Finite Element Analysis of Tie Beams under the Effect of Differential Settlement of Isolated Footings

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Abstract

The main objective of this paper is numerical analysis of the tie beams of isolated footings under the effect of differential settlement. 3-D simulation of the tie beams connected to isolated footings system by the finite element method was conducted. The numerical results were compared with the results of mathematical simulation of the tie beams given by previous research to determine the coefficient of semi rigid connection. The numerical result illustrate that the bending moments carried by the tie beam increase as the isolated footing width increased. Charts presented in the current paper show the effect of each parameter in the straining actions of tie beams that can be simply used by design engineers.

Keywords: Tie Beam; Differential Settlement; Isolated Footings; Soil Reaction; Finite Element.

1. Introduction

Tie beams are usually connecting isolated and strip footings at the foundation level to improve their settlement resistance and increase the foundation system stiffness. It used to carry the ground floor partitions, decreasing the differential settlement between footings and increase the foundation system stiffness. However tie beams are widely used around the world, there is no closed form for analysis or design of tie beams. But there are a common practice criteria and some codes and standards suggestion [1-5].

Indian Standards IS 1893 (2002) state that isolated footings or pile caps shall be interconnected with ties in some specific seismic zones for soil types other than rocks. These ties shall be able to carry an axial force equal to a seismic factor Ah/4 times the larger of the column or pile cap load [6]. Massachusetts state building code and the Egyptian code require that all ties shall be capable of resisting, in tension or compression, a force equal to 10% of the larger total column load [7, 8].

International Building Code 2009 state that the force to be resisted by the ties equals to the lesser of the product of the larger footing design gravity load times a seismic coefficient, SDS, divided by 10 and 25 percent of the smaller footing design gravity load [9].

Al-Omari et al. (2008) investigated numerically by using ANSYS software ,the effect of tie beams on settlement, moments and shear of footings. The results indicated that the tie beams reduce the total and differential settlements of footings but this restriction is often on the expense of increasing the shear and moment particularly in the central footing. However, the settlement reduction may be considered invaluable in view of avoiding the excessive stresses in beams and slabs of the superstructure [10].

Almasri et al. (2012), using finite element analysis of three dimensional structural models, presented that how the structural resistance to settlement can be improved by tie beams between footings. Also, seismic analysis is performed to investigate the tie beams behavior under dynamic loading in improving structural performance of foundation system. Results indicate that tie beams can reduce differential settlement greatly under both static and dynamic conditions [11].

M. A. El Sideek (2014) used finite element method to investigate the effect of tie beams in improving foundation resistance to settlement under static loading. Major of international codes still lacks adequate provisions on design of the tie beams and there is a shortage in research in this field. So; these need to be developed [12].

The major aim of the present paper is the concentration in the tie beams design as a structural element under the differential settlement effect directly taking all the parameters into consideration.

2. Structural Model

The tie beam can be represented as prismatic element connecting points A and B as shown in Figure 1. These connections at A and b are semi rigid connections that van be represented by rotational spring, on which the applied moment is equal to the product of its stiffness and the angle of rotation as stated by Wang 2014 [13].
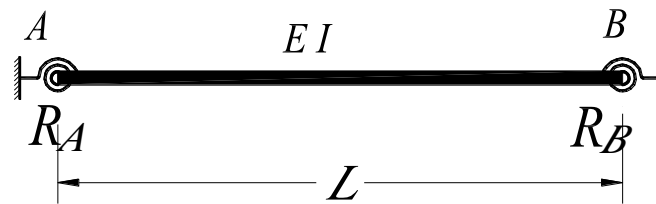

Figure 1. Structural model

$$M_A = R_A \frac{EI}{L} \varphi_A \qquad M_B = R_B \frac{EI}{L} \varphi_B \tag{1}$$

Where, R_A and R_B are non-dimensional quantities which are measure of degree of rigidity.

The relation between acting end moments M_A and M_B on the semi rigid connections and the fixed end moments M_A^F and M_B^F can be obtained as follows:

$$\begin{pmatrix} M_A \\ M_B \end{pmatrix} = \begin{pmatrix} s_{ii} & s_{ij} \\ s_{ji} & s_{jj} \end{pmatrix} \begin{pmatrix} +\frac{1}{3} M_A^F - \frac{1}{6} M_B^F \\ -\frac{1}{6} M_A^F + \frac{1}{3} M_B^F \end{pmatrix} \tag{2}$$

Where

$$s_{ii} = \frac{4 R_A R_B + 12 R_B}{R_A R_B + 4(R_A + R_B) + 12} , s_{jj} = \frac{4 R_A R_B + 12 R_A}{R_A R_B + 4(R_A + R_B) + 12} \quad and \; s_{ij} = s_{ji}$$

$$= \frac{2 R_A R_B}{R_A R_B + 4(R_A + R_B) + 12} \tag{3}$$

$$M_A^F = M_B^F = \frac{6EI}{L^2} \Delta \tag{4}$$

Thus,

$$M_A = \frac{R_A R_B + 2 R_B}{R_A R_B + 4(R_A + R_B) + 12} \left(\frac{6EI}{L^2} \Delta \right) \tag{5}$$

And

$$M_B = \frac{R_A R_B + 2 R_A}{R_A R_B + 4(R_A + R_B) + 12} \left(\frac{6EI}{L^2} \Delta \right) \tag{6}$$

Where; Δ is the differential settlement between footings.

As stated by Wang; the end moments of the semi rigid connections equal to a ratio of the fixed end moments [13]. This ratio depends on many factors such as stiffness of the tie beam; stiffness of the isolated footing and soil stiffness. This expression can be written as follow;

$$M_A = \alpha_A \ \frac{6EI}{L^2}\Delta = \alpha_A M_A^F; \qquad (7)$$

$$M_B = \alpha_B \ \frac{6EI}{L^2}\Delta = \alpha_B M_B^F \qquad (8)$$

Where, α_A and α_B are coefficient of semi rigid connections which equal to ratio between the end moments of the semi rigid connections and the fixed end moments.

3. Numerical Model

Three dimensional models of two square footings with a tie beam are modeled by SAP program. Solid elements were used to model the footing and the tie beam as shown in Figure 2. The model is meshed using an 8-node linear element with a mesh size 0.20 m in horizontal direction and 0.30 m in vertical direction to get accurate results. All materials assumed to be homogeneous. Table 1. summarizes the material properties for isolated footing and soil.
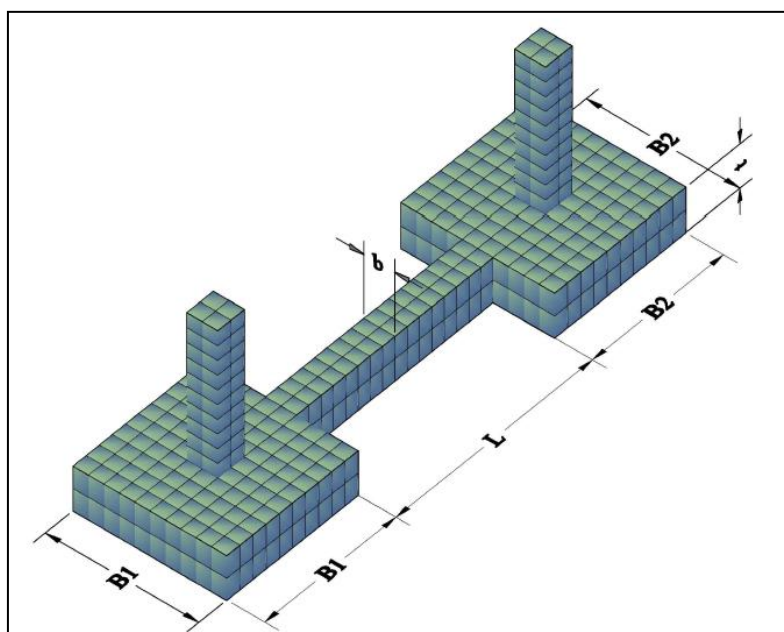

Figure 2. Structural finite element model with tie beam in three dimensional

The modulus subgrade reaction is developed on the assumption that the soil reaction of the isolated footings is proportional to the deflection of the beam at every point. The vertical settlement is represented by means of identical, independent, closely spaced, discrete and linearly elastic springs. This spring is known as the modulus of subgrade reaction as stated by Winkler, in 1867.

The numerical model based on the following assumption:

- There is no contact between the tie beam and the supporting soil;

- All material are homogeneous, isotropic and obeys Hooke's Law.

All material are homogeneous, isotropic and obeys Hooke's Law Cross-section of the tie beam is 0.40 in width and 0.60 m in thickness. The tie beam length was varied in three values 2.00 m and 3.00 m; 4.00 m. Footing width equal to the tie beam thickness. All footing are square where their width varied between 1.00 m to 3.00 m with increase 0.50 m. Dimensions of all elements are presented in Table 2.

Table 1. Material properties

Property	Concrete	Soil
Poisson ratio	0.20	0.30
Unit weight (kN/m^3)	25	17.50
Modulus of elasticity (KN/m^2)	22000000	13000
Bearing capacity q_a (KN/m^2)	----	100, 200
Modulus of subgrade reaction K = 10 q_a (KN/m^3)	----	1000, 2000

Table 2. Model geometry and dimensions

Property	value
Tie beam width (m)	0.40
Tie beam thickness; b (m)	0.60
Tie beam length ; L (m)	2.00; 3.00; 4.00
Isolated footing thickness (m)	0.60
Isolated footing width; B1 and B2 (m)	1.00; 1.50; 2.00; 2.50; 3.00
Column dimension (m)	0.40×0.40

Figure 3. shows sample of the obtained normal stress distribution contours considering different values of the parameters that affect in the design of these tie beams (tie beam stiffness, footings dimensions, soil bearing capacity and the differential settlement value). The result indicates that the position of the maximum end moment for the tie at contact between the tie beam and the footing. Therefore; the clear length of the tie beam is considered as a critical parameter.
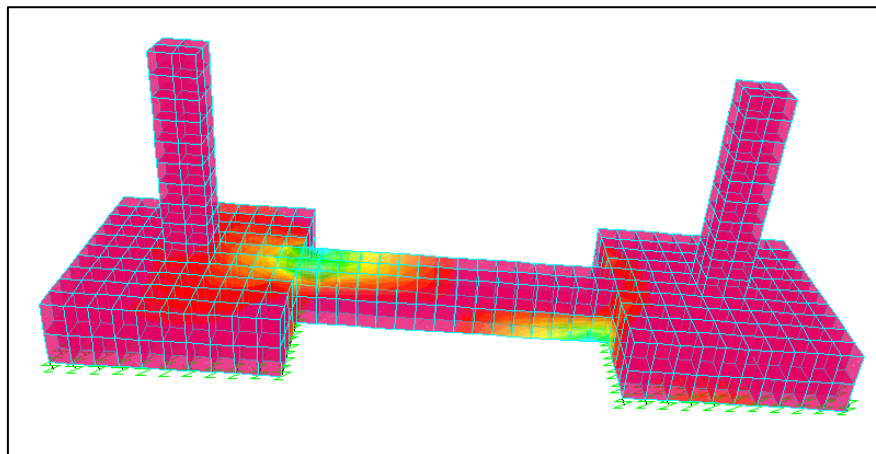
Figure 3. Sample of normal stress distribution contours

4. Numerical Results

4.1. Effect of the Tie Beam Length

Figure 4 and 5. show the effect of the tie beam length in case the tie beam connected to two equal square footing at different bearing capacity. The vertical axis represent the maximum end moment in case of semi rigid connection. As can be drawn from result; the maximum moment proportional with the tie length. The maximum moment increase as footing width increase in case of the two footing is identical in the dimension. From Figure 4; the maximum moment approximately equal 4.30 m.t when width the isolated footing equal to 3.0 m. From Figure 5; the maximum moment approximately equal 8.00 m.t when width the isolated footing equal to 3.0 m. Based on this result; the modulus of subgrade reaction.
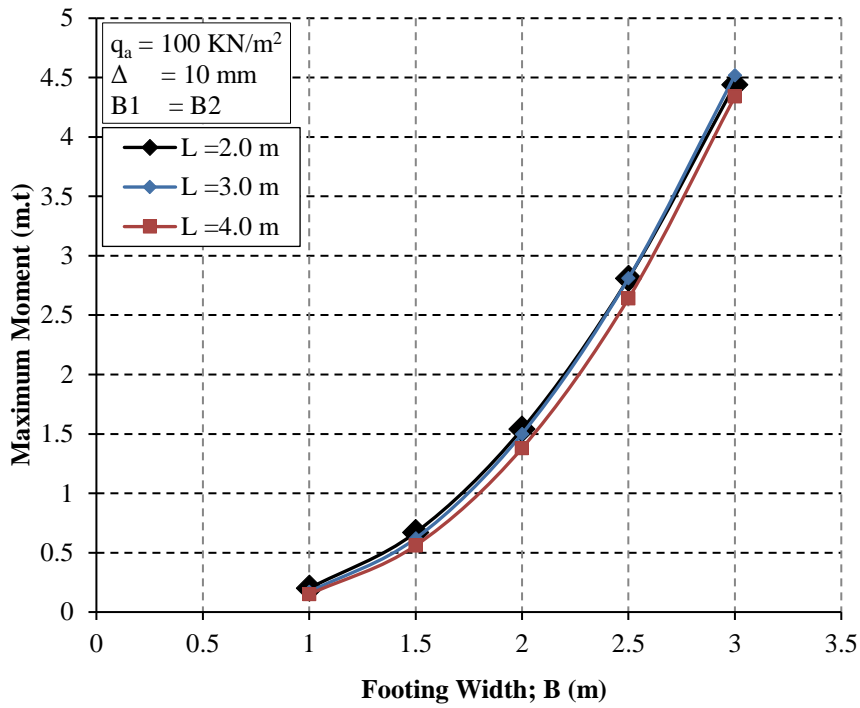
Figure 4. Relation between footing width and end moment of the tie beam at different values the tie beam length at differential settlement= 10 mm and soil bearing capacity =10 t/m²
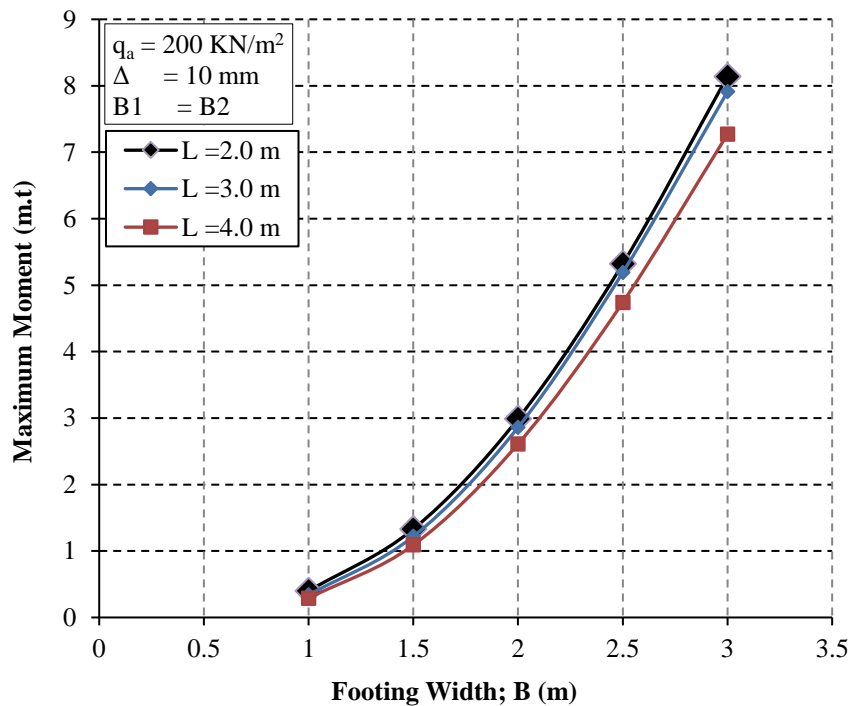

Figure 5. Relation between footing width and end moment of the beam at different values the tie beam length at differential settlement = 10 mm and soil bearing capacity =20 t/m²

4.2. Effect of the Footing Width

The relationship between maximum end moment of tie beam and the footing width considering differential practical values of other parameters may be affect in the tie beam moment (tie beam stiffness, soil bearing capacity and the differential settlement value) are shown in Figures 6 to 13.
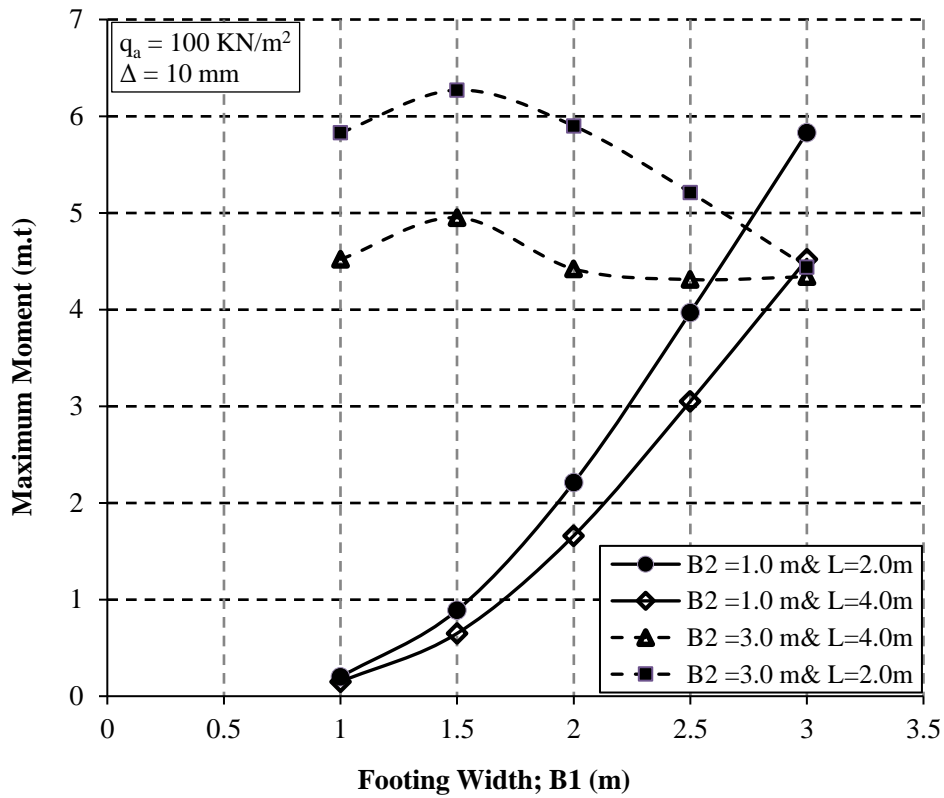
Figure 6. Relation between footing width and end moment of the beam at different values the tie beam length at differential settlement = 10 mm and soil bearing capacity 10 t/m²
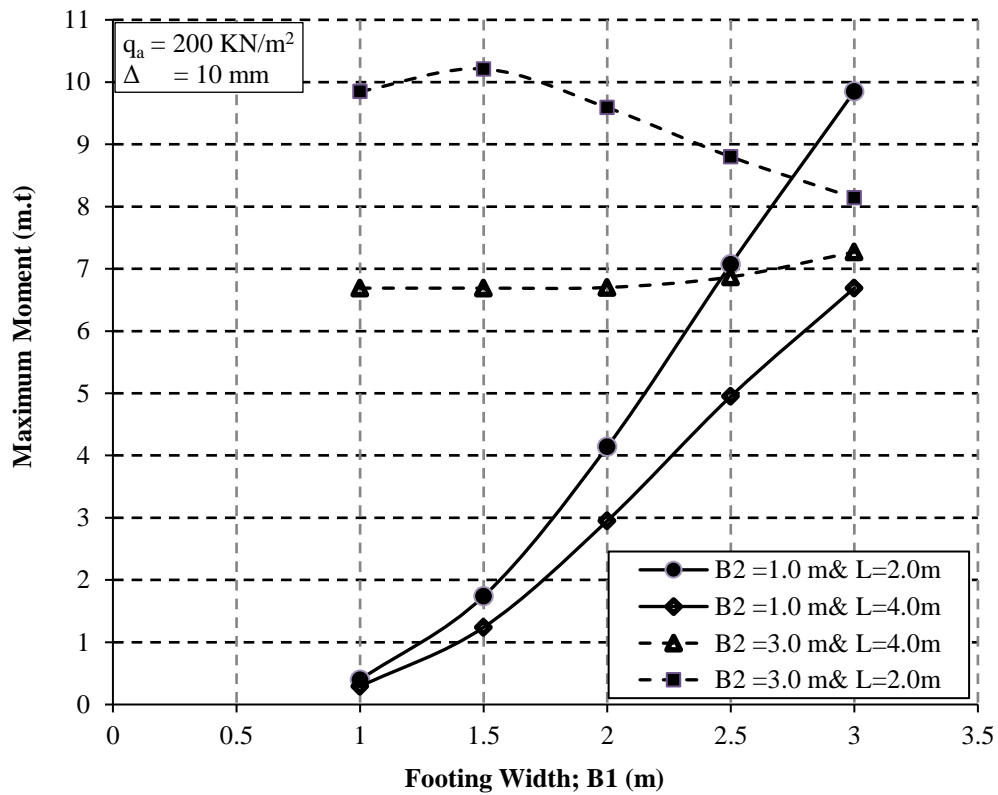

Figure 7. Relation between footing width and end moment of the beam at different values the tie beam length at differential settlement = 10 mm and soil bearing capacity 20 t/m²
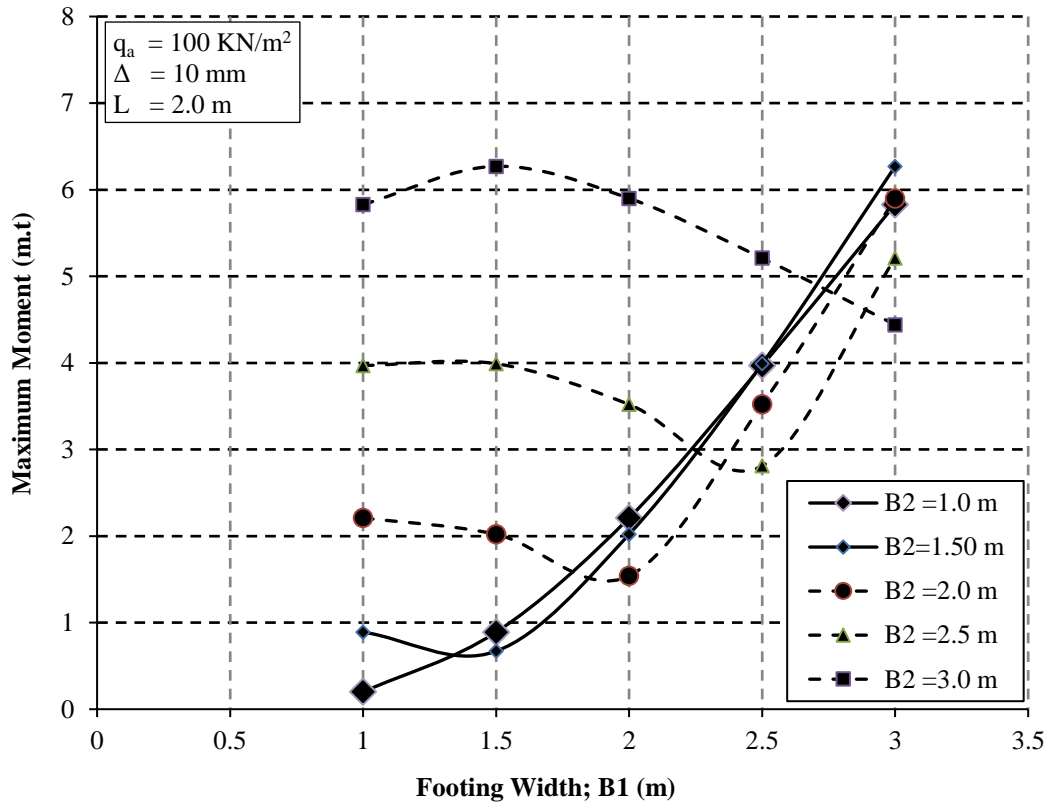
Figure 8. Relation between footing width and end moment of the beam at differential settlement = 10 mm, soil bearing capacity 10 t/m² and the tie beam length = 2.0 m
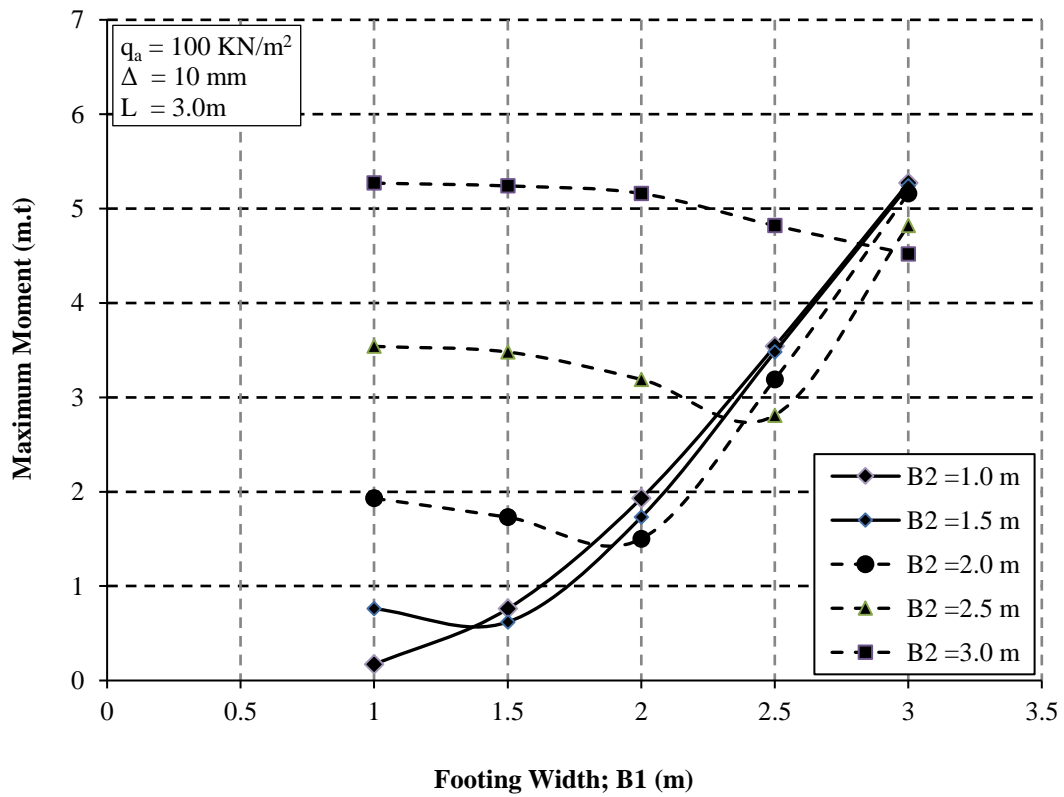

Figure 9. Relation between footing width and end moment of the beam at differential settlement = 10 mm, soil bearing capacity 10 t/m² and the tie beam length = 3.0 m

Figure 10. Relation between footing width and end moment of the beam at differential settlement = 10 mm, soil bearing capacity 10 t/m² and the tie beam length = 4.0 m

Figure 11. Relation between footing width and end moment of the beam at differential settlement = 10 mm, soil bearing capacity 20 t/m² and the tie beam length = 2.0 m

Figure 12. Relation between footing width and end moment of the beam at differential settlement = 10 mm, soil bearing capacity 20 t/m² and the tie beam length = 3.0 m
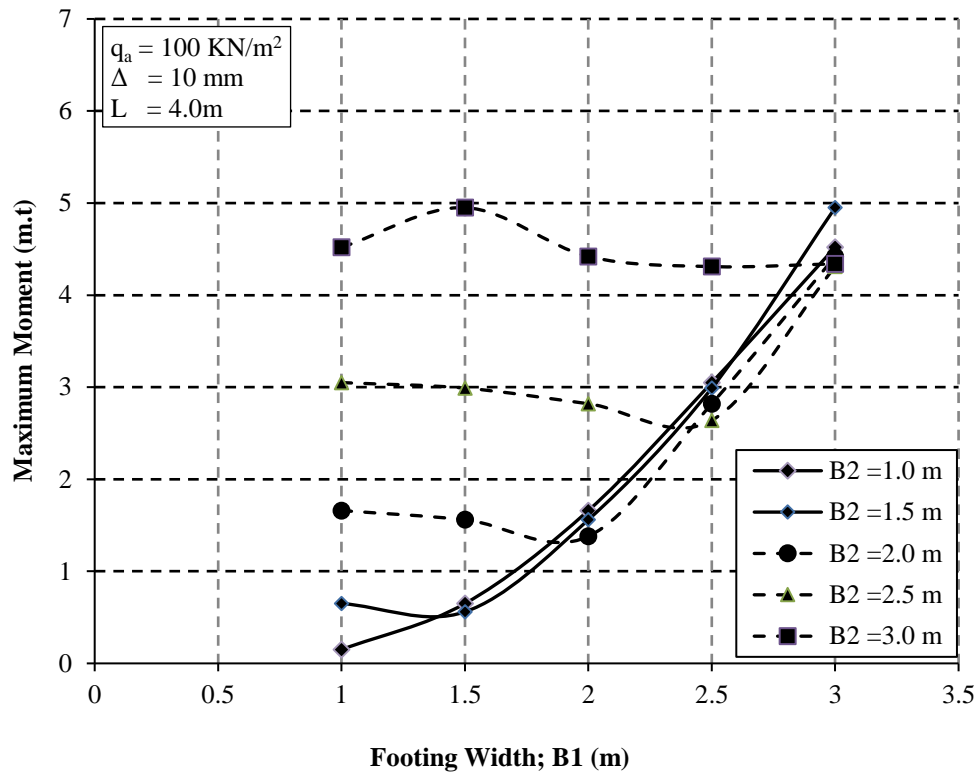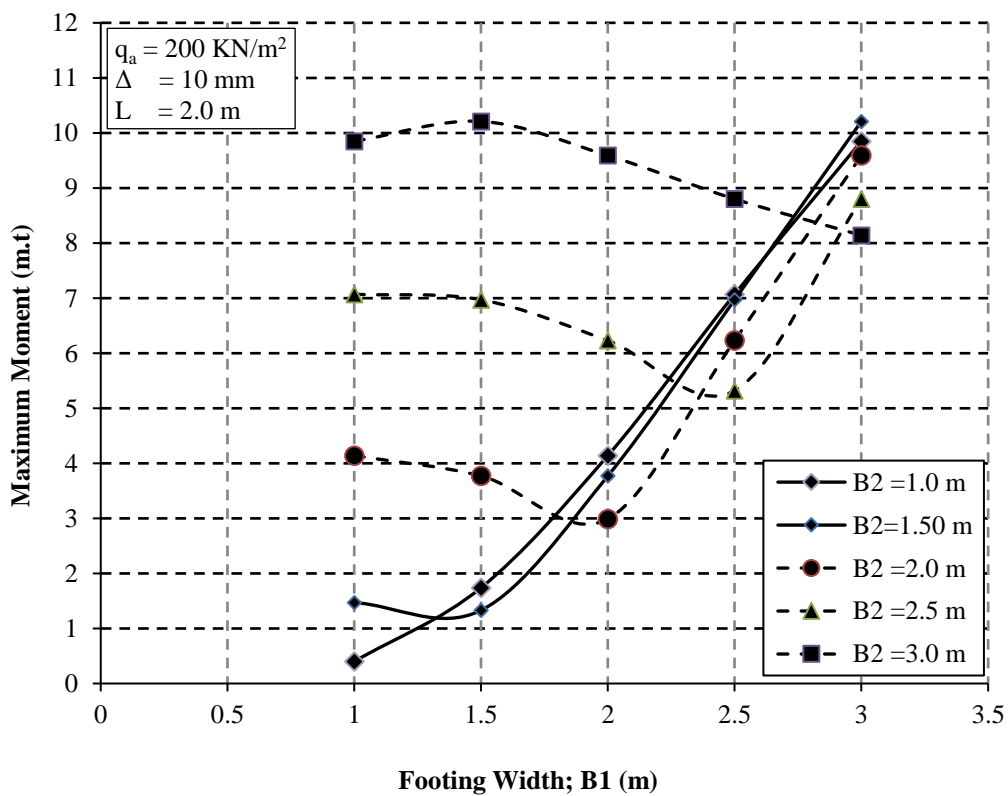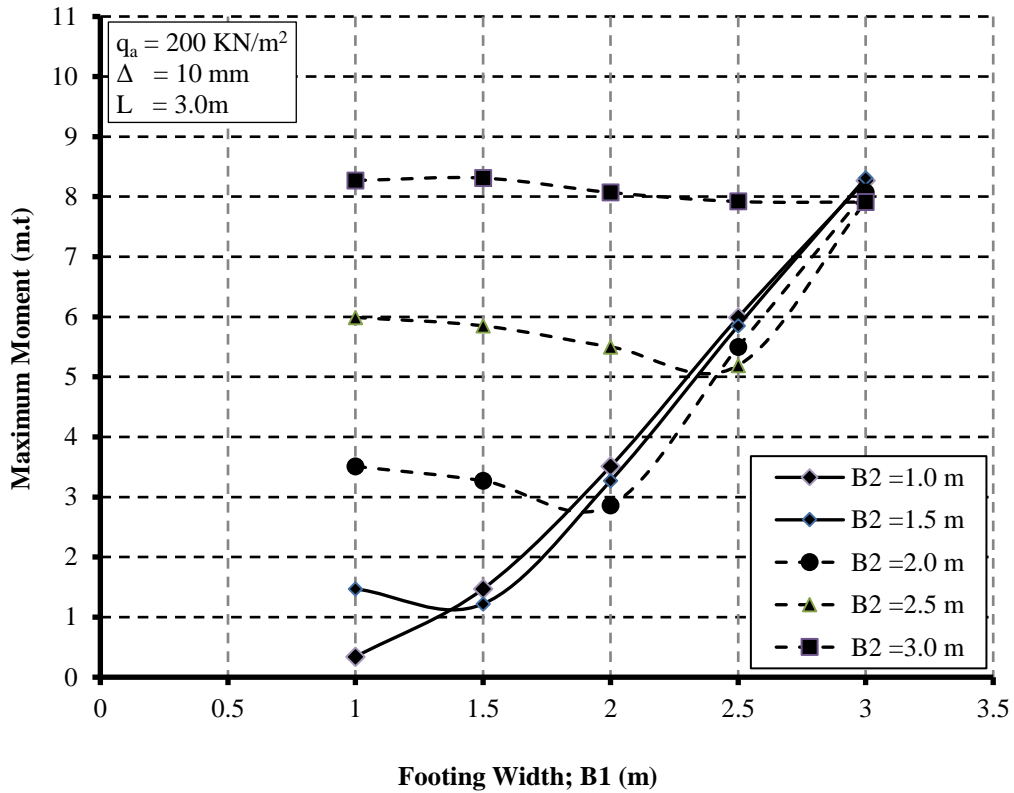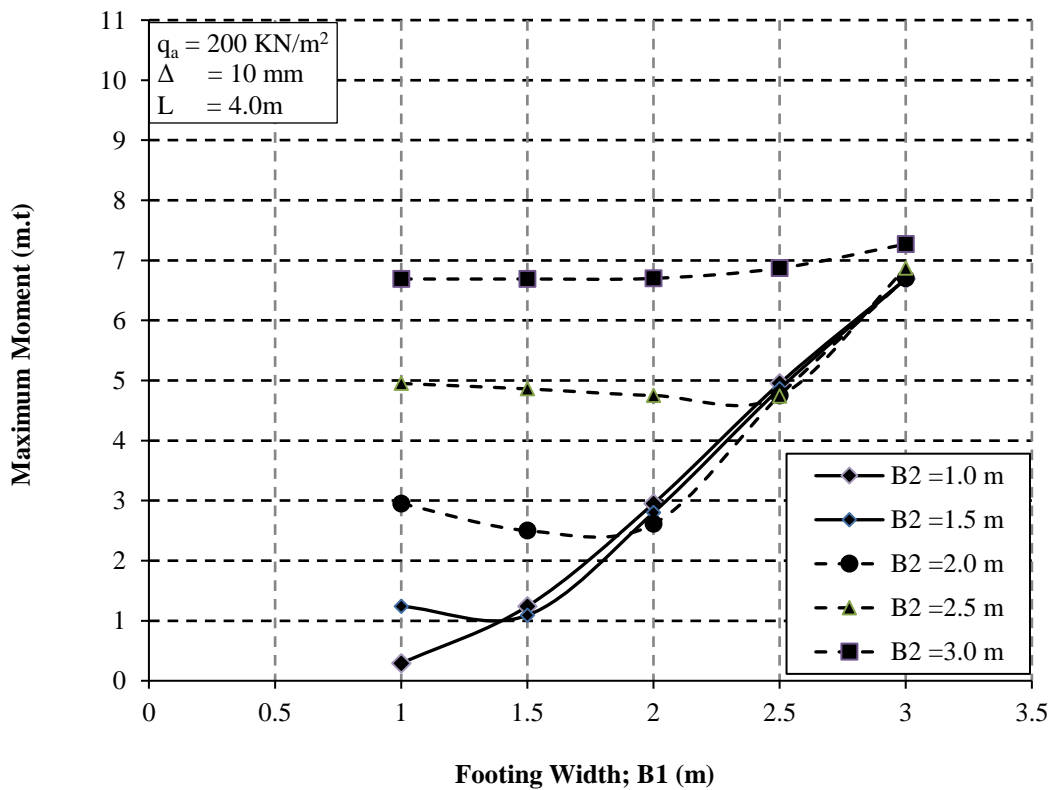

Figure 13. Relation between footing width and end moment of the beam at differential settlement = 10 mm, soil bearing capacity 20 t/m² and the tie beam length = 4.0 m

Generally, from the previous figures, we can draw the following notes:

- As expected, maximum moment in tie beam is linear directly proportional to the differential settlement that agree with the theoretical study shown above.

- Also, maximum moment in tie beam is directly proportional to soil bearing capacity, footing dimensions and stiffness of tie beam (i.e. directly proportional to the moment of inertia of the tie beam cross section and the modulus of elasticity of the used concrete but inversely proportional to the tie beam length).

- The relation between the bending moment of tie beam and the parameters under study may be not enough clear because the stiffness of the assumed rotational springs, as discussed in the theoretical study, depend on all the parameters together. The created stiffness of the assumed rotational spring at one footing is sensitive to the dimensions of the other footing and soil bearing capacity but is slightly sensitive to the tie beam stiffness.

5. Coefficient of Semi Rigid Connection

Table 3 compares numerical and mathematical results. The fixed end moments related linearly with the modulus of elasticity of concrete, the moment of inertia and the differential settlement and inversely with the tie length as stated by Wang 2014 [13]. Figure 14. present relation between the coefficient of semi rigid connection and the footing width.

Based on result summarized in Table 3 and shown in Figure 14; the coefficient of semi rigid connection range from 0.003 to 0.13. The coefficient of semi rigid connection has a linear relation with bearing capacity. It increases as the bearing capacity increases. In case of small footing width; the maximum moment has insignificant effect in design of the tie beam. In this case, the shear straining action has significant effect in behavior of the tie beam.

Table 3. Coefficient of semi rigid connection

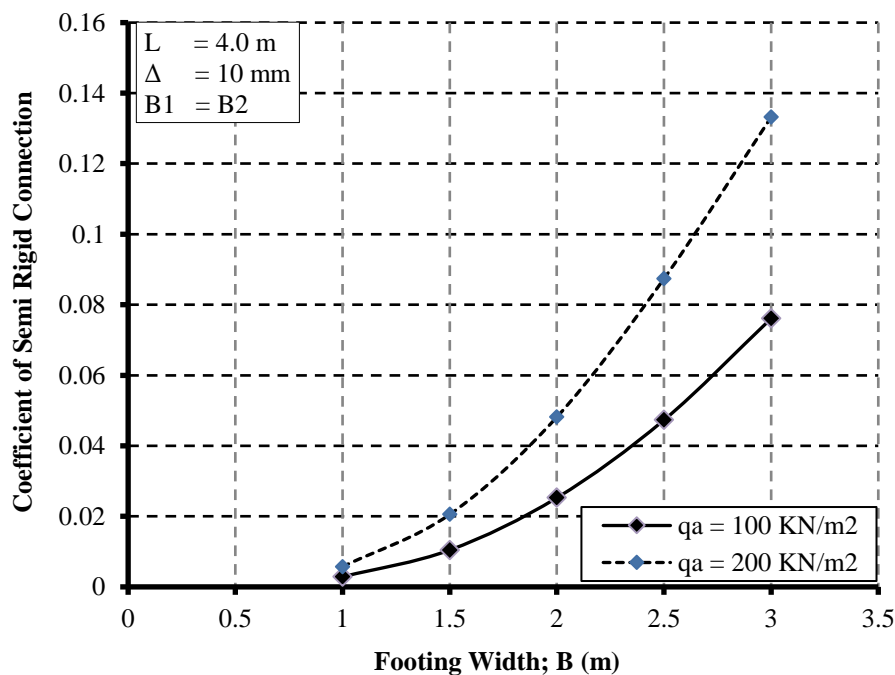| Tie beam length; L (m)	Bearing capacity q_a (KN/m²)	Footing width at B1=B2 (m)	1	1.5	2	2.5	3
4	100	$M_A = M_B$	0.17	0.62	1.5	2.81	4.52
		$M_A^F = M_B^F$	59.4	59.4	59.4	59.4	59.4
		$\alpha_A = \alpha_B$	0.0029	0.01044	0.0253	0.0473	0.0761
	200	$M_A = M_B$	0.34	1.22	2.86	5.19	7.91
		$M_A^F = M_B^F$	59.4	59.4	59.4	59.4	59.4
		$\alpha_A = \alpha_B$	0.0057	0.0205	0.0482	0.0874	0.1332

Figure 14. Relation between footing width and coefficient of semi rigid connection

6. Conclusion

In this paper; three-dimensional finite element method with solid element was applied to study the tie beams behavior connected to the isolated footings under the effect of differential settlement. The following conclusion can be drawn:

- Position of the maximum end moment of the tie beam at the contact face between the isolated footing and the tie beam.

- The maximum moment of the tie beam proportional linearly with the modulus of subgrade reaction.

- The results obtained from this study are introduced in many curves to clear the relation between tie beam straining actions and each parameter may affect in the design of these tie beams (the clear length between footings, moment of inertia of tie beam, footings dimensions, soil bearing capacity and the differential settlement value).

- The analysis indicates that the bending moments carried by the tie beam increase as the isolated footing dimension increased.

Finally, the obtained curves can be of great help for design purposes of the tie beams.

7. References

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