(-1, 1)	0.2 for N = 2 0.4 for N = 4
22.	Cross-in-tray Function	Rody P. S. O. (2009)	2	(-10, 10)	-2.06261187
23.	Cross-leg table Function	Rody P. S. O. (2009)	2	(-10, 10)	-1
24.	Crowned Cross Function	Rody P. S. O. (2009)	2	(-10, 10)	0.0001
25.	Deceptive Functions	Marcin Molga (2005)	2 - N	(0, 1)	
26.	Dekkers and Aarts Function	Dekkers and Aarts (1991)	2	(-10, 10)	-24777
27.	Dixon and Price Function		1 – N	(-10, 10)	
28.	Drop Wave Functions	Marcin Molga (2005)	2	(-5.12, 5.12)	-1
29.	Easom's Functions	Michalewicz (1996)	2 – N	(-60, 60)	-1
30.	Eggholder Function	Ernesto P. Adorio (2005)	2 – N	(-512, 512)	-959.64 for N = 2
31.	Epistatic Michalewicz Function	Second ICEO	5, 10	(0, pi)	-4.687658 for N = 5
32.	Exponential Function	Breiman and Culter (1993)	1 – N	(-1, 1)	1
33.	Giunta Function	Rody P. S. O. (2009)	2	(-1, 1)	0,0644704205
34.	Goldstein-Price's function	Dixon and Szego (1978)	2	(-2, 2)	3
35.	Griewank 's Function	Griewank (1981)	1 – N	(-600, 600)	0
36.	Gulf Research Problem	Himmelblau (1972)	3	(0.1, 100)	0
37.	Hartman 3 Problem	Dixon and Szego (1978)	3	(0, 1)	-3.86278214782
38.	Hartman 6 Problem	Dixon and Szego (1978)	6	(0,1)	-3.32236801141
39.	Helical Valley Problem	Wolfe (1978)	3	(-10, 10)	0
40.	Himmelblau Function	Ernesto P. Adorio (2005)	2	(-6, 6)	0
41.	Holder Table Function	Rody P. S. O. (2009)	2	(-10, 10)	-19.2085025678
42.	Hosaki Function	Benke and Skinner (1991)	2	(0, 6)	-2.3458
43.	Hyper-ellipsoid function	Storn & Price (1997)	1 – N	(-5.12, 5.12)	0
44.	Kowalik Function	Jansson and Knuppel (1995)	4	(0, 0.42)	0.00030748
45.	Kwon Function	Kwon (2003)	2	(-1, 1)	-16.0917200
46.	Langermann's Function	Marcin Molga (2005)	2	(0, 10)	-4.15581
47.	Levy and Montalvo 1 Function	Levy and Montalvo (1985)	1 – N	(-10, 10)	0
48.	Levy and Montalvo 2 Function	Levy and Montalvo (1985)	1 – N	(-10, 10)	0
49.	Lyapunov Exponents	Wang (2001)	2	(20, 30)	
50.	Matyas Function	Ernesto P. Adorio (2005)	2	(-10, 10)	0
51.	McCormik Function	McCormik (1982)	2	(-1.5, 4), (-3, 3)	-1.9133
52.	Meyer and Roth Function	Wolfe, 1978	3	(-10, 10)	0.0004
53.	Michalewicz's Function	Second ICEO	2 – N	(0, pi)	-1.8013 for N = 2 -4.6876 for N = 5 -9.660 for N = 10
54.	Miele and Cantrell Function	Wolfe, 1978	4	(-1, 1)	0

55.	Modified Langerman Function	Second ICEO	5, 10	(0, 10)	-0.965
56.	Muller-Brown Surface Function	K. Muller, (1979)	2	(-1.5, 1)	-146. 699
57.	Multi-Gaussian Function	Benke & Skinner, (1991)	2	(-2, 2)	1.29695
58.	Multimod Function	Ernesto P. Adorio (2005)	1 – N	(-10, 10)	0
59.	Neumaier 2 Function	Neumaier (2003)	4	(0, 4)	0
60.	Neumaier 3 Function	Neumaier (2003)	2 – N	(-N^2, N^2)	-(N*(N+4)*(N-1))/6
61.	Odd Square Function	Second ICEO	2 – 20	(-15, 15)	-1.143833
62.	Paviani Function	Kaj Madsen, 2000	10	(2.001, 9.999)	-45.77847
63.	Peaks Function	Mathworks, 2011	2	(-4, 4)	-6.55113
64.	Penholder Function	Rody P. S. O. (2009)	2	(-11, 11)	-0.9635348327265
65.	Periodic Function	Price (1977)	2	(-10, 10)	0.9
66.	Perm Function	Xin-She Yang (2010)	2 - N	(-N, N)	0
67.	Perm0 Function	Xin-She Yang (2010)	2 - N	(-N, N)	0
68.	Powell's Quadratic Function	Wolfe (1978)	4	(-10, 10)	0
69.	Price's Transistor Modelling Function	Price (1977)	9	(-10, 10)	0
70.	Quartic Function	Ali, M. et. Al (2009)	2 – N	(-1.28, 1.28)	Random
71.	Rastrigin's Function	Torn and Zilinskas (1989)	1 – N	(-5.12, 5.12)	0
72.	Rosenbrock's Function	Schwefel (1995)	2 – N	(-2.048, 2.048)	0
73.	Rotated Hyper-Ellipsoid Function	Marcin Molga (2005)	1 – N	(-65.536, 65.536)	0
74.	Salomon Function	Salomon (1995)	2 – N	(-100, 100)	0
75.	Sawtoothxy Function	Mathworks, 2011	2	(-20, 20)	0
76.	Schaffer 1 Function	Michalewicz (1996)	2 – N	(-100, 100)	0
77.	Schaffer 2 Function	Michalewicz (1996)	2 – N	(-100, 100)	0
78.	Schwefel's Function	Muhlenbein (1991)	1 – N	(-500, 500)	-418.9829N
79.	Shekel 5 Function	Dixon and Szego (1978)	4	(0, 10)	-10.1531996790582
80.	Shekel 7 Function	Dixon and Szego (1978)	4	(0, 10)	-10.4029405668187
81.	Shekel 10 Function	Dixon and Szego (1978)	4	(0, 10)	-10.5364098166920
82.	Shekel's Foxholes	Marcin Molga (2005)	2	(-65.536, 65.536)	0.998004
83.	Shekel's Foxholes Function 5d	Dixon and Szego (1978)	5, 10	(0, 10)	-10.5046, fot N = 5
84.	Shubert's Function	Levy and Montalvo (1985)	2 – N	(-10, 10)	-186.7309, for N = 2
85.	Shubert's Function 2	R. Thangaraj et al. (2012)	2 – N	(-10, 10)	-24.062499
86.	Sinusoidal Function	Zabinsky (1992)	2 – N	(0, 180)	-(A+1), default A = 2.5
87.	Sphere Function	Storn & Price (1997)	1 – N	(-5.12, 5.12)	0
88.	Step Function	R. Thangaraj et al. (2012)	1 – N	(-1, 1)	0
89.	Sum of Different Power Functions	Marcin Molga (2005)	1 – N	(-1, 1)	0
90.	Storn's Tchebychev Function	Price (2002)	9	(-128, 128)	0
91.	Testtube holder function	Rody P. S. O. (2009)	2	(-10, 10)	-10.872299901558
92.	Trefethen4 Function	Ernesto P. Adorio (2005)	2	(-6.5, 6.5); (-4.5, 4.5)	-3.30686865
93.	Trid Function	M.J.Hirsch (2006)	2 – N	(-N^2, N^2)	-50 for N = 6 -210 for N = 10
94.	Tripod Function	Ali, M. et. Al (2009)	2	(-100, 100)	0
95.	Whitley Function	A. Georgieva (2009)	2 – N	(-100, 100)	0
96.	Wood Function	Wolfe (1978)	4	(-10, 10)	0
97.	Xin-She Yang's Function 1	Xin-She Yang (2010)	1 – N	(-2pi, 2pi)	0
98.	Xin-She Yang's Function 2	Xin-She Yang (2010)	1 – N	(-10, 10)	-0.6065
99.	Xin-She Yang's Function 3	Xin-She Yang (2010)	2 – N	(-20, 20)	-1 for beta = 15
100.	Xin-She Yang's Function 4	Xin-She Yang (2010)	1 – N	(-10, 10)	-1
101.	Xin-She Yang's Function 5	Xin-She Yang (2010)	2	(0, 10)	Random
102.	Zakharov Function	Xin-She Yang (2010)	2 – N	(-10, 10)	0
103.	Zettl Function	Rody P. S. O. (2009)	2	(-10, 10)	-0.0037912372204