Lauryna ŠIAUDINYTĖ

# RESEARCH AND DEVELOPMENT OF METHODS AND INSTRUMENTATION FOR THE CALIBRATION OF VERTICAL ANGLE MEASURING SYSTEMS OF GEODETIC INSTRUMENTS 

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## Supervisor

Assoc Prof Dr Domantas BRUČAS (Vilnius Gediminas Technical University, Measurement Engineering - 10T).
The Dissertation Defense Council of Scientific Field of Measurement Engineering of Vilnius Gediminas Technical University:

## Chairman

Prof Dr Eimuntas Kazimieras PARŠELIŪNAS (Vilnius Gediminas Technical University, Measurement Engineering - 10T).

## Members:

Prof Dr Pasquale DAPONTE (University of Sannio, Italy, Measurement Engineering - 10T),
Dr Darius GAILIUS (Kaunas University of Technology, Measurement Engineering - 10T),
Prof Dr Mindaugas JUREVIČIUS (Vilnius Gediminas Technical University, Measurement Engineering - 10T),
Prof Dr Habil Vladas VEKTERIS (Vilnius Gediminas Technical University, Mechanical Engineering - 09T).
The dissertation will be defended at the public meeting of the Dissertation Defense Council of Measurement Engineering in the Senate Hall of Vilnius Gediminas Technical University at $\mathbf{2}$ p. m. on 12 November 2014.
Address: Sauletekio al. 11, LT-10223 Vilnius, Lithuania.
Tel.: +370 5274 4956; fax +3705270 0112; e-mail: doktor@vgtu.lt
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lauryna.siaudinyte@vgtu.lt

# GEODEZINIUৃ PRIETAISŲ VERTIKALIUJUU KAMPU MATAVIMO SISTEMU KALIBRAVIMO METODŨ BEI [RENGINIUֻ TYRIMAS IR TOBULINIMAS 

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## Vadovas

doc. dr. Domantas BRUČAS (Vilniaus Gedimino technikos universitetas, matavimų inžinerija - 10T).

Vilniaus Gedimino technikos universiteto Matavimų inžinerijos mokslo krypties disertacijos gynimo taryba:

## Pirmininkas

prof. dr. Eimuntas Kazimieras PARŠELIŪNAS (Vilniaus Gedimino technikos universitetas, matavimų inžinerija - 10T).

## Nariai:

prof. dr. Pasquale DAPONTE (Sannio universitetas, Italija, matavimų inžinerija - 10T),
dr. Darius GAILIUS (Kauno technologijos universitetas, matavimų inžinerija - 10T), prof. dr. Mindaugas JUREVIČIUS (Vilniaus Gedimino technikos universitetas, matavimų inžinerija - 10T),
prof. habil. dr. Vladas VEKTERIS (Vilniaus Gedimino technikos universitetas, mechanikos inžinerija - 09T).

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Adresas: Saulėtekio al. 11, LT-10223 Vilnius, Lietuva.
Tel.: (8 5) 274 4956; faksas (85) 270 0112; el. paštas doktor@vgtu.lt
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## Abstract

This research deals with the methods and instrumentation for calibration of vertical angle measuring systems of geodetic instruments. Two different methods are proposed in the thesis. First method is based on the vertical angle measuring system calibration using trigonometric method where 1 m reference scale with 1 mm grating is utilized. Two ways of application of this method are analyzed in the thesis as well as uncertainty sources and their impact on measurement results are provided in the table of the uncertainty budget.

Another method for vertical angle measuring system calibration is based on the new setup of the reference means. New proposed apparatus is designed to fit the instrument under calibration in horizontal position. Therefore, this setup enables to perform calibration of vertical angle measuring systems using horizontal angle measuring system calibration techniques. The special mirror mount was attached to the telescope of the calibrated instrument and the change of the telescope position was measured by the electronic autocollimator. The analysis of the uncertainty budget is presented in this thesis.

The dissertation consists of introduction, 3 chapters, general conclusions and references.

The introduction reveals the topicality of the thesis, investigated problem and object of the research. The aim and tasks as well as research methodology, scientific novelty, practical significance of the results and defended statements are also presented in the introduction.

Chapter 1 revises scientific papers on the subject of the dissertation. Analysis of standards, methods and instrumentation for the calibration of angle measuring systems are provided in this Chapter.

Chapter 2 describes the main principles of two proposed methods. The instrumentation and measurement procedure are analyzed as well as uncertainty evaluation model is designed.

Chapter 3 is focused on the experiment of the practical application of both proposed methods. The uncertainty sources are analyzed and specified in the tables of uncertainty budgets. The experimental results of the calibration of vertical angle measuring systems of the total station are revealed.

Research results are presented in 7 publications of scientific journals: 3 publications in journals indexed in ISI Web of Science data base with the impact factor, 4 - in other international scientific journals indexed in SCOPUS, Compendex databases. 5 papers are published in the proceedings of international conferences. 1 national patent regarding method for calibration of vertical angle measuring systems using reference scale was registered in the State Patent Bureau of the Republic of Lithuania.

## Reziumè

Disertacijoje detaliai nagrinėjami vertikaliujų kampų matavimo sistemų kalibravimo metodai ir priemonės. Disertacijoje siūlomi du skirtingi vertikaliuju kampų matavimo sistemų kalibravimo metodai. Pirmasis metodas igyvendinamas vertikaliuju kampų matavimo sistemų kalibravimui naudojant etaloninę 1 metro ilgio skalę, sudalintą 1 mm padalomis. Disertacijoje nagrinèjami du šio metodo igyvendinimo variantai - keičiant etaloninès skalès padètị ir išlaikant skalę vienoje padètyje.

Kitas siūlomas vertikaliuju kampų matavimo sistemų metodas pagristas nauju etaloniniu prietaisų išdėstymu. Sukurtas irenginys, leidžia kalibruojamaji prietaisą tvirtinti horizontalioje padètyje. Naudojant ši irengini vertikaliuju kampų matavimo sistemą galima kalibruoti naudojant horizontaliụju kampų matavimo sistemų kalibravimo principus. Veidrodèlis su specialiu laikikliu tvirtinamas prie kalibruojamojo prietaiso žiūrono, o atliekant kampų matavimus, pakitusi veidrodèlio padètis nustatoma elektroniniu autokolimatoriumi. Disertacijoje išnagrinėti paklaidų šaltiniai, darantys itaką matavimo rezultatų tikslumui ir nustatytos jų neapibrezžtys.

Disertaciją sudaro ịvadas, trys skyriai, išvados ir literatūros sąrašas.
Ivade nagrinejjamas darbo aktualumas, problema bei tyrimo objektas. Taip pat ìvade pateikiamas darbo tikslas, uždaviniai, tyrimo metodika, mokslinis naujumas, praktine darbo rezultatų reikšmė bei ginamieji teiginiai.

Pirmajame skyriuje nagrinėjama mokslinė literatūra, susijusi su disertacijos tematika. Analizuojami standartai, metodai ir priemonès susiję su kampų matavimo sistemų kalibravimu.

Antrajame skyriuje išdėstomi pagrindiniai dviejų siūlomų vertikaliụjų kampų matavimo sistemų kalibravimo metodų principai bei pasiūlytas neapibrež̌ties ịvertinimo modelis.

Trečiajame skyriuje pateikiama detali informacija apie eksperimentini tyrima, kurio metu išbandyti abu siūlomi vertikaliụjų kampų matavimo sistemų kalibravimo metodai.

Disertacijos tema paskelbti 7 moksliniai straipsniai, iš kuriụ 3 - mokslo žurnaluose cituojamose ISI Web of Science duomenų bazèje ir turinčiuose citavimo rodikli, 4 - kituose tarptautiniuose mokslo žurnaluose, cituojamuose SCOPUS bei Compendex duomenų bazėse. Disertacijos tema tarptautinėse mokslinėse konferencijose skaityti 5 pranešimai, išspausdinti konferenciju pranešimų rinkiniuose. Gautas LR Valstybinio patentų biuro patentas.

## Notations

## Symbols

m - meter
mm - millimeter
nm - nanometer
$\mu \mathrm{m}$ - micrometer
cm - centimeter
rad - radian
${ }^{\circ}$ - degree
' - arc minute
" - arc second
u - standard uncertainty
$\mathrm{u}_{\mathrm{c}}$-standard combined uncertainty
c - sensitivity coefficient
k - coverage factor
U - expanded uncertainty

## Abbreviations

KRISS - Korea Research Institute of Standards and Science
NIST - National Institute of Science and Technology
ESRF - European Synchrotron Radiation Facility

PTB - Physikalisch - Technischen Bundesanstalt (eng. National Metrology Institute of Germany)
VCC - Vertical Circle Comparator
TS - Total Station
GUM - Guide to the expression of Uncertainty in Measurement
ISO - International Organization for Standardization
LI - Laser Interferometer
AC - Autocollimator
TPM - Theodolite Testing Machine
NMI - National metrology institute
AIST - National Institute of Advanced Industrial Science and Technology

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## Introduction

## The Investigated Problem

Precision angle measurements and instrumentation are a few of the main factors ensuring quality in most fields of industry - civil engineering and survey, machinery, laboratory measurements, etc. Total stations are very widely used in the field of geodesy. Moreover, because of the good optics, high resolution angle measuring systems and the ability to perform distance and angle measurements while using only one instrument, they can be used in laboratory measurements too. There are two angle encoders embedded into total stations for horizontal and vertical angle measurements. Therefore, there is a need to calibrate these instruments in order to define their systematic errors, eccentricity of the encoders, etc. like any other angle encoder. Horizontal and vertical angle measurements have some specific features and needs specific arrangements for the calibration, especially this concerns vertical angle calibration. As previous research showed, most of the methods in angle metrology deal with the flat angle calibration. However, calibration of vertical angle measuring systems is an interesting task for scientists.

According to ISO 17123 part 3 regarding theodolite angle measurements it is stated that vertical angle measurements should be performed in the field using
only four targets. Angles measured between these targets are not sufficient to determine systematic errors of vertical angle measuring system. Such procedure would give only approximate results concerning angle measurement accuracy.

Therefore, there is a need to create new vertical angle measuring system testing methods and instrumentation for the indoor calibration of total stations. This research is based on creating and developing methodology and instrumentation for vertical angle measuring system calibration of total stations. Uncertainty evaluation is an essential part for such measurements helping to define error sources and their impact on measurement results. This research deals with analysis of combined uncertainty components as well as evaluation of expanded uncertainty.

## Importance of the Thesis

Many opto-electronic digital instruments, such as rotary encoders, theodolites, total stations, laser trackers, etc. are used in machine engineering and instrumentation, geodesy, surveying, robotics and other branches of industry. Most optical - electronic geodetic measuring instruments consist, among the other elements, of the circular scales and angular transducers for angle determination in two perpendicular planes - horizontal and vertical. Accuracy of the instrument mostly depends on the accuracy of these embedded angle measuring systems. Metrology of the optical instruments suited for horizontal and vertical angle measurements has some specific features and needs specific arrangements for their calibration, especially this concerns vertical angle calibration.

There are two new different competitive methods proposed in this thesis for the calibration of vertical angle measuring systems under laboratory conditions. Further development of these methods would reduce measurement uncertainty and enable to perform time saving measurement procedure.

## The Object of the Thesis

The object of this research is accuracy of vertical angle measuring systems of the total station.

## The Goal of the Thesis

The goal of this research is to propose new means and methods for calibration of vertical angle measuring systems and to determine error sources influencing measurement accuracy.

## The Tasks of the Thesis

The following tasks have to be carried out to achieve the goal of the work:

1. To determine the most relevant means and methods used for calibration of angle measuring systems of geodetic instruments under laboratory conditions.
2. To propose a trigonometric method for calibration of vertical angle measuring systems of geodetic instruments under the laboratory conditions.
3. To propose a new space efficient setup for calibration of vertical angle measuring systems of geodetic instruments.
4. To perform realization, measurement uncertainty evaluation and comparison of the proposed methods.

## Research Methodology

The methods and instrumentation used in World famous metrology institutes for angle measuring system calibration under laboratory conditions are analyzed. Standard means were used for development of calibration methods as well as new arrangement for the calibration of vertical angle measuring systems was proposed. In order to compare both approaches, the uncertainty evaluation of each method was performed by analyzing error sources and their influence on measurement results.

## Scientific Novelty

The scientific novelty was carried out by the following results:

1. A new trigonometric method using the linear scale in order to perform the calibration of vertical angle measuring systems under laboratory conditions was created and registered in the State Patent Bureau of the Republic of Lithuania.
2. A new instrumental setup for calibration of vertical angle measuring systems using horizontal angle calibration principle is proposed in the thesis.
3. A weight balanced apparatus for the control of total station position and convenience of vertical angle measuring system calibration procedure is proposed.

## Practical Significance of Achieved Results

Proposed methods are easy to perform. Moreover, they can be easily applied in laboratories using standard instrumentation available in most metrology laboratories. Proposed apparatus designed for the calibration of vertical angle measuring systems stabilizes the position of a total station making the measurement process more convenient. Practical realization of proposed methods requires significantly smaller premises due to smaller operating range.

## The Defended Statements

1. Calibration of vertical angle measuring system of a total station can be performed under the laboratory conditions by using reference means.
2. Calibration of vertical angle measuring system of a total station can be performed by using modified means for the calibration of horizontal angle measuring systems.
3. Statistical uncertainty evaluation can be applied for the quality control and development of both proposed methods.

## Approval of the Results

Research results are presented in 7 scientific publications -3 publications in ISI Web of Science data base with the impact factor, 4 - in other international scientific journals (indexed by SCOPUS, Compendex databases). 5 papers are published in the proceedings of international conferences:

- $8^{\text {th }}$ International Conference "Environmental Engineering", May 19-20, 2011, Vilnius, Lithuania.
- International Conference "Metrologia 2011", September 27-30, Natal, Brazil.
- XX IMEKO World Congress: Metrology for Green Growth, September 9-14, 2012, Busan, Republic of Korea.
- $9^{\text {th }}$ International Conference "Mechatronic Systems and Materials", July 1-3, 2013, Vilnius, Lithuania.
- $13^{\text {th }}$ IMEKO TC10 Workshop on Technical Diagnostics "Advanced Measurement Tools in Technical Diagnostics for Systems' Reliability and Safety", June 26-27, Warsaw, Poland.
1 national patent regarding method for the calibration of vertical angle measuring systems using reference scale was registered in the State Patent Bureau of the Republic of Lithuania.


## Dissertation Structure

The dissertation consists of introduction, 3 chapters, general conclusions and references. The volume of the thesis is 121 pages, 59 formulae, 57 figures, 7 tables and 95 references.

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## 1

## Angle Measurements - Standards, Methods and Instrumentation

In this Chapter scientific papers regarding methods and means of angle measuring system calibration as well as international standards are discussed in order to define the capabilities for calibration of vertical angle measuring systems under the laboratory conditions.

The material provided in this Chapter was published in scientific journals and proceedings (Bručas et al. 2010; Šiaudinyte et al. 2011; Rybokas et al. 2011; Rybokas et al. 2013; Giniotis et al. 2013).

### 1.1. Standards and Calibration in Angle Metrology

There are many methods and regulations for high quality angle measurements. They are all described in national and international standards as well as reports of national metrology institutes. However, there are only a few standards describing angle measurements of geodetic instruments. Regarding vertical angle measurements in ISO 12857 the procedure of vertical angle measurements using the grating of an invar geodetic staff as a target is described. Due to time consuming data processing the new standard and simplified field procedure was suggested in ISO 17123. DIN 18723 is German national standard
analogous to ISO 12857 which is focused on precision testing of the instruments (Zeiske 2001, ISO 2001).

In ISO 17123 part 3 it is stated that testing of vertical angles should be performed in the field with 50 m distance between the instrument and the target. It is also stated that vertical angle measurements should cover the range of $30^{\circ}\left(90^{\circ} \pm 15^{\circ}\right)$ with the number of sets $\mathrm{n}=3$ in both faces of an instrument. All these requirements for the vertical angle measurements can lead to an approximate measurement results and increase inaccuracies in uncertainty evaluation. However, there is no international standard covering the methodology of the calibration of geodetic instruments under the laboratory conditions. Although the radian is SI unit, in ISO 17123-3 examples preferred units for evaluation of accuracy parameters of geodetic instruments are angles, arc minutes and arc seconds. Therefore, there are no strict regulations for units used in angle calibration (Emerson 2005). In part 5 (ISO 2005 ${ }^{\text {b }}$ ) regarding electronic tacheometers the coordinate determination principle is analyzed and data processing techniques are suggested for repeatability determination under field test conditions. However, it is stated that for uncertainty determination one should refer to GUM (Guide to the expression of uncertainty in measurement) (ISO 2001; ISO 2002; ISO 2004).

Guide to the expression of uncertainty in measurement is the main standard determining the uncertainty evaluation of measurements. This set of rules is very widely used in metrology institutes Worldwide to perform international comparisons of measurement results. The accuracy parameters are defined in ISO 5725-1. Accuracy of measurement results depend on many factors, such as operator, the equipment used, calibration of the equipment, environmental conditions and the time between measurements. The term accuracy is used to refer to trueness and precision. The trueness of the measurement method can be investigated by comparing the accepted reference value with the level of the results given by the measurement method and expressed in terms of bias which is the difference between the expectation of the test results and an accepted reference value. Trueness refers to the closeness of agreement between the arithmetic mean of test results and the accepted reference value. Precision refers to the closeness of agreement between test results. Therefore, it is possible for test results to be precise but not accurate. The term accepted reference value is defined as a value that serves as an agreed-upon reference for comparison and which is derived as a theoretical or established value based on scientific principles or an assigned or certified value, based on experimental work of national or international organization (ISO 1994; ISO $2005^{\text {a }}$ ).

To sum up, there is a lack of information providing principles and methods for the calibration of vertical angle measuring systems. Moreover, testing of
geodetic instruments under laboratory conditions should be analyzed more in detail. Therefore, it leads to the conclusion that new method should be developed for the calibration of vertical angle measuring systems as well as error sources and measuring principle analyzed.

### 1.2. Errors, Error Sources and Uncertainty Evaluation

### 1.2.1. Errors of Geodetic Instruments

Errors in measurement caused by imperfections of measurement instruments, measurers and natural environment influences are unavoidable. These errors can be of systematic or random nature. Systematic errors, caused by lack of calibration of instruments as well as maladjustment or the effects of the environment are generally described as errors that follow mathematical or physical laws. Such errors, if they are discovered, can be quantified by testing or calibrating devices and understanding principles and laws of nature that have effect on the measurements. This leads to the possibility of correcting systematic errors. Such errors tend to occur due to equipment flaws or problems with the design of the experiment. While systematic errors, unlike random errors, tend to always shift the results in one direction they cannot be estimated by repeating the experiment using the same equipment. Therefore, such errors are harder to estimate compared to random errors. Systematic error location and minimization involves deep analysis and design of the test conditions and procedures, comparison of results to other independently achieved results using different equipment techniques. It is also achievable by carrying out an experimental procedure involving known reference values and procedure adjustment until desired results are obtained.

Errors that follow random patterns are considered to be random errors. These errors are unavoidable as well. They tend to have varying mathematical signs. Such errors occur in unknown but definable magnitude. Random errors are generated by imperfections of instruments and measurers as well as uncertainties of environment effect determination.

Errors caused by people performing the experiment are called personal errors. These errors are most commonly random and are caused by human's personal inability to achieve absolute exactness. Therefore, there is always a certain level of inaccuracy in any tasks performed, starting with centering of instruments over ground points and reading rods and scales. Instead of considering random errors as mistakes, they are considered as minor deviations. Unevaluated adjustment changes of instruments as well as loose parts or components of the instrument may cause the appearance of such errors. This
leads to a conclusion why systematic errors can never be fully eliminated from the measurements. It is due to additional random and personal errors that tend to add uncertainties that are unquantifiable. Such errors are controlled by quality control process, not post-measurement elimination. They can be detected and minimized through repeated measurement statistical analysis (Buckner 1997). The bias $\left(\Delta_{i}\right)$, expressed in (1.1) is the total systematic error which can be estimated as a difference between the arithmetic mean of the experiment results and the accepted reference value.

$$
\begin{equation*}
\Delta_{i}=\overline{q_{i}}-\mu \tag{1.1}
\end{equation*}
$$

where $\overline{q_{i}}$ - arithmetic mean of the experiment results, $\mu$ - accepted reference value.

The errors that occur while performing the experiment depend on the instruments and the method chosen for the experiment. This research is based on angle measurements, therefore specific angle measuring instrumentation as well as the error sources are needed to be analyzed. It is important to evaluate every possible error source because they are the components of the measurement uncertainty (Stone at al. 2004).

Since no instruments can be produced without any errors instrumental errors must be calculated or compensated while performing measurements in order to reduce or eliminate them. There are four main errors dependent on distance measurements: the zero error, the cyclic error, the scale error and optical pointing error.

Optical pointing error is a random error related to the magnification of the total station. Instrument pointing error is caused by the misalignment of the EDM signal and the collimation axis. This error is influenced by focusing, optical qualities of the telescope, target design and size, operator bias and atmospheric conditions if measurements are performed in the field. The instrument pointing error for field measurements can be expressed:

$$
\begin{equation*}
\sigma_{p}=\frac{45^{\prime \prime}}{M} \tag{1.2}
\end{equation*}
$$

where $\sigma_{p}$ - instrument pointing error and $M$ - objective lens magnification.
Performing measurements with a number of repetitions reduces the standard deviation. When measuring angles every repetition consists of two pointings, therefore, angle pointing error is expressed:

$$
\begin{equation*}
\sigma_{\alpha p}=\frac{\sigma_{p} \sqrt{2}}{\sqrt{n}} \tag{1.3}
\end{equation*}
$$

where $\sigma_{\alpha p}-$ angle pointing error; $\sigma_{p}$ - pointing error which is characterized by the standard deviation of every measured angle. Pointing error can be assumed the same for each repetition; n - number of repetitions in the same setup.

In long distance measurements pointing error can be significant to both horizontal and vertical angle measurements. This error is minimized by using the technique of averaging sets of angles or using greater magnification lens. This technique helps to reduce atmospheric influence if the measurements are performed outdoors. The source of pointing error is misalignment of the EDM signal and collimation axis. After the determination of scale coefficient all measured distances must be multiplied to correct the readings.

Manufacturers quote the estimated combined pointing and reading precision for an individual direction measured with both faces of the instrument in terms of standard deviations. According to the standards DIN 18723 or ISO 12857 angle reading and pointing error while measuring with a total station is expressed:

$$
\begin{equation*}
\sigma_{o p r}=\frac{2 \sigma_{D I N}}{\sqrt{n}} \tag{1.4}
\end{equation*}
$$

where $\sigma_{\text {apr }}-$ angle reading and pointing error; $\sigma_{\text {DIN }}-$ DIN value published in DIN 18723 standard depending on total station accuracy written in specification; n - number of observations. Multiple readings using both total station faces compensate systematic errors and increase the precision. Therefore, it is always better to use the method of repetition (Coan 2011; Engineer manuals 2002; Ghilani 2010). The scale error is a systematic error proportional to the measured distance. This error is caused by the drift in frequency of the quartz crystal oscillator in the instrument, emitting and receiving diodes, mechanical aging of the instrument components or incorrect values of temperature, humidity and pressure measurements (Rüeger 1996). For EDM scale coefficient determination it is recommended to perform a series of linear measurements over certified baselines with known distances and compare them against the known ones or compare calibrated instrument directly against frequency testing apparatus. The unknown scale coefficient can be determined as follows:

$$
\begin{equation*}
k=\frac{D}{S} \tag{1.5}
\end{equation*}
$$

where k - unknown scale coefficient; S - measured distance; D - known distance.

Zero error or additive constant is also very important in distance measurements with total stations. It is a systematic error which occurs when EDM and prism measurements are performed. The calibration of zero error is based on comparison of measured distance with total station and prism with a
known distance measured with interferometer. There are special benches designed for zero error determination. Determined offset has to be added to all measured distances (Martin, Gatta 2006).

Another systematic error in EDM of total stations is the cyclic error often called short periodic error which occurs in EDM devices due to the carrier wave and phase measurement amplitude non-linearity. The magnitude and sign of cyclic error varies depending on the measured length due to the tendency of such error to repeat itself within a measured distance for each unit length. Cyclic error tends to increase along with ageing of the components of the device. Cyclic error is calibrated to determine instruments' behavior while measuring different distances. Usually the total station is placed on the pillar and reflector is placed at certain precisely measured distance. Then reflector is moved further to the point with known parameters. Cyclic errors apply to the distance meter and reflector pair. Measured distances are compared to the ones measured with the reference means. After the measurements the graph of readings is sinusoidically approximated to determine the influence of cyclic error (Skeivalas 2004).

Determination of another systematic error of collimation axis is based on pointing the telescope of very precisely leveled total station to the point close to the horizon and taking horizontal angle readings of both instrument faces. Collimation error is a deviation between optical axis of TS and its line of sight (Fig. 1.1). Theoretically, if there was no collimation error, the difference between two faced measurements should be exactly $180^{\circ}$. The computed difference of readings is called double collimation error and is expressed in formula (1.6) (GKTR 2000):

$$
\begin{equation*}
c=\frac{r_{F 1}-\left(r_{F 2} \pm 180^{\circ}\right)}{2} \tag{1.6}
\end{equation*}
$$

where $\mathrm{r}_{\mathrm{F} 1}$ - reading of horizontal angle in one face position; $\mathrm{r}_{\mathrm{F} 2}$ - reading of horizontal angle in opposite face position; c - collimation error. There is an automatic compensation system for temperature deviations and optical and electrical system fluctuations embedded in the angle measuring systems. Biaxial compensator of the total stations is designed to reduce measurement errors in both directions. There is a semiconductor light diode (Gallium and Arsenic diode) integrated in an EDM as a light source.


Fig. 1.1. Determination of collimation error

Vertical index error indicates the angle between the zenith direction and the zero reading of the vertical circular scale. This error needs to be checked daily for more accurate measurements (Zeiske 2000). This systematic error can be checked by pointing the cross hairs of the telescope of very precisely leveled total station to the point in both faces. Vertical index error is similar to collimation error just in vertical plane and it can be eliminated by measuring in both instrument faces. It can be determined as follows:

$$
\begin{equation*}
\sigma_{v i}=\frac{360^{\circ} n-\left(\sum r_{F 1}+\sum r_{F 2}\right)}{2 n} \tag{1.7}
\end{equation*}
$$

where $\sigma_{\mathrm{vi}}-$ vertical index error; $\Sigma \mathrm{r}_{\mathrm{F} 1}$ - sum vertical angle readings in one face position; $\Sigma r_{\mathrm{F} 2}$ - sum of vertical angle readings in opposite face position; n number of $\mathrm{r}_{\mathrm{F} 1}$ and $\mathrm{r}_{\mathrm{F} 2}$ pairs.

Glass circular scales embedded in angle encoders are very important components of total station angle measuring systems. Since every additional component of the system can produce additional errors, some of them are related to glass circles. Glass circle eccentricity error is caused by incorrect disk position in the instrument. Therefore, the misalignment of total station's vertical axis and horizontal circle as well as horizontal (tilting) axis and vertical circle appears. Glass circle graduation error appears when there is an eccentricity in graduations of circular scale. This error refers to the position of graduations as well as equal spacing between them. Graduation error is minimized by a very precise photoetching technique which is based on the photo - reduced master scale image projection on the circle. Due to the encoder scanning system, both eccentricity and graduation errors can be eliminated or compensated by performing measurements in both instrument faces. Every instrument can be tested individually to determine the sine curve of the circle error and the determined correction factor is applied to every measured angle (Engineer manuals 2007, Martin et al. 2003).

During the measurements total station must be very precisely leveled to avoid leveling error and to ensure the perpendicularity between the instrument exes. While rotating total station around the wobble error which cannot be eliminated by taking measurements in both faces may occur. However, biaxial compensator along with specific compensator can reduce both leveling and wobble errors to negligible.

Abbe error is one of the most common uncertainty sources appearing due to the tilt of measured object. Such tilt may arise during the motion of the object or bad alignment. Abbe errors can be determined by an autocollimator. There are special angle control loops embedded in length measurement machines to reduce the Abbe error. After corrections it is possible to reduce this error down to 20 nm
(Koening et al. 2007). Main errors regarding total station angle measurements are shown in Figure 1.2.


Fig. 1.2. Main Total Station errors regarding angle measurements

### 1.2.2. Uncertainty

ISO 17025:2005 "General Requirements for the Competence of Testing and Calibration Laboratories" is an international standard representing the set of requirements which have to be met by the checking and testing laboratories before their accreditation procedure. This standard deals with the laboratory management system requirements as well as technical requirements including measurement methods, equipment, staff and result reporting requirements. For the estimation of uncertainty of measurement it is required to use appropriate methods of data analysis as it is stated in GUM (Guide to the Expression of Uncertainty in Measurement, issued by BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML) (ISO 17025:2005).

Therefore, the main reference for uncertainty evaluation is ISO/IEC Guide 98:1993 Guide to the expression of uncertainty in measurement (GUM) or the modified version by Joint Committee for Guides in Metrology „Evaluation of measurement data - Guide to the expression of uncertainty in measurement".

Every measurement result has an uncertainty. Any measurement consists of the measurement method, the measuring instrumentation and the operator which generally are the main error sources. It is possible to control or reduce uncertainty by employing experienced metrologist, modern precision instrumentation and reliable method together. However, it is impossible to eliminate it completely. The uncertainty needs evaluation in order all possible error sources to be determined as well as their magnitude and influence to the final result.

Uncertainty of measurement in ISO GUM is defined as a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand. Uncertainty should not be mixed with an error because uncertainty always has a positive value unlike error which might have both negative and positive values.

There are few types of uncertainties described further in this Chapter. In general, standard uncertainty is defined as the uncertainty of the measurement result expressed as a standard deviation. Sometimes the term of systematic standard uncertainty is used to specify the nature of uncertainty components appearing due to the influence of systematic errors which affect every measurement. Although uncertainty and error are not the same terms, all uncertainty components are arising from random or systematic effects (as errors) and can be evaluated by both uncertainty evaluation types. Combined standard uncertainty is the combination of uncertainties due to random and systematic errors while the variances and covariances of the components are evaluated. Expanded uncertainty is reported as an interval and expressed as a combined standard uncertainty multiplied by a coverage factor which refers to the level of confidence. Expanded uncertainty shows the interval in which the values that can be attributed to the measurand according to the best estimate of the value are distributed (Taylor, Kuyatt 1994; JCGM 2008).

General flowchart for uncertainty evaluation is presented in Figure 1.3.


Fig. 1.3. General flowchart for uncertainty evaluation

There are two main methods for the uncertainty evaluation:

- Type A evaluation of standard uncertainty is performed by statistical methods using series of observations. During this procedure the standard deviations of all parameters are estimated and analysis of variance is carried out to identify and quantify the parameter arising from random or systematic effects.
- Type B evaluation of standard uncertainty is performed using other means when series of observations are not available. It might be specifications or other documentation as well as evaluation based on previous experience and scientific knowledge.
Type A method is based on statistical evaluation of measurement results. First of all, the arithmetic mean of the results of independent observations is estimated as showed in (1.8):

$$
\begin{equation*}
\overline{\mathrm{q}}=\frac{1}{\mathrm{n}} \sum_{k=1}^{n} q_{k} \tag{1.8}
\end{equation*}
$$

where $\bar{q}$ - arithmetic mean; n - number of independent observations; $q_{k}$ - the value of individual observation.

According to the definition repeatability is the precision under conditions where independent test/ measurement results are obtained with the same method on identical test/measurement items in the same test or measuring facility by the same operator using the same equipment within short intervals of time (ISO 5725-1:1994). The experimental standard deviation of random observations is a parameter related to the repeatability of the measurements and shows the dispersion of observed values about their arithmetic mean and is calculated as follows (1.9):

$$
\begin{equation*}
s^{2}\left(q_{k}\right)=\frac{1}{n-1} \sum_{j=1}^{n}\left(q_{j}-\bar{q}\right)^{2} \tag{1.9}
\end{equation*}
$$

where $\bar{q}$ - arithmetic mean; $n$ - number of independent observations; $q_{j}-$ observed values. An estimate of the standard uncertainty is the experimental standard deviation of the mean which is expressed in (1.10) and computed for every data set:

$$
\begin{equation*}
u^{2}=s^{2}(\bar{q})=\frac{s^{2}\left(q_{k}\right)}{n} \tag{1.10}
\end{equation*}
$$

where $\mathrm{s}^{2}\left(\mathrm{q}_{\mathrm{k}}\right)$ - experimental standard deviation; n - number of independent observations.

After evaluating the standard uncertainties of all error sources by using Type A or Type B evaluation methods, the sensitivity coefficients must be evaluated as the partial derivatives. Sensitivity coefficients describe how the output estimates vary with changes in the values of the input estimates and can be calculated as shown in (1.11):

$$
\begin{equation*}
c_{i}=\frac{\partial f}{\partial x_{i}} \tag{1.11}
\end{equation*}
$$

where $\mathrm{c}_{\mathrm{i}}-\operatorname{sensitivity~coefficient;~} \frac{\partial f}{\partial x_{i}}$ - the partial derivative of a function $f$ with respect to the variable $x_{i}$.

Combined standard uncertainty is expressed as square root of the sum squares of standard uncertainties of combined uncertainty components multiplied by their sensitivity coefficients:

$$
\begin{equation*}
u_{c}^{2}(y)=\sum_{i=1}^{N}\left[c_{i} u\left(x_{i}\right)\right]^{2} \tag{1.12}
\end{equation*}
$$

where $c_{i}-$ sensitivity coefficient of the $i^{\text {th }}$ component of standard combined uncertainty; $u\left(x_{i}\right)$ - standard uncertainty of the $i^{\text {th }}$ component of standard combined uncertainty; N - number of standard combined uncertainty components.

If the input quantities are correlated, then correlation coefficient $(-1 \leq r \leq 1)$ should be computed as follows (1.13):

$$
\begin{equation*}
r\left(x_{i}, x_{j}\right)=\frac{u\left(x_{i}, x_{j}\right)}{u\left(x_{i}\right) u\left(x_{j}\right)} \tag{1.13}
\end{equation*}
$$

where $u\left(x_{i}, x_{j}\right)$ - standard uncertainty of the input quantity; $u\left(x_{i}\right) u\left(x_{j}\right)$ - the product of standard uncertainties of correlated components.

In case of correlated input quantities combined uncertainty is expressed:

$$
\begin{equation*}
u_{c}^{2}(y)=\sum_{i=1}^{N} c_{i}^{2} u^{2}\left(x_{i}\right)+\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{i} c_{j} u\left(x_{i}\right) u\left(x_{j}\right) r\left(x_{i}, x_{j}\right) \tag{1.14}
\end{equation*}
$$

where $c_{i}, c_{j}-$ sensitivity coefficients of the combined standard uncertainty components; $r\left(x_{i}, x_{j}\right)$ - correlation coefficient between correlated input quantities; $u$ - standard uncertainty of combined standard uncertainty components; $N$ number of combined standard uncertainty components.

Coverage factor (k) is related to the probability with which the best estimate of the value falls into the certain interval. The coverage factor depends upon the
type of probability distribution of the output quantity in a measurement model and on the selected coverage probability. For the normal distribution, $k=1$ produces an interval having a level of confidence of approximately $68 \%, k=2$ produces an interval having a level of confidence of approximately $95 \%$, and $k=$ 3 produces an interval having a level of confidence of approximately $99 \%$. For the rectangular distribution $\mathrm{k}=1.65$ to reach the confidence level of $95 \%$. However, for Student's or t -distribution the coverage factor k depends on the degrees of freedom (DOF). For type A evaluation the degrees of freedom $v_{i}=n-1$ are determined for a single quantity estimated by the arithmetic mean of $n$ independent observations. For type B evaluation the degrees of freedom can be determined as shown in (1.15):

$$
\begin{equation*}
v_{i}=\frac{1}{2}\left(\frac{100}{R}\right)^{2} \tag{1.15}
\end{equation*}
$$

where $R$ - relative standard uncertainty in percent.
It can also be calculated as shown in (1.16).
DOF as it appears in the $t$-distribution is a measure of the uncertainty of the variance. The distribution of the variable may be approximated by a t-distribution with an effective degrees of freedom $v_{\text {eff }}$ obtained from the Welch-Satterthwaite formula (1.16). Effective degrees of freedom are used to determine combined degrees of freedom of combined uncertainty components in order to determine the coverage factor from the table of t -distribution.

$$
\begin{equation*}
v_{e f f}=\frac{u_{c}^{4}(y)}{\sum_{i=1}^{N} \frac{u_{i}^{4}(y)}{v_{i}}} \tag{1.16}
\end{equation*}
$$

where $u_{c}$ - combined standard uncertainty; $u_{i}$ - standard uncertainty of $i^{\text {th }}$ component of combined uncertainty; $v_{i}$ - degrees of freedom of $\mathrm{i}^{\text {th }}$ component of combined uncertainty; $N$ - number of combined uncertainty components.

After the determination of the coverage factor the expanded uncertainty of the measurements can be evaluated as shown in (1.17). Expanded uncertainty expresses previously available information in the form which describes the interval and is used in reporting measurement results.

$$
\begin{equation*}
U=k \cdot u_{c}(y) \tag{1.17}
\end{equation*}
$$

where k - coverage factor; $\mathrm{u}_{\mathrm{c}}(\mathrm{y})$ - combined standard uncertainty.
The result of the measurement should be stated in such form as shown in (1.18) including the best estimate of the value (y) expanded uncertainty (U) (JCGM 2008).

$$
\begin{equation*}
Y=y \pm U \tag{1.18}
\end{equation*}
$$

Sometimes relative uncertainty is required to report the measurement results and is used in degrees of freedom of uncertainty calculations. The relative uncertainty is expressed as percentage of the ratio between the modulus of best estimate of the value and the uncertainty. If uncertainty used in the formula (1.19) is standard uncertainty $(u)$ then it would be standard relative uncertainty $\left(u_{r}\right)$ calculated, if the uncertainty used in formula is combined $\left(u_{c}\right)$ or expanded $(U)$ then it would be combined standard relative $\left(u_{c, r}\right)$ or expanded relative uncertainty $\left(U_{r}\right)$ respectively (Taylor 1994).

$$
\begin{equation*}
u_{r}=\frac{u\left(x_{i}\right)}{\left|x_{i}\right|} ; \quad u_{c, r}=\frac{u_{c}\left(y_{i}\right)}{\left|y_{i}\right|} ; \quad U_{r}=\frac{U\left(y_{i}\right)}{\left|y_{i}\right|} \tag{1.19}
\end{equation*}
$$

During uncertainty evaluation it is recommended to fill the table of an uncertainty budget where all the parameters such as standard uncertainties, combined uncertainties, sensitivity coefficients, probability distributions and degrees of freedom of all combined uncertainty components would be clearly stated (Rabinovich 2010).

### 1.3. Angle Measuring Systems

### 1.3.1. Angle Measurements in Geodesy and Surveying

Rotary encoders, total stations, laser trackers and other opto-electronic digital instruments are used in fields of robotics, surveying, machine engineering and many others. Circular scales and angular transducers for angle determination in horizontal and vertical planes are commonly the key components of such optoelectronic geodetic measuring instruments. The accuracy of such instruments is directly dependent on the accuracy of the embedded angle measuring instruments. Measurements using such equipment are specific and require certain arrangements for instrument calibration and especially for measurements in vertical plane.

The main instruments used in geodetic measurements are total stations often called or tacheometers (Fig. 1.4).

Tacheometry (gr. tacheos - fast, metreo - measure) is the geodetic measurement method for the determination of the Earth's surface point position in three coordinates ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ). During the measurements horizontal and vertical angles are measured to make a relation between measured points. Total stations are irreplaceable in survey and civil engineering for angle, distance, height difference measurements.


Fig. 1.4. A cross section of a total station (Zogg et al. 2009)
Total station basically consists of theodolite and electronic distance measurement device (EDM). There are two angle measuring systems for both horizontal and vertical angle measurements embedded in a total station. Angle encoders are the most important components of these angle measuring systems. While rotating a total station around the vertical axis, the horizontal angle is being measured and while rotating TS telescope around horizontal axis, the vertical angle is being measured. There is an automatic compensation system for elimination of temperature deviations as well as fluctuations of optical system and electrical circuit embedded into total station. Biaxial electronic compensator integrated in a total station reduces measurement errors to minimum. As a source of light in EDM the semiconductor diode (GaAs - gallium and arsenic) is used (Skeivalas 2004). Angles are recorded digitally and previewed on the screen using image processing and pattern recognition methods. The main components
of angle and distance measuring systems as well as tilting system of a total station are shown in Fig. 1.4.

### 1.3.2. Angle Encoders

Rotary encoders are widely used in precision angle measuring instrumentation to determine the position of the rotational axis. The main component of rotary encoder is circular scale with the grating of various pitches which is scanned by a reading head unit. There are few reasons which have a big impact on measurement accuracy. Nonuniformity of the scale pattern and the eccentricity of the pattern or irregular shape of the scale can cause big measurement errors as well as other factors such as scale installation on the measured rotary axis, the scanning head alignment and signal processing circuits. The eccentricity of the scale can be reduced by adding additional reading heads to the encoder (Lu, Trumper 2006).

Rotary encoder is an electromechanical device used to encode angular movement of the shaft or axis of the measuring system to a certain analogue or digital signal (Siaudinyte et al. 2012). The modulation of the light beam crossing the raster scale and indication scale is the main principle of operating the rotary encoder. A circular scale is an efficient and reliable mean of getting and passing information about the angular position of an object or axis of the measuring instrument. Scales are made of various materials depending on operational conditions, accuracy and price (Giniotis, Grattan 2002; Bručas et al. 2013 ${ }^{\text {b }}$ ). There are two widely used types of angle encoders: absolute and incremental. The main difference between an absolute and incremental encoder is that the absolute encoder always has a fixed zero position. However, this position varies in the incremental encoder showing the difference between the previous and present position of the encoder. A digital encoder generates a unique digital binary code for each turn of an axle. The rotary encoder has a circular raster scale mounted on the shaft and divided into many parts that define angular degrees, minutes, seconds and the decimal parts of seconds of arc (depending on the discretion of the required data). The coded scale consists of the parts of the circle covered by the layer of a black or white color and has a preset binary value 1 or 0 (Fig. 1.5). During operation, the angle encoder is turned to the needed angle position and stops at a certain combination of the black and white parts of the scale. Each part has a unique binary code that is recognized as angle reading which is transmitted to further data processing devices. An optical disk of an absolute angle encoder is intended to assign digital codes to a certain position of a shaft (i.e. if a circular scale consists of 8 tracks with engraved marks, this scale is able to generate 256 different positions or angular movements with the accuracy of 1.406 degrees equal to $360 / 256$ ). Most common numerical encoding
for this type of encoders is based on the position of the black and white parts and binary or Gray coding systems. Upon rotation of the circular scale and shaft on the axle, photo detectors read the pattern and generate a digital code. During the revolution of a digital code generating the angle encoder, all elements change their position. All track readings of the code encoder have outputs separately from each other. The way the glass circle is divided into black and white blocks is not completely standardized and depends on a manufacturer. Using the red light from LED (Light Emitting Diode) and a mirror, the marks are projected onto the CCD (Charge - Coupled Device) linear array (Šiaudinyte et al. 2011; Fraden 2010).


Fig. 1.5. Types of circular scales used in
a) incremental encoders; b) absolute encoders (Šiaudinytè, Giniotis 2011)

Many angular transducers or encoders are used in industry and machine engineering for the position and displacement measurement. The accuracy of angular position fixed by means of these devices reaches $0.1^{\prime \prime}-0.3^{\prime \prime}$. Control of their accuracy parameters is complicated task consisting from some high requirements needed for rotation, positioning, signal processing, object adjustment and data processing. These elements are commonly used in complex angle measuring systems automatically controlled by computer (Giniotis et al. 2013; Rybokas et al. 2013).

German enterprise Heidenhain is famrous for manufacturing best quality high resolution angle encoders which are often used for the calibration of angle measuring instruments in metrology institutes of all over the world. Recently Heidenhain presented a new type of angle encoders which have completely different components from previous angle encoders. It is an absolute encoder, although it has two graduation tracks. The absolute position is determined while an absolute track and higher resolution incremental track is scanned. The scanning signals of the incremental fine track are interpolated for the position value and are processed together with the information from the serial code track to obtain absolute position values of high resolution. This single field scanning
reduces sensitivity of contamination and has more advantages than four field scanning which increases deviations of scanning signals according to the contaminated areas of the circular scale. Scanning principle of the angle encoder is shown in Fig. 1.6 (Heidenhain 2008, Heidenhain 2011).


Fig. 1.6. One field scanning principle of the angle encoder (Heidenhain, 2011)
Angle encoder is a system that converts the angular motion created by the changes of physical parameters occurred due to the displacement of the system elements in the sensitive element to the signals of certain accuracy of information about the displacement magnitude and direction. Photoelectric angle encoders consist of three main component groups - mechanical, optical and electronic. The mechanical part of the angular encoder assures the rotation of the shaft as well as the circular scale being attached to it in respect of the stationary part of the device with certain accuracy. The stationary part of the encoder consists of the circular raster scale, illumination and signal recording elements. Optical part includes light emitting diode, raster disk and lens. This part is responsible for illumination of the raster disk and directing light signals to the photo elements. Electronic part of the encoder converts impulses to the rectangular shaped electronic signals and amplifies them. Signal compatibility with the software and digital indication systems is also performed by the electronic part. The encoder error can be caused by mechanical, optical and electrical factors. Even dust on the grating might distort the fringe pattern which result in wrong reading. Intensity of light and mechanical rotations may also have an impact on the produced signal. Such raster discs can be calibrated by using reference means such as another raster scales' pattern or special benches including rotary tables and microscopes (Brea, Morlanes 2008; Giniotis, Rybokas 2010).

The photoelectric angular encoder operation is based on the radiant flux crossing two raster elements, one of which is a rotating raster disk and the other
is stationary. Rotation of the encoder shaft causes changes of the combinations of possible ways for the radiant flux to go through that the interaction of two disks generate. These combinations, recorded by the photodiodes, are converted to electric signals of a defined shape. The increment of the signal enables the rotational movement magnitude determination (Histand, Alciatore 1999).

Angle encoders vary in diameter depending on their resolution. The biggest in diameter angle encoders are commonly used as the main components in manufacturing of high accuracy angle comparators and angle generators. Such comparators are motorized and have multiple reading heads ( 8 to 12 or more). By using such comparators a final readout has a resolution of $0.05^{\prime \prime}$ can be achieved and the expanded uncertainty is minimized to $0.03{ }^{\prime \prime}$. These instruments also may perform measurements at a very small measurement step up to $0.0012^{\prime \prime}$ with standard uncertainty to $0.005^{\prime \prime}$. High resolution horizontal angle comparators are used ta well known National metrology institutes such as KRISS (Republic of Korea), PTB (Germany) and NIST (U.S.A) (Kim et al. 2013; Probst et al. 1998). The latest angle encoders are often used as a reference in angle metrology. Modern, self-calibrated encoders with ten (or more) reading heads, reduced eccentricity and improved scale graduation can reduce measurement uncertainty to minimum. Since angle encoders are calibrated against each other, they are often used for flat angle calibration of total stations. The angle encoder developed in AIST (National institute of advanced Industrial Science and Technology) is used as a national standard for an angle in Japan. This standard encoder performs measurements with the angle deviation small as 0.11 and uncertainty of 0.01 " (Watanabe, 2008).

### 1.3.3. Rotary Tables

Modern rotary tables evolved from mechanical, accurate worm - and - gear devices which generate angle by dividing the circle. Rotary tables are created to perform fluent continuous rotation and this is the biggest difference from indexing tables where positions of angles are fixed according to their serrated tooth indexer pitch. Biggest errors for rotary tables are angle errors and eccentricity errors. The rotation is highly affected by the uniformity of the oil film and a heavy asymmetrical workpiece because then oil film tends to be thicker on one side and influences the rotation. Rotary tables for trueness of rotation are calibrated in a horizontal and in a vertical position (Moore 1970).

In the picture below (Fig.1.7.) the exploded view of the warm gear rotary table which was developed by Sherline Company is shown (Sherline Products Inc 2013). The table is marked every $5^{\circ}$ and its handwheel of $5^{\circ}$ is divided into 50 parts which leads to the pitch of 6 arc minutes. This table can be used in both
horizontal and vertical planes. Main components of such rotary table are presented in Table 1.1.


Fig. 1.7. The view of disassembled rotary table (Sherline Products Inc 2013)

Table 1.1. Main components of ordinary rotary table

| Part <br> No. | Description | Part <br> No. | Description | Part <br> No. | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Bearing | 10 | Oiler | 19 | Cap screw |
| 2 | T-nuts | 11 | Preload Nut | 20 | Cap screw |
| 3 | Set screw | 12 | Lock Pin | 21 | Headstock bearing |
| 4 | Hold down clamp | 13 | Upright | 22 | Cap screw |
| 5 | Chuck adaptor | 14 | Right angle base | 23 | Cup point set screw |
| 6 | Rotary table base | 15 | Button cap screw | 24 | Cone point set screw |
| 7 | Table | 16 | Hold down tab | 25 | Washer |
| 8 | Worm housing | 17 | Button cap screw | 26 | Cap screw |
| 9 | Worm gear | 18 | Handwheel assembly | 27 | Pointer |

Rotary tables may be used as the component of other complex machines (CNC, CMM etc.).

There are three main types of rotary tables - worm gear rotary tables, roller gear rotary tables and direct drive motor rotary tables. During the research of comparison of worm gear and ball gear rotary tables they were calibrated using personal computer with DSP board, servo amplifier, servo motor with rotary encoder, and rotary table with an encoder of resolution of $0.0001^{\circ}$ attached to the
output axis. Rotary tables were controlled by computer and their values were compared with the values of the encoder. Measurements were repeated 5 times in clockwise and counter clockwise directions. The positioning accuracy and repeatability of rotary tables are very important indicators which express the accuracy level of them. Accuracy is the difference between the actual position and the position measured by a reference measurement device. Repeatability is defined as the range of positions attained when the system is repeatedly commanded to one location under identical conditions. The rotational fluctuation is the deviation of rotation angle of the table from that of the motor. This can be categorized as a systematic deviation. This occurs due to the pitch error of the driving mechanism. According to the research results, roller gear rotary table showed a better performance (repeatability of $3.6^{\prime \prime}$, accuracy of $10,6^{\prime \prime}$ without identified rotational fluctuation) than the rotary table driven by the warm gear (repeatability $34.5^{\prime \prime}$, accuracy $38.6^{\prime \prime}$ with an existing rotational fluctuation and identified influence of the unbalanced mass) (Dassanayake et al. 2008). Three different types of rotary tables are displayed in Figure 1.8.


Fig. 1.8. Rotary table types: worm gear rotary table (Sherline Products Inc 2013), roller gear rotary table (Schaffler Technologies 2013), and direct drive rotary table (CyTe Systems 2013)

The main difference between rotary tables and Indexing tables is that rotary tables ensure smooth movement in whole circle at the smallest pitch depending on the resolution of their angle encoder. Indexing tables, however, can perform the discrete rotation and the pitch size depends on the size of the serrated teeth.

### 1.3.4. Indexing Tables

The angle can be generated by using two methods - the sine principle and dividing the circle. The sine principle uses the ration of the length of two sides of a right triangle in deriving right triangle however this principle is dependent on an established system of length measurement. The principle of circle division is independent from length standards and is based on dividing whole circle by even
parts until it is closed. 1440 Precision Index is considered a principal angle standard and is based on the circle division into 1440 even parts with the pitch of $15^{\prime}$ as it is shown in Fig.1.9 (Moore 1970). The instrument is guaranteed to be accurate to within $\pm 0.1^{\prime \prime}$ at any of the 1440 indexed positions. Such accuracy is attained by controlling all the components of the rotary table. The serrated tooth divider which was developed by William Schabot Bernard and later modified is the crucial component of Moore's 1440 Precision Index (Bernard, 1960). This divider employs two face gears of identical shape and spacing of teeth. One member is displaced axially to disengage the teeth and then rotated radially to the desired angle. When two opposed faces of the gears are brought into forced engagement, they become locked in place, preventing rotation or side movement.


Fig. 1.9. The serrated - tooth divider of Moore's 1440 precision Index (Moore 1970)

This Moore's 1440 Precision Index is used to calibrate circle-dividing instruments such as rotary tables as well as angle gauge blocks. Precision Index can be calibrated by using another Precision index in conjunction with mirror and autocollimator (Moore 1970). Moore'e Special Index is very difficult to automate therefore it is usually operated manually by the operator.

Moreover, this indexing table is very widely used in world famous laboratories due to its irreplaceable accuracy of $\pm 0.1^{\prime \prime}$. In the latest EURAMET 725 Report calibration of two 1440 Precision Index tables of which one is equipped with a small angle divider are analyzed. Bilateral comparison was performed by the National Metrology Institute in Belgium (SMD) and French National metrology and test laboratory (LNE) using various techniques described in Subchapter 1.4. Results have shown that deviation of Index table without the small angle divider was up to $0.1^{\prime \prime}$ with the uncertainty $U=0.15^{\prime \prime}(k=2)$, however measurements of Index table with the small angle divider showed bigger deviation (up to $0.41^{\prime \prime}$, with the uncertainty of $U=0.15^{\prime \prime}(k=2)$. It was proven that additional component can influence the measurement accuracy as well as measurement methods and devices used during calibration methods. Although,
deviation of the Index table with the small angle divider needs to be minimized, both Moore Indexing Tables can be used as standards (Pirée 2013).

Index tables in combination with rotary tables can present very accurate results due to their modern structure and automation of the process.

There are also precision index tables designed to perform vertical angle measurements. A company from Taiwan "Topsdisk" presented vertical hydraulic index (Fig. 1.10) which is designed to perform measurements in a vertical plane. The advantage of this index is that it can withstand the weight of 125 kg and hydraulic clamping system assures smooth rotation, so could be used for measurements, however stated indexing accuracy is $\pm 5^{\prime \prime}$ (Spintop Machinery Co Ltd. 2014).


Fig. 1.10. Modern Vertical Indexing table (Spintop Machinery Co Ltd. 2014)

One of the latest inventions is precision angular indexing system designed for calibration of rotary tables. Generally, indexing tables contain of two step operations for indexing (i.e. lift up and rotate), but this new type of indexing table employs one step operation, lifting up and rotating simultaneously by using a camshaft and motor. The indexing table employed a pindisc to transform the rotation motion into lift-up and rotation motion of the disk. The repeatability of this table was measured by fixing calibrated table with the laser optics on the reference high resolution rotary encoder. The laser display and control box were connected to a computer which was programmed to control the rotation and direction of automatic indexing table and takes the laser display reading. This table is designed to perform measurements in both horizontal and vertical directions. Measurements were performed in clockwise and counter-clockwise directions by taking 3 readings at every position of 5 degree interval. As results showed, the repeatability of this indexing table in vertical direction was 0.05 " and 0.03 " in horizontal direction (Taek 2012). Although reasons of measurement accuracy of indexing table in a vertical orientation are need to be investigated, so
far this calibration system can be used for high accuracy automatic calibration of rotary tables.

### 1.3.5. Gauge Blocks and the Polygon

Gauge blocks are the type of length or angle standards. Gauge blocks can be made of steel, ceramic, tungsten carbide, chromium carbide however the influence of the temperature and elastic compression has to be evaluated according to the material which tea gauge blocks are made of. Gauge blocks need to be calibrated as any other reference mean. The first set of angle gauges was invented by Tomlinson in 1939. It consisted of 12 blocks having included angles of $3^{\prime \prime}, 9^{\prime \prime}$ and $27^{\prime \prime} ; 1^{\prime}, 3^{\prime}, 9^{\prime}$ and $27^{\prime}$ and $1^{\circ}, 3^{\circ}, 9^{\circ}, 27^{\circ}, 41^{\circ}$. Gauge blocks are very finely polished and can be joined together to form any angle (up to $1^{\prime \prime}$ ) without additional magnet. Such gauge blocks as well as precision polygons are obligatory equipment for laboratory to be accredited. All devices must be calibrated and the uncertainty has to be evaluated according to „Guide to the Expression of Uncertainty in Measurement"(Moore 1970; Faison, Brickenkamp 2003).

The polygon is not an angle gauge block and belongs to separate category. The polygon (sometimes called optical polygon or multi - angular prism) is considered to be a primary standard and the most accurate device for calibration of circle - divided instruments. General view of the polygon and its main parts is shown in Figure 1.11 (Japanese Industrial Standard 2006).


Fig. 1.11. The view of the polygon (JIS 2006)
The polygon has faces covered with the very flat mirror. The number of divisions (mirror faces) varies from 6 to 72 while still maintaining adequate reflectivity and flatness. The reflecting surface of the polygon is used to indicate the nominal angle which depends on the number of mirror faces. Polygons are classified into the two grades depending on their angular precision and reflecting surface flatness. Grade 0 polygons must have angular precision within $2^{\prime \prime}$,
flatness of effective reflecting surface should be less than $0.0025 \mu \mathrm{~m}$ and size of effective reflecting surface $30 \times 20 \mathrm{~mm}$, however grade 1 polygons have to meet requirements respectively $5^{\prime \prime}, 0.05 \mu \mathrm{~m}$ and diameter of 20 mm . Polygons are made of high quality crown glass, quartz glass with no inner strains or a material equivalent or higher quality (tungsten carbide). Reflecting aluminum film is applied on the reflecting surfaces which have to be free from flaws, cloudiness and discoloration and the polygon must be resistant to the changes of temperature and humidity. The perpendicularity of the center hole to the base of the polygon must be within 1 ' and body of the polygon should be marked with the nominal angles to be viewed from upper surface (JIS, 2006). The most popular method for polygon calibration described in various international standards is to calibrate polygon using one or two high precision electronic autocollimators and mounting the polygon on the rotary table. Autocollimator, reference rotary table and the polygon are set to zero position. Then rotary table is rotated by the angle of polygon and the readings of an autocollimator are taken. The reading of an autocollimator shows the deviation from nominal angle of the polygon. In case of using two autocollimators which can be pointed to different polygon faces. After full circle, one autocollimator is moved to another position in a line with different polygon face. This way accuracy can be determined by direct comparison of autocollimator measurements and measurements obtained by the encoder of the rotary table. Research has shown that the standard deviation of 0.151 " is possible to achieve. There is also a self-calibration system designed for polygon calibration without a necessity of autocollimator measurements. The polygon is calibrated by comparison of the readings of two angle encoders in respect of each other. This method is based on the graduation position relation of the rotary encoders and the time scale of timing that each reading head detects a graduation signal (Bručas et al. 2010; Bručas, Giniotis 2010; Watanabe et al. 2003). Precision polygon is mostly used in horizontal angle metrology however the principle of autocollimator measurements may ensure high precision performance in calibration of vertical angle measuring systems of total stations. The detailed description of the method and machine for the calibration of vertical angle measuring systems can be found in the Subchapter 1.4.2.

### 1.3.6. Comparators

There are only very few comparators designed for vertical circle calibration. Such a vertical circle comparator (VCC) was created as the standard in order to perform precise calibration of vertical angle measuring system of robotic total stations and laser trackers in ESRF, France. The schematic view of the vertical circle comparator is shown in Fig. 1.12 (Martin 2010).


Fig. 1.12. Schematic view of vertical circle comparator designed in ESRF, France (Martin 2010)

As it is presented - "VCC is composed of a motorized 2.5 m long linear motion guide with carriage fixed to a 3 m long aluminum structural rail and an interferometer. The interferometer system is positioned at one end of the rail while the motorization driving the carriage is at the opposite end. Its reflector is placed on the carriage. The full system is placed on a heavy duty adjustable height stand". According to this description, instrument to be calibrated is placed against the vertical circle comparator and the reflector is mounted on the adjustable part of the comparator. This comparator is designed to measure readings of vertical angle measuring system in the range of $90^{\circ} \pm 45^{\circ}$ and $270^{\circ} \pm 45^{\circ}$ because this is the range mostly used while performing vertical angle measurements. During the calibration procedure vertical circle readings are compared with the vertical displacements of its spherically mounted retroreflector which are measured by the interferometer. The distance between comparator and the total station is measured by using distance meter calibration bench. The expanded uncertainty of vertical angle measuring system calibration using this this vertical circle comparator is 1.65 " with coverage factor $\mathrm{k}=2$ (Martin 2010).

Another machine for the calibration of vertical angle measuring system of a total station was developed by Leica. This machine is unique by its structure and precision. It is a comparator based on the principle theodolite in theodolite and fulfils the condition of orthogonality of the axes. This machine has two reference
indexes in horizontal and vertical planes. The main parts of the theodolite testing machine are shown in Figure 1.13.


Fig. 1.13. Main components of theodolite testing rig (Lippuner 2006)

The total station (5) is fixed to the horizontal index (8), roller bearings (7) and a horizontal drive (6). The mirror attachment (3) is mounted on the telescope of the TS. Vertical index and angle sensor (9) controls the movement of an electronic autocollimator (2) which is fixed to the special frame and vertical drive (4) to perform the rotation around horizontal axis of a total station. Electronic autocollimator is pointed to the mirror fixed on the telescope of TS and measurements can be performed in all the usable part of the vertical circle of a total station. Whole system is fixed to the granite frame (1) for stability (Lippuner 2006).

### 1.3.7. Autocollimators

The working principle of an autocollimators is based on the meaning of the word collimate. The light beam is travelling along autocollimator's optical axis in parallel lines until it reaches the mirror and reflects.

Autocollimators are the devices used in angle metrology for non-contact small angle measurements, alignment of the devices and calibration of angle measuring systems. These optical instruments measure the deviation of the light beam reflected from the mirrored target. The working principle of an autocollimator is shown in Fig. 1.14 (Yandayan el.al 2013).


Fig. 1.14. The principle of the autocollimator (Yandayan et al. 2013)
The object reticle is illuminated and through the beam splitter projected to infinity by the collimator objective. When the light beam reaches the mirror it is reflected in different path through the collimator objective to the image plane or the digital camera with the sensors (Charged Coupled Device) in electronic autocollimators. The difference between initial and latter positions of the light beam is measured. The angle at which the reflective target is tilted can be expressed as shown in (1.20):

$$
\begin{equation*}
\alpha=\frac{s}{2 f} \tag{1.20}
\end{equation*}
$$

where $\alpha$ - tilted angle; s - shift of the light beam; f - focal length of an autocollimator.

An electronic autocollimator shows the tilted angles in the display in two planes ( x and y ) (Yandayan el.al 2013). The modern electronic autocollimators comes with the resolution of 0.001 " which means that such device can be used in measurements where very high accuracy is required such as creating reference angles (Estler 1998). There is new equipment for using autocollimators for spatial angle measurements under development which shows that this instrument
is unreplaceable in non-contact angle metrology and surface flatness measurements due to its precise optics (Geckeler et al. 2012). The main error sources for measurements with an autocollimator are:

1. Scale error which is proportional to the size of the measured angle.
2. Pyramid error when the value of $y$ axis of an autocollimator is stable and x value varies. This error is usually caused by misalignment or tilted mirror.
3. Eccentricity related errors are also very common when the mirror axis doesn't match the rotation axis as well as errors due related to the flatness and geometry (size) of measured mirror.
4. Vibration, measurement noise and strong airflow can also significantly affect the measurement results
Autocollimator is very widely used at the angle calibration laboratories for small angle measurements. However, interferometric measurements are getting popular as an alternative for precise angle measurements because it is easier to quantify the interferometer measurement errors compared to autocollimator measurements (Stone et al. 2004).

### 1.3.8. Laser Interferometers

The meter is defined as the length of the path travelled by light in vacuum during the time interval of $1 / 299792458$ of a second. It is a Si (fr. Système Intérnational d'Unités) unit of length (Taylor 1991). According to this definition, the half of the length of the light path can be expressed in (1.21):

$$
\begin{equation*}
L=\frac{c t}{2 n} \tag{1.21}
\end{equation*}
$$

where L - distance between the laser ant the target; c - the speed of light in vacuum ( $\mathrm{c}=299792458 \mathrm{~m} / \mathrm{s}$ ); t - the time of the round trip of the laser beam; n refractive index of air.

Michelson interferometer was invented in 1893 and it changed the mechanical length standard to optical length standard. The principle of Michelson interferometer is based on the light beam split by the beam splitter into two beams travelling different paths and their interference after they are recombined together. Movable distance of the target mirror is measured while the reference mirror is fixed. While moving the mirror, the frequency of the reflected light is shifted. There is a sensor inside the interferometer for the determination of changed beam intensity. By using this interferometer it is possible to determine the wavelength of the gas. The spectrum the wavelengths of visible electromagnetic radiation (visible light) varies from 390 nm to 700 nm . The color of light is determined by its frequency or wavelength. The best way to have a
stable and controlled light is to use the laser light. It is used as a standard because of its high stability of the wavelength (frequency uncertainty is less than $1 \cdot 10^{-10}$ ). The word LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. Helium-Neon has a well - known wavelength ( 633 nm ), therefore it is commonly used as a light source and iodine stabilized lasers are the most stabile ones (Layer 2011).

There are a few main types of laser interferometers used in laboratory measurements - homodyne and heterodyne laser interferometers are the most commonly used. Homodyne interferometers are based on the interference of two beams (one split beam) of the same frequency. The optics of homodyne interferometers is similar to Michelson interferometer‘s, however it produces a low signal due to the high noise ratio compared to heterodyne interferometer. Heterodyne interferometers are based on two beams with different frequencies (weak and strong) and different polarization mixed with each other and in nonlinear combination creating two new frequencies (heterodynes). The two frequency interferometers measure the relative displacement of two reflectors by splitting the beam. Then 2 beams are directed to different retroreflectors and resultant signals are returned to a photodetector (Paschotta 2012; Kneppers 1991). The working principle of laser interferometer is shown is Figure 1.15.


Fig. 1.15. The principle of laser interferometer (Laser Interferometer Implementation 2012)

Heterodyne interferometers are very sensitive and usually more accurate than homodyne interferometers.

An interferometer is usually used for linear displacement measurements however it can be used for angle measurements as well. Angle interferometers can be both homodyne and heterodyne. The main principle of such interferometer
is analogous to previously mentioned ones. However, angle interferometer has special optics which includes two corner cubes instead of one. As it is shown in Fig. 1.16 (Chapman et al. 2013) angle determination is performed by measuring total path of the split beams of the laser, both arms between the beam splitter and each angular reflector and the fixed distance ( S ) between two corner cubes angular reflectors.


Fig. 1.16. Angular interferometer (Chapman et al. 2013)
The difference of two different laser paths can be expressed as follows:

$$
\begin{equation*}
\Delta L=S \sin \theta \tag{1.22}
\end{equation*}
$$

where $\Delta L$ - difference of $A r m 2$ and $\operatorname{Arm} 1 ; \theta$ - position of angular reflector. Therefore, the angle can be expressed:

$$
\begin{equation*}
\theta=\arcsin \frac{\Delta L}{S} \tag{1.23}
\end{equation*}
$$

Such angle interferometers can be used for flatness measurements to determine the slope of the surface as a measured angle. Although this angle measurement principle is very similar to the angle measurements performed by angle generators, the latter are more commonly used because of the rotation ability. However, angle generators can generate only horizontal angles and angle interferometer can measure both vertical and horizontal angles.

There are three main categories of the errors in the interferometric measurements (Castro 2007):

- Intrinsic (laser wavelength accuracy, measurement resolution, optics nonlinearity);
- Environmental (atmospheric compensation, material expansion, optical thermal drift);
- Installation (dead path error, cosine error, abbe error).


### 1.3.9. Standard Scales

In 1889 the definition of the meter was based on the distance between two lines. Therefore, the few copies of the international platinum-iridium temperature resistant meter prototype bars were made in France and sold to many countries. The meter (m) is the SI unit of length and since 1983 it is defined as the length of the path travelled by light in vacuum during the time interval of 1/299 792458 of a second. After the laser light was announced as a length standard, the calibration laboratories stopped using 1 meter bars. However, similar graduated 1 meter length scales still can be used for precise measurements. Such scales have a grating every 1 mm and are made of invar, steel, brass, glass, silicon and quartz in the shape of $H, U$ or modified $X$ (Fig. 1.17). These are the shapes that are the most difficult to break or damage. The grating lines are mostly cut by diamond and photo-etched. These 1 meter scales can be calibrated against each other by setting two microscopes in both ends of the meter bar. After replacing meter bar with another bar the deviation of the lines is measured by the microscopes. Such system is developed in BIPM (Beers 1987; Layer 2011). Although such scales are commonly used for linear measurements, they can be employed for vertical angle measurements by using precisely engraved marks on its mirror surfaces as targets.


Fig. 1.17. Reference scale with 1 mm grating

### 1.3.10. Laser Trackers

Laser trackers are length and angle measuring devices which have the structure similar to theodolite where the telescope is replaced by the laser source. According to B89.4.19 standard, the special tests for length and angle measurements should be performed by measuring lengths in 33 and measuring angles in 36 predetermined positions in both faces. Laser trackers have angle
encoders embedded in their measuring system, therefore, calibration is necessary. Angle measuring systems are calibrated by measuring different distances (closer to the tracker and further) and using certain expressions for horizontal and vertical angle encoder eccentricity determination (Muralikrishnan et al. 2009).

### 1.4. Methods for Calibration of Angle measuring Systems

### 1.4.1. Horizontal Angle Measuring System Calibration Methods

The Centre of Applied Sciences and Technological Development of Mexico has presented the method of calibration of multiangular prism (polygon) using only one autocollimator. The experiments performed at the Centre has shown that the errors between measurements performed using two autocollimators and one autocollimator vary from $0.0^{\prime \prime}$ to $0.8^{\prime \prime}$ at $90^{\circ}$ nominal angle. The measurement accuracy was determined by minimizing the standard deviation and uncertainty values up to $\pm 1.0^{\prime \prime}$. This method is performed by using mirror sided polygon (optical polygon) placed on rotary table, one autocollimator directed to one of the mirror sides of polygon. After that, a beam splitter is placed between both devices obtaining "A" and "B" beams. One of them, A, goes directly towards the 0 degree face of polygon, while the other, the beam B , is directed towards an adjacent face aided through a flat mirror as showing in Fig. 1.18 (Sandoval, Uribe 2003).


Fig. 1.18. Principle of polygon calibration using one autocollimator (Sandoval, Uribe 2003)

To improve measurement accuracy instead of one autocollimator two autocollimators can be used. The combined method for calibration of horizontal angle measuring system of total station is based on usage of total station, two autocollimators, 72 sided polygon, mirror and rotating plateau. The polygon is placed on the top of rotating table and above it the total station attached to the platform is concentrically mounted. The mirror is attached to the total station near its telescope. The telescope of one autocollimator is pointed to the side of the polygon and the other is raised to the height of a mirror placed near total station's telescope. The readings are taken at an initial position and whole system is rotated at $5^{\circ}$ angle (to the closest side of the polygon). Afterwards the total station is turned back by the same angle and the reading of total station shows the difference between both positions. In this case, the standard is created by using polygon and autocollimator to be compared with total station readings. The standard deviation of these measurements is 0.87 ", although, the main drawback of this method is the minimal calibration step of $5^{\circ}$ which limits the possibility to investigate the full circular scale at a desired angle (Ježko 2007).

In industrial Technology Research Institute of Taiwan the cross calibration method by using precision goniometer with a small angle interferometer was presented. Flat angle measurements are performed by using special system combined of small angle interferometer, precision goniometer, autocollimator and polygon. Heidenhain RON905 incrementral encoder and goniometer are coaxialy mounted on the same shaft. The polygon to be calibrated is mounted in the center of goniometer. The goniometer and polygon are placed in zero positions and the autocollimator is pointed to the zero face of the polygon. On both sides of rotation bar two retro - reflectors were mounted for interferometer to measure their relative displacement. Goniometer readings are compared to encoder readings and autocollimator readings can be compared to the angle determined by interferometric measurements. The principle of autocollimator and interferometric measurement combination is shown in Fig. 1.19. By using 24 sided polygon, the goniometer deviation was $0.12^{\prime \prime}$ with the uncertainty of 0.06 ". To measure very small angles some special facilities are being developed for the calibration of autocollimators which are widely used in angle metrology. This facility is based on angles being generated by a precision rotary table with Heidenhain RON905 encoder and measured by the sine-bar on which the mirror together with interferometric optics is placed ont the top of the rotary table. The rotating angle can be determined as follows:

$$
\begin{equation*}
\sin \theta=\frac{d}{h} \tag{1.24}
\end{equation*}
$$

where $\theta$ - rotating angle; $d$ - the optical path difference between two retroreflectors $\left(\mathrm{d}=\mathrm{d}_{2}-\mathrm{d}_{1}\right) ; \mathrm{h}$ - distance between separated interferometer measurement arms (Liou et al. 2006; Eves 2013).


Fig. 1.19. The principle of autocollimator calibration using interferometric system and sine-bar mounted on the top of rotary table (Eves 2013)

To ensure high measurement accuracy, indexing tables and autocollimators are widely used in precision angle metrology. However, the indexing table has limited resolution and manual operation, and a small angle generator is only applied to the calibration of an autocollimator because of limited measurement range. Therefore, some NMIs (National Metrology Institutes) developed angle comparators to generate very fine and accurate angular position within the full circle range. The new precision angle comparator using self - calibration of scale errors based on the equal - division - averaged method was developed in Korea Research Institute of Standards and Science. This method is based on twelve encoder heads mounted around the circular scale and grouped in pairs by two. Averaging the readout of each set helps to compensate the scale error. While performing experiments it was determined that such angle encoder generates the circular motion with resolution of $0.005^{\prime \prime}$. After implementing such angle encoder in an angle comparator and using autocollimator for the accuracy determination the uncertainty of $0.05^{\prime \prime}$ was reached (Kim at al. 2011).

Although laser tracker is used for alignment and distance measurement the angle encoder plays an important role in laser trackers accuracy. An indirect method fo error determination of rotary encoder is based on measuring distances from different azimuthal positions to the targets which are symetrically placed in front of the laser tracker at its height. Performing measurements with theodolite type laser tracker both targets are measured in two different possitions - front face and back face. Using this two - faced method the influence of odd order harmonics can be removed by averaging between both faces measurements.

However, even order harmonic appear and they may be eliminated by the calculations of Fourier analysis to enhance the devices's accuracy. This method offers simple technique and instrumentation, however this method doesn't provide a direct error map of rotary encoders scale and it is quite complicated to determine higher order harmonics (Muralikrishnan et al. 2010).

In the European Synchrotron Radiation Facility (ESRF) accelerator laboratory in Grenoble, France, the Group of Alignment and Geodesy has recentrly developed two instrument standards such as Horizontal Circle Comparator (HCC) and Vertical Circle Comparator (VCC). These instruments are used for the calibration of total stations. Total station is mounted on the standard rotary table and directed to the reflector placed at nominal distance from the device. The distance between the reflector and the calibrated instrument is precisely measured. Then the rotary table is rotated at a desired angle and afterwors the total station is turned back by the same angle. This action is repeated for several times to increase the accuracy of the procedure. The main purpose of this method is the determination of the difference between the HCC and Total station angle readings. This method is very good for indoor calibration and the horizontal circle of the total station can be calibrated throughout 360 degrees. Although, the main disadvantage of this method is the lack of accuracy of determination of total station's position before and after the rotation. The uncertainty of this horizontal circle calibration method following the GUM is $\pm 0.98$ " (Martin, Chetwynd 2009).

The main component of the angle measuring system is angle encoder which is embedded into devices which perform precise rotation. One of such devices is laser tracker which contains of such angle measuring system. At the National Accelerator Laboratory in California, US a special bench for angle measurement system calibration is designed. Laser tracker is placed on the top of precisely calibrated and leveled rotary table. Rotary table is calibrated using two autocollimators placed at particular angle and pointed to the different mirror faces of the polygon to create the standard angle which is compared with the reading of the precision angle encoder with four reading heads embedded into the rotary table. Results have shown that angle determination accuracy can be achieved up to $\pm 0.5^{\prime \prime}$ by using this system. Moreover, this system is used to calibrate the angle encoder of laser tracker's. Laser tracker is placed on the top of the rotary table. Rotary table is turned at a desired angle together with the laser tracker, however trackers head is pointed to the mirror and remains stationary. This procedure gives two readings (rotary table reading and laser tracker reading) which are compared to each other. Laser tracker results have shown that errors of horizontal angle encoder of laser tracker vary from - 2.4 "to 1.5 "and addiditonal errors such as wobble error, collimation type errors, centering and leveling errors are minimized to negligible by performing measurements in both device faces
and using rotary precision horizontal angle calibration system (Gassner, Ruland 2008). Modern self calibrating precision angle measuring systems have angle sensors of 0.001 "resolutiondue to which the uncertainty of whole system can be minimized down to $0.2^{\prime \prime}$ (Kojima, Wakiwaka 2007; Wantanabe 2008).

Angle measuring system precision index tables need to be calibrated as well. One of the main methods for this procedure, presented in the reports of European Association of National Metrology Institutes is mounting the index table on a vertical axis rotation angular reference table along with a polygon in front of the autocollimator. This enables to monitor angles of two systems. A clockwise rotation along the reference system between two positions is performed with a subsequent counter-clockwise rotation of the same nominal value. Given difference between the initial position and the final position gives the value of deviation. The 0 value of the mobile part is then matched with the same value of the fixed part near the handle. The recommendation is to perform at least 3 cycles with $15^{\circ}$ interval.

The second method is based on stacking two tables with a fixed polygon positioned on top and one autocollimator. In this case all $15^{\circ}$ interval combinations result a $24 \times 24$ measurement result matrix of polygon faces. The benefit of this method is the ability to obtain error separation matrix with deviations from nominal values of two index tables and a polygon.

The third method facilitates two autocollimators, positioned to read angles at $15^{\circ}$ from each other along with a rotary table with a polygon mounted on top. The index table that needs to be calibrated has a mirror mounted on top. The mirror is positioned in front of the first autocollimator and the value of this autocollimator is recorded. Afterwards this index table is rotated to face the second autocollimator and the value of this autocollimator is recorded. After this the bottom table is rotated by $15^{\circ}$ for the mirror to face the first autocollimator again. The value of this autocollimator is recorded. Afterwards the top table is rotated to face the second autocollimator (that is by $30^{\circ}$ ) and the value is recorded. These steps are repeated until the bottom index table has made a full turn. Such angle measurement system calibration techniques by using Moore 1440 Precision Index are commonly used and analyzed (Pirée 2013; Taek 2012; Estler 1998).

At Vilnius Gediminas Technical University the angle testing rig for horizontal angle calibration of geodetic instruments was developed. This testing rig is based on combination of various angle measuring techniques for an angle encoder error determination. Worm gear rotary table in combination of photoelectric angle encoder, autocollimator with the polygon and the microscope was used to calibrate horizontal angle measuring system of the total station while performing three independent and different measurements. Automation of the whole system makes it time saving and easy to operate. After uncertainty
evaluation it turned out, that the smallest uncertainty of 0.273 arc seconds of was achieved of encoder angle positioning determined by microscope/scale measurements (Bručas 2008).

### 1.4.2. Vertical Angle Measuring System Calibration Methods

At ESRF the vertical circle comparator (VCC) is developed for the calibration of total station's vertical circular scale. An interferometric system is mounted perpendiculary to the direction of total station observation axis at $90^{\circ}$. The reflector is placed on one end of the interferometric system while interferometer is placed on the other. Horizontal distance between total station ant the interferometric system is calibrated and vertical distance between the reflector and interferometer is measured by interferometer. While changing the position of the reflector different vertical angles can be measured. The VCC calibration procedure compares the total station's vertical circle readings with the angles determined based on vertical displacements of its reflector and the calibrated distance between the VCC and device to be calibrated. The uncertainty of vertical angle measuring system calibration was determined $\pm 1.4$ " (Martin 2010). The main advantage of this method is that vertical displacement of the reflector is measured by interferometric system. Distance between the total station and VCC is measured by total station and controlled by interferometric system displaced on the opposite side of the VCC. Both of these distance measurements are traceable to the metre. However, this method has some drawbacks as well. This Vertical circle comparator is designed to calibrate vertical angle measuring systems of the robotic total stattions, therefore, recommended operating distance is around 6 meters which reduces the vertical angle calibration range to $90^{\circ} \pm 10.4^{\circ}$. This distance can be reduced to 2.5 meters (minimum distance for reflector to work) which increases vertical angle operational range to $90^{\circ} \pm 23.75^{\circ}$. Another drawback of this method is the uncertainty caused by horizontal distance measurements between the total station and VCC measured by a total station. Shorter distance increases vertical angle measurement range, however uncertainty increases as well (Martin, Chetwynd 2009).

Another method for vertical angle measurement system calibration is developed in Leica, Switzerland. This method is based on the mirror polygon and autocollimator measurements. Theoretically, the best method to calibrate vertical angle encoder of a total station is to mount mirror polygon in vertical direction instead of TS telescope coaxially with the vertical angle encoder of TS and point an autocollimator towards it. Then the polygon could be rotated and the readings of an autocollimator ant vertical encoder could be compared. Practically, this principle was implemented by fixing a special mirror to the telescope of a total
station which is mounted to special high precision theodolite testing machine on the top the reference horizontal rotary encoder. This machine has a frame which enables autocollimator rotation around the total station to be calibrated in a vertical plane. The position of an autocollimator is controlled by high precision indexing of the test machine. After measuring vertical angles the readings of TS‘s vertical encoder are compared to the reference vertical high precision indexing readings. Reference encoder is mounted in parallel plane to the calibrated encoder. This method is implemented only by using special Theodolite Testing Machine (TPM) which was fully automated and based on a principle theodolite in theodolite. The standard deviation of 0.058 " for horziontal angles and 0.091" for vertical angles was achieved while performing angle measurements in both horizontal and vertical planes (Ingensand 1990; Lippuner, Scherrer 2005; Lippuner 2006). The principle of Theodolite Testing Machine is shown in Figure 1.20.


Fig.1.20. The principle of TPM (Lippuner, Scherrer 2005)
In Korea Research Institute for Standards and Science (KRISS) there is an apparatus for vertical angle measurement system calibration under development. The new method is realised by positioning total station horizontally and fixing it to the indexing table. The telescope is pointed to the collimator and the crosshairs of the autocollimator and total station is aligned. Then index table is rotated at a desired angle and the total station is tuned back to the previous position. During this procedure the total station vertical angle readings are compared to Moore 1440 Precision Indexing Table's readings.

At National Metrology Center in Singapore the vertical angle calibration method where indexing table and a collimator is used was developed. A special fixture is used to ease the alignment process and minimise effect from compound angle. The expanded uncertainty obtained by this method is 2.0 " $(\mathrm{k}=2)$ and the vertical angle calibration range of $90 \pm 30^{\circ}$ is possible to achieve (Tan et al. 2011).

Recently a very high bar is set to the requirements for the measurement quality and convenience of the measurement process. Therefore, it is becomming popular to use indexing tables for calibration of vertical angle measuring systems. There are developed some angle measuring systems, which can perform calibration in a vertical plane. Usually it is automatic complex systems which contain of indexing table, rotary table, laser optics and computer for rotation control and data processing (Taek 2012).

According to International Standard 17123-3 vertical angle measuring system should be calibrated outside the laboratory. The theodolite should be set up 50 m from the tall building. At this building marks or point should be selected to cover a range of the vertical angle approximately $30^{\circ}$. Measurements should consist of 3 sets and has to be performed in both faces (ISO 17123, 2001). However, to obtain more reliable results these instruments can be calibrated under the laboratory conditions while minimizing the atmospheric influence.

There are only two methods patented for the indoor calibration of vertical angle measuring systems of total stations. Although these methods are traceable to international standards, they are very expensive and not available for smaller laboratories. Therefore, there is a need to develop new relatively cheap, space efficient and convenient methods for vertical angle measuring system calibration.

### 1.5. Conclusions of the Chapter 1 and Formulation of the Tasks of the Thesis

1. The literature review revealed that there are more methods developed for horizontal angle measuring system calibration than for vertical angle measuring system calibration.
2. The measurement uncertainty depends on the instrumentation used for the calibration of angle measuring systems, therefore, reference means should be used to ensure high quality measurements.
3. Calibration of vertical angle measuring systems is complicated because of the structure and design of geodetic angle measuring instruments.
4. There are possibilities to perform calibration of vertical angle measuring systems by using means for horizontal angle measuring system calibration.
The following tasks were formulated to achieve the aim of the work:
5. To develop new methods for the calibration of vertical angle measuring systems of geodetic instruments under the laboratory conditions.
6. To use primary standards such as indexing table and laser interferometer for obtaining reference angle.
7. To perform uncertainty evaluation in order to determine uncertainty sources and their impact on measurement results.

## 2

## Proposed Methods for Vertical Angle Measuring System Calibration

A present scientific and technical background validates the concept of development of the standard measure for calibrating the wide range of angular readings from optical instruments and consisting from thousands of angular values in compliance with the requirements stated in their technical specifications. Generally there are several groups of plane angle measurement principles (Giniotis 2005):

1. Solid angular gauge method:
a) polygons (multiangular prisms);
b) angular prisms;
c) angle gauges, etc.
2. Trigonometric method (angle determination by means of linear measurements);
3. Goniometric method (plane angle determination by means of a circular scale):
a) full circle (limb, circular code scales etc.);
b) non-full circle (sector scales).

Calibration of vertical angle measures of the geodetic instruments has usually been performed by facilitating a special bench consisting of autocollimators attached at different preset vertical angles to calibrate the
instrument. Although this method was widely used, it also showed some major drawbacks: the physical properties of the equipment tend to be inconvenient for the performance in this case the entire test bench often is very bulky. Another drawback of this method is a narrow field of operation - it is able to measure only very limited number of vertical angles (Walser 2004).

Such problems lead the research towards a new approach to the problem. Presence of new technology, including precise angle encoders provided the basis of a new method, incorporating a precise angle encoder. In this method a vertical angle reference is created. While it is possible to create unlimited number of reference angle values using this method, the equipment tends to be expensive, therefore, complicating the accessibility to usage of the method to the ones who need it (Ingensand 1990).

The overview of previously analyzed angle calibration methods shows that there is a need for further research in this field due to the features and limitations of the existing methods which tend to limit the user either for quality or costs of the calibration.

It is most common for geodetic instruments to have two angle measuring devices embedded - one for horizontal and one for vertical angle measurement. While there is a number of methods for calibration of the horizontal angle measuring instruments implemented on practice, vertical angle measuring instrument calibration is still in a developing stage. Generally, measurement methods can be grouped in two categories - surface measurements and measurements using reticle of the telescope for pointing to an object. Further in this Chapter of the thesis two different methods for the calibration of vertical angle measuring systems are presented as well as their advantages and drawbacks are analyzed.

The material provided in this Chapter was published in scientific journals and proceedings of International Conferences (Bručas et al. 2013 ${ }^{\text {a }}$; Bručas et al. 2014; Giniotis et al. 2009; Giniotis et al. 2012; Šiaudinyte et al. 2011; Šiaudinyte, Giniotis 2011; Šiaudinyté et al. 2012; Suh, Šiaudinyte 2014)

### 2.1. Method for Vertical Angle Measuring System Calibration Using Graduated Reference Scale

There are not many methods and devices created for vertical angle measuring system calibration of total stations. A few of them described in previous Chapter are ensuring standard deviation of measurements of less than $0.1 "$. However, these methods and devices are very expensive. The need of cost saving methods is constantly increasing along with bigger requirements and higher standards. There are two main principles used in order to determine the
angular position errors of circular scales or an angle measuring system. First principle is based on a comparison between the angle measured by calibrated device and the nominal angle created by using other high accuracy equipment. Second principle is based by rotating both calibrated and reference circular scales and performing direct comparison by expressing the angle difference by their angle means. Usually vertical angle measuring systems of total stations are calibrated using big angles ( $30^{\circ}$ ), however such a relatively big - interval measurements cannot fully express encoder's accuracy in a full circle (Giniotis 2005). As it is stated in ISO 17123-3 vertical angle measurements of theodolites should be arranged outdoors 50 meters away from the target. 4 targets should be chosen to cover vertical angle measurement range of $90^{\circ} \pm 15^{\circ}$. The procedure should consist of 3 sets of measurements to 4 targets using both theodolite faces. After the measurements the evaluations of the standard uncertainty is required (ISO 2001).

The method for calibration of vertical angle measuring systems of geodetic instruments was developed in Institute of Geodesy of Vilnius Gediminas Technical University. This method proposes the arrangement to create the reference standard for angle measurement suitable for vertical angle calibration purposes in laboratory environment (Giniotis et al. 2009, Giniotis et al. 2012). This method is suitable for relatively small angle measurements. This method is based on trigonometric determination of the reference angle by using standard means. It is a comparison of the angle measured by total station and the reference angle determined by measuring two distances - horizontal (distance between TS vertical axis and vertically placed scale) and vertical (distance between the lines of the vertically placed reference linear scale). The principle of the method is shown in Fig. 2.1 (Bručas et al. 2013 ${ }^{\text {a }}$; Bručas et al. 2014).


Fig. 2.1. Principle of vertical angle calibration method

The reference angle determination based on measuring horizontal and vertical distances can be expressed:

$$
\begin{equation*}
\varphi=\arctan \frac{\Delta h}{l} \tag{2.1}
\end{equation*}
$$

where $\Delta \mathrm{h}$ - vertical distance determined between the scale grating; 1 - horizontal distance between the axis of the TS and the reference scale.

Using this method, reference 1 meter graduated scale bar is placed vertically using its original mount for stability and leveling on the carriage. The graduated scale must be perpendicular to the optical axis of the total station at initial position. Therefore, it is precisely leveled and aligned by using laser level, leveling screws and the cross-line of the total station's telescope. The carriage is then mounted on the distance measurement base rails. In order to ensure smooth movement along the rails, the carrier has to be mounted on the rails only allowing movement along the rails and minimized friction (smooth) movement must be ensured.

The total station is then placed and leveled on the stable vibration proof mount at the end of the rails. It has to be placed at such height that the center of the cross-line of its telescope in horizontal position would match the center of the central line of the vertically placed graduated scale. For the experiment reference 1 meter H shape invar scale (Gaertner Scientific Corporation Chicago. No. 244 A.U) with 1 mm grating pitch was chosen. After leveling the total station, the position of the scale is double-checked and readjusted in order to make the grating lines of the scale parallel to the horizontal line of cross-line of the total station's reticle. This condition has to be checked at both end marks of the graduated line scale. The amount of light in the experimental area has to be adjusted so that all of the scale lines are clearly visible without any shadows or other obstacles. The horizontal distance between the total station and the reference scale ( $l$ ) must fit the focusing range of the total station (TS). The closer vertical reference scale is to the total station, the bigger range of TS vertical angle encoder can be calibrated (Šiaudinytè et al. 2011; Šiaudinyte et al. 2012).

The particular angle is observed by pointing the telescope of the total station to the line of the scale and then this angle is compared to the reference angle which is determined by using standard means such as reference scale and laser interferometer.

This method has two approaches which are analyzed in the following Subchapters. If the measurements are performed while the reference scale is stationary, the reference angle is determined according to two distances - vertical and horizontal. In another approach the measured angle is fixed with the telescope of the TS and the reference scale is moved until another line of the reference scale is matched with the reticle center of the TS. As it is seen from the Fig. 2.1 the reference angle ( $\varphi^{\prime}$ ) can be expressed:

$$
\begin{equation*}
\varphi^{\prime}=\arctan \frac{\Delta h^{\prime}}{\Delta l} \tag{2.2}
\end{equation*}
$$

where $\Delta \mathrm{h}$ ' - known vertical distance between two calibrated scale lines; $\Delta \mathrm{l}$ horizontal displacement of the reference scale.

Considering that corresponding angles are equal if two parallel lines are crossed by a transversal, the angles $\varphi$ and $\varphi$ ' are equal (Šiaudinyte, Giniotis 2011). These two approaches and the instrumentation as well as the alignment of the devices and measurement procedure are discussed further in this thesis.

### 2.1.1. Vertical Angle Measurement System Calibration Method Using Displaced Target Technique

The total station is placed on top of the table (bench) under which the interferometer is pointed directly to its retro-reflector fixed on the lower part of the carriage under the reference vertical scale. The total station is placed at such height that its telescope would be pointed to the exact center line of the vertically placed scale. The scale is fixed to its mount on the carriage and leveled precisely. The carriage is positioned to be not less than the focusing distance of the total station (which in this case is 1.6 m ). The long handle is fixed on the carriage to move it in a linear way. The alignment of the devices is done by using cross line laser plumb which produces lines in two perpendicular planes (Fig. 2.2). The reflections of these lines are used to observe the biaxial tilt of the reference scale. The scale is leveled by matching these reflections with the initial laser beam lines. However, the uncertainty due to the reflected line width is unavoidable because the width of the laser beam line expands within the distance.


Fig. 2.2. Cross-line laser used for the alignment of the reference scale (Hersey et al. 2009)

The main parts of the frame are shown in Figure 2.2 where 1 - vertical alignment indicator, 2 - horizontal alignment indicator, 3 - support frame, 4 projection unit, 5 - gimbal mount, 6 - u-shaped rigid portion, 7 - sidewalls, 8 upper portion, 9 - base, 10 - flat mirror by which laser beam emitting from the laser emitting diode is deflected, 11 - magnet, 12 - damping plate, 13 - vertical projection module, 14 - horizontal projection module. The wavelength of current laser plumb is $630-650 \mathrm{~nm}$ (Hersey et al. 2009).

After leveling and alignment, the telescope of the TS is pointed to a desired line of the scale and fixed. After fixing the telescope of the TS, the readings of vertical angle and interferometer are taken. By using the long handle the carriage with the linear scale is moved until the center of TS telescope reticle matches another line of the vertical scale (Fig. 2.3.). Only focus can be adjusted by using focusing screw of total station. After the motion of the carriage the reading of the laser interferometer is taken. The difference between two interferometer readings is considered to be a horizontal displacement of the scale ( $\Delta l^{\prime}$ ). Vertical distance ( $\Delta h^{\prime}$ ) is the distance between the two calibrated lines of reference vertical linear scale (lines before and after the motion of the carriage). An electronic level can be used for the observation of the position of the carriage however it is problematic to determine a small tilt of the reference scale during the measurements.

The reference angle is determined by two distances (horizontal and vertical) measured by using the interferometer and graduated reference scale. The reference angle is compared to the angle measured by TS. Measured angle is an angle measured between the direction to the center line of the vertical scale and the direction to any other line of the reference scale.


Fig. 2.3. Principle of vertical angle calibration method using target displacement technique

The main advantage of this approach is the ability measure angles of various magnitudes by adjusting the horizontal distance as well as the usage of very precise instrumentation such as an interferometer, reference linear scale which have very small uncertainty for the reference angle determination. However, the drawbacks such as motion of the reference scale can lead to certain errors and tilt of the reference scale as well as difficulties in adjusting the light can increase the uncertainty.


Fig. 2.4. Internal structure of HP Laser System 5519A Laser tube assemblies (Goldwasser 2013)

For the horizontal displacement measurements Hewlett Packard Laser System 5519 A (Fig. 2.4) was chosen because a helium neon laser of this system offers exceptional stability. The system contains Zeeman-split two-frequency output. With the beam diameter of 6 mm this interferometer can perform 80 m long range distance measurements.

### 2.1.2. Vertical Angle Measurement System Calibration Method Using Stationary Target Technique

Another approach of vertical angle measurement system calibration method is proposed to avoid the movement of the vertical reference scale. The TS is placed at the same position as mentioned previously for its telescope reticle center to match the center line of the reference scale. The sight axis of TS and scale vertical axis must be perpendicular to each other. The horizontal distance to the scale should fit TS focus range.

The principle of this approach is shown in Fig. 2.5.


Fig. 2.5. Arrangement for total station vertical angle measuring system calibration

As can be seen from the picture an instrument to be calibrated is placed at a certain distance $l$ (Fig. 2.5) from the reference scale. The telescope of the instrument is declined at the angle $\varphi\left(\varphi_{1}, \varphi_{2}, \varphi_{3}\right.$ or $\left.\varphi_{4}\right)$ of which it must be calibrated. The reading $h$ from the scale is taken and the differences between the scale grating $\left(\Delta h_{1}, \Delta h_{2}, \Delta h_{3}\right.$ or $\left.\Delta h_{4}\right)$ are determined. The angle of interest $\left(\varphi_{i}\right)$ is expressed:

$$
\begin{equation*}
\varphi_{i}=\arctan \left(\frac{\Delta h_{i}}{l}\right) \tag{2.3}
\end{equation*}
$$

where $\Delta h_{i}$ - difference between the center and any other line of the calibrated reference scale, $l$-distance between the TS and the reference scale graduated surface (Rybokas et al. 2011).

After performing such measurements reference scale might be moved using previous approach of the method and measurements can be repeated for the better control to avoid rough measurement mistakes.

The main advantage of this approach is the elimination (or reduction to minimum) the movement of the reference scale.

There are several drawbacks related to the uncertainty due to limited resolution of TS while measuring horizontal distance as well as uncertainty of measuring all the components of horizontal distance between total station and reference scale (scale depth and prism constant).

### 2.1.3. Uncertainty Evaluation of the Vertical Angle Measuring System Calibration Method Where the Reference Scale is Used

For uncertainty evaluation it is very important to analyze all the components influencing measurement accuracy. All error sources are analyzed separately and later the accuracy is reported in the form of combined and expanded uncertainties.

The reference angle is determined according to the horizontal and vertical distances as it is shown in formula 2.3. When the scale is displaced the reference angle can be expressed as follows (2.4):

$$
\begin{equation*}
\varphi^{\prime}=\arctan \left(\frac{\Delta h^{\prime}}{\Delta l^{\prime}}\right) \tag{2.4}
\end{equation*}
$$

where $\Delta h^{\prime}$ - known calibrated distance between the grating of the scale; $\Delta l^{\prime}-$ distance measured by the laser interferometer between two scale positions.

Therefore, the correction value $(B)$ (or measurement result accuracy) can be expressed:

$$
\begin{equation*}
B=\arctan \left(\frac{\Delta h^{\prime}}{\Delta l^{\prime}}\right)-\theta_{T S} \tag{2.5}
\end{equation*}
$$

where $\theta_{T S}$ - angle measured by the total station.
The uncertainty of the correction value is expressed as follows:

$$
\begin{equation*}
u_{c}^{2}(B)=c_{h}^{2} u^{2}\left(\Delta h^{\prime}\right)+c_{l}^{2} u^{2}\left(\Delta l^{\prime}\right)+c_{\theta_{T S}}^{2} u^{2}\left(\theta_{T S}\right) \tag{2.6}
\end{equation*}
$$

where c - sensitivity coefficients; $u$ - combined standard uncertainties due to vertical distance $\left(\Delta h^{\prime}\right)$, horizontal distance $\left(\Delta l^{\prime}\right)$ and angle measured with TS $\left(\theta_{T S}\right)$. The uncertainty due to vertical distance is dependent on the accuracy, tilt, thermal expansion and compression of the reference scale and can be expressed:

$$
\begin{equation*}
u^{2}\left(\Delta h^{\prime}\right)=c_{h}^{2}\left\{u^{2}\left(\Delta h_{\text {Scal }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {tilt }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {therm }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {comp }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {point }}^{\prime}\right)\right\} \tag{2.7}
\end{equation*}
$$

where $u^{2}\left(\Delta h_{\text {Scal }}^{\prime}\right)$ - standard uncertainty due to the reference scale; $u^{2}\left(\Delta h_{\text {tilt }}^{\prime}\right)$ standard uncertainty due to the tilt of the scale; $u^{2}\left(\Delta h_{\text {therm }}^{\prime}\right)$ - standard uncertainty due to thermal expansion; $u^{2}\left(\Delta h_{\text {comp }}^{\prime}\right)$ - standard uncertainty due to compression of the scale; $u^{2}\left(\Delta h_{p o i n t}^{\prime}\right)$ - standard uncertainty due to pointing to the center of the scale line. The uncertainty due to pointing must be evaluated according to the different widths of the cross line of the telescope and the reference scale. This uncertainty $u\left(\Delta h_{\text {point }}^{\prime}\right)$ is analyzed in the experimental part of this research. Uncertainty due to thermal expansion can be evaluated according to thermal expansion coefficient. The uncertainty due to thermal expansion of the scale
$\mathrm{u}\left(\Delta h^{\prime}{ }_{\text {therm }}\right)$ can be determined for every measured pitch and for all the length of the scale as thermal expansion:

$$
\begin{equation*}
\Delta h_{\text {therm }}=\gamma \Delta t h \tag{2.8}
\end{equation*}
$$

where $\gamma$ - thermal expansion coefficient $\left(\gamma=1 \cdot 10^{-6}\right) ; \Delta t-$ deviation from $20^{\circ} \mathrm{C}$ ( $\Delta \mathrm{t}=0.5^{\circ} \mathrm{C}$ ); $h$ - length of the scale ( $h=1.0 \mathrm{~m}$ ); u( $\left.\Delta h_{\text {therm }}^{\prime}\right)$ - uncertainty due to thermal expansion of the scale was determined as $\Delta h_{\text {therm }}^{\prime}=0.5 \mu \mathrm{~m} / \mathrm{m}$ for 1 meter scale.

The reference scale is 1 m long and it is used in vertical orientation. Therefore, the correction of the compression due to gravity has to be evaluated (Legendre et al. 2000). The expression for the correction due to compression in this case can be assumed as the uncertainty due to compression of the scale because the standard uncertainties of the 2.9 formula's components are unknown. Therefore, the uncertainty due to compression of the scale can be evaluated as follows (2.9):

$$
\begin{equation*}
u\left(\Delta h_{\text {comp }}\right)=500 \frac{\rho g}{E}(L)^{2} \tag{2.9}
\end{equation*}
$$

where $\rho-$ density of the gauge block material $\left(\rho g=0.291\left(\mathrm{Lb} / \mathrm{in}^{3}\right) .1\right.$ psi $\left(\mathrm{lb} / \mathrm{in}^{2}\right)=6.895 \times 10^{-3} \mathrm{~N} / \mathrm{mm}^{2} ; \mathrm{E}-$ Young's modulus of elasticity of the gauge block material $\left(\mathrm{E}=20.5 \mathrm{Mpsi}, 1 \mathrm{~Pa}=1.4504 \times 10^{-4} \mathrm{psi}\right) ; \mathrm{g}-$ acceleration of gravity; L - length of the gauge block ( $\mathrm{L}=1000 \mathrm{~mm}$ ). The uncertainty due to the compression of the reference scale was evaluated as $u\left(\Delta h_{c o m p}^{\prime}\right)=7.1 \mathrm{~nm}$.

The uncertainty due to horizontal distance in the approach of displaced target technique depends on laser interferometer measurements and can be expressed as follows 2.10:

$$
\begin{equation*}
u^{2}\left(\Delta l^{\prime}\right)=c_{l}^{2}\left\{u^{2}\left(\Delta_{L \text { Lcal }}^{\prime}\right)+u^{2}\left(\Delta \Delta_{\text {Lrep }}^{\prime}\right)+u^{2}\left(\Delta \Delta_{\text {LIres }}^{\prime}\right)\right\} \tag{2.10}
\end{equation*}
$$

where $u^{2}\left(\Delta l_{\text {Llcal }}^{\prime}\right)$ - standard uncertainty due to the laser interferometer; $u^{2}\left(\Delta l_{\text {Llreep }}^{\prime}\right)$ - standard uncertainty due to repeatability of the laser interferometer; $u^{2}\left(\Delta l_{\text {LIres }}^{\prime}\right)-$ standard uncertainty due to limited display resolution of the laser interferometer.

The uncertainty due to the total station angle measurements (2.11) contains uncertainty due to limited display resolution of the device $u\left(\theta_{T \text { Sres }}\right)$ and uncertainty due to repeatability $u\left(\theta_{\text {TSrep }}\right)$ :

$$
\begin{equation*}
u^{2}\left(\theta_{T S}\right)=c_{\theta_{T S}}^{2}\left\{u^{2}\left(\theta_{T S \text { res }}\right)+u^{2}\left(\theta_{T S r e p}\right)\right\} \tag{2.11}
\end{equation*}
$$

The uncertainties due to limited display resolution of the devices can be evaluated as shown in 2.12:

$$
\begin{equation*}
u\left(\theta_{T S r e s}\right)=\frac{R}{2 \sqrt{3}} \tag{2.12}
\end{equation*}
$$

where R - display resolution of the total station. Formula 2.12 can be used for the uncertainty due to limited resolution determination of any measurement instrumentation. Uncertainties due to repeatability (2.21) are evaluated by determining standard uncertainties (1.10) of the measurement sets and then calculating pooled standard deviation (2.22).

The correction value for the approach without the movement of the scale can be expressed (2.13):

$$
\begin{equation*}
B=\arctan \left(\frac{\Delta h}{l}\right)-\theta_{T S} \tag{2.13}
\end{equation*}
$$

where $\Delta \mathrm{h}$ - calibrated distance between two lines of the reference scale; $l-$ measured distance between the TS and the graduated plane of the reference scale.

In the approach where the reference scale remains stationary, uncertainty due to vertical distance has the same components however uncertainty due to horizontal distance $(l)$ can be expressed as follows:

$$
\begin{equation*}
u^{2}(l)=c_{l}^{2}\left\{u^{2}\left(l_{T S}\right)+u^{2}\left(l_{p}\right)+u^{2}\left(l_{M}\right)+u^{2}\left(l_{S}\right)\right\} \tag{2.14}
\end{equation*}
$$

where $u^{2}\left(l_{T S}\right)$ - standard uncertainty due to TS measurements of the distance between the TS and the prism; $u^{2}\left(l_{p}\right)$ - standard uncertainty due to prism measurements of the prism constant determination; $u^{2}\left(l_{M}\right)$ - standard uncertainty due to the measurements to the mirror for prism constant determination; $u^{2}\left(l_{S}\right)$ standard uncertainty due to reference scale depth measurements.

To evaluate the uncertainty due to the TS distance measurements $u\left(l_{T S}\right)$, the uncertainty due to limited display resolution of the distance measurement readings $u\left(l_{\text {TSres }}\right)$ and uncertainty due to the TS distance measurement repeatability $u\left(l_{T S r e p}\right)$ have to be taken into consideration for three separate distance measurements as well as micrometer's used for the depth measurements parameters. Therefore, the uncertainty due to horizontal distance measurements with the TS can be determined as shown in (2.15):
$u^{2}(l)=c_{l}^{2}\left\{\begin{array}{l}u^{2}\left(l_{\text {TSres } 1}\right)+u^{2}\left(l_{\text {TSrep } 1}\right)+u^{2}\left(l_{\text {TSres } 2}\right)+u^{2}\left(l_{\text {TSrep } 2}\right)+u^{2}\left(l_{\text {TSres } 3}\right)+ \\ +u^{2}\left(l_{\text {TSrep } 3}\right)+u^{2}\left(l_{\mu \text { mcal }}\right)+u^{2}\left(l_{\mu \text { mrep }}\right)+u^{2}\left(l_{\mu \text { mres }}\right)\end{array}\right\}$
where $u\left(l_{\text {Tsres }}\right), u\left(l_{\text {Tsres } 2}\right), u\left(l_{\text {Tsres } 3}\right)-$ standard uncertainties due to limited TS display resolution for distance measurement; $u\left(l_{\text {Tsrepl }}\right)$ - standard uncertainty due to repeatability of distance measurements between the TS and the prism mounted
on the scale; $u\left(l_{\text {Tsrep } 2}\right)$ - standard uncertainty due to repeatability of distance measurements between the TS and the prism mounted on the mirror (prism constant determination); $u\left(l_{\text {Ssep }} 3\right)$ - standard uncertainty due to repeatability of distance measurements between the TS and the mirror (prism constant determination); $u\left(l_{\mu m c a l}\right)$ - standard uncertainty due to the depth micrometer; $u\left(l_{\mu m r e p}\right)$ - standard uncertainty due to repeatability of the depth micrometer; $u\left(l_{\mu m r e s}\right)$ - standard uncertainty due to limited display resolution of the depth micrometer.

The distance measurements to the scale were performed using the same total station and prism, therefore, the uncertainty due to measurements of the prism constant should be very similar.

The evaluation of the uncertainty due to the angle measurements by TS can be evaluated analogically as in the previous approach, however the new data sets from the new approach must be used with the new standard uncertainty due to repeatability of angle measurements.

### 2.2. Method for Vertical Angle Measuring System Calibration Using Proposed Apparatus

The proposed angle measuring system is based on well-known and reliable angle measuring technique described in standards, latest angle metrology related papers and official reports of famous metrology institutes. The main principle of angle calibration is comparison of reference angle and measured angle. The reference angle can be obtained by using various techniques and instrumentation. One of the main techniques used for horizontal angle measuring system calibration of total station is comparing measured angle with the reference angle created by indexing table. After aligning and leveling all devices to be used in calibration, total station is fixed to the special frame mounted on the indexing table, set to its initial position and pointed to the target. The upper part of the indexing table is lifted by a small handle and rotated by an eligible angle $\left(\theta_{l}\right)$ with all the system. Afterwards, the telescope of the total station is returned to its previous position $\left(\theta_{T S}\right)$ and pointed to the same target. The measurement principle is shown in Fig. 2.6. The difference between the mean of measurements and the reference value is bias, or systematic error of calibrated system. Measurements are repeated every $10^{\circ}$ of the full circle for six times in clockwise and six times in counter clockwise directions. Uncertainty of measurements is evaluated according to GUM (Guide to the expression of uncertainty in measurement).


Fig. 2.6. The measurement principle of the new setup

The main component of this angle measuring system is a special apparatus fixed on the top of Moore's 1440 Precision Index (detailed description available in Subchapter 1.3.4). This apparatus has a special weight balanced structure and special frame which is designed to fit total station in horizontal position. The special mount for fixing total station's tribrach is installed in this system as well as six adjustment screws, three on each side, of the frame to support and level upper part of a total station. Upper handle of a total station should be removed to fit in the frame. The main parts of this angle measuring system for the calibration of vertical angle measuring system of a total station are shown in Figure 2.7.


Fig. 2.7. The main components of angle measuring system

To experiment and to develop angle measuring system two approaches were carried out by performing measurements using slightly different instrumentation. First approach was realized by using angle measuring system composed of angle measuring apparatus, total station Leica Tc 2003, Moore's Special 1440 Index and manual autocollimator Nikon 6B.

The second approach was realized with electronic autocollimator MollerWedel Elcomat and the mirror with its mount fixed to the telescope of the total station.

### 2.2.1. Calibration of Vertical Angle Measuring System with the Apparatus and the Manual Autocollimator

Total station Leica TC2003 is mounted to its tribrach, leveled and fixed horizontally to the vertical angle measurement system calibration apparatus which is placed on the Moore's 1440 Precision Indexing table in front of the leveled manual autocollimator. Whole system is set upon the leveled granite surface plate. The telescopes of both total station and autocollimator are aligned and coaxially pointed to each other. Autocollimator Nikon 6B is used to establish the best total station telescope position by matching the crosshairs of the devices ${ }^{\text {6 }}$ telescopes. In an alignment stage the telescope of the total station is set to the infinity focus and pointed to the telescope of the autocollimator. During the experiment the line produced by the cross-line laser was used to align the telescopes of both total station and autocollimator.


Fig. 2.8. The cross lines of the devices are matched by an operator

After this procedure the both crosshairs of both devices should be seen through both telescopes. If the total station has the autocollimation function then it is easier to align both telescopes. Leica TPS 2003 is not provided with autocollimation function therefore, the alignment was performed by using infinity focus of the telescope and adjusting the light source to see the crosshairs
through the telescope of the autocollimator. After that, the telescope of the autocollimator is adjusted to make the crosshairs of both telescopes parallel to each other as it is shown in Fig. 2.9.


Fig. 2.9. The view of parallel crosshairs

The position of total station's telescope is controlled by six adjustment screws embedded in the apparatus which are designed to perform both support and leveling functions. The alignment is finished when the crosshairs of both telescopes are parallel to each other and both vertical lines are coincide.

Moore 1440 Precision Index is considered to be international angle standard. The accuracy of $0.1^{\prime \prime}$ is provided by this tool but only at $15^{\prime}$ interval. This means that calibration of angles smaller than 15 ' is not possible by using Moore‘s Precision Index. Totally 1440 angles can be measured by this indexing table. First, the total station is set to its initial position and the vertical lines of autocollimator and total station telescopes are matched by an operator (Fig. 2.8). The vertical angle reading of the total station is taken. Subsequently, the indexing table in conjunction with vertical angle calibration apparatus is rotated by 10 degrees and fixed. It can be rotated by any desired angle $\geq 0.25^{\circ}$. If any other indexing table is used, the whole system can be rotated by any angle depending on the resolution of the indexing table. Then the telescope of the total station is rotated backwards until total station's vertical crosshair line matches autocollimator's vertical crosshair line. Total station vertical angle readings are taken again. This vertical angle measuring system calibration apparatus enables to measure vertical angles of the total station in the range of $40^{\circ}$ to $140^{\circ}$ $\left(90^{\circ} \pm 50^{\circ}\right)$ and $220^{\circ}$ to $320^{\circ}\left(90^{\circ} \pm 50^{\circ}\right)$ which is twice as big as it can be achieved by the vertical angle comparator designed ESRF, France (Martin 2010). Measurement procedure is repeated for 12 times ( 6 times measuring while turning indexing table clockwise and 6 times turning indexing table counterclockwise). Total station readings are compared with the indexing table readings
while using the crosshair lines of the autocollimator as the reference position. After performing such measurements the components of combined uncertainty are identified and their influence on measurements is determined.

The alignment of devices for this approach was not complicated and did not require much time. Moreover, no additional expensive devices were used for alignment. However, this approach has some drawbacks. Every reading is read by an operator after matching crosshairs of both telescopes. This makes this approach time - consuming. In addition, while performing measurements, the influence of vibration has been noticed. In the second approach of the method all the remarks were taken into account and improvements were made.

### 2.2.2. Calibration of Vertical Angle Measuring System with the Apparatus, Electronic Autocollimator and the Mirror

To perform further investigations, some improvements were implemented and another approach of the method was realized. Special granite surface plate stand was filled with sand for stabilization in order to minimize vibrations which could affect measurement results.

The leveled total station is mounted horizontally to the vertical angle measurement system calibration apparatus on the indexing table. The mirror and its mount is fixed to the telescope of the total station by using small screws to adjust its position and leveled electronic autocollimator is pointed directly to it as it is shown in Figure 2.10. The telescopes of the electronic autocollimator and the total station are aligned to be on the same sight axis by using laser plumb. To ensure better alignment devices can be aligned by matching cross-hairs of the telescopes before mounting a mirror. In this case the remote interferometer (Hewlett - Packard 10565B) was used to align the telescopes of the electronic autocollimator and total station by using the autocollimator's beam. During the alignment stage the remote interferometer is placed between two telescopes for their sight axes to meet in remote interferometer's retro reflector. When the autocollimator's beam reflects from the mirror mounted on the total station's telescope, two separate crosses can be observed in the remote interferometer. Alignment is finished when these two crosses are matched by using adjustment screws of the apparatus and the reading of the electronic autocollimator doesn't change after turning total station by $180^{\circ}$.

After the alignment, the angle measuring system, total station and the electronic autocollimator are set to the initial position. The indexing table is rotated by a desired angle and the telescope of the total station is turned backwards by the same angle. Autocollimator readings show the angle of changed mirror position which means that there was a mismatch between the readings of the indexing table and the total station. The procedure is repeated 12
times ( 6 times clockwise and 6 times counter - clockwise) in the range of $40^{\circ}$ to $140^{\circ}$ and $220^{\circ}$ to $320^{\circ}$ and the readings of total station vertical angle as well as autocollimator readings of both ( $x$ and $y$ ) axes are taken.


Fig. 2.10. The setup of instrumentation for the vertical angle measuring system calibration with a mirror and autocollimator

In this approach it is very important to fix the mirror on the telescope very precisely as well as align both devices to be on the same sight axis. If this condition is not fulfilled then measurement uncertainty will increase due to biaxial change of mirror position.

After reducing vibration of granite plate stand, the effect was noticeable in stable readings of the electronic autocollimator. Comparing with the first approach this approach was faster and time saving. However, time consuming alignment of the devices and difficulties of precisely mounting the mirror on the telescope of the total station are the weaknesses of this approach.

### 2.2.3. Uncertainty Evaluation of the Vertical Angle Measuring System Calibration Method Where the Apparatus is Used

To analyze the measurement methods where complex instrumentation is used there is a need to evaluate measurement uncertainty. Therefore the components of combined uncertainty as well as their significance have to be determined. Using formulas and uncertainty determination flowchart displayed in Subhapter 1.2.2. The uncertainty of vertical angle measurements can be evaluated as described below in this Chapter.

The measurement function of such measurements is expressed as it is shown in (2.16). The correction value is added algebraically to the uncorrected result of a measurement to compensate for systematic error.

The correction has to be evaluated considering all the instrumentation used for the measurements. Therefore, rotated angle of indexing table is expressed:

$$
\begin{equation*}
\theta_{I}=\theta_{0}+\theta_{I . C} \tag{2.16}
\end{equation*}
$$

where $\theta_{\mathrm{I}}$ - rotated angle of indexing table; $\theta_{0}-$ nominal angle; $\theta_{\mathrm{IC}}-$ correction value for the indexing table.

The rotated angle of total station is expressed:

$$
\begin{equation*}
\theta_{T S}=\theta_{T S r}+B \tag{2.17}
\end{equation*}
$$

where $\theta_{T S}$ - rotated angle of the total station; $\theta_{T S r}$ - vertical angle reading of the total station; $B$ - correction value for the vertical angle readings of total station.

The reading of an autocollimator depends on both indexing table and total station rotated angles. Therefore, the reading of an autocollimator can be expressed as follows:

$$
\begin{equation*}
\theta_{A C}=\left(\theta_{0}+\theta_{I . C}\right)-\left(\theta_{T S r}+B\right) \tag{2.18}
\end{equation*}
$$

where $\theta_{A C}$ - the reading of the autocollimator; $\theta_{0}$ - nominal angle; $\theta_{I C}$ correction value for the indexing table; $\theta_{T S r}$ - vertical angle reading of the total station; $B$ - correction value for the vertical angle readings of total station.

In this method the total station, indexing table and the autocollimator are three main error sources influencing the magnitude of the uncertainty. The combined uncertainty of the correction value (2.21) can be expressed as the sum of squares of the uncertainty due to the indexing table $u^{2}\left(\theta_{\mathrm{IC}}\right)$, uncertainty due to the total station $\mathrm{u}^{2}\left(\theta_{\mathrm{TS}}\right)$ and uncertainty due to the autocollimator $\mathrm{u}^{2}\left(\theta_{\mathrm{AC}}\right)$ multiplied by their sensitivity coefficient squares $\left(c_{i}^{2}\right)$.

$$
\begin{equation*}
u_{c}^{2}(B)=c_{I . C .}^{2} u^{2}\left(\theta_{I . C}\right)+c_{T S}^{2} u^{2}\left(\theta_{T S}\right)+c_{A C}^{2} u^{2}\left(\theta_{A C}\right) \tag{2.19}
\end{equation*}
$$

Uncertainties due to total station and autocollimator readings have two more components each. Therefore, uncertainties due to resolution and repeatability of both instruments have to be included in whole uncertainty budget.

The final equation for the combined uncertainty of the correction value describing uncertainty budget is:

$$
\begin{align*}
& u_{c}^{2}(B)=u^{2}\left(\theta_{\text {ICCal }}\right)+u^{2}\left(\theta_{\text {TSrep }}\right)+u^{2}\left(\theta_{\text {TSres }}\right)+u^{2}\left(\theta_{\text {TSshift }}\right)+  \tag{2.20}\\
& +u^{2}\left(\theta_{\text {ACcal }}\right)+u^{2}\left(\theta_{\text {ACrep }}\right)+u^{2}\left(\theta_{\text {ACres }}\right)
\end{align*}
$$

This combined uncertainty is evaluated by using both Type A and Type B evaluation methods. The best way to analyze each component of combined uncertainty is to fill uncertainty budget table. The uncertainty due to repeatability of the total station $\left(\mathrm{u}\left(\theta_{\text {TSrep }}\right)\right)$ is 0 because the total station every time was rotated
to the position were exactly the same angle reading was shown on TS's display (Suh, Siaudinytė 2014).

Uncertainties due to the limited display resolution of the total station $\left(\mathrm{u}\left(\theta_{\mathrm{TS} \text { Ses }}\right)\right)$ and due to the limited display resolution of the autocollimator ( $u\left(\theta_{\text {ACress }}\right)$ ) have rectangular distributions because all the readings have the same probability to be displayed according to the device's rounding system. Such type of uncertainty was evaluated by the difference of upper and lower resolution limits of the device divided by 2 times square root of 3 as expressed in (2.21):

$$
\begin{equation*}
u\left(\theta_{\text {res }}\right)=\frac{R}{2 \sqrt{3}} \tag{2.21}
\end{equation*}
$$

where R - display resolution of the device.
The resolution of the total station Leica TC 2003 is $\mathrm{R}=0.1^{\prime \prime}$ and resolution of an electronic autocollimator (Möller-Wedel) is $\mathrm{R}=0.05$ ". Uncertainty due to repeatability of the autocollimator was evaluated by calculating standard uncertainties (Subchapter 1.2.2.) of every data set of every measured angular position. The best way to evaluate the uncertainty when all individual data sets have their own uncertainties is to determine pooled standard uncertainty as shown in (2.22):

$$
\begin{equation*}
s_{p}=\sqrt{\frac{\sum_{i=1}^{N} s_{i}^{2}}{N}} \tag{2.22}
\end{equation*}
$$

where $s_{i}$ - standard uncertainty of every data set; $N$ - number of data sets.
Then the standard uncertainty due to repeatability $\left(u\left(\theta_{\text {ACrep }}\right)\right)$ of the autocollimator can be evaluated as follows:

$$
\begin{equation*}
u=\frac{s_{p}}{\sqrt{n}} \tag{2.23}
\end{equation*}
$$

where $s_{p}$ - pooled standard uncertainty; $n$ - number of observations in a data set.
Uncertainties due to the indexing table $\left(\mathrm{u}\left(\theta_{\mathrm{I} \cdot \mathrm{Cal}}\right)\right.$ ) and due to the autocollimator ( $\mathrm{u}\left(\theta_{\text {ACcal }}\right)$ ) can be evaluated using type B uncertainty evaluation method by taking uncertainties directly from the calibration certificates of the devices.


Fig. 2.11. The possible shift of the total station's telescope
Uncertainty due to the possible telescope shift while in horizontal position $\mathrm{u}\left(\theta_{\mathrm{TSshift}}\right)$ needs to be analyzed deeper because its influence can be significant on measurement results. In this method the total station is used horizontally. Therefore, in this position there is a possible biaxial shift of the telescope as shown in Figure 2.11. Upon a query, manufacturer of the total station affirmed the presents of the special mechanisms embedded in the total station for compensation of such influence and the measurement errors due to position of the total station are negligible. Such kind of uncertainty source should be investigated in further research to prove the stability of inner total station components while it is in horizontal position.

### 2.3. Conclusions of the Chapter 2

1. A novel trigonometric method for calibration of vertical angle measuring systems is proposed in this Chapter. Simple implementation of this method enables calibration of vertical angle measuring systems under laboratory conditions.
2. A new setup for calibration of vertical angle measuring system calibration is based on horizontal angle measuring system calibration principles. This setup is space efficient and provides bigger calibration range than other researched methods.
3. For the realization of proposed methods reference means such as laser interferometer, Moore's Special Index and electronic autocollimator with the mirror are used. Therefore, it ensures the credibility of determination of the reference angle.
4. Calibration under stabile laboratory conditions reduces uncertainty and allows to change and control measurement pitch as well as increase the number of measurements in order to achieve more accurate results.

## 3

## Experimental Evaluation of the Proposed Methods

Theoretical modelling provides good knowledge about the principles of the method evaluation. However, it is not able to reflect the practical implementation issues. Therefore, practical approach is needed to define the issues, uncovered by theoretical modelling. There are no measurements without errors. Therefore, uncertainty determination is necessary. Uncertainty evaluation gives the information about the error sources and their influence on the measurement results.

The experimental evaluation of proposed methods for the calibration of vertical angle measuring systems of the total station were performed at Korea's Research Institute of Standards and Science. The laboratories and instrumentation of Centre of Length of Physical Metrology Division were used for the practical realization of proposed methods. The material provided in this Chapter was published in the proccedings of International Conference (Suh, Šiaudinyte 2014).

### 3.1. The Setup of the Experiment, Data Processing and Uncertainty Evaluation of the Calibration Method Using the Reference Scale

The total station has to be placed in such position that the telescope in its horizontal position would be pointed to the center line of the scale and the measurements would be performed at the distance not shorter than the focusing range of the TS. Total Station Leica TC 2003 which has a focusing range of 1.6 $m$ was used for the experiment. The TS is fixed to the stable steel pillar on the table under which the interferometer is placed more than 1.6 m away from the reference scale to meet this requirement. After precisely leveling TS and the reference linear scale instruments have to be prepared for the measurements.

The reference scale is fixed to its own special mount and placed on the carriage (Fig. 3.1). For the experiment an invar Gaertner Scientific reference scale No. 244 A.U. was used.


Fig. 3.1. Arrangement of the system for the calibration of vertical angle measuring systems using reference scale

To align the devices PLS180 cross line laser is placed in a position for its horizontal and vertical beams to cross the optical axis of the telescope. The scale is adjusted so that the same beam could cross the vertical axis of it. The vertical position of the scale is adjusted with the screws according to the laser beam position and spherical level on the mount of the reference scale. The mirror surface of the invar scale produces the reflection of the laser level's beam. If the scale is not perpendicular to the axis of the telescope, two lines of the same laser beam and its reflection can be seen. The position of the reference scale is adjusted to match these two lines of the laser beams.


Fig. 3.2. Alignment of the reference scale
The scale was aligned by using PLS180 cross line laser. The beams reflected from the reference scale were matched with the laser beams to ensure perpendicularity of the scale as it is shown in Fig. 3.2. However, there is a need to evaluate the uncertainty due to the tilt of the scale $u\left(\Delta h_{\text {tilt }}\right)$ which affects vertical distance. Laser beam is thinner than its reflection because of the increased distance. Therefore, while aligning the scale and matching these beams the reflection would overlap the original beam. The best way to determine the largest uncertainty is to use rectangular distribution and the half width of the beam reflection line. The width of the laser beam reflection is $W_{L B}=0.0015 \mathrm{~m}$. The uncertainty due to tilt of the reference scale is expressed:

$$
\begin{equation*}
u\left(h_{t i l t}\right)=\frac{W_{L B 0.5}}{\sqrt{3}} \tag{3.1}
\end{equation*}
$$

where $W_{L B 0.5}$ - the half width of the laser beam reflection.
The alignment is continued by adjusting special leveling screws of the scale mount. The position of the vertical scale bar is observed by aligning reticle of the TS telescope and the lines of the reference scale. The alignment is finished when the grating of the scale is parallel to the horizontal lines and perpendicular to the vertical lines of the telescope crosshairs in the full length of the scale from line 0 to the line 100 .

When the devices are aligned the calibration of the vertical angle measuring system of the total station can be started. There are two approaches of this method discussed in Subchapter 2.1. By using first approach the interferometer (Hewlett Packard Laser System 5519A) and its retroreflector mounted on the carriage under the reference scale is used for the displacement measurements of
the scale. When the telescope of the total station is pointed to the center line (50) of the scale, the vertical angle reading showed in the display should be $90^{\circ} 0^{\prime} 0.0^{\prime \prime}$ or $270^{\circ} 0^{\prime} 0.0^{\prime \prime}$ (depending on the measuring face of the TS). Then the telescope is pointed to any line of the scale and fixed. Both the vertical angle reading of the TS and the interferometer readings are taken. By using a long handle fixed to the carriage the scale is moved until the horizontal line of the reticle cross line matches another line of the reference scale as it is showed in Fig. 3.7. This movement gives both vertical and horizontal displacements for the angle determination.

Since the telescope of the TS is fixed, the vertical angle reading haven't changed, however the interferometer reading has to be taken again. The difference between two interferometer readings shows the horizontal distance by which the reference scale was moved. Vertical distance is known as a difference between two lines (before and after the movement) from the reference scale. Lower and upper parts of the scale are measured 6 times in both faces (totally 12 times).

The initial attempts of the experiment were performed to every centimeter of the scale. However, after such measurements it was determined that the standard deviation of the results can be reduced by grouping angles. The mismatch between the angle determined by the reference means and the measured angle was combined of two, four and five neighboring angles and standard deviation was calculated. As it is shown in Fig. 3.3 the standard deviation reduced after increasing sets of angle measurements.


Fig. 3.3. Influence of sets of angle measurements

It was decided to use lines at every $10^{\text {th }}$ centimeter of the scale for further angle measurements. After performing such measurement and having all necessary readings, measured angle is compared with the reference angle
determined trigonometrically by using reference means. The uncertainty evaluation of this approach is presented in Subchapter 2.1.3.

In the second approach, the vertical reference linear scale remains stationary. It is very complicated to measure the distance to the surface of the scale where the grating is directly. However, horizontal distance to the scale can be measured by a total station and the reflector. As it is shown in Fig. 3.4 there is still a distance from the prism to the grating surface of the scale which needs to be measured. The total horizontal distance between the total station and scale grating $(l)$ consists of measured distance by the total station $\left(l_{T S}\right)$, length according to the prism constant of the reflector $\left(l_{p}\right)$, the width of the magnet $\left(l_{m}\right)$ which is needed to attach the prism to the scale and the depth of the scale $\left(l_{s}\right)$.


Fig. 3.4. Determination of horizontal distance
After pointing the reticle of TS telescope to the center line of the scale the magnet mounted reflector (prism) is attached to the center of the reference scale that crosshairs of the telescope would be pointed directly to the center of the prism. This distance can be measured by a total station. However, the prism constant and the depth of the scale have to be measured separately to determine the horizontal distance between the axis of the TS and the lines of the reference scale. To determine the constant of the prism the reflector is mounted to the magnet and attached to the precisely leveled mirror. The telescope of the TS is pointed to the reflector and the reading of horizontal distance is taken (Fig. 3.5). 20 readings are taken and averaged. Then the prism with its magnet is removed from the mirror and the distance between the total station and the mirror is measured by the total station. As previously mentioned 20 readings of horizontal
distance are taken. The total distance of the prism constant and the magnet is the difference between averaged horizontal distances between these two measurements.


Fig. 3.5. Prism constant determination
There is no need to determine the width of the magnet and prism constant separately. So horizontal distance of prism constant $\left(l_{p}\right)$ and magnet width $\left(l_{m}\right)$ is $l_{p}+l_{m}=0.01178 \mathrm{~m}$. Unfortunately, the mirrored surface of the reference scale is narrow and not sufficient to measure horizontal distance to its surface directly by the total station.

To determine the depth of the reference scale the Mitutoyo depth micrometer was used as it is shown in Fig. 3.6.


Fig. 3.6. Scale depth measurements

The depth of the scale was measured 3 times at pitch of 10 cm . The average depth of the scale was determined $l_{s}=12.427 \mathrm{~mm}$ with standard uncertainty $\mathrm{u}=4.83 \cdot 10^{-5} \mathrm{~m}$.

Although horizontal distance measurements in both approaches have different uncertainty components, the determination of vertical distance remains the same. Uncertainty due to vertical distance consists of such components as uncertainty due to the reference scale $u\left(\Delta h_{\text {Scal }}\right)$, uncertainty due to thermal expansion of the scale $u\left(\Delta h_{\text {therm }}\right)$, uncertainty due to compression of the scale $u\left(\Delta h_{\text {comp }}\right)$ and uncertainty due to pointing $u\left(\Delta h_{\text {point }}\right)$.

Since the reference scale is calibrated, its uncertainty can be evaluated using type B evaluation method (value can be obtained from calibration certificate or other documentation). Uncertainty due to pointing $u\left(\Delta h_{\text {point }}\right)$ was analyzed deeper. Since the widths of graduation lines of the reference scale and the reticle crosshair of TS telescope don't match they have to be measured separately (Fuhe, Dezheng 1996). As it is shown in Fig. 3.7 the center of TS's reticle is pointed to the line center of the reference scale. The vertical angle is measured between two line centers of the reference scale with the vertical distance ( $h$ ) between them. Zoom in view of the Fig. 3.7 shows that width of the reference scale line $\left(W_{S}\right)$ and the cross line of the TS's reticle ( $W_{T S}$ ) differs. Therefore, this uncertainty has to be evaluated.


Fig. 3.7. General and zoomed - in views of the TS's telescope pointed to the reference scale

To determine the width of the telescope crosshair width, the lens resolution chart (Fig. 3.8) was used. The numbers in this chart indicates the scale of the line width which let us know how many line pairs of dark and bright lines fits in one millimeter.


Fig. 3.8. Lens resolution chart
For the determination of the reticle crosshair width Edmund scientific resolution chart which implies NIST 1010A standard and ISO Test Chart \#2 pattern was used. The chart is covered with mirror and has transparent spaces to indicate the width of the line. The chart and the backlight were placed 2 meters away from the total station and the telescope was pointed to the chart. The view of the lens resolution chart and the reticle of the telescope is shown in Fig. 3.9.


Fig. 3.9. The view of the lens resolution chart and the reticle

It was determined that width of the reticle matches the lines in the section with the scale factor 16. It is stated that there are 32 lines in 1 mm . There are 9 horizontal lines displayed in this resolution chart. As it is shown in (3.2) the width of the crosshair of the reticle (D) can be determined by using the ratio of the of the width of the cross line and width of 9 resolution chart lines measured in the image multiplied by the real width of 9 lines in the resolution chart.

$$
\begin{equation*}
D=d_{\text {real }} \frac{d_{1 \text { meas }}}{d_{\text {9meas }}} \tag{3.2}
\end{equation*}
$$

where $d_{\text {real }}$ - real width of 9 neighboring lines in the resolution chart $\left(d_{\text {real }}=9 / 32=0.28125 \mathrm{~mm}\right) ; d_{\text {Imeas }}$ - measured width of the cross line of the telescope in the image $\left(d_{1 \text { meas }}=0.6 \mathrm{~mm}\right)$; $d_{9 \text { meas }}$ - measured width of 9 neighboring lines in the resolution chart $\left(d_{9_{\text {meas }}}=7.5 \mathrm{~mm}\right)$.

The determined width of the cross line of the reticle $\mathrm{D}=0.0225 \mathrm{~mm}=2.3 \mu \mathrm{~m}$.
$\theta_{C L}=1.15 \cdot 10^{-5} \mathrm{~m}=1.7 \mathrm{rad}$ and can be converted to arc seconds by multiplying by conversion factor $\left(\rho=2.06 \cdot 10^{5}\right)$. Therefore, the possible pointing with the center of the cross line error $\theta_{C L}=3.51 " / 10=0.35^{\prime \prime}$.

The line width of the reference scale is $W=5 \mu \mathrm{~m}$ and it was measured with the measuring microscope Mitutoyo AT115-100, No.09BAA441-A (Fig. 3.10).


Fig. 3.10. Measuring line width of the reference scale

The reticle cross line is thinner than the scale line, however the latter reduces within the distance and the former remains the same.

The uncertainty due to pointing to the line center of reference scale $u\left(h_{\text {poinn }}\right)$ varies depending on the distance between the scale and the telescope. It can be expressed based on triangular distribution as follows:

$$
\begin{equation*}
u\left(h_{p o \text { int }_{i}}\right)=\frac{W_{S_{i}}-W_{T S}}{\sqrt{6}} \tag{3.3}
\end{equation*}
$$

where $u\left(h_{\text {point } i}\right)$ - uncertainty due to pointing to the line center of the reference scale; $W_{S i}$ - line half-width of the reference scale in the image plane depending on the distance between the device and the reference scale; $W_{T S}$ - constant halfwidth of the reticle line of the TS telescope.

The recalculated line width of the scale in the image plane of the TS $\left(W_{S i}\right)$ can be expressed:

$$
\begin{equation*}
W_{S i}=W_{S m} M \tag{3.4}
\end{equation*}
$$

where $W_{S m}$ - measured line width of the reference scale $\left(W_{S m}=5 \mu \mathrm{~m}\right)$; $M$ - magnification factor of the telescope.

The magnification factor (M) can be determined based on the expression of the focal length of the telescope (Frade, 2003):

$$
\begin{equation*}
\frac{1}{f}=\frac{1}{a_{i}}+\frac{1}{b} \tag{3.5}
\end{equation*}
$$

where $f$ - focal length of the TS telescope; $b$ - distance between the lens and the image (effective focal length of the telescope (constant)); $a_{i}$ - varying distance between the object and the lens determined as a difference between measured distance to the scale $\left(a_{m}\right)$ and half of the effective focal length of the telescope (b). It is expressed:

$$
\begin{equation*}
a_{i}=\left(\frac{\Delta h}{\tan \theta}\right)-\left(\frac{b}{2}\right) \tag{3.6}
\end{equation*}
$$

where $\theta$ - vertical angle reading of the TS; $\Delta h$ - vertical distance known from the reference scale; $b$-distance between the lens and the image (effective focal length of the telescope (constant $b=0.135 \mathrm{~m}$ )). According to (3.6), the magnification factor of the telescope lens is expressed:

$$
\begin{equation*}
M=\frac{b}{a_{i}} \tag{3.7}
\end{equation*}
$$

The uncertainty due to pointing to the line center of the reference scale was evaluated using the line width of the reference scale in the image plane of the TS where the largest uncertainty is. Parameters for uncertainty due to pointing determination while the scale is stationary are shown in Table 3.1.

Table. 3.1. Parameters for uncertainty due to pointing determination when the scale is stationary

| No. | Parameter | Value |
| :---: | :---: | :---: |
| 1. | $a$ | 2.095859 m |
| 2. | $b$ | 0.135 m |
| 3. | $(\Delta h /$ tan $\theta)$ | 2.16335945 m |
| 4. | $M$ | 0.064 |
| 5. | $W_{S m}$ | $5 \cdot 10^{-6} \mathrm{~m}$ |
| 6. | $W_{T S}$ | $2.3 \cdot 10^{-6} \mathrm{~m}$ |
| 7. | $u\left(h_{\text {point }}\right)$ | $5.5 \cdot 10^{-5} \mathrm{~m}$ |

In the approach where the reference scale is moved the distance between the telescope lens (a) differs with every measured angle by moving the scale to the new position. Therefore, the distance (a) after the scale movement can be determined:

$$
\begin{equation*}
a_{i}=\left(\frac{\Delta h}{\tan \theta}+\Delta l_{i}\right)-\left(\frac{b}{2}\right) \tag{3.8}
\end{equation*}
$$

where $\Delta h$ - difference between calibrated distances of the reference scale after thermal expansion evaluation; $\theta$ - average of the angles measured by the TS; $\Delta l_{i}$ - average of horizontal displacements of the reference scale measured by the laser interferometer.

According to the expression of the measurement function, in the approach with the moving scale the sensitivity coefficients for the uncertainty due to horizontal distance measurements can be expressed:

$$
\begin{equation*}
c_{l i}=\left(\frac{\partial b}{\partial \Delta l_{i}}\right)=\frac{\partial b}{\partial \Delta l_{i}}\left(\tan ^{-1}\left(\frac{\Delta h}{\Delta l}\right)-\theta_{T S}\right)=-\frac{\Delta h}{\Delta l_{i}^{2}+\Delta h^{2}} \tag{3.9}
\end{equation*}
$$

where $\Delta l_{i}$ - average of horizontal distances measured by interferometer, $\Delta h$ vertical distance between two lines of the reference scale.

Sensitivity coefficients for the uncertainty due to vertical distance determination can be expressed:

$$
\begin{equation*}
c_{h i}=\left(\frac{\partial b}{\partial \Delta h}\right)=\frac{\partial b}{\partial \Delta h}\left(\tan ^{-1}\left(\frac{\Delta h}{\Delta l}\right)-\theta_{T S}\right)=\frac{\Delta l_{i}}{\Delta l_{i}^{2}+\Delta h^{2}} \tag{3.10}
\end{equation*}
$$

Sensitivity coefficient for angle measurements by the TS is:

$$
\begin{equation*}
c_{\text {GTS }}=\left(\frac{\partial b}{\partial \theta_{T S}}\right)=\frac{\partial b}{\partial \theta_{T S}}\left(\tan ^{-1}\left(\frac{\Delta h}{\Delta l}\right)-\theta_{T S}\right)=-1 \tag{3.11}
\end{equation*}
$$

According to the expression of the correction value (2.4) uncertainty components are displayed in Table 3.2.

In the approach with the moving scale eight displacements of the reference scale were measured by the laser interferometer. Therefore, there were eight sensitivity coefficients determined as well as eight combined uncertainties for the correction values. Uncertainty parameters regarding sensitivity coefficients are displayed in the Table 3.3.
Table 3.2. Uncertainty budget for the calibration method with the displaced reference scale

| Source of uncertainty | Standard uncertainty $u\left(x_{i}\right)$ | Sensitivity coefficient $c_{i}$ | Uncertainty contribution $\left\|c_{i}\right\| \cdot u\left(x_{i}\right)$ | Probability distribution | Degrees of freedom $v$ | Effective degrees of freedom $v_{\text {eff }}$ | Coverage factor k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Combined uncertainty $\mathrm{u}\left(\Delta \mathrm{l}^{\prime}\right)$ | $1.102 \cdot 10^{-4} \mathrm{~m}$ | $-\frac{\Delta h}{\Delta l_{i}^{2}+\Delta h^{2}}$ | $1.102 \cdot 10^{-4} \cdot\left(-\frac{\Delta h}{\Delta l_{i}^{2}+\Delta h^{2}}\right)$ | t-(Student‘s) |  | 13 |  |
| Uncertainty due to the laser interferometer u( $\left.\Delta \mathrm{l}_{\mathrm{LLcal}}^{\prime}\right)$ | $1.0 \cdot 10^{-6} \mathrm{~m}$ |  |  | Normal | $\infty$ |  |  |
| Uncertainty due to repeatability of the laser interferometer $u\left(\Delta l_{\text {LIrep }}^{\prime}\right)$ | $1.063 \cdot 10^{-4} \mathrm{~m}$ |  |  | t-(Student‘s) | 11 |  |  |
| Uncertainty due to limited display resolution of the laser interferometer $\qquad$ | $2.89 \cdot 10^{-5} \mathrm{~m}$ |  |  | Rectangular | $\infty$ |  |  |
| Combined uncertainty $\mathrm{u}\left(\Delta \mathrm{h}^{\prime}\right)$ | $4.398 \cdot 10^{-4} \mathrm{~m}$ | $\frac{\Delta l_{i}}{\Delta l^{2}{ }_{i}+\Delta h^{2}}$ | $4.39810^{-4}\left(\frac{V_{i}}{\Delta_{i}^{2}+\Delta h^{2}}\right)$ | Rectangular |  | 53 |  |
| Uncertainty due to reference scale $u\left(\Delta h^{\prime}\right.$ scal $)$ | $7.7 \cdot 10^{-5} \mathrm{~m}$ |  |  | Normal | $\infty$ |  |  |
| Uncertainty due to thermal expansion of the reference scale $\mathrm{u}\left(\Delta \mathrm{h}_{\text {stherm }}^{\prime}\right)$ | $5.0 \cdot 10^{-7} \mathrm{~m}$ |  |  | Rectangular | 50 |  |  |
| Uncertainty due to compression of the reference scale $\mathrm{u}\left(\Delta \mathrm{h}_{\text {Scomp }}\right)$ | $7.1 \cdot 10^{-9} \mathrm{~m}$ |  |  | Rectangular | $\infty$ |  |  |
| Uncertainty due to pointing $\mathrm{u}\left(\Delta \mathrm{h}_{\text {point }}\right)$ | $5.5 \cdot 10^{-7} \mathrm{~m}$ |  |  | Triangle | 50 |  |  |

Table 3.2 (continued)

| Source of uncertainty | Standard uncertainty $u\left(x_{i}\right)$ | Sensitivity coefficient $c_{i}$ | Uncertainty contribution $\left\|c_{i}\right\| \cdot u\left(x_{i}\right)$ | Probability distribution | Degrees of freedom $v$ | Effective degrees of freedom $\mathrm{v}_{\text {eff }}$ | Coverage factor k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Uncertainty due to tilt of the scale $\mathrm{u}\left(\Delta \mathrm{h}_{\text {tilt }}^{\prime}\right)$ | $4.33 \cdot 10^{-4} \mathrm{~m}$ |  |  | Rectangular | 50 |  |  |
| Combined uncertainty $\mathrm{u}\left(\theta_{\mathrm{TS}}\right)$ | 0.294" | -1 | 0.294" | t-(Student‘s) |  | 11 |  |
| Uncertainty due to limited display resolution of angle readings of the $\mathrm{TS} \mathrm{u}\left(\theta_{\mathrm{TSres}}\right)$ | 0.029" |  |  | Rectangular | $\infty$ |  |  |
| Uncertainty due to repeatability <br> of TS (angle measurements) <br> $\mathrm{u}\left(\theta_{\text {TSrep }}\right)$ | 0.293" |  |  | t-(Student's) | 11 |  |  |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{1}\right)$ |  |  | 0.294" | t-(Student ${ }^{\text {s }}$ ) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{2}\right)$ |  |  | 0.294" | t-(Student ${ }^{\text {s }}$ ) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{3}\right)$ |  |  | 0.294" | t-(Student's) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{4}\right)$ |  |  | 0.294" | t-(Student's) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{5}\right)$ |  |  | 0.294" | t-(Student's) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{6}\right)$ |  |  | 0.294" | t-(Student's) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{7}\right)$ |  |  | 0.294" | t-(Student's) |  | 11 | 2 |
| Uncertainty of the correction value $\mathrm{u}_{\mathrm{c}}\left(\mathrm{B}_{8}\right)$ |  |  | 0.294" | t-(Student ${ }^{\text {s }}$ ) |  | 11 | 2 |

According to GUM, when the number of effective degrees of freedom is $v \geq 10$ it is allowed to approximate it as normal distribution with coverage factor $\mathrm{k}=2$. The Expanded uncertainty is $\mathrm{U}_{95 \%}(\mathrm{~B})=0.59^{\prime \prime}(\mathrm{k}=2)$ or $\mathrm{U}_{95 \%}(\mathrm{~B})=1.4254 \cdot 10^{-6}$ rad $(k=2)$. To convert radians to arc seconds the conversion factor $\rho=206264.806$ was used.

Table 3.3. Parameters of combined uncertainties of the correction values

| Correction value | Sensitivity coefficient $c_{h i}$ | Sensitivity coefficient $\mathcal{C l}_{l i}$ | Uncertainty contribution $u(h)$, rad | Uncertainty contribution $u(l)$, rad | Combined uncertainty of the correction value $u_{c}\left(B_{i}\right),(")$ | Expanded uncertainty $U_{i 95 \%}(k=2)$, <br> (") |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{B}_{1}$ | 0.002314369 | -0.000567879 | $1.01785 \cdot 10^{-6}$ | $-6.25567 \cdot 10^{-8}$ | 0.294000365 | 0.59 |
| $\mathrm{B}_{2}$ | 0.001776763 | -0.000326338 | $7.81409 \cdot 10^{-7}$ | $-3.5949 \cdot 10^{-8}$ | 0.294000215 | 0.59 |
| $\mathrm{B}_{3}$ | 0.001205740 | -0.000147558 | $5.30277 \cdot 10^{-7}$ | $-1.62548 \cdot 10^{-8}$ | 0.294000099 | 0.59 |
| $\mathrm{B}_{4}$ | 0.000607412 | $-3.70321 \cdot 10^{-5}$ | $2.67136 \cdot 10^{-7}$ | $-4.0794 \cdot 10^{-9}$ | 0.294000025 | 0.59 |
| $\mathrm{B}_{5}$ | 0.000607475 | $-3.70398 \cdot 10^{-5}$ | $2.67164 \cdot 10^{-7}$ | $-4.08025 \cdot 10^{-9}$ | 0.294000025 | 0.59 |
| $\mathrm{B}_{6}$ | 0.001205649 | -0.000147536 | $5.30237 \cdot 10^{-7}$ | $-1.62523 \cdot 10^{-8}$ | 0.294000099 | 0.59 |
| $\mathrm{B}_{7}$ | 0.001775925 | -0.000326020 | $7.81041 \cdot 10^{-7}$ | $-3.59139 \cdot 10^{-8}$ | 0.294000214 | 0.59 |
| $\mathrm{B}_{8}$ | 0.002314254 | -0.000567819 | $1.01779 \cdot 10^{-6}$ | $-6.25501 \cdot 10^{-8}$ | 0.294000365 | 0.59 |

After analyzing the uncertainty budget it is obvious that the largest uncertainty comes from total station measurements. Since uncertainty due to repeatability is the largest it was decided to analyze total station measurements deeper. The deviations from average of TS Face I and Face II measurements are shown in Fig. 3.11 and Fig. 3.12 respectively:


Fig. 3.11. Deviations from the average of Total Station Face I measurements in the method with the displaced reference scale


Fig. 3.12. Deviations from the average of Total Station Face II measurements in the method with the displaced reference scale

As it is shown in the figures above, the measurements with the biggest deviations were performed to the $100^{\text {th }}$ line and line number 300 . In the following graph (Fig. 3.13) the difference between the reference and average of 12 times in both faces measured vertical angles as well as their standard uncertainties are displayed.


Fig. 3.13. Difference between the reference and measured angle in the method with the displaced reference scale

Fig. 3.14 shows standard deviations of the angles measured by using both faces of the total station the largest standard deviation appeared while performing face II measurements to $100^{\text {th }} \mathrm{mm}$ of the reference scale. Although standard deviations of measurements with both faces to 800 mm of the reference scale are moderate compared to others, the difference between the reference and measured angle shown in Fig. 3.13 is biggest of all measurements performed in the approach with the displaced target technique. It is clear that measurements to this particular line of the reference scale were precise but not accurate. This leads to an assumption that systematic error is presented in these measurements. However, the source of this error remains unknown. It could have been caused by the used instrumentation, the light reflection from this line or by the operator.


Fig. 3.14. Standard deviations of the angles measured with both faces of the Total Station in the method with the displaced reference scale

Another way to perform this method is apply stationary reference scale technique. In this approach there is no movement of the reference scale which can increase measurement errors. The uncertainty budget for this approach was analyzed and the components of the combined uncertainties are presented in the Table 3.4.

Since the number of effective degrees of freedom is $v \leq 10$, coverage factor $k$ has to be determined according to t-distribution table with effective degrees of freedom $v=6$ and the level of confidence of $95 \%$. Expanded uncertainty of correction value for the approach with the stationary reference scale was determined $U_{95 \%}(B)=0.24^{\prime \prime}(k=2.447)$ or $U_{95 \%}(B)=1.186 \cdot 10^{-6} \mathrm{rad}$.
Table 3.4. Uncertainty budget for calibration method using stationary reference scale

| Source of uncertainty | Standard uncertainty | Sensitivity coefficient | Uncertainty contribution $\left\|c_{i}\right\| \cdot u\left(x_{i}\right)$ | Probability distribution | $\begin{gathered} \hline \text { Degrees } \\ \text { of } \\ \text { freedom } \\ v \\ \hline \end{gathered}$ | Effective degrees of freedom $v_{\text {eff }}$ | Coverage factor k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Combined uncertainty $\mathrm{u}(\Delta \mathrm{h})$ | $4.398 \cdot 10^{-4} \mathrm{~m}$ | $-1.068 \cdot 10^{-5}$ | $2.03 \cdot 10^{-7} \mathrm{rad}$ | Rectangular |  | 53 |  |
| Uncertainty due to the reference scale $\mathrm{u}\left(\Delta \mathrm{h}_{\text {scal }}\right)$ | $7.7 \cdot 10-5 \mathrm{~m}$ |  |  | Normal | $\infty$ |  |  |
| Uncertainty due to thermal expansion of the reference scale $u\left(\Delta \mathrm{~h}_{\text {Stherm }}\right)$ | $5 \cdot 10^{-7} \mathrm{~m}$ |  |  | Rectangular | 50 |  |  |
| Uncertainty due to compression effect of the reference scale $u\left(\Delta h_{\text {Scomp }}\right)$ | $7.1 \cdot 10^{-9} \mathrm{~m}$ |  |  | Rectangular | $\infty$ |  |  |
| Uncertainty due to pointing $u\left(\Delta h_{\text {point }}\right)$ | $5.5 \cdot 10^{-7} \mathrm{~m}$ |  |  | Triangular | 50 |  |  |
| Uncertainty due to the tilt of the scale $\mathrm{u}\left(\Delta \mathrm{~h}_{\text {tilt }}\right)$ | $4.33 \cdot 10^{-4} \mathrm{~m}$ |  |  | Rectangular | 50 |  |  |
| Combined uncertainty u(l) | $6.738 \cdot 10^{-5} \mathrm{~m}$ | $4.620 \cdot 10^{-4}$ | $7.2 \cdot 10^{-10} \mathrm{rad}$ | Normal |  | 21 |  |
| Uncertainty due to limited display resolution of the TS (distance measurements between TS and the prism) $u\left(l_{\text {TSresi }}\right)$ | $2.88675 \cdot 10^{-6} \mathrm{~m}$ |  |  | Rectangular | $\infty$ |  |  |
| Uncertainty due to repeatability of TS (distance measurements between TS and the prism) $u\left(l_{\text {TSrep1 }}\right)$ | $5.0 \cdot 10^{-5} \mathrm{~m}$ |  |  | T (Student‘s) | 9 |  |  |
| Uncertainty due to limited display resolution of the TS (distance between TS and the prism measurements - prism constant determination) $\mathrm{u}\left(1_{\text {TSres } 2}\right)$ | $2.88675 \cdot 10^{-6} \mathrm{~m}$ |  |  | Rectangular | $\infty$ |  |  |
| Uncertainty due to repeatability of TS (distance between TS and the prism measurements - prism constant determination) $\mathrm{u}\left(1_{\mathrm{TS} \text { rep } 2}\right)$ | $2.0 \cdot 10^{-5} \mathrm{~m}$ |  |  | T (Student‘s) | 9 |  |  |

Table 3.4 (continued)

| Source of uncertainty |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

The situation in the method where the reference scale remains stationary is similar to the previously analyzed because the source of the largest uncertainty is repeatability of TS angle measurements. Measurement deviations from the average of 6 measurements with both TS faces are presented in Fig. 3.15 and Fig. 3.16 respectively.


Fig. 3.15. Deviations from the average of Total Station Face I measurements in the method with the stationary reference scale


Fig. 3.16. Deviations from the average of Total Station Face II measurements in the method with the stationary reference scale

As it is shown in Fig. 3.15 and Fig. 3.16 by remaining the reference scale stationary it was possible to reduce measurement deviations down to $\pm 1.2^{\prime \prime}$. The Face II of the total station gives out smaller deviations than Face I which can be seen in Fig. 3.17.


Fig. 3.17. Standard deviations of Total Station measurements in the method with stationary reference scale

The largest standard deviation of Face I remains at $800^{\text {th }} \mathrm{mm}$ of the reference scale as it was in previous approach and it verifies the assumption that the systematic error exists. In the approach with the stationary reference scale the horizontal distance was different from the previous approach therefore the angle measured by the TS had a different value. This leads to the conclusion that such error could have appeared due to the particular line of the scale. It might have been caused by manufacturing imperfections or specific environmental conditions such as light reflection at this specific point.


Fig. 3.18. The difference between reference and measured angle in method with the stationary reference scale

Fig. 3.18 shows that Side I has bigger measurement errors than Side II. The average error of Side I measurements is $\Delta_{\text {Sidel }}=-1.08^{\prime \prime}$ while of $\Delta_{\text {Side II }}=-0.77$ ". Averaged differences between the reference and measured angles and standard uncertainties of both faces measurements are shown in Fig. 3.19.


Fig. 3.19. Averaged differences between the reference and measured angles of both faces

Uncertainty sources are similar in both of described approaches with the reference scale however the latter has smaller uncertainty because of the movement elimination. The comparison of combined uncertainty components are shown in Fig. 3.20.


Fig. 3.20. Comparison of combined uncertainty components of calibration methods with the reference scale

Uncertainties due to repeatability and resolution of the total station have highest impact on measurement results as well as uncertainties due to the reference scale and tilt of the reference scale. Horizontal distance measurements have the most combined uncertainty components in method with the stationary reference scale. In the method with the displaced reference scale, uncertainty due to horizontal distance measurements is very small compared to another approaches. However, the expanded uncertainty is the most influenced by repeatability of TS measurements which have bigger standard deviation due to the motion of the reference scale.

### 3.2. The Experimental Setup, Data Processing and Uncertainty Evaluation of the Method Using Vertical Angle Measuring System Calibration Apparatus

Special granite surface plate stand was filled with the sand for stabilization in order to minimize vibrations which could affect measurement results. The leveled total station is mounted horizontally to the apparatus on the indexing table. The mirror and its mount is fixed to the telescope of the total station and the telescopes of both the total station and the autocollimator are aligned and coaxially pointed to each other. After the alignment, the angle measuring system, total station and the electronic autocollimator are set to the initial position. The indexing table is rotated by a desired angle with the apparatus and fixed. Then the telescope of the total station is turned backwards by the same angle. Autocollimator readings show the angle of changed mirror position which means that there was a mismatch between the angles of the indexing table and the total station. The procedure is repeated 12 times ( 6 times clockwise and 6 times counter - clockwise) in the range of $40^{\circ}$ to $140^{\circ}$ and $220^{\circ}$ to $320^{\circ}$ and the readings of total station vertical angle as well as autocollimator are taken. In this approach it is very important to fix the mirror on the telescope very precisely as well as align both devices to be on the same sight axis. If this condition is not fulfilled then measurement uncertainty will increase due to biaxial change of mirror position. To perform high quality measurements a special mount for the mirror (3.21) was designed to fit TS's telescope.


Fig. 3.21. Axonometric view of the mirror mount

This special mirror mount has four adjustment screws as well as screws for fixing the mount to the telescope of the total station. The mirror is fixed to a special recess designed for it. The adjustment screws are in the plane parallel to the mirror located in four points around the mirror in order to adjust the mirror in all positions. There are two spaces ( 1 mm each) designed to adjust the mirror position within this range. The use of such a mirror mount for the experiment is beneficial for better alignment of the devices. The alignment is begun by leveling both total station and the autocollimator. Then TS is horizontally fixed to the apparatus for the calibration of vertical angle measuring systems and two telescopes of the total station and an autocollimator are pointed against each other. The horizontal and vertical beams of the laser level ares used to determine primary coaxial position of the both telescopes.

Since both devices have reticles, the alignment is continued by matching them while looking though the eyepiece of the total station. After that the alignment is checked by rotating apparatus with the indexing table at $180^{\circ}$ and turning back the TS telescope until the cross line of the autocollimator is seen through the eyepiece. If there is a mismatch between the cross lines, TS position is adjusted by half of this mismatch with the adjustment screws of the apparatus. This procedure is repeated until this mismatch is minimized as possible. The final step of alignment is done by fixing the mirror mount to the telescope of the total station. The mirror position is adjusted by the adjustment screws of the mirror mount and observing the readings of the autocollimator while rotating the system at the pitch of $50^{\circ}$.

When the system is aligned the measurements are performed by pointing the mirror the autocollimator and taking the readings of both the total station and autocollimator. Then the apparatus with the indexing table is rotated by $10^{\circ}$ and the telescope of the total station is rotated back until the previous vertical angle reading is showed in the display of the total station. The readings of the
autocollimator are taken as a measurement result in order to determine the deviation between pervious and latest position of the mirror. Measurements are repeated six times at the pitch of $10^{\circ}$ rotating the system in clock-wise and counter-clockwise directions.

Data processing and uncertainty evaluation is the essential part to complete the experiment and analyze the results. At first experimental standard deviation and standard uncertainties are determined for every data set. In this experiment the number of independent observations in a data set is $n=6$. The best way to evaluate the uncertainty when all individual data sets have their own uncertainties is to determine pooled standard uncertainty as showed in (2.22).

There were $N=11$ data sets during the experiment. The uncertainty due to repeatability of the autocollimator $\left(u\left(\theta_{A}\left(c_{e v}\right)\right)\right.$ can be evaluated as a ratio of pooled uncertainty $\left(s_{p}\right)$ and square root of the number of observations in a data set ( $n=6$ ) using type A uncertainty evaluation method as showed in (2.23).

In this experiment an operator is reading autocollimator readings while rotating the TS‘s telescope according to vertical angle readings of the total station. These devices have different and finite resolution, therefore standard uncertainties due to the limited display resolution of the total station $\left(U\left(\theta_{\text {TSres }}\right)\right)$ and due to the limited display resolution of the autocollimator $\left(U\left(\theta_{A C r e s}\right)\right)$ which have rectangular distributions have to be evaluated separately. The display resolution of angle reading of total station Leica TC2003 is $R_{T S}=0.1$ " and the display resolution of the autocollimator Moller-Wedel Elcomat is $R_{A C}=0.05^{\prime \prime}$.

The final equation for the combined uncertainty due to correction value is expressed as follows:

$$
\begin{align*}
& u_{c}^{2}(B)=c_{I \text { C.al }}^{2} u^{2}\left(\theta_{I \text { ICal }}\right)+c_{\text {TSres }}^{2} u^{2}\left(\theta_{\text {TSres }}\right)+c_{A \text { Ccal }}^{2} u^{2}\left(\theta_{A C \text { cal }}\right)+  \tag{3.12}\\
& +c_{A C \text { rep }}^{2} u^{2}\left(\theta_{A C \text { rep }}\right)+c_{A C \text { res }}^{2} u^{2}\left(\theta_{A C \text { Cres }}\right)
\end{align*}
$$

The uncertainty budget influencing the measurement accuracy is shown in Table 3.5.

Table 3.5. Uncertainty budget of the method using vertical angle measuring system calibration apparatus

| Source of uncertainty | Standard uncertainty $u\left(x_{i}\right)$ | Sensitivity coefficient $c_{i}$ | Uncertainty contribution $u \cdot\left\|c_{i}\right\|$ | Probability distribution | Degrees of freedom $v$ | Effective <br> degrees <br> of <br> freedom <br> $v_{\text {eff }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Uncertainty due to the Indexing table $\mathrm{u}\left(\theta_{\mathrm{I} . \mathrm{CaI}}\right)$ | 0.050" | 1 | 0.05" | Normal | $\infty$ |  |
| Uncertainty due to the limited display resolution of the total station $\mathrm{u}\left(\theta_{\text {TSres }}\right)$ | 0.029" | 1 | 0.029" | Rectangular | $\infty$ |  |
| Uncertainty due to autocollimator $\mathrm{u}\left(\theta_{\mathrm{ACcal}}\right)$ | 0.2" | 1 | 0.2" | Normal | $\infty$ |  |
| Uncertainty due to repeatability of the autocollimator $\mathrm{u}\left(\theta_{\text {ACrep }}\right)$ | 0.049" | 1 | 0.049" | t-(Student‘s) | 5 |  |
| Uncertainty due to limited display resolution of the autocollimator $\mathrm{u}\left(\theta_{\mathrm{ACres}}\right)$ | 0.014" | 1 | 0.014" | Rectangular | $\infty$ |  |
| Uncertainty of the correction value $u_{c}(B)$ | 0.262" |  |  | Normal |  | 6 |

The expanded uncertainty of this setup is $U_{95 \%}=0.52^{\prime \prime}(k=2)$.
The measurement results of 6 data sets are shown in Fig. 3.22.


Fig. 3.22. Angle measurement errors of vertical angle encoder

As it is shown in Fig. 3.22 there is a slight difference between errors of the measurements performed in the range of $40^{\circ} \pm 140^{\circ}$ (Side I) and $220^{\circ} \pm 320^{\circ}$ (Side II) of the total station vertical encoder. The average deviation from the mean of Side I is $0.22^{\prime \prime}$ while of Side II is $0.33^{\prime \prime}$. Although there is a visual symmetry in the distribution of standard deviation values, the bigger error is noticeable in Side II measurements. Such a difference can be a result caused by an influence of vibration or the slight tilt of the mirror mounted on the telescope of the total station. The possibility that this error of vertical angle measuring system of the total station can be caused due to the shift while the device was in a horizontal position cannot be absolutely ignored, however it needs further investigations. Fig. 3.22 also shows the range in which angle calibration of vertical angle measuring system of the total station can be done. Comparing to other previously analyzed vertical angle calibration methods, this enlarges the measurement range up to $90^{\circ} \pm 50^{\circ}$ and $270^{\circ} \pm 50^{\circ}$ and it is one of the main advantages of this setup (Suh, Šiaudinyte 2014).


Fig. 3.23. Combined uncertainty components of the method with the apparatus

Combined uncertainty components of the method with the apparatus are shown in Fig. 3.23. The uncertainty due to electronic autocollimator has the greatest impact on the accuracy of measurement results. The standard deviations of the approach with the electronic autocollimator are shown in Fig. 3.24.


Fig. 3.24. Standard deviations of six measurements of the vertical angle measuring system at $10^{\circ}$ pitch

The comparison of the measurement results of approach using autocollimator Nikon GB No. 78155 as a reference point and approach using the mirror and electronic autocollimator Moller - Wedel Elkomat is shown in Fig. 3.25. The average standard deviation of Side I was $1.8^{\prime \prime}$ and Side II $1.6^{\prime \prime}$ of the total station encoder. Such a big difference between the results of two approaches was caused by a human interaction, vibration influence as well as uncertainty due to autocollimator Nikon ( $u_{\text {Nikon }}=0.5^{\prime \prime}$ ), uncertainty due to pointing which is influenced by different widths of the cross lines of both telescopes.

After evaluation of all shortcomings, stability of the granite surface was improved and second approach with the mirror and the electronic autocollimator was performed.


Fig. 3.25. Standard deviation of both approaches using vertical angle measuring system calibration apparatus

Angle measurement deviations from the average and measurement standard uncertainties of the method using apparatus are shown in Fig. 3.26.


Fig. 3.26. Deviations from the average readings of autocollimator

It is very important to perform further research in order to investigate the behavior and measurement accuracy of vertical angle measuring system while the total station is in horizontal position. This could be one of the main
uncertainty sources as well as limited display resolution and the repeatability of the devices.

The setup with the mirror and electronic autocollimator is time saving and uncertainty reducing because readings are read directly from the electronic output of the devices. Although the vibration is reduced, the procedure of the mirror mounting on the telescope and alignment of the devices might be time consuming.

### 3.3. Analysis and Comparison of Experimental Results of Vertical Angle Measuring System Calibration Methods

The experimental evaluation was performed applying two different methods for vertical angle measuring system calibration. The method with the reference scale is based on pointing the telescope to the target by matching the cross line of the reticle with the line of the reference scale. Another method was based on mirror surface measurements while taking the readings of the electronic autocollimator. Surface measurements are considered to be more accurate and more often are used in metrology laboratories. However, such methods are more expensive and require specific laboratory conditions (i.e. autocollimator should be protected from direct air flow). The advantages and drawbacks of both proposed methods are summed up in the table 3.6.

Table 3.6. Advantages and drawbacks of both proposed methods

| Parameters | Angle measurements with the <br> apparatus | Angle measurements with <br> the reference scale |
| :--- | :--- | :--- |
| Calibration <br> range | $90^{\circ} \pm 50^{\circ}$ | $90^{\circ} \pm 17^{\circ}$ |
| Expanded <br> uncertainty, <br> U | $\mathrm{U}_{95 \%}=0.52$ " $(\mathrm{k}=2)$ | Stationary: $\mathrm{U}_{95 \%}=0.24 "(\mathrm{k}=2.447)$ <br> Displaced: $\mathrm{U}_{95 \%}=0.59^{\prime \prime}(\mathrm{k}=2)$ |
| Advantages | - Smaller standard deviation <br> - Easier to operate | - Very small angles can be measured <br> depending on the scale grating <br> Cheaper than method with the <br> apparatus |
| Shortcommings | - Expensive (indexing table, <br> electronic AC, apparatus, <br> mirror mount) <br> - Time consuming alignment | - Small measurement range <br> - Unavoidable uncertainty due to the <br> tilt of the scale |

The experiment of this research showed that method with the reference scale has more uncertainty components however their magnitudes were smaller compared to the calibration method with the apparatus. The experimental results leads to the conclusion that these two different methods for vertical angle measuring system calibration can equally compete. Like any other measuring smethods, proposed ones also have their advantages and shortcomings. The measurement procedure using calibration apparatus is time saving and convenient because the readings are taken from the electronic autocollimator. However, the alignment and mirror mounting can be time consuming. One of the biggest advantages of this method is the calibration range. While in the other method it is possible to reach the calibration range on $90^{\circ} \pm 17^{\circ}$ (depending on the horizontal distance), in this method the range is stable $90^{\circ} \pm 50^{\circ}$. Another difference of these two methods is the measurement pitch. In the method with the apparatus the smallest measurement pitch depends on the resolution of the indexing table. If Moore's Special Index is used, the minimal pitch is 15 '. In the method where the reference scale is used very specific angles can be calibrated depending on the grating of the reference scale - this is a big advantage of this method. The latter method is also less time consuming, however the possible tilt of the reference scale is very difficult to control.

### 3.4. Conclusions of Chapter 3

1. It was determined that expanded uncertainty for the method using displaced reference scale is $U_{95 \%}=0.59$ " and using stationary reference scale $U_{95 \%}=0.24$ ". This leads to a conclusion that the motion of the scale increases the uncertainty by 2.5 times.
2. It was determined that the expanded uncertainty for the method using proposed apparatus is $U_{95 \%}=0.52$ " which is smaller compared to the uncertainty determined at ESRF ( $\mathrm{U}_{95 \%}=1.40^{\prime \prime}$ ).
3. It was determined that the main source for the largest uncertainty of both proposed methods is repeatability of TS measurements.

## General Conclusions

1. A novel method for the calibration of vertical angle measuring systems is proposed in the thesis and patented in the State Patent Bureau of the Republic of Lithuania. The reference angle is determined trigonometrically according to the horizontal distance between the calibrated instrument and vertical distance between two measured lines of the reference scale.
2. Research showed that the new arrangement of modified equipment for horizontal angle measuring system calibration can be applied for the calibration of vertical angle measuring systems.
3. After development of the instrumentation for the calibration of vertical angle measuring systems, comparing with the method applied in ESRF laboratory, calibration range was expanded 2.5 times from $90^{\circ} \pm 20^{\circ}$ and $270^{\circ} \pm 20^{\circ}$ up to $90^{\circ} \pm 50^{\circ}$ and $270^{\circ} \pm 50^{\circ}$.
4. Uncertainty evaluation was performed for both proposed methods and the expanded uncertainty was determined as follows for the:
a) Method with the stationary reference scale is $U_{95 \%}=0.24$ " $(k=2.447)$, with the displaced reference scale is $\mathrm{U}_{95 \%}=0.59$ ", $(\mathrm{k}=2)$. The motion of the scale increases the uncertainty by 2.5 times.
b) Method with the apparatus is $U_{95 \%}=0.52$ " $(\mathrm{k}=2)$. Determined uncertainty is smaller compared to ESRF which is $\mathrm{U}_{95 \%}=1.4^{\prime \prime}$.
5. It was determined that biggest sources of uncertainty in the method with the reference scale are the tilt of the reference scale $u\left(\Delta h_{\text {tilt }}\right)=4.33 \cdot 10^{-4} \mathrm{~m}$ and distance as well as angle measurements performed by the total station $u\left(\theta_{\mathrm{TS}}\right)=0.099^{\prime \prime}$. In the method with the apparatus the measurement results are influenced by the uncertainty due to electronic autocollimator $u\left(\theta_{\text {ACcal }}\right)=0.200^{\prime \prime}$ and indexing table $u\left(\theta_{\text {I.Cal }}\right)=0.050 "$.

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## The List of Scientific Author's Publications on the Subject of the Dissertation

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Giniotis, V.; Šiaudinyté, L.; Bručas, D. 2012. The calibration method of vertical angle measuring systems of geodetic instruments. LT patent No. LT 5818 B, Int. Cl: G01B 5/00.

## Summary in Lithuanian

## Ivadas

## Mokslo problemos formulavimas

Preciziniai matavimai ir priemonės yra pagrindiniai veiksniai užtikrinantys kokybę daugumoje pramonės šakų - civilinės inžinerijos, geodezijos, pramoninės irangos gamybos bei laboratorinių matavimų srityse. Elektroniniai tacheometrai - dažniausiai naudojami prietaisai geodeziniams matavimams atlikti. Dėl unikalios konstrukcijos, optinės sistemos ir didelès raiškos kampų matavimo sistemų, šiais prietaisais galima atlikti atstumo, horizontaliųų ir vertikaliujų kampų matavimus vienu metu. Tacheometrus taip pat galima panaudoti laboratoriniams matavimams.

Šių instrumentų viduje įmontuota kampų matavimo sistema, kurios kalibravimas būtinas norint užtikrinti aukštą matavimų tikslumą ir nustatyti prietaiso sistemines paklaidas. Horizontaliụjų ir vertikaliujų kampų matavimo sistemų kalibravimui reikalinga specifinė irranga. Kadangi dauguma laboratorinių metodų skirti horizontaliujų kampų matavimo sistemų kalibravimui, vertikaliųjų kampų matavimo sistemos yra mažiau ištirtos ir tai yra įdomi užduotis mokslininkams.

Pagal tarptautinio standarto ISO 17123 trečiają dali, teodolitų vertikaliujų kampu matavimo sistemų kalibravimas turètų būti atliekamas lauke naudojant 4 taikinius tarp kurių matuojami kampai. Vykdant tokią kalibravimo procedūra, matuojami dideli kampai ir didžioji kampų keitiklio skalès dalis lieka neištirta. Todèl yra didelis poreikis
kurti laboratorinius vertikaliuju kampų matavimo sistemų metodus ir tobulinti irangą jiems atlikti.

## Darbo aktualumas

Tobulinant matavimo įrangą nuolat didejjantys reikalavimai matavimų tikslumo didinimui yra neatsiejama metrologijos mokslo progreso dalis. Disertacijoje siūlomi du nauji vertikaliưjų kampų matavimo sistemų kalibravimo metodai igyvendinami laboratorinemis salygomis ir didinantys geodezinių prietaisų matavimo tikslumą. Šie metodai skirti elektroninių tacheometrų vertikaliujų kampų matavimo sistemoms kalibruoti, tačiau panaudojus siūlomus kalibravimo principus, juos galima taikyti ir lazerinių matuoklių kalibravimui. Pasiūlyti būdai yra nesudėtingai igyvendinami ir nereikalauja didelių patalpų. Disertacijoje nagrinėjami paklaidų šaltiniai, darantys itaka matavimo rezultatų tikslumui.

## Tyrimų objektas

Disertacijos tyrimų objektas yra geodezinių prietaisų vertikaliu̧jų kampų matavimo sistemos tikslumas.

## Darbo tikslas

Ištirti ir tobulinti geodezinių prietaisų vertikaliuju kampų matavimo sistemų kalibravimo metodus ir priemones bei îvertinti paklaidų šaltiniư darančių itaką matavimo rezultatų tikslumui, neapibrėžtis.

## Darbo uždaviniai

Darbo tikslui pasiekti ir mokslinei problemai spręsti suformuluoti šie uždaviniai:

1. Ištirti ir parinkti kampų matavimo sistemų kalibravimo būdus ir priemones tinkamus geodezinių prietaisų vertikaliuju kampų matavimo sistemų laboratoriniam kalibravimui.
2. Pasiūlyti trigonometrinị geodezinių prietaisų vertikaliųjų kampų matavimo sistamų kalibravimo metoda.
3. Pasiūlyti naują nedidelių patalpų reikalaujantí prietaisų derinị geodezinių prietaisų vertikaliujų kampų matavimo sistemai kalibruoti.
4. Taikyti ir palyginti pasiūlytus metodus, ìvertinant ju matavimo neapibrèžties komponentes ir jų itaką matavimo rezultatų tikslumui.

## Tyrimu metodika

Analitinėje dalyje išanalizuoti pasaulio ižymmiausių metrologijos institutų naudojami kampų matavimo sistemų kalibravimo būdai ir i̇ranga. Pasiūlyti du vertikaliujų kampų matavimo sistemų kalibravimo metodai, kuriuose taikomos etaloninès priemonės aukštam matavimo tikslumui užtikrinti. Atlikus abiejų metodų eksperimentinius bandymus, matavimų rezultatai palyginti tarpusavyje tiriant matavimų neapibréžties komponentes.

## Darbo mokslinis naujumas

Darbo mokslinis naujumas pagrịstas šiais rezultatais:

1. Pasiūlytas ir LR Valstybiniame patentų biure patentuotas naujas trigonometrinis geodezinių prietaisų vertikaliųjų kampų matavimo sistemų laboratorinis kalibravimo būdas naudojant linijinę skalę.
2. Pasiūlytas naujas geodezinių prietaisų vertikaliujų kampų matavimo sistemų kalibravimo metodas, pagrįstas horizontaliųjų kampų matavimo sistemų kalibravimo principais.
3. Pasiūlytas irenginys skirtas elektroninio tacheometro padèties nustatymui (reguliavimui) vertikaliujų kampų matavimo sistemų kalibravimo metu.

## Darbo rezultatų praktinė reikšmė

Disertacijoje siūlomiems vertikaliuju kampų matavimo sistemų kalibravimo metodams taikoma daugumoje metrologijos laboratorijų naudojama etaloninė iranga. Taikant disertacijoje pasiūlytą ịrenginị elektroninio tacheometro padètis stabilizuojama ir matavimo procesas tampa patogesnis. Abu siūlomi nesudètingai igyvendinami metodai gali būti pritaikomi țvairaus dydžio kalibravimo laboratorijose.

## Ginamieji teiginiai

1. Elektroninio tacheometro vertikaliu̧ju kampų matavimo sistema galima kalibruoti laboratorinemis salygomis taikant etalonines priemones.
2. Vertikaliujų kampų matavimo sistemos kalibravimą galima atlikti taikant patobulintą horizontaliujų kampų matavimo sistemų kalibravimo įranga.
3. Kalibravimo metodų kokybės kontrolei ir tobulinimui galima taikyti statistinị matavimų neapibrěžties įvertinima.

## Darbo rezultatu aprobavimas

Disertacijos tema paskelbti 7 moksliniai straipsniai, iš jų 3 leidiniuose, referuojami Thomson Reuters ISI Web of Science duomenų bazėje, 4 - kituose tarptautiniuose mokslo žurnaluose, referuojamuose SCOPUS, Compendex duomenų bazėse. Geodezinių prietaisų vertikaliujų kampų matavimo sistemų kalibravimo būdas taikant etaloninę skalę patentuotas LR Valstybiniame patentų biure. Disertacijos tema skaityti 5 pranešimai tarptautinėse mokslinėse konferencijose:

- 8-oji tarptautinė konferencija "Environmental Engineering", 2011 m. gegužės 19-20, Vilnius, Lietuva;
- Tarptautiné konferencija "Metrologia 2011", 2011 m . rugsèjo 27-30, Natal, Brazilija;
- Pasaulinis kongresas "XX IMEKO World Congress: Metrology for Green Growth", 2012 m . rugsèjo 9-14, Busan, Korėjos Respublika.
- 9-oji tarptautine konferencija "Mechatronic Systems and Materials", 2013 m. liepos 1-3, Vilnius, Lietuva.
- Tarptautine konferencija " $133^{\text {th }}$ IMEKO TC10 Workshop on Technical Diagnostics: Advanced Measurement Tools in Technical Diagnostics for Systems' Reliability and Safety", 2014 m. birželio 26-27, Varšuva, Lenkija.


## Disertacijos struktūra

Disertacija sudaro ívadas, trys skyriai, bendrosios išvados, literatūros šaltinių sąrašas, autoriaus publikacijų disertacijos tema sarašas, santrauka lietuvių kalba. Darbo apimtis - 121 puslapis neskaitant priedu, tekste yra 59 formulės, 57 paveikslai, 7 lentelès. Rašant disertaciją panaudoti 95 literatūros šaltiniai.

## 1. Kampu matavimai - standartai, būdai ir priemonès

Pirmajame skyriuje nagrinėjamos mokslinės publikacijos ir tarptautiniai norminiai dokumentai, kuriuose aprašomi kampų bei ilgių etalonai, matavimo procedūros bei jų ypatumai, taip pat matavimų rezultatų apdorojimo bei neapibrėžties ívertinimo algoritmai. Nagrinėjant tarptautinius standartus analizuojama vertikaliúju kampų matavimo sistemų kalibravimui siūloma metodika. Gilinantis $\mathfrak{i}$ tarptautines didžiausių pasaulio metrologijos institutų palyginamasias ataskaitas nagrinėjama ilgių bei kampų matavimo proceso metodika, iranga bei specifinès salygos, galinčios daryti itaka matavimų rezultatų tikslumui. Išanalizavus geodezinių prietaisų kalibravimo procedūras pateikiamas apibendrintas GUM (Guide to the expression of uncertainty in measurement) siūlomas neapibrėžties įvertinimo algoritmas

Šiame skyriuje nagrinèjama kampų matavimo sistemos samprata, kampų matavimo sistemas savyje talpinančių įrenginių konstrukcija bei atskiri kampų matavimo sistemų elementai. Nagrinėjami kampų keitiklių, sukamuju bei indeksavimo staliukư, matavimo blokų ir daugiakampio veidrodinio poligono, kampų komparatoriư, autokolimatoriu, linijinių bei apskritiminių skalių, lazerinių interferometrų veikimo principai bei šios irangos pritaikomumas horizontaliuju ir vertikaliuju kampų matavimo sistemų kalibravimo srityje.

Disertacijoje nagrinėjami vertikaliujų ir horizontaliujų kampų matavimo sistemų kalibravimo metodai naudojami žinomų metrologijos institutų kalibravimo laboratorijose. Detaliai išnagrinėti Leica ir ESRF patentuoti elektroninių tacheometrų vertikaliưjų kampų matavimo sistemų kalibravimo būdai, igyvendinami naudojant automatizuotas etalonines matavimo sistemas.

Atlikus kampų matavimo sistemų kalibravimo metodų analizę nustatyta jog horizontaliųjų kampų matavimo sistemų kalibravimo metodų ir irangos pasiūla yra didesnė nei vertikaliųų kampų matavimo sistemų. Vertikaliujų kampų matavimo sistemų kalibravimo įranga užtikrinanti didžiausią tikslumą yra sunkiai prieinama nedidelėms kalibravimo laboratorijoms dè didelių kaštų. Geodezinių prietaisų vertikaliưjų kampų matavimo sistemų kalibravimo būdai aprašyti tarptautiniuose standartuose neleidžia detaliai ištirti kalibruojamojo prietaiso, todèl būtina ieškoti naujų sprendimų ju kalibravimui. Visa tai parodo, kad geodezinių prietaisų vertikaliujų kampų matavimo sistemų kalibravimo sritis yra aktuali ir tobulinant metodus bei irangą juos galima pritaikyti ir nedidelėse kalibravimo laboratorijose.

## 2. Siūlomi vertikaliuju kampu matavimo sistemu kalibravimo metodai

Antrajame disertacijos skyriuje detaliai nagrinėjami du pasiūlyti skirtingi geodezinių prietaisų vertikaliųjų kampų matavimo sistemų kalibravimo metodai.
Pirmasis metodas igyvendinamas naudojant invarinę skalę. Elektroninis tacheometras montuojamas tokiame aukštyje, kad jo žiūronui esant horizontalioje padètyje jo horizontalusis siūlelių tinklelio siūlelis sutaptų su vertikaliai pastatytos skalès centrine padala. Skalè tvirtinama vertikaliai nuo kalibruojamojo prietaiso atstumu, nemažesniu nei elektroninio tacheometro fokusavimo nuotolis. Skalė tvirtinama ant išilgai bėgiais judančios karietèlės specialiame stove ir kruopščiai gulščiuojama (S1 pav.). Irengus lazerinio interferometro sistemą galima išmatuoti karietėlès padèties pokytị ( $\Delta l^{\prime}$ ). Kai visi prietaisai sulygiuoti ir išgulščiuoti, kalibruojamojo prietaiso žiūronas nukreipiamas ị pasirinktą skalės padalą ir vertikalusis kampas užfiksuojamas specialiu sraigtu. Tuomet karietèle stumiama bėgiais tolyn kol žiūrono horizontalusis siūlelių tinklelio siūlelis sutapdinamas su kita skales padala ir taip nustatomas vertikalusis atstumas tarp skalès padalų ( $\Delta h^{\prime}$ ). Pamatinis kampas išreiškiamas (S1):

$$
\begin{equation*}
\varphi_{i}=\arctan \left(\frac{\Delta h^{\prime}}{\Delta l^{\prime}}\right) \tag{S1}
\end{equation*}
$$

čia $\Delta h^{\prime}$ - vertikalusis atstumas tarp skalès padalư; $\Delta l^{\prime}$ - horizontalusis atstumas tarp pradinès ir galinės skalès padèčių.


S1 pav. Vertikaliujų kampų matavimo sistemos kalibravimo principas keičiant skalės padèti
Šis metodas gali būti igyvendinamas ir nekeičiant skalès padèties. Prietaisai lygiuojami ir gulščiuojami. Taikant šit variantą kalibruojamuoju prietaisu matuojami kampai tarp krypčių í centrinę bei pasirinktąsias skalès padalas. Išmatuotieji kampai yra lyginami su trigonometriškai nustatytu pamatiniu kampu.

Pamatinio kampo nustatymui trigonometriniu metodu reikia nustatyti horizontaluji atstumą tarp kalibruojamojo prietaiso ašies iki skalės padalų (l) ir vertikaluji atstumą $\left(\Delta h_{i}\right)$ tarp skalès padalų (S2 pav.).


S2 pav. Vertikaliụjų kampų matavimo sistemos kalibravimo principas nejudinant skalès
Matavimams naudojama "H" formos skalè, todè nustatyti atstuma tarp kalibruojamojo prietaiso ašies ir skalès padalų nèra paprasta. Šis uždavinys buvo sprendžiamas ties skalès centrine padala tvirtinant prizmę. Horizontalusis atstumas matuojamas elektroniniu tacheometru. Prizmès konstanta nustatoma atskirai, o skalès gylio matavimai atliekami mikrometru.

Ivertinant neapibrezzti, labai svarbu ištirti visus matavimo metu naudotus prietaisus ir įvertinti jų parametrus. Sudėtine neapibrėžtis, kai keičiama skalės padètis išreikšta (S2):

$$
\begin{equation*}
u_{c}^{2}(B)=c_{h}^{2} u^{2}\left(\Delta h^{\prime}\right)+c_{l}^{2} u^{2}\left(\Delta l^{\prime}\right)+c_{\theta_{T S}}^{2} u^{2}\left(\theta_{T S}\right) \tag{S2}
\end{equation*}
$$

čia $c$ - jautrumo koeficientas, $u\left(\Delta h^{\prime}\right)$ - kombinuota standartinė neapibrėžtis dèl vertikalaus atstumo nustatymo, $u\left(\Delta l^{\prime}\right)$ - kombinuota standartinė neapibrėžtis dèl horizontalaus atstumo matavimo, $\mathrm{u}\left(\theta_{\mathrm{TS}}\right)$ - kombinuota standartiné neapibrèžtis dèl elektroniniu tacheometru atliktų matavimų.

Kombinuota standartiné neapibrezztis dè vertikalaus atstumo matavimo yra sudaryta iš penkių komponenčių ir išreiškiama ( S 3 ):

$$
\begin{equation*}
u^{2}\left(\Delta h^{\prime}\right)=c_{h}^{2}\left\{u^{2}\left(\Delta h_{S c a l}^{\prime}\right)+u^{2}\left(\Delta h_{\text {tilt }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {therm }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {comp }}^{\prime}\right)+u^{2}\left(\Delta h_{\text {point }}^{\prime}\right)\right\} \tag{S3}
\end{equation*}
$$

čia $u^{2}\left(\Delta h_{\text {Scal }}^{\prime}\right)$ - standartinė neapibrėžtis dèl skalės; $u^{2}\left(\Delta h_{\text {tilt }}^{\prime}\right)$ - standartinė neapibréžtis dèl skalės posvyrio; $u^{2}\left(\Delta h_{\text {therm }}^{\prime}\right)$ - standartinė neapibréžtis dèl terminio skalės plėtimosi; $u^{2}\left(\Delta h_{\text {comp }}^{\prime}\right)$ - standartiné neapibrėžtis dèl skalès gniuždymo; $u^{2}\left(\Delta h_{\text {point }}^{\prime}\right)$ - standartinė neapibrèžtis dèl vizavimo ị skalès padalos centra.

Skalès padèties pokyčiui matuoti buvo naudojamas lazerinis interferometras, todèl kombinuota standartinė neapibrėžtis dėl horizontalaus atstumo matavimų sudaryta iš trijų komponenčiu, nusakančių lazerinio interferometro parametrus ir išreiškiama (S4):

$$
\begin{equation*}
u^{2}\left(\Delta^{\prime}\right)=c_{l}^{2}\left\{u^{2}\left(\Delta_{\text {LIcal }}^{\prime}\right)+u^{2}\left(\Delta_{\text {LIrep }}^{\prime}\right)+u^{2}\left(\Delta_{\text {LIres }}^{\prime}\right)\right\} \tag{S4}
\end{equation*}
$$

čia $u^{2}\left(\Delta l_{\text {LIcal }}^{\prime}\right)$ - standartinė neapibrėžtis dè lazerinio interferometro; $u^{2}\left(\Delta l_{\text {LIrep }}^{\prime}\right)$ standartiné neapibrėžtis dèl lazerinio interferometro atskaitų stabilumo; $\mathrm{u}^{2}\left(\Delta \mathrm{~h}_{\text {LIres }}\right)$ standartinė neapibrėžtis dèl lazerinio interferometro ribotos rodomų atskaitų rezoliucijos. Standartinė neapibrěžtis dèl matavimo elektroniniu tacheometru išreiškiama (S5):

$$
\begin{equation*}
u^{2}\left(\theta_{T S}\right)=c_{\theta_{T S}}^{2}\left\{u^{2}\left(\theta_{T S r e s}\right)+u^{2}\left(\theta_{T S r e p}\right)\right\} \tag{S5}
\end{equation*}
$$

$u^{2}\left(\theta_{\text {TSres }}\right)$ - standartinė neapibrežtis dè elektroninio tacheometro ribotos vertikaliuju kampų atskaitų rezoliucijos; $u^{2}\left(\theta_{T S r e p}\right)$ - standartinė neapibrèžtis dè elektroninio tacheometro vertikaliujų kampų atskaitų stabilumo.

Igyvendinant ši metodą nekeičiant skalès padèties, neapibrėžties komponentés išlieka panašios, tačiau neapibrėžtị dèl horizontalaus atstumo matavimo interferometru, keičia neapibrėžtys dè elektroniniu tacheometru bei mikrometru matuojamo horizontalaus atstumo, kuri išreikšta (S6):

$$
\left.u^{2}(l)=c_{l}^{2}\left\{\begin{array}{l}
u^{2}\left(l_{\text {TSres } 1}\right)+u^{2}\left(l_{\text {TSrep } 1}\right)+u^{2}\left(l_{\text {TSres } 2}\right)+u^{2}\left(l_{\text {TSrep } 2}\right)+u^{2}\left(l_{\text {TSres } 3}\right)+  \tag{S6}\\
+u^{2}\left(l_{\text {TSrep } 3}\right)+u^{2}\left(l_{\mu \text { mcal }}\right)+u^{2}\left(l_{\text {}}^{\text {umrep }}\right.
\end{array}\right)+u^{2}\left(l_{\mu \text { mres }}\right), ~\right\}
$$

čia $u\left(l_{\text {Tsresi }}\right), u\left(l_{\text {Tsres } 2}\right), u\left(l_{\text {Tsres } 3}\right)-$ standartinės neapibrėžtys dèl ribotos elektroninio tacheometro rodomų ilgių atskaitų rezoliucijos matuojant horizontaluji atstuma; $u\left(l_{\text {Tsrep } 1}\right)$ - standartiné neapibrėžtis dèl elektroninio tacheometro ilgių atskaitų stabilumo matuojant atstumą tarp prietaiso ir prizmės; $u\left(l_{\text {Tsrep2 }}\right)$ - standartiné neapibrėžtis dèl elektroninio tacheometro ilgių atskaitų stabilumo matuojant atstuma tarp prietaiso ir prizmés nustatant prizmés konstanta; $u\left(l_{\text {Tsrep } 3}\right)$ - standartinė neapibréžtis dèl elektroninio tacheometro ilgių atskaitų stabilumo matuojant atstumą tarp prietaiso ir veidrodèlio nustatant prizmės konstanta; $u\left(l_{\mu m c a l}\right)$ - standartiné neapibrėžtis dèl gylio mikrometro; $u\left(1_{\mu m r e p}\right)$ - standartiné neapibrèžtis dè gylio mikrometro atskaitu stabilumo; $u\left(l_{\mu m r e s}\right)$ standartinė neapibrėžtis dèl ribotos gylio mikrometro rodomų atskaitur rezoliucijos.

Antrasis šiame disertacijos skyriuje siūlomas vertikaliujų kampų matavimo sistemų kalibravimo metodas pagrịstas horizontaliuju kampų matavimo sistemų kalibravimo principais. Šis metodas igyvendinamas naudojant specialų ǐrengini, ì kurị tvirtinamas kalibruojamasis prietaisas. Ant stabilaus antivibracinio pagrindo montuojama kalibravimo sistema, kurią sudaro „Moore‘s 1440 Special Index" indeksavimo staliukas su ant jo tvirtinamu irenginiu kalibruojamajam prietaisui montuoti bei elektroninis autokolimatorius (S3 pav.). Elektroninis tacheometras gulščiuojamas ir montuojamas horizontaliai $\mathfrak{i}$ specialų írengini, pritvirtinta prie indeksavimo staliuko. Ant kalibruojamojo prietaiso žiūrono tvirtinamas veidrodèlis, ì kurị nukreipiamas elektroninis autokolimatorius. Prietaisai kruopščiai lygiuojami naudojant irenginyje esančius sraigtus. Sulygiavus prietaisus nustatoma pradinė padėtis ir pasirenkamas vertikaliujų kampų matavimo sistemos kalibravimo žingsnis. Pamatinis kampas sukuriamas sukant indeksavimo staliuką kartu su irenginiu, kuriame itvirtintas kalibruojamasis prietaisas. Tuomet elektroninio tacheometro žiūronas grązinamas í pradinę padètí sukant tokiu pačiu kampu pagal atskaitas jo ekranèlyje. Tai atlikus fiksuojamos elektroninio autokolimatoriaus atskaitos, parodančios veidrodèlio, pritvirtinto prie elektroninio tacheometro žiūrono padèties pokytị. Procedūra kartojama
kol ištiriamas visas apskritimas. Tokiu principu elektroninio tacheometro vertikalios kampų matavimo sistemos rodmenys lyginami su indeksavimo staliuko rodmenimis.


S3 pav. Vertikaliujų kampų matavimo sistemų kalibravimo ̨̣renginys
Igyvendinant šit metoda, nustatytos trys pagrindinès neapibrezžies komponentès elektroninis tacheometras, autokolimatorius bei indeksavimo staliukas.
Sudètinė šio metodo neapibrėžtis išreikšta (S7) formule:

$$
\begin{equation*}
u_{c}^{2}(B)=c_{I, C .}^{2} u^{2}\left(\theta_{I . C}\right)+c_{T S}^{2} u^{2}\left(\theta_{T S}\right)+c_{A C}^{2} u^{2}\left(\theta_{A C}\right) \tag{S7}
\end{equation*}
$$

čia $u^{2}\left(\theta_{\text {I.C }}\right)$ - standartinė indeksavimo staliuko neapibrėžtis; $u^{2}\left(\theta_{\mathrm{TS}}\right)$ - standartinė elektroninio tacheometro neapibrěžtis; $u^{2}\left(\theta_{\mathrm{AC}}\right)$ - standartinè autokolimatoriaus neapibrèžtis; $\left(\mathrm{c}_{\mathrm{i}}{ }^{2}\right)$ - jautrumo koeficientai. Standartinès elektroninio tacheometro bei autokolimatoriaus neapibrėžtys turi savo komponentes todèl galutine sudėtine šio metodo neapibrezžtis išreiškiama (S8)

$$
\begin{align*}
& u_{c}^{2}(B)=u^{2}\left(\theta_{\text {ICal }}\right)+u^{2}\left(\theta_{\text {TSrep }}\right)+u^{2}\left(\theta_{\text {TSres }}\right)+u^{2}\left(\theta_{\text {TSshift }}\right)+  \tag{S8}\\
& +u^{2}\left(\theta_{\text {ACcal }}\right)+u^{2}\left(\theta_{\text {ACrep }}\right)+u^{2}\left(\theta_{\text {ACres }}\right)
\end{align*}
$$

čia $u^{2}\left(\theta_{\text {I.Cal }}\right)$ - standartinė indeksavimo staliuko neapibrěžtis; $u^{2}\left(\theta_{\text {TSrep }}\right)$ - standartinė neapibrėžtis dèl elektroninio tacheometro vertikaliujų kampų atskaitų stabilumo lygi 0 , nes kiekvieną kartą prietaisas buvo sukamas tiksliai ì prieš tai buvusią padèti.; $u^{2}\left(\theta_{\text {TSres }}\right)$ - standartiné neapibrėžtis dè elektroninio tacheometro ribotos vertikaliuju kampú atskaitų rezoliucijos; $u^{2}\left(\theta_{\text {TSshift }}\right)$ - standartiné neapibrėžtis dèl elektroninio tacheometro galimo matavimo sistemos poslinkio dèl horizontalios padèties; $u^{2}\left(\theta_{\text {ACcal }}\right)$ - standartinė autokolimatoriaus neapibrèžtis; $u^{2}\left(\theta_{\text {ACres }}\right) \quad-\quad$ standartinė neapibrėžtis dè autokolimatoriaus ribotos vertikaliuju kampu atskaitų rezoliucijos; $u^{2}\left(\theta_{\text {ACrep }}\right)$ standartinė neapibrěžtis dèl autokolimatoriaus atskaitų stabilumo.

Disertacijoje nagrinėjamas ir kitas šio metodo igyvendinimo variantas naudojant autokolimatoriú ir nemontuojant veidrodèlio ant elektroninio tacheometro žiūrono. Tokiu atveju, operatorius pradine padèti nustato pats, sutapdindamas kalibruojamojo prietaiso ir autokolimatoriaus siūlelius ir atskaitydamas autokolimatoriaus atskaita. Grązinant elektroninio tacheometro žiūroną ì pradinę padėtị prietaisų siūlelių tinklelių siūleliai vèl sutapdinami.

Abiejų siūlomų metodų neapibrezžtys ịvertintos naudojant A ir B tipo neapibrėžčių ivertinimo metodus. Siūlomų metodu patikimumą pabrėžia jų igyvendinimui naudojamos tarptautiniuose norminiuose dokumentuose įvardintos etaloninės priemonės.

## 3. Pasiūlytų metodų eksperimentinis ivertinimas

Teoriniai modeliai suteikia žinių apie pagrindinius siūlomų metodų igyvendinimo principus, paklaidų šaltinius bei neapibrėžties įvertinimo galimybes. Eksperimentinis ivertinimas yra būtinas siekiant atskleisti siūlomų metodų ypatumus, kuriuos sunku ìvertinti atliekant teorinius tyrimus.

Trečiajame disertacijos skyriuje detaliai analizuojami ir aprašomi siūlomų vertikaliưjų kampų matavimo sistemų kalibravimo metodai, pateikiami matavimų rezultatai bei ịvertintos neapibrėžtys.

Siūlomų metodų praktinė realizacija igyvendinta Pietų Korèjos nacionaliniame metrologijos instituto (KRISS - Korea Research Institute of Standards and Science) akredituotoje ilgių bei kampų metrologijos skyriaus laboratorijoje. Taikant elektroninio tacheometro vertikaliuju kampu matavimo sistemu kalibravimo būda, kuriame naudojama linijinė skalė, ypatingas dèmesys skiriamas prietaisų lygiavimui ir gulščiavimui. Skalè tvirtinama specialiame stove ant karietèlès nemažesniu nei $1,6 \mathrm{~m}$ atstumu nuo elektroninio tacheometro ir gulščiuojama ( S 4 pav.). Ekperimentiniam šio siūlomo būdo ìvertinimui naudotas elektroninis tacheometras Leica TC 2003, kurio fokusavimo nuotolis $1,6 \mathrm{~m}$, invarinè 1 m skalè Gaertner Scientific No. 244 A.U., sudalinta 1 mm padalomis, lazerinis interferometras HP Laser System 5519A. Atliekant matavimus kai skalès padėtis nebuvo keičiama, naudota prie skalès pritvirtinta prizmė bei gylio mikrometras horizontaliajam atstumui tarp kalibruojamojo prietaiso ir skalės išmatuoti.


S4 pav. Prietaisų išdestymas laboratorijoje

Minėtuju metodų variantų neapibrėžčių komponentės skiriasi dėl naudojamos irangos, tačiau neapibrěžtis, kylanti dèl invarinės 1 m skalès naudojimo išliko vienoda.

Nustatyta neapibrěžtis dèl terminio skalès plėtimosi, gniuždymo (skalė buvo statoma vertikaliai). Skale buvo kalibruojama ir nustatytos patikimiausios atstumu tarp skalės padalų vertès. Taip pat nustatyta, jog kalibruojamojo prietaiso siūlelių tinklelio viduriniojo siūlelio plotis ( $W_{T S}$ ) skiriasi nuo invarinės skalės padalos pločio ( $W_{S}$ ) (S5 pav.). Tai yra svarbi neapibrėžties komponenté matuojant vertikaluji atstuma ( $\Delta h$ ). Kintant atstumui tarp elektroninio tacheometro ir skalès, keičiasi tikimybé operatoriui pataikyti ì padalos centrą. Sudėtinès neapibrėžties įvertinimui naudota didžiausia neapibrėžtis, kai dèl atstumo tarp ̨̨renginių skalės padalos bei siūlelio pločiai sutampa.


S5 pav. Bendrasis ir padidintas skalės padalos bei tacheometro siūlelių tinklelio vaizdai
S6 paveiksle pavaizduoti skirtumai tarp pamatinių bei išmatuotų kampų taikant vertikaliưjų kampų matavimo sistemų kalibravimo būdą kai skalès padėtis nekeičiama. S6 paveikslas atspindi dviejų elektroninio tacheometro vertikaliosios apskritiminės skalės pusių bei atskaitų rodmenis. Nustatyta, kad vertikaliosios apskritiminės skalės pirmosios pusès paklaidos yra $\Delta_{\text {Sidel }}=-1,08^{\prime \prime}$, o antrosios pusès $\Delta_{\text {Side II }}=-0,77^{\prime \prime}$.


S6 pav. Skirtumai tarp pamatinių bei išmatuotų kampų taikant metoda, kuriame naudojama skalè

Neapibrėžtys $\mathfrak{\text { ǐvertintos laikantis tarptautiniame standarte ISO/IEC Guide 98-3 }}$ (GUM - Guide to the expression of uncertainty in measurement) nurodytos procedūros naudojant A ir B tipo neapibrėžčių įvertinimo metodus. Kai efektyvių laisvės laipsnių skaičius $v \geq 10$, leidžiama Stjudento skirstinị aproksimuoti kaip normalujj skirstinị su aprépties koeficientu $\mathrm{k}=2$. Metode su keičiama skalės padėtimi skaičiuojami 8 skirtingi jautrumo koeficientai kiekvienai skalès padečiai.

S7 paveiksle pavaizduoti dviejų vertikaliosios skalès padečių skirtumų tarp pamatinio ir išmatuoto kampų vidurkiai skalès atžvilgiu bei matavimo standartinės neapibrėžtys.


S7 pav. Skirtumų tarp pamatinių bei išmatuotų kampų prie dviejų žiūrono padèčių vidurkiai

Iš S7 paveikslo matyti, jog didesni nukrypimai nuo pamatinio kampo nustatyti apatinėje skalès dalyje tarp 0 mm ir 500 mm padalư, tačiau didesnės neapibrėžtys vyrauja viršutinėje skalės dalyje tarp 500 mm ir 1000 mm padalų. Tokiems rezultatams itaką galèjo daryti skalès padètis. Taip pat skalès gamybos netobulumai (padalų įėžų nevienodumas) bei konkrečios padalos apšvietimas matavimų proceso metu galėjo daryti itaką matavimo rezultatų tikslumui.

Abiejų šių metodų variantų neapibrěžties komponentès panašios, tačiau būdo, kuriame skalès padėtis nekeičiama, neapibrėžtis mažesnė, nes skalès padėtis yra stabili ir taip išvengiama galimo skalès pokrypio.

Vertikaliưjų kampų matavimo sistemų kalibravimo būdų, kai keičiama skalės padėtis ir kai skalės padėtis išlieka stacionari, neapibrėžčių komponenčių palyginimas pateiktas S8 paveiksle.


S8 pav. Neapibrěžčiu komponenčių palyginimas, taikant metodus, kuriuose naudojama skalè

Neapibrezžtys dè elektroninio tacheometro atskaitų stabilumo, ribotos atskaitų rezoliucijos bei invarines skales daro didžiausią ịtaką matavimo tikslumui. Iš S8 paveikslo matyti, kad horizontalaus atstumo matavimo neapibrežtis keičiant skalès padėtí yra labai nedidelè lyginant su kitu variantu. Išpléstinės neapibrěžties didžiausia komponenté yra elektroniniu tacheoemetru atliekamų matavimų atskaitų stabilumo neapibrèžtis, kuri išauga dèl invarinės skalès padèties pokyčio.

Taikant kitą siūlomą vertikaliujų kampų matavimo sistemos kalibravimo metoda, naudojamas „Moore's Special Index" indeksavimo staliukas ant kurio montuojamas tacheometro itvirtinimo irenginys, autokolimatorius bei ant tacheometro žiürono pritvirtintas veidrodèlis.

S9 paveiksle pavaizduoti elektroniniu autokolimatoriumi išmatuoti vertikaliuju kampu matavimo sistemos nuokrypiai nuo pamatinio kampo. Analizuojant grafiką pastebima, kad didesnès paklaidos išryškèja antrojoje elektroninio tacheometro keitiklio pusèje $\left(220^{\circ} \pm 320^{\circ}\right)$. Nustatyta, jog pirmosios keitiklio pusès ( $40^{\circ} \pm 140^{\circ}$ ) vidutinis standartinis nuokrypis yra $\sigma=0,22^{\prime \prime}$, o antrosios $\sigma=0,33^{\prime \prime}$. Šios paklaidos galèjo atsirasti dè nedidelio veidrodèlio laikiklio, pritvirtinto prie kalibruojamojo prietaiso žiūrono, pokrypio. Šiems matavimų rezultatams ittaka galėjo daryti horizontali elektroninio tacheometro padétis. Lyginant su anksčiau analizuotais vertikaliuju kampų matavimo sistemų metodais, šis būdas išsiskiria kalibravimo diapazonu, kuris yra $90^{\circ} \pm 50^{\circ}$ ir $270^{\circ} \pm 50^{\circ}$ ir tai yra didžiausias šio metodo privalumas.


S9 pav. Vertikaliujų kampų keitiklio paklaidos


S10 pav. Autokolimatoriaus atskaitų nuokrypiai nuo vidurkių

Matavimų rezultatu, gautų taikant vertikaliujų kampų kalibravimo metoda, kuriame naudojamas pasiūlytas įrenginys, nuokrypiai nuo vidurkio ir standartinès neapibrėžtys pateikti S10 paveiksle. Didžiausia neapibréžtis pastebima ties $240^{\circ}$ vertikaliưjų kampų atskaita.

Atlikus abiejų pasiūlytų metodų eksperimentinius tyrimus nustayta, kad matavimų tikslumui didžiausią įtaką daro elektroninio tacheometro vertikaliuju kampų matavimo sistemos atskaitų stabilumas. Metode, kuriame naudojamas elektroninio tacheometro itvirtinimo įrenginys, matavimo tikslumas priklauso nuo autokolimatoriaus atskaituc stabilumo bei skiriamosios gebos.

Abu siūlomi metodai yra nesudetingai igyvendinami, ju realizavimui nereikalingos didelès patalpos, o pamatiniai kampai kuriami naudojant etalonine ịranga. Taip pat taikant siūlomų metodų principus gali būti kalibruojami ne tik elektroniniai tacheometrai bet ir lazeriniai matuokliai.

## Bendrosios išvados

1. Sukurtas ir LR Valstybiniame patentų biure patentuotas naujas laboratorinis geodezinių prietaisų vertikaliưjų kampų matavimo sistemų kalibravimo būdas, kuriame pamatinis kampas nustatomas trigonometriškai pagal horizontaluji atstumą tarp prietaiso ir skalès bei vertikaluji atstumą tarp skalės padalų tarp kurių matuojamas kampas.
2. Pritaikius horizontaliưjų kampų matavimo sistemų kalibravimo principus, pasiūlytas naujas irenginys, skirtas vertikaliuju kampų matavimo sistemų kalibravimui.
3. Taikant horizontaliujų kampų matavimo sistemų kalibravimo priemones, vertikaliujų kampų matavimo sistemų kalibravimo amplitudé, lyginant su ESRF laboratorijoje taikomais būdais, išplėsta 2,5 karto nuo $90^{\circ} \pm 20^{\circ}$ bei $270^{\circ} \pm 20^{\circ}$ iki $90^{\circ} \pm 50^{\circ}$ bei $270^{\circ} \pm 50^{\circ}$.
4. Ivertinus vertikaliưjų kampų matavimo sistemų kalibravimo metodus statistiniais matavimų neapibrėžties metodais nustatyta, kad:
a) būdo, kuriame naudojama skalès padėtis nekeičiama, išplėstinė neapibrėžtis yra $\mathrm{U}_{95 \%}(\mathrm{~B})=0,24 "(\mathrm{k}=2,447)$, o būdo, kuriame skalės padétis keičiama $U_{95 \%}(B)=0,59^{\prime \prime}, \quad(k=2)$. Keičiant skalès padėti išplėstinė matavimų neapibrèžtis padidèjo 2,5 karto.
b) būdo, kuriame naudojamas specialus ̧̣renginys, išplėstinė neapibrėžtis yra $\mathrm{U}_{95 \%}=0,52^{\prime \prime} \quad(\mathrm{k}=2)$. Tai yra 2,7 karto mažesné išplėstine matavimų neapibrėžtis lyginant su ESRF laboratorijoje nustatytaja, kuri yra $\mathrm{U}_{95 \%}=1,4^{\prime \prime}(\mathrm{k}=2)$.
5. Nustatyta, kad didžiausią ittaką metodo, kuriame naudojama skalė, matavimo tikslumui daro elektroninio tacheometro parametrai $u\left(\theta_{\mathrm{TS}}\right)=0,099{ }^{\prime \prime}$ bei vertikaliosios skalès posvyris $\mathrm{u}\left(\Delta \mathrm{h}_{\text {tilt }}\right)=4,33 \cdot 10^{-4} \mathrm{~m}$. Metodo, kuriame naudojamas spec. írenginys, didžiausios sudėtinės neapibrěžties komponentès yra indeksavimo staliuko $u\left(\theta_{\text {I.Cal }}\right)=0,050$ " bei elektroninio autokolimatoriaus $u\left(\theta_{\text {ACcal }}\right)=0,200 "$ standartinės neapibrėžtys.

## Annexes ${ }^{1}$

Annex A. The Co-authors Agreements to Present Publications for the Dissertation Defence
Annex B. Copies of Scientific Publications by the Author on the Subject of the Dissertation

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Lauryna Šiaudinytè
RESEARCH AND DEVELOPMENT OF METHODS AND INSTRUMENTATION FOR THE CALIBRATION OF VERTICAL ANGLE MEASURING SYSTEMS OF GEODETIC INSTRUMENTS
Doctoral Dissertation
Technological Sciences,
Measurement Engineering (10T)
Lauryna Šiaudinyté
GEODEZINIU PRIETAISUU VERTIKALIUUJUU KAMPU MATAVIMO SISTEMŲ
KALIBRAVIMO METODŨ BEI IRENGINIŲ TYRIMAS IR TOBULINIMAS
Daktaro disertacija
Technologijos mokslai,
Matavimu inžinerija (10T)
```

201410 10. 11,25 sp. I. Tiražas 20 egz.
Vilniaus Gedimino technikos universiteto
leidykla „Technika",
Saulètekio al. 11, 10223 Vilnius,
http://leidykla.vgtu.lt
Spausdino UAB „Baltijos kopija"
Kareiviu g. 13B, 09109 Vilnius


[^0]:    ${ }^{1}$ The annexes are supplied in the enclosed compact disc

