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WEBIRA - Comparative Analysis of Weight Balancing Method

A. Krylovas, N. Kosareva, E.K. Zavadskas

Aleksandras Krylovas, Natalja Kosareva*,
Edmundas Kazimieras Zavadskas
Vilnius Gediminas Technical University
Sauletekio al. 11, LT-10223 Vilnius, Lithuania
aleksandras.krylovas@vgtu.lt, natalja.kosareva@vgtu.lt,
edmundas.zavadskas@vgtu.lt
*Corresponding author: natalja.kosareva@vgtu.lt

Abstract: The attributes weight establishing problem is one of the most important MCDM tasks. This study summarizes weight determining approach which is called WEBIRA (WEight Balancing Indicator Ranks Accordance). This method requires to solve complicated optimization problem and its application is possible by carrying out non trivial calculations. The efficiency of WEBIRA and other MCDM methods – SAW (Simple Additive Weighting) and EMDCW (Entropy Method for Determining the Criterion Weight) compared for 4 different data normalization methods. The results of the study revealed that more sophisticated WEBIRA method is significantly efficient for all considered numbers of alternatives. Efficiency of all methods decreases with increasing number of alternatives, but WEBIRA is still applicable, while application of other methods is impossible as the number of alternatives is greater than 11. WEBIRA is the least affected by the data normalization, while EMDCW is the most affected method.
Keywords: WEBIRA, SAW, EMDCW, multi-attribute decision making (MADM), entropy, KEMIRA.

1 Introduction

From the large diversity of MCDM methods some are very simple to use methods, and the other – complex, requiring more effort and computing resources. This article analyses the attributes weighting task, which is solved by different methods. One of the main and well known multiple criteria decision making (MCDM) methods is calculation of weighted averages of the the performance values of alternatives evaluated in terms of attributes (criteria):

$$S^{(j)}(W,R)=\sum_{i=1}^{n} w_i r_i^{(j)},\ j=1,2,\ldots,m, \tag{1}$$

here $W=(w_1,w_2,\ldots,w_n)$, $0\leqslant w_i\leqslant 1$, $\sum\limits_{i=1}^{n} w_i=1$ is vector of weights, n – number of attributes, $R=\begin{pmatrix} r_1^{(1)} & r_2^{(1)} & \ldots & r_n^{(1)} \\ r_1^{(2)} & r_2^{(2)} & \ldots & r_n^{(2)} \\ \ldots & \ldots & \ldots & \ldots \\ r_1^{(m)} & r_2^{(m)} & \ldots & r_n^{(m)} \end{pmatrix}$ – matrix of alternatives $j\in\{1,2,\ldots,m\}$ estimates, which elements $0\leqslant r_i^{(j)}\leqslant 1$ are obtained by certain measurements or expert assessments applying different calculation procedures, which are usually referred to as normalization methods (see [1, 2]). For example, in this article maximum method (Max), sums method (Sum), minmax method (MinMax) and vector normalization (Vctr) will be used.

The mentioned MCDM method is known as WSM (Weighted Sum Model). It was noticed by Churchman in 1954 [3] that "course of action that maximizes the expected total weighted efficiency (effectiveness) is optimum". This method proposed for solving MCDA problems in [4]. Zionts and Wallenius in 1976 [5] emphasysed that the basis of this method is the intuitive understanding that the "overall utility function is assumed to be implicitly a linear function, and more generally a concave function of the objective functions". Triantaphyllou in 2000 [6] noticed that in the maximization case, the best alternative is the one that yields the maximum total performance value (1). Though the method is rather old it is still relevant and frequently used, the articles with its applications appearing in solid academic journals nowadays. The article [7] proposes a modified version of the Weighted Sum Model that takes into account decision-maker preferences and provides a possibility of higher interactivity in the selection of the most suitable alternative. A mathematical model for a Dynamic Weighted Sum Method (DWSM) is presented in [8]. In Hwang and Yoon [9] this method was called Simple Additive Weighting (SAW). The name stuck, and today one can find a number of articles in which this method is actually called as SAW. Triantaphyllou in [10] drew attention to aggregating of benefit and cost criteria in four different MCDA methods. In [11] Simple Additive Weighting is proposed as a metamodel for other MCDA methods. Generalized SAW under fuzzy environment and the relative preference relation proposed in [12] to easily and quickly solve FMCDM problems. A new comprehensive overview of multiple attribute decision-making techniques and their applications is presented in [13]. In the review [14] of MCDM literature it was noted that the Analytic Hierarchy Process (AHP) in the individual tools and hybrid MCDM in the integrated methods were ranked as the first and second methods in use.

MacCrimmon in [15] noticed that 1) as the number of relevant attributes and alternatives increases, the ability of the decisionmaker to handle the problem decreases; 2) using a combination of MCDM methods frequently may be more feasible than using any one method separately. A study [16] presents a hybrid MCDM method combining SAW, Techniques for Order Preference by Similarity to an Ideal Solution (TOPSIS) and Grey Relational Analysis (GRA) techniques. The ranking results show that multiple MCDM methods are more trustworthy than those generated by a single MCDM method. A new COmbinative Distance-based ASsessment (CODAS) method to handle MCDM problems is proposed in [17]. To improve the accuracy of weighted sum and weighted product models (WSM and WPM), in [18] the Weighted Aggregated Sum Product Assessment (WASPAS) method was applied as an aggregation operator on WSM and WPM. In the paper [19], an extended version WASPAS-IVIF method is proposed which can be applied in uncertain decision making environment. In [20] authors revealed that MCDM methods work fairly well in estimating the number of clusters in a data set.

Another aspect of the MCDM methods – weight coefficients W selection, which can be accomplished by using *a priori* information (expert's estimates, subject specific knowledge) or *a posteriori* information of matrix R itself. The latter are sometimes called objective weight determining methods [21].

In the articles [22–25] a new weight determining approach which is applicable to the tasks when matrix R is composed of two or more components is proposed. All MCDM methods using this methodology can be assigned to the group of weights balancing methods, which hereinafter we call WEBIRA (WEight Balancing Indicator Ranks Accordance).

The main idea of this approach is maximizing compatibility of the two (or more) sets of attributes which are treated as independent. Optimization task is being solved and criteria weights are sought throughout the weight balancing procedure. WEBIRA so far have not been compared with other MCDM methods, thus relevant is the question when it is appropriate to use. WEBIRA is suitable for a very important economic benefit carrying tasks, it has both scientific and practical meaning. For example, developers constructing sustainable products and technologies must pay attention to three main components such as economic development, social development and environmental protection. MCDM problems of sustainability could be solved by applying

WEBIRA for 3 sets of attributes – economic, social and environmental components of sustainable development.

A major criticism of MADM is that different techniques may yield different results when applied to the same problem. It does not exist multiple criteria evaluation method which is "best under any circumstances". Therefore the comparative analysis of various MCDM methods determining which method is best for a particular case is relevant and important task. The performance of eight methods: ELimination and Choice Expressing REality (ELECTRE), TOPSIS, Multiplicative Exponential Weighting (MEW), SAW, and four versions of AHP investigated in simulation experiment in [26]. Simulation parameters are the number of alternatives, criteria and their distribution. In general, all AHP versions behave similarly and closer to SAW than the other methods. ELECTRE is the least similar to SAW. The following performance order of methods was established: SAW and MEW (best), followed by TOPSIS, AHPs and ELECTRE. The comparative analysis of MCDA methods SAW and COPRAS (Complex Proportional Assessment) describing their common and diverse characteristics is proposed in [27]. The paper [28] presents an empirical application and comparison of six different MCDM approaches (between them WSM, AHP, TOPSIS, COPRAS) for the purpose of assessing sustainable housing affordability. TOPSIS, SAW, and Mixed (Rank Average) for decision-making as well as AHP and Entropy for obtaining the weights of attributes have been compared in [29]. Mixed method as compared to TOPSIS and SAW is the preferred technique, moreover, AHP is more acceptable than Entropy for weighting. The comprehensive study [30] carried out the comparative analysis among well-known and widely-used methods WPM, WSM, TOPSIS, AHP, PROMETHEE, ELECTRE, when applied to the reference problem of the selection of wind turbine support structures for a given deployment location. The outcomes of this research highlight that more sophisticated methods, such as TOPSIS and Preference Ranking Organization METHod for Enrichment Evaluation (PROMETHEE), better predict the optimum design alternative.

WEBIRA method requires to solve complicated optimization task and therefore it relates to executing non trivial computer calculations. Naturally, the question arises as to when it would appear reasonable to apply WEBIRA, and when – the less sophisticated approaches. In this article Monte Carlo-type experiments are performed and WEBIRA is compared with two simple objective weight determining methods: AVRG – the simple arithmetic average, i. e. the weighted sum with equal weights and the Entropy Method for Determining the Criterion Weight (EMDCW) described in [21]. The Shannon entropy method [31] is one of the most famous approach for determining the objective attribute weights. Entropy measures the uncertainty associated with a random variable, i.e. the expected value of the information transmitted to the decision maker. The authors of the paper [21] have combined the best features of the entropy method and the CILOS (the Criterion Impact Loss) approach to obtain a new method – Integrated Determination of Objective CRIteria Weights, or (IDOCRIW). In [32] three methods have been used for estimating criteria weights: Entropy, CILOS and IDOCRIW, while for the selection of priority well-known and widely used MCDM methods SAW, TOPSIS and COPRAS have been used in MCDM analysis of operating of rotor systems with tilting pad bearings.

Another problem the article dealt with – the comparative analysis of efficiency of some data normalization methods. A state-of-the-art survey on the influence of normalization techniques in ranking is proposed in [33]. Thirty-one methods were identified, classified and evaluated for use in materials selection problems. Review of normalization methods used in construction engineering and management, and their applications presented in [34].

The article is organized as follows. In Chapter 2 the algorithm of solving the optimization problem and the case study of its application is proposed. In Chapter 3 random matrices generating scheme is described. In Chapter 4 the transformation formulas for matrix of estimates proposed. Chapter 5 describes the process of numerical experiments and methods of efficiency comparison. Chapter 6 describes the statistical analysis of the results of numerical experiments, summarizes and proposes recommendations for application of various MCDM methods and normalizing procedures.

2 Algorithm of WEBIRA method

Suppose, that matrix R is composed of two components
$R = (P|Q)$, $P = \left(p_i^{(j)}\right)_{m\times n_p}$, $Q = \left(q_i^{(j)}\right)_{m\times n_q}$, $n_p + n_q = n$.

Two weighted sums are calculated

$$S_P^{(j)} = \sum_{i=1}^{n_p} w_{Pi} p_i^{(j)},\ S_Q^{(j)} = \sum_{i=1}^{n_q} w_{Qi} q_i^{(j)},\ j = 1, 2, \ldots, m. \tag{2}$$

Coefficients $W_P = \left(w_{p1}, w_{p2}, \ldots, w_{pn_p}\right)$, $W_Q = \left(w_{q1}, w_{q2}, \ldots, w_{qn_q}\right)$ satisfy monotonicity conditions:

$$1 \geqslant w_{p1} \geqslant w_{p2} \geqslant \cdots \geqslant w_{pn_p} \geqslant 0, \quad 1 \geqslant w_{q1} \geqslant w_{q2} \geqslant \cdots \geqslant w_{qn_q} \geqslant 0. \tag{3}$$

Inequalities (3) are set from expert estimates, when k experts line up attributes p_{ie}, q_{ie} according to their importance:

$$p_{i_1e} \succ p_{i_2e} \succ \cdots \succ p_{i_{n_p}e},\ q_{i_1e} \succ \cdots \succ q_{i_2e} \succ q_{i_{n_q}e},\ e = 1, 2, \ldots, k. \tag{4}$$

When we have *a priori* information about experts evaluations (4), inequalities (3) could be obtained by different methods. In the article [22] Kemeny median has been adapted for this purpose and the name KEMIRA (KEmeny Median Indicator Ranks Accordance) proposed for the method. In the articles [23–25] inequalities (3) were set by calculating entropy values or by application of voting theory methods. As the inequalities (3) indicating the weight preferences are established, all of the mentioned methods can be assigned to the objective weight determining, because there is not need of further information to set the weights of the attributes. We consider the task when inequalities (3) already established and the weights W_P, W_Q determining task is being solved. The task is formulated as minimization of a certain distance or measure:

$$s\left(W_P, W_Q\right) = \min_{W_P, W_Q} \sqrt{\frac{1}{m}\sum_{j=1}^{m}\left(S_P^{(j)} - S_Q^{(j)}\right)^2}, \tag{5}$$

where the weights W_P, W_Q are satisfying the inequalities (3). So, all the mentioned MCDM methods [22–25] can be assigned to the group of weight balancing methods, which we call WEBIRA.

In this article we analyze only the benefit type attributes, i.e. whose higher value is better. When optimization task (5),(3) is already solved, the ranks can be assigned to the alternatives $j \in \{1, 2, \ldots, m\}$ depending on the size of the weighted sums $S_P^{(j)}$ and $S_Q^{(j)}$. Suppose that j_1^P, j_2^P, …, j_m^P, j_1^Q, j_2^Q, …, j_m^Q are such numbers of alternatives that the weighted sums (2) are satisfying the inequalities:

$$S_P^{j_1^P} \geqslant S_P^{j_2^P} \geqslant \cdots \geqslant S_P^{j_m^P}, \quad S_Q^{j_1^Q} \geqslant S_Q^{j_2^Q} \geqslant \cdots \geqslant S_Q^{j_m^Q}. \tag{6}$$

Denote a set of the best k alternatives according to the first n_p attributes $A_k^P = \{j_1^P, j_2^P, \ldots, j_k^P\}$, according to the last n_q attributes – $A_k^Q = \{j_1^Q, j_2^Q, \ldots, j_k^Q\}$ and their intersection $A = A_k^P \cap A_k^Q$.

The meaning of the sets A_k^P and A_k^Q is selecting the best k alternatives according to the attributes P and Q respectively while the meaning of the set A – the best alternatives according to the both attributes. The purpose of WEBIRA method is to balance weights so that a number of elements $|A|$ of the set A would be not less than the certain number. So, it is required to find a sufficient number of the best alternatives according to both attributes P and Q. In the articles [22–25] the minimizing tasks (5),(3) have been solved together with $\max|A|$ under various parameter k values of the sets A_k^P and A_k^Q. In the current article we limit ourselves to the case $k = 1$ and require $A \neq \emptyset$, i. e. we will search the only one the best alternative according to the both attributes P and Q.

This additional condition can be formulated as follows:

$$S_P^{(j_0^P)} = \max_{j=1,2,\dots,m} \{S_P^{(j)}\}, \quad S_Q^{(j_0^Q)} = \max_{j=1,2,\dots,m} \{S_Q^{(j)}\}, \ j_0^P = j_0^Q. \tag{7}$$

Algorithm of solving the problem (5),(3),(7) is as follows.

1. By random re-selection among weights W_P, W_Q satisfying conditions (3) the weights satisfying an additional condition (7) are being searched.

2. If the weights W_P^0, W_Q^0 were not found after $iter_0$ iterations, algorithm is finishing work and it is concluded that the weights can not be determined.

3. If the weights W_P^0, W_Q^0 are set, the loss value (5) is fixed as s^0, directions ΔW_P, ΔW_Q are selected at random and the new weights $W_P^1 = W_P^0 + h\Delta W_P$, $W_Q^1 = W_Q^0 + h\Delta W_Q$ calculated, here h is the predetermined value.

4. The correction of weights W_P^1, W_Q^1 is performed as follows $W \to \tilde{W}$:
   $\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)$, $\tilde{w}_i = \begin{cases} 1, \text{if } w_i \geqslant 1, \\ 0, \text{if } w_i \leqslant 0, \\ w_i, \text{else,} \end{cases}$
   then the weights are normalized $w_i = \frac{\tilde{w}_i}{\sum_{i=1}^{n} \tilde{w}_i}$.

5. Checking whether the new weights W_P^1, W_Q^1 satisfy the condition (7). If they satisfy and the number of iterations does not exceed the established limit $iter_1$ the algorithm moves to the Step 7.

6. If the number of iterations exceeds the established limit $iter_1$, algorithm finishes its work with the determined weights W_P^0, W_Q^0.

7. The loss value (5) is calculated $s^1\left(W_P^1, W_Q^1\right)$. If $s^1 \leqslant s^0$, we substitute $W_P^0 = W_P^1$, $W_Q^0 = W_Q^1$, $s^0 = s^1$ and then the new weights W_P^1, W_Q^1 are calculated with the same directions ΔW_P, ΔW_Q. Go to the weight correction procedure to the Step 4.

8. If $s^1 \geqslant s^0$, the directions ΔW_P, ΔW_Q are changed randomly, i. e. go to the Step 3 of algorithm.

Note. Algorithm parameters $iter_0$, $iter_1$ and h are set empirically and their values were set respectively to 225, 200 and 0.05.

Example. Provide the case study of the described algorithm application.
Set parameter values: $m = 6$, $n_x = 5$, $n_y = 4$, $iter_0 = 225$, $iter_1 = 100$, $h = 0.05$,

$$P = \begin{pmatrix} 0.8333 & 1.0000 & 0.6667 & 0.8000 & 0.3333 \\ 0.6667 & 0.2000 & 0.6667 & 1.0000 & 1.0000 \\ 0.8333 & 0.8000 & 1.0000 & 0.2000 & 0.1667 \\ 1.0000 & 0.8000 & 0.8333 & 0.6000 & 1.0000 \\ 0.1667 & 0.4000 & 0.3333 & 1.0000 & 0.3333 \\ 0.3333 & 0.2000 & 1.0000 & 0.6000 & 0.8333 \end{pmatrix},$$
$$Q = \begin{pmatrix} 1.0000 & 1.0000 & 0.3333 & 0.4000 \\ 0.3333 & 0.6000 & 0.6667 & 0.8000 \\ 0.3333 & 0.4000 & 0.3333 & 0.6000 \\ 0.3333 & 1.0000 & 0.3333 & 1.0000 \\ 0.5000 & 0.6000 & 0.5000 & 0.8000 \\ 0.1667 & 0.6000 & 1.0000 & 1.0000 \end{pmatrix}. \tag{8}$$

The initial point: $(N = 1)$ $W_P = (0.5556, 0.1111, 0.1111, 0.1111, 0.1111)$, $W_Q = (0.3636, 0.3636, 0.1818, 0.0910)$, $S_P = (0.7741, 0.6889, 0.7037, 0.9148, 0.3222, 0.4778)$, $S_Q = (0.8242, 0.5333, 0.3818, 0.6363, 0.5636, 0.5515)$. The ranks of the alternatives: $Rangs_P = (4, 1, 3, 2, 6, 5)$, $Rangs_Q = (1, 4, 5, 6, 2, 3)$. We see, that condition (7) is not satisfied, i. e. two criteria P and Q differently determine the best alternative.

Let's skip some of the checked weights W_P, W_Q and provide only some of the calculations results: $(N = 119)$ $W_P = (0.3478, 0.3478, 0.1739, 0.0870, 0.0435)$, $W_Q = (0.2500, 0.2500, 0.2500, 0.2500)$. $S_P = (0.8377, 0.5478, 0.7667, 0.8667, 0.3565, 0.4478)$, $S_Q = (0.6833, 0.6000, 0.4166, 0.6666, 0.6000, 0.6917)$. The ranks of the alternatives are: $Rangs_P = (4, 1, 3, 2, 6, 5)$, $Rangs_Q = (6, 1, 4, 5, 2, 3)$ and again the condition (7) is not satisfied.

The initial point was found when $(N = 169)$, $W_P = (0.5000, 0.5000, 0.0000, 0.0000, 0.0000)$, $W_Q = (0.3636, 0.3636, 0.1818, 0.0910)$, $S_P = (0.9166, 0.4334, 0.8167, 0.9000, 0.2833, 0.2667)$, $S_Q = (0.8242, 0.5333, 0.3818, 0.6363, 0.5636, 0.5515)$.

The ranks of the alternatives: $Rangs_P = (1, 4, 3, 2, 5, 6)$, $Rangs_Q = (1, 4, 5, 6, 2, 3)$ already responding the condition (7).

The initial loss calculated by the formula (5) is $s^0 = 0.660953$. Randomly determined directions: $\Delta_P = (0.30, -0.30, -0.22, -0.13, -0.05)$, $\Delta_Q = (-0.41, 0.31, -0.20, -0.08)$. The function (5) is increasing in this direction, therefore, the opposite direction was chosen and the weights were set as follows: $W_P^1 = W_P^0 - h \cdot \Delta_P$, $W_Q^1 = W_Q^0 - h \cdot \Delta_Q$. Then the weights were adjusted (Step 4 of algorithm). The minimum value $s^1 = 0.653260$ of the function (5) in the direction Δ_P, Δ_Q was obtained with the weights $W_P = (0.4900, 0.4900, 0.0111, 0.0067, 0.0022)$, $W_Q = (0.3772, 0.3416, 0.1882, 0.0930)$.

The second iteration of the algorithm (random change in direction) is as follows. The direction vectors are: $\Delta_P = (0.35, 0.24, -0.22, -0.16, 0.04)$, $\Delta_Q = (-0.40, 0.39, -0.20, 0.00)$. In this case the function declines in the direction $W_P^1 = W_P^0 - h \cdot \Delta_P$, $W_Q^1 = W_Q^0 - h \cdot \Delta_Q$ and the minimum value $s^1 = 0.648413$ of the function (5) is obtained when the weights are $W_P = (0.4813, 0.4813, 0.0224, 0.0147, 0.0003)$, $W_Q = (0.3932, 0.3188, 0.1962, 0.0918)$.

We present some further iterations results:
$Iter = 17$, $s^1 = 0.575767$, $W_P = (0.4384, 0.4321, 0.0779, 0.0482, 0.0033)$, $W_Q = (0.3942, 0.3456, 0.1468, 0.1134)$.

$Iter = 29$, $s^1 = 0.508748$, $W_Q = (0.4119, 0.4057, 0.0832, 0.0804, 0.0188)$,
$W_Q = (0.4094, 0.4035, 0.0935, 0.0935)$.
$Iter = 65$, $s^1 = 0.450759$, $W_P = (0.3447, 0.3447, 0.1488, 0.1488, 0.0130)$,
$W_Q = (0.4454, 0.4380, 0.0583, 0.0583)$.
$Iter = 93$, $s^1 = 0.388068$, $W_P = (0.2453, 0.2453, 0.2453, 0.2453, 0.0187)$,
$W_Q = (0.3954, 0.3954, 0.1227, 0.0866)$.

Notice that $Iter_1 = 100$ iterations were accomplished, but we failed to reduce the value $s^1 = 0.388068$ of the loss function, i. e. in all randomly selected directions the function (5) increased.

Given the weights obtained in the 93-th iteration the values of criteria are
$S_P = (0.8158, 0.6402, 0.6982, 0.8119, 0.4723, 0.5389)$,
$S_Q = (0.8662, 0.5201, 0.3828, 0.6546, 0.5655, 0.5124)$.

The ranks of the alternatives: $Rangs_P = (1, 4, 3, 2, 6, 5)$, $Rangs_Q = (1, 4, 5, 2, 6, 3)$.

Please note that for the algorithm realization it was necessary to apply a relatively complicated computer program which was realized in C++. However, one can easily check the calculations and this may be done in each step of the algorithm independently of other steps.

3 Random matrices estimates generation

The elements $x_i^{(j)}$, $y_i^{(j)}$ of the estimates matrices $X = \left(x_i^{(j)}\right)_{m\times n_x}$ and $Y = \left(y_i^{(j)}\right)_{m\times n_y}$ are the integers simulating the scores of the expert estimates $x_i^{(j)} \in \{1, 2, \dots, b_i^X\}$, $y_i^{(j)} \in \{1, 2, \dots, b_i^Y\}$ of the alternatives $i \in \{1, 2, \dots, m\}$. Each row of matrices X and Y corresponds to one alternative

$$X = \begin{pmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_{n_x}^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_{n_x}^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ x_1^{(m)} & x_2^{(m)} & \cdots & x_{n_x}^{(m)} \end{pmatrix}, Y = \begin{pmatrix} y_1^{(1)} & y_2^{(1)} & \cdots & y_{n_y}^{(1)} \\ y_1^{(2)} & y_2^{(2)} & \cdots & y_{n_y}^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ y_1^{(m)} & y_2^{(m)} & \cdots & y_{n_y}^{(m)} \end{pmatrix}. \tag{9}$$

The columns of matrices (9) arranged in descending order of attributes priorities. The first line of matrix is generated with preset probabilities P_{ik}^X, P_{ik}^Y:

$$\begin{aligned} &P\{x_i^{(1)} = b_i^X - k\} = P_{ik}^X,\ k = 0, 1, 2, \dots, b_i^X - 1, \\ &P\{y_i^{(1)} = b_i^Y - k\} = P_{ik}^Y,\ k = 0, 1, 2, \dots, b_i^Y - 1. \end{aligned} \tag{10}$$

Antecedent probabilities P_{ik}^X, P_{ik}^Y chosen in such way, that the first alternative should have on average higher estimates. Other alternatives estimates generated with the equal probabilities:

$$\begin{aligned} &P\{x_i^{(j)} = b_i^X - k\} = \tfrac{1}{b_i^X},\ k = 0, 1, \dots, b_i^X - 1,\ i = 1, 2, \dots, n_x, \\ &P\{y_i^{(j)} = b_i^Y - k\} = \tfrac{1}{b_i^Y},\ k = 0, 1, \dots, b_i^Y - 1,\ i = 1, 2, \dots, n_y, \\ &j = 2, 3, \dots, m. \end{aligned} \tag{11}$$

Therefore, the second and all other alternatives are treated as a kind of noise making heavy recognition of the first – the best alternative. The more alternatives we have, the more difficult is the task of identification.

The experiments were carried out with the following parameter values:
$b_i^X = 6$, $i = 1, 2, 3, 4, 5$, $b_i^Y = 6$, $i = 1, 2, 3, 4$. Probabilities of the first alternative estimates:

$$P\{x_{1,2}^{(1)} = 6\} = P\{x_{1,2}^{(1)} = 5\} = 0.5;\ P\{x_{1,2}^{(1)} = l\} = 0., l = 1, 2, 3, 4.$$

$$P\{x_{3,4,5}^{(1)} = 6\} = 0.;\ P\{x_{3,4,5}^{(1)} = l\} = 0.25, l = 2,3,4,5;\ P\{x_{3,4,5}^{(1)} = 1\} = 0.$$

Probabilities of other alternatives estimates are equal:

$$P\{x_i^{(j)} = l\} = \frac{1}{6},\ j = 2,3,\dots,m,\ i = 1,2,3,4,5,\ l = 1,2,3,4,5,6.$$

Similarly selected estimates probabilities of the first and other alternatives according to Y:

$$P\{y_{1,2}^{(1)} = 6\} = P\{y_{1,2}^{(1)} = 5\} = 0.5;\ P\{y_{1,2}^{(1)} = l\} = 0., l = 1,2,3,4.$$

$$P\{y_{3,4}^{(1)} = 6\} = 0.;\ P\{y_{3,4}^{(1)} = l\} = 0.25, l = 2,3,4,5;\ P\{y_{3,4}^{(1)} = 1\} = 0.$$

$$P\{y_i^{(j)} = l\} = \frac{1}{6},\ j = 2,3,\dots,m,\ i = 1,2,3,4,\ l = 1,2,3,4,5,6.$$

The example of generated matrix with the best first alternative:

$$(X|Y) = \left(\begin{array}{ccccc|cccc} 5 & 6 & 3 & 4 & 2 & 6 & 5 & 4 & 4 \\ 4 & 4 & 4 & 2 & 3 & 1 & 2 & 5 & 3 \\ 3 & 4 & 5 & 1 & 3 & 2 & 2 & 5 & 5 \\ 3 & 3 & 6 & 6 & 5 & 6 & 4 & 4 & 4 \\ 4 & 3 & 4 & 3 & 5 & 6 & 5 & 6 & 4 \\ 2 & 1 & 5 & 5 & 2 & 4 & 6 & 3 & 1 \\ 1 & 2 & 3 & 2 & 2 & 2 & 5 & 6 & 4 \\ 4 & 2 & 2 & 3 & 1 & 6 & 2 & 5 & 6 \end{array}\right). \tag{12}$$

4 Transformations of estimates matrix

Recall that randomly generated matrices X, Y elements $x_i^{(j)}$, $y_i^{(j)}$ are the integers while input data of WEBIRA method – matrices P, Q elements acquire real values in the interval $[0,1]$. They are the normalized values of matrices X, Y elements. In this article four transformation (normalization) methods $(X,Y) \to (P,Q)$ are applied:

$$\begin{aligned} &\text{Max method:} && p_i^{(j)} = \frac{x_i^{(j)}}{\max\limits_{j\in\{1,2,\dots,m\}} x_i^{(j)}},\ q_i^{(j)} = \frac{y_i^{(j)}}{\max\limits_{j\in\{1,2,\dots,m\}} y_i^{(j)}}, \\ &\text{Sum method:} && p_i^{(j)} = \frac{x_i^{(j)}}{\sum\limits_{j=1}^{m} x_i^{(j)}},\ q_i^{(j)} = \frac{y_i^{(j)}}{\sum\limits_{j=1}^{m} y_i^{(j)}}, \\ &\text{MinMax method:} && p_i^{(j)} = \frac{x_i^{(j)} - \min\limits_{j\in\{1,2,\dots,m\}} x_i^{(j)}}{\max\limits_{j\in\{1,2,\dots,m\}} x_i^{(j)} - \min\limits_{j\in\{1,2,\dots,m\}} x_i^{(j)}}, \\ & && q_i^{(j)} = \frac{y_i^{(j)} - \min\limits_{j\in\{1,2,\dots,m\}} y_i^{(j)}}{\max\limits_{j\in\{1,2,\dots,m\}} y_i^{(j)} - \min\limits_{j\in\{1,2,\dots,m\}} y_i^{(j)}}, \\ &\text{Vector normalization:} && p_i^{(j)} = \frac{x_i^{(j)}}{\sqrt{\sum\limits_{j=1}^{m}\left(x_i^{(j)}\right)^2}},\ q_i^{(j)} = \frac{y_i^{(j)}}{\sqrt{\sum\limits_{j=1}^{m}\left(y_i^{(j)}\right)^2}}. \end{aligned} \tag{13}$$

Notice that formulas (13) applicable when all attributes are the benefit type of optimization direction, (i. e., the higher value is better, see, for example, [1]). Another case – cost type criteria, (i.e., the lower value is better) will be not discussed in this article.

Suppose that matrices P, Q and their concatenation – matrix

$R=(P|Q)=\left(r_i^{(j)}\right)_{m\times(n_x+n_y)}$, i. e. $r_i^{(j)}=\begin{cases} p_i^{(j)}, & \text{if } i\leqslant n_x,\\ q_{i-n_x}^{(j)}, & \text{if } n_x+1\leqslant i\leqslant n_x+n_y\end{cases}$

are obtained from randomly generated matrices X, Y by one of the four methods (13).

WEBIRA method will be compared with two MCDM methods – simple arithmetic average:

$$\text{AVRG: } S^{(j)}=\frac{1}{n_x+n_y}\left(\sum_{i=1}^{n_x}p_i^{(j)}+\sum_{i=1}^{n_y}q_i^{(j)}\right) \tag{14}$$

and EMDCW (Entropy Method for Determining the Criterion Weight, see [21]:

$$\begin{aligned} &\text{EMDCW: } S^{(j)}=\sum_{i=1}^{n_x+n_y}w_ir_i^{(j)},\ w_i=\frac{1-e_i}{n_x+n_y-\sum\limits_{i=1}^{n_x+n_y}e_i},\\ &e_i=-\frac{1}{m}\sum_{j=1}^{m}\tilde{r}_i^{(j)}\cdot\ln\left(\tilde{r}_i^{(j)}\right),\ \tilde{r}_i^{(j)}=\frac{r_i^{(j)}}{\sum\limits_{j=1}^{m}r_i^{(j)}} \end{aligned} \tag{15}$$

Next, we provide the case study of formulas (14) and (15) application. The result of matrix (12) transformation using Max method:

$$\begin{pmatrix} 1.0000 & 1.0000 & 0.5000 & 0.6667 & 0.4000 & 1.0000 & 0.8333 & 0.6667 & 0.6667\\ 0.8000 & 0.6667 & 0.6667 & 0.3333 & 0.6000 & 0.1667 & 0.3333 & 0.8333 & 0.5000\\ 0.6000 & 0.6667 & 0.8333 & 0.1667 & 0.6000 & 0.3333 & 0.3333 & 0.8333 & 0.8333\\ 0.6000 & 0.5000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.6667 & 0.6667 & 0.6667\\ 0.8000 & 0.5000 & 0.6667 & 0.5000 & 1.0000 & 1.0000 & 0.8333 & 1.0000 & 0.6667\\ 0.4000 & 0.1667 & 0.8333 & 0.8333 & 0.4000 & 0.6667 & 1.0000 & 0.5000 & 0.1667\\ 0.2000 & 0.3333 & 0.5000 & 0.3333 & 0.4000 & 0.3333 & 0.8333 & 1.0000 & 0.6667\\ 0.8000 & 0.3333 & 0.3333 & 0.5000 & 0.2000 & 1.0000 & 0.3333 & 0.8333 & 1.0000 \end{pmatrix}$$

Weighting sums (15) of 8 alternatives obtained by EMDCW method are as follows:

$$0.7747; 0.4847; 0.5110; 0.8141; 0.7650; 0.5522; 0.4435; 0.5803.$$

Weighting sums calculated by AVRG method (14) are:

$$0.1470; 0.1069; 0.1135; 0.1550; 0.1521; 0.1084; 0.1004; 0.1164.$$

Thus, both methods assign the fourth as the best alternative and we treat it as a mistake, because the best is considered the first alternative. WEBIRA method was applied with initial weights values

$$\begin{aligned} W_x&=(0.3314, 0.1953, 0.1581, 0.1581, 0.1571),\\ W_y&=(1.0000, 0.0000, 0.0000, 0.0000). \end{aligned}$$

Weighted averages (2) calculated with these weights are

$$\begin{aligned} S_X^j&=(0.7739, 0.6476, 0.5813, 0.7698, 0.7043, 0.4914, 0.3259, 0.4933),\\ S_Y^j&=(1.0000, 0.1667, 0.3333, 1.0000, 1.0000, 0.6667, 0.3333, 1.0000). \end{aligned}$$

The initial loss (5) in this case is $s^0=0.8785$. After 134 iterations WEBIRA method allowed to reduce this value to $s^1=0.7147$ and the following weights were found:

$$\begin{aligned} W_x&=(0.4623, 0.1373, 0.1373, 0.1373, 0.1258),\\ W_y&=(0.5901, 0.4099, 0.0000, 0.0000). \end{aligned}$$

Weighted averages are:

$$S_X^j = (0.8100, 0.6741, 0.5816, 0.7464, 0.7245, 0.4869, 0.3029, 0.5551),$$
$$S_Y^j = (0.9316, 0.2349, 0.3333, 0.8633, 0.9316, 0.8032, 0.5382, 0.7267).$$

Ranks of alternatives according to the X: 1, 4, 5, 2, 3, 8, 6, 7 and according to the Y: 1, 5, 4, 6, 8, 7, 3, 2. So, WEBIRA method set as the best the first alternative and we treat this as the right decision. Notice that all three methods set the same three best alternatives: 1, 4 and 5.

Submit normalized matrices, calculated by other methods. MinMax method:

$$\begin{pmatrix} 1.0000 & 1.0000 & 0.2500 & 0.6000 & 0.2500 & 1.0000 & 0.7500 & 0.3333 & 0.6000 \\ 0.7500 & 0.6000 & 0.5000 & 0.2000 & 0.5000 & 0.0000 & 0.0000 & 0.6667 & 0.4000 \\ 0.5000 & 0.6000 & 0.7500 & 0.0000 & 0.5000 & 0.2000 & 0.0000 & 0.6667 & 0.8000 \\ 0.5000 & 0.4000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.5000 & 0.3333 & 0.6000 \\ 0.7500 & 0.4000 & 0.5000 & 0.4000 & 1.0000 & 1.0000 & 0.7500 & 1.0000 & 0.6000 \\ 0.2500 & 0.0000 & 0.7500 & 0.8000 & 0.2500 & 0.6000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.2000 & 0.2500 & 0.2000 & 0.2500 & 0.2000 & 0.7500 & 1.0000 & 0.6000 \\ 0.7500 & 0.2000 & 0.0000 & 0.4000 & 0.0000 & 1.0000 & 0.0000 & 0.6667 & 1.0000 \end{pmatrix}.$$

EMDCW and AVRG methods determined as the best the fifth alternative, WEBIRA – the first alternative.

Matrix transformated by Sum method:

$$\begin{pmatrix} 0.1923 & 0.2400 & 0.0937 & 0.1538 & 0.0869 & 0.1818 & 0.1612 & 0.1052 & 0.1290 \\ 0.1538 & 0.1600 & 0.1250 & 0.0769 & 0.1304 & 0.0303 & 0.0645 & 0.1315 & 0.0967 \\ 0.1154 & 0.1600 & 0.1562 & 0.0384 & 0.1304 & 0.0606 & 0.0645 & 0.1315 & 0.1612 \\ 0.1154 & 0.1200 & 0.1875 & 0.2307 & 0.2173 & 0.1818 & 0.1290 & 0.1052 & 0.1290 \\ 0.1538 & 0.1200 & 0.1250 & 0.1153 & 0.2173 & 0.1818 & 0.1612 & 0.1578 & 0.1290 \\ 0.0769 & 0.0400 & 0.1562 & 0.1923 & 0.0869 & 0.1212 & 0.1935 & 0.0789 & 0.0322 \\ 0.0384 & 0.0800 & 0.0937 & 0.0769 & 0.0869 & 0.0606 & 0.1612 & 0.1578 & 0.1290 \\ 0.1538 & 0.0800 & 0.0625 & 0.1153 & 0.0434 & 0.1818 & 0.0645 & 0.1315 & 0.1935 \end{pmatrix}.$$

In this case, as in another – vector normalization method the obtained results coincide with the Max method, i. e. EMDCW and AVRG methods set as the best the fourth alternative, while WEBIRA – the first.

Matrix transformated by Vector normalization:

$$\begin{pmatrix} 0.5103 & 0.6155 & 0.2535 & 0.3922 & 0.2222 & 0.4615 & 0.4240 & 0.2917 & 0.3442 \\ 0.4082 & 0.4103 & 0.3380 & 0.1961 & 0.3333 & 0.0769 & 0.1696 & 0.3646 & 0.2581 \\ 0.3061 & 0.4103 & 0.4225 & 0.0980 & 0.3333 & 0.1538 & 0.1696 & 0.3646 & 0.4303 \\ 0.3061 & 0.3077 & 0.5070 & 0.5883 & 0.5556 & 0.4615 & 0.3392 & 0.2917 & 0.3442 \\ 0.4082 & 0.3077 & 0.3380 & 0.2941 & 0.5556 & 0.4615 & 0.4240 & 0.4375 & 0.3442 \\ 0.2041 & 0.1025 & 0.4225 & 0.4902 & 0.2222 & 0.3076 & 0.5089 & 0.2187 & 0.0860 \\ 0.1020 & 0.2051 & 0.2535 & 0.1961 & 0.2222 & 0.1538 & 0.4240 & 0.4375 & 0.3442 \\ 0.4082 & 0.2051 & 0.1690 & 0.2941 & 0.1111 & 0.4615 & 0.1696 & 0.3646 & 0.5163 \end{pmatrix}.$$

5 Numerical experiments

Numerical experiments were conducted as follows. Random matrices X, Y generated in such a way that on average more often the best alternative is the first. Matrices P, Q are calculated by four normalization methods and the best alternative is determined by three methods EMDCW, AVRG and WEBIRA. When the best is the first alternative the result of the experiment is marked with (+) and recorded to the table. If the best is any other (not the first) alternative (−) is recorded to the table. It is possible that WEBIRA can not set the best alternative. We then record (n). Notice, that in the cases of EMDCW and AVRG methods such experimental result is impossible. In the Table 1 the results of 5 experiments are presented. Methods EMDCW, AVRG, WEBIRA are denoted respectively as (E), (A), (W).

Table 1: Fragment of experimental results.

	Max method			MinMax method			Sum method			Vector normalization		
Nr.	(E)	(A)	(W)	(E)	(A)	(W)	(E)	(A)	(W)	(E)	(A)	(W)
1	+	+	+	-	+	+	+	+	+	+	+	+
2	+	+	+	+	+	+	+	+	n	+	+	n
3	-	-	+	+	-	+	-	-	+	-	-	+
4	-	+	+	-	+	+	-	+	+	-	+	+
5	-	-	n	-	-	n	-	-	n	-	-	n

After a series of experiments, we calculate the number of pluses (+) denoted as p in each of the 12 columns of the Table 1, the number of minuses (-) denoted as m and undetected cases (n). WEBIRA method peculiarity compared to AVRG and EMDCW – possible non zero values of parameter n. It means that WEBIRA quite often eliminates cases when it can not detect the best alternative. Consider the following indicator to compare methods performance:

$$E_n = \frac{p-m}{p+m+n}. \tag{16}$$

Indicator E_n shows reliability of the correspondent method. Our purpose is the detection of significantly different average values of E_n in the groups.

100 series of Monte Carlo experiments were carried out by 100 in each series. The common number of experiments was 10000. Random matrices estimates generation procedure is described in Chapter 3. The number of alternatives varied $m = 3, 4, ..., 50$. Table 2 presents the average values of E_n dependence on the number of alternatives m, MCDM and data normalization methods.

Conclusions and future research

WEBIRA method allows quite effectively separate the cases when it is not possible to select the best alternative. Otherwise, when the method is applicable its efficiency is significantly higher compared to the two selected simple methods: AVRG and EMDCW. This is true for all four matrices normalization methods: Max, MinMax, Sum and Vector normalization. Efficiency decreases with increasing number of alternatives, but it is still applicable for WEBIRA method, while application of AVRG and EMDCW is impossible as the number of alternatives is greater than 11, as their efficiency became negative value.

In the Figure 1 graphs average values of efficiency indicator E_n depending on the MCDM method are presented. For the Max normalization and $m = 1, 2, \ldots, 30$ the average efficiency of WEBIRA is significantly higher than efficiency of EMDCW and AVRG methods. For MinMax, Sum and Vector normalizations all 3 MCDM methods: EMDCW, AVRG and WEBIRA mutually significantly differ comparing the average values of efficiency indicator E_n when $m = 1, 2, \ldots, 15$, while WEBIRA is significantly efficient than EMDCW and AVRG for all considered numbers of alternatives.

Table 2: Average values of E_n dependence on the number of alternatives m, MCDM and data normalization methods.

	Max method			MinMax method			Sum method			Vctr method		
m	(E)	(A)	(W)	(E)	(A)	(W)	(E)	(A)	(W)	(E)	(A)	(W)
3	0.7224	0.7268	0.8837	0.4736	0.6086	0.8398	0.6944	0.7344	0.8820	0.7018	0.7286	0.8820
4	0.5762	0.5896	0.8122	0.3298	0.4868	0.7781	0.5358	0.5934	0.8006	0.5468	0.5890	0.8039
5	0.4760	0.4860	0.7721	0.2746	0.4082	0.7479	0.4288	0.4842	0.7591	0.4438	0.4770	0.7599
6	0.3870	0.4010	0.7354	0.2220	0.3358	0.7059	0.3426	0.3958	0.7075	0.3552	0.3918	0.7055
7	0.2968	0.3308	0.6694	0.1580	0.2758	0.6539	0.2556	0.3180	0.6449	0.2682	0.3200	0.6472
8	0.2060	0.2236	0.6318	0.0824	0.1744	0.6074	0.1606	0.2166	0.6003	0.1782	0.2110	0.6016
9	0.1520	0.1816	0.5999	0.0626	0.1470	0.5797	0.1134	0.1732	0.5637	0.1272	0.1724	0.5662
10	0.0998	0.1200	0.5644	0.0300	0.0884	0.5430	0.0574	0.1064	0.5314	0.0718	0.1082	0.5300
11	0.0404	0.0628	0.5275	-0.0412	0.0414	0.5149	0.0108	0.0494	0.4906	0.0168	0.0500	0.4905
13	-0.0646	-0.0374	0.4745	-0.0992	-0.0556	0.4653	-0.0776	-0.0490	0.4282	-0.0724	-0.0556	0.4274
15	-0.1356	-0.1032	0.4349	-0.1602	-0.1016	0.4293	-0.1572	-0.1208	0.3943	-0.1534	-0.1220	0.3903
20	-0.2824	-0.2512	0.3427	-0.2988	-0.2424	0.3378	-0.2930	-0.2786	0.2990	-0.2864	-0.2826	0.2989
30	-0.4726	-0.4368	0.2639	-0.4840	-0.4272	0.2606	-0.4818	-0.4660	0.2233	-0.4764	-0.4678	0.2183
50	-0.6592	-0.6250	0.1658	-0.6572	-0.6172	0.1725	-0.6604	-0.6530	0.1482	-0.6616	-0.6524	0.1488

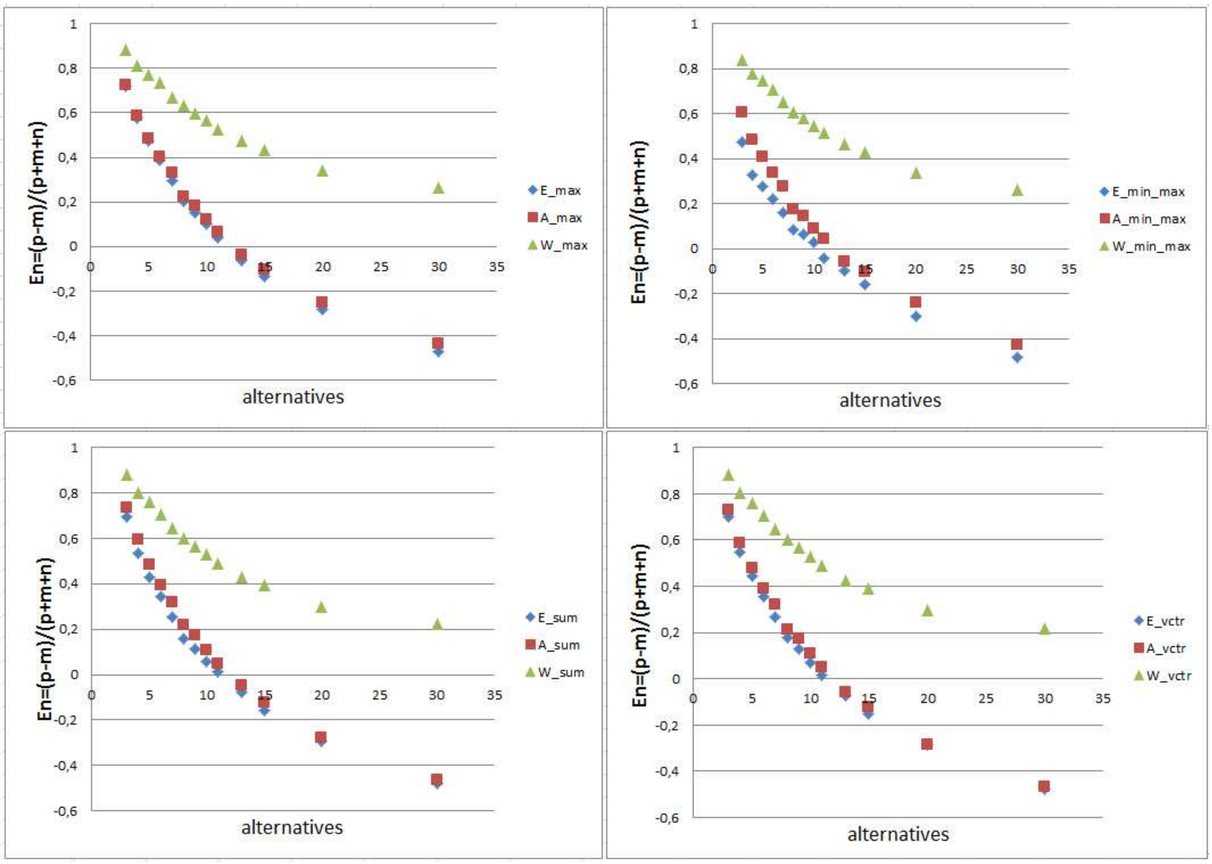

Figure 1: The dependence of E_n average values on the number of alternatives and MCDM method for Max, MinMax, Sum and Vector normalization respectively.

In the Figure 2 graphs of indicator E_n average values depending on the data normalization method are presented. WEBIRA is the least affected by the data normalization method, while EMDCW is the most dependent on normalization. One Way ANOVA was performed to compare

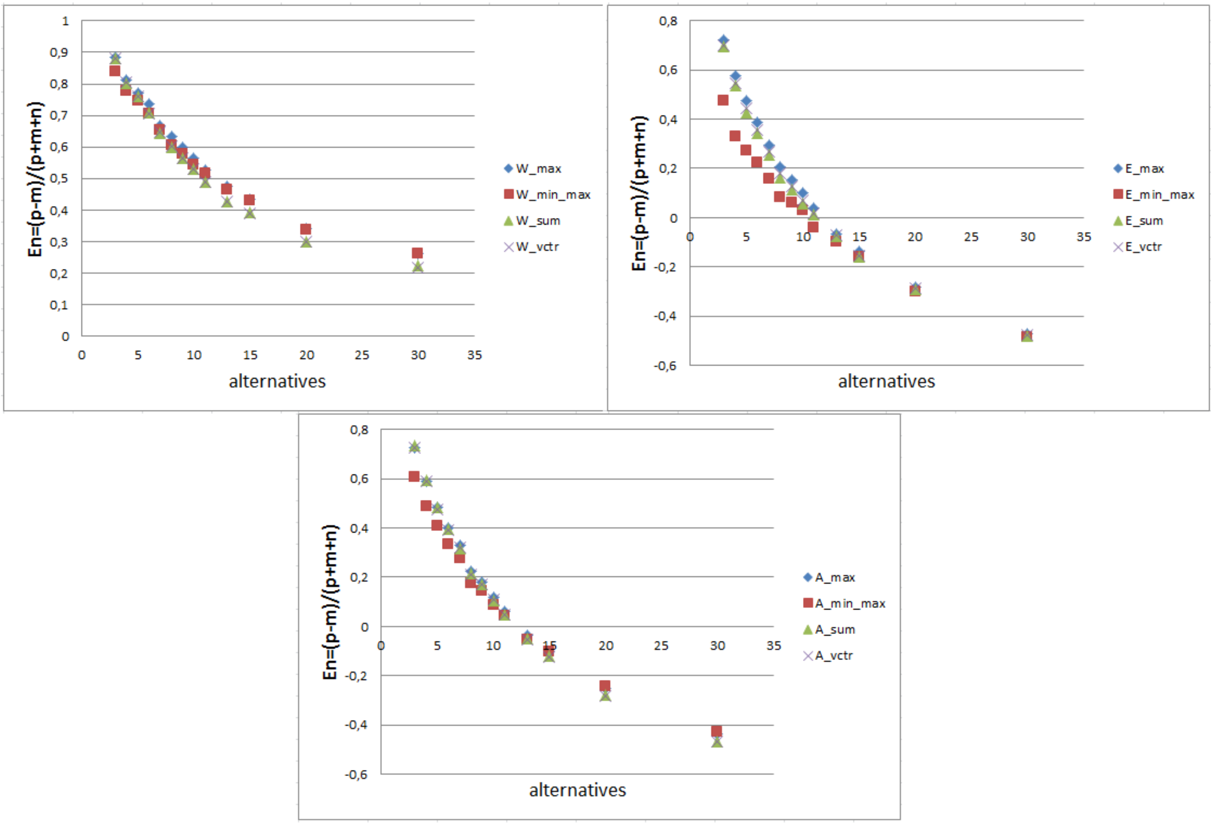

Figure 2: The dependence of E_n average values on the number of alternatives and normalization method for WEBIRA (W), EMDCW (E) and AVRG (A) method respectively.

average values of indicator E_n for the fixed m values at significance level 0.05. In the case of EMDCW (E) method it was established that E_n average values for MinMax data normalization are significantly lower than the average values obtained by Max, Sum and Vector methods when $m \leqslant 11$, and that Max normalization is significantly more efficient than MinMax and Sum methods. Hence, MinMax normalization reduces the efficiency of the EMDCW method.
In the case of AVRG (A) method average value of E_n obtained by MinMax data normalization are significantly lower than E_n averages received by the three other data normalization methods. In the case of WEBIRA (W) method at a low number of alternatives Max data normalization significantly increases the average efficiency of the method.

Random matrices generated in the article are simulating repeated expert evaluations of the same alternatives and depend on *a priori* probabilities (10)–(11). These probabilities can be such that the recognition of the best alternative will be very easy or almost impossible. It is obvious that in the first case application of WEBIRA method is irrational, and in the second – any method will not determine the best alternative. In these cases the alternatives separation requires further research. This article is limited to the case when the task of the best alternative recognition is of medium difficulty.

References describe more matrices transformation methods such as Max, MinMax, Sum and Vector normalization. Their efficiency could depend on the expert evaluation scales. In this article all attributes were assessed in 6-point scale, i. e. $x_i^{(j)}, y_i^{(j)} \in \{1,2,3,4,5,6\}$.

There are many other simple alternative MCDM methods similar to AVRG and EMDCW. Their efficiency comparison using indicator E_n proposed in the article is a separate interesting task. It is appropriate to look for the most efficient methods and investigate the cases when it makes sense to apply the method WEBIRA.

WEBIRA method is extended and applicable in the case of three or more subgroups of evaluating criteria. The first direction of our next research is to elaborate WEBIRA metodology for solution of practical problems where several groups of criteria naturally arise. For example, for solving sustainable management tasks where several interconnected domains such as ecology, economics, politics and environment are considered. Our other research area is the comparison of the proposed WEBIRA method with other existing methods used for solving this type of problems, i.e. when there are several natural groups of evaluation criteria. The third task is to prepare software for practical MADM problems solving by applying WEBIRA approach.

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