



Finite Element Modeling of Post-Tensioned Two-Way Concrete Slabs under Flexural Loading

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Abstract

Post-Tensioned (PT) method is a widely used technique to prevent cracking and to minimize the deflection which is resulted by loads. In this method, stress is applied after concrete placing and reach adequate hardening and strength. This paper investigates the structural behaviour of PT two-way concrete slabs. The main objective of this study involves a detailed flexural behavior analytical investigation of PT concrete two-way slab with the different bonded tendon layout. This will be achieved by non-linear Finite Element (FE) analysis programs method, to choose the most effective and optimum position of tendon layout with different number of tendons and applied load on the concrete two-way slab. A parametric study was conducted to investigate the effect of tendons layout on the overall behavior of post-tensioned two-way concrete slab. The result obtained from finite element analysis showed that the failure load in PT in both directions increased about 89 % as compared with slab PT in one direction.

Keywords: Bonded Tendon; Post Tensioned Concrete; Two-Way Slab; Nonlinear; Finite Element; Computer Modelling.

1. Introduction

Concrete is a structural engineering construction material used by different methods. Although concrete is strong in compression, it is weak in tension. Concrete tensile strength is changing and differs between 8 and 14 percent of its strength in compression. Cracks appear in tension zones after applying loadings, PT techniques are widely used in concrete to prevent cracking and minimize the deflection which is resulted by externally applied load. Stresses are transferred after concrete pouring and reach required hardening and strength. The tendons are placed inside ducts before pouring of concrete, after the hardening of concrete, stressing jacks must be used to stress each tendon to the required load. To ensure the initial-posttensioning forces, all tendons must be anchored at the member ends [1].

A 3-D FE analysis is the most used and complete technique, which is used for static and dynamic analyses, controlling all aspects that affect the response of structures. It becomes the numerical method of choice in many engineering and applied science areas [2].

The behaviors of post-tensioned concrete elements were studied experimentally by Williams and Waldron [3], Yang et al. [4], Ranzi et al. [5], Bailey and Ellobody [6, 7] and others. A collection of studies by several researchers explains the FE analysis of the behavior of reinforced concrete structures such as shear failure of slabs, cyclic loading of columns, and the behavior of structure to seismic and bond models between concrete and steel.

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El Semelawy [8] developed a numerical model that can design the optimum pre-stressed concrete slab using a new heuristic searching algorithm. The best result must contain all constraints of behavior required by the selected design code. The developed numerical tool utilizes FE analysis with prestressing model employing tendon element. Ellobody and Bailey [9] studied the structural behaviour of unbonded post-tensioned one-way concrete slab at elevated temperature. A nonlinear finite element model was developed for the PT analysis of unbonded concrete slab in fire.

Package ABAQUS program was used to model the PT structures with bonded and unbonded tendons by Kang and Huang [10]. Three modeling approaches had been used to simulate unbonded and bonded tendons, compared all approaches with the analytical and experimental results. The first approach was based on contact technic, the second approach was a multiple-spring system and the third approach was the model using a contact formulation (surface-to-surface contact). Yang [4] experimentally studied 12 partially PT one-way slabs made from lightweight concrete under a symmetrical top two- concentrated forces and a symmetrical top one- concentrated force.

Panfilov [11] used a numerical investigation of the monolith beam floor and slab with reinforcement of pre-stressed wire rope. The reinforced concrete slab and wire rope connections were studied; moreover, the study evaluated the reinforcement of pre-stressed wire rope efficacy when used in beam floor and slab. Lou et al. [12] presented a parametric study to predict the behavior of continuous two-span pre-stressed concrete beams with internally unbonded steel tendons and fiber reinforced polymer. A nonlinear model was verified by the laboratory experiments. Their analytical results confirmed that the unbonded tendon type did not affect the cracks mode and the moment redistributions.

Mohammed and Tayşi [13] developed a nonlinear FE model to analyze post-tensioned slabs at elevated temperature. The interface between the tendons and surrounding concretes was also modeled. The load–deflection behavior, load–force behavior in tendons, and the failure mode were evaluated. The numerical analysis was performed using ANSYS software and was performed in two different one-way concrete slabs taken from literature. The research aimed to study the behavior of post-tensioned concrete slabs under effect of many parameters; such as tendon bonding effects, thermal loading effects and tendon profile effects.

Mohammed et al. [14] studied the nonlinear behaviour of the PT concrete cantilever beam using various tendon profiles. ANSYS program was used for beam analysis. The authors studied the effect of various tendon profiles on the bonded PT reinforced concrete cantilever beam flexural behaviour. Six various tendon profiles models were investigated. Load-deflection response, mid-span deflections, and failure loads for all models were discussed.

Kim and Lee [2] studied experimentally the behavior and deflection of post-tensioned flat plate based on the layout of the tendon. Two PT concrete flat plates and one reinforced concrete flat plate were constructed and tested. Two-way post-tensioning layout and one-way post-tensioning layout were taken into consideration. The modes of crack and load-deflection response were also presented by the authors.

However, FE models studying the behavior of PT concrete two-way slabs under flexural loading have not been thoroughly studied. Very few studies exist in literature related to the effect of tendon layout on the behaviour of PT two-way concrete slabs especially the numerical analysis. The current study presents a finite element using ANSYS for modeling the bonded PT concrete slabs. The developed model was validated with prior experimental work conducted by Kim and Lee [2] and was used to study the effects of the tendons layout on the behavior of bonded post-tensioned two-way concrete flat slabs. In addition, this work was studied the parameters that affect the overall behaviour of PT two-way concrete slab such as tendons layout.

2. Finite Element Modeling

The ANSYS was used to analyze all modeled slabs. SOLID65 (or 3-D reinforced concrete solid) in ANSYS was used to model the concrete. Eight nodes exist in this element. At every node, three degrees of freedom exist, which means that concrete can have three orthogonal directions cracks, creep and plastic deformation. The nonlinear isotropic material is the behaviour of this element.

LINK8 is a spare element and a discrete model, which is used for modeling links, trusses, springs, and sagging cables. This element is a uniaxial tension-compression and a 3-D spare element. At each node, LINK8 has 3 degrees of freedoms. No bending of the element is taken into considerations, i.e. nodes in the x, y and z directions are translated [15]. To maintain internal forces, the tendons should be anchored at both ends of the slab members, which create high stress at both ends. The shell plates should be applied at the ends to avoid the ends from crushing. To analyze thin to moderately thick shells, SHELL181 is adequate.

The concrete-tendon contact was modeled by (CONTACT PAIR MANAGER). TARGE170 was utilized to model different 3-D "target" surfaces for the associated CONTA175. The contact elements themselves model the shell, solid or line elements describing the deformable body boundary. These contact elements are potentially modeling the contact between the target surfaces, defined by TARGE170.

The PT material components such as concrete, reinforcements, tendons, and contact surfaces were modeled by ANSYS elements. The element types required input properties depending on the analysis types. The material properties may be linear or nonlinear, isotropic, anisotropic or orthotropic. The Solid65 element needs linear isotropic besides multilinear isotropic material properties for appropriate concrete modeling.

In the current study, for the compression of concrete, the multi-linear relationship of stress-strain is adopted. The considered stress-strain relation depends on the study conducted by Desayi and Krishnan [16]; as shown by the Figure 1(a). For modeling the reinforcing steel bars of the current study, the bilinear relationship of stress-strain showed in Figure 1(b). is taken into consideration. Steel bars were assumed to transmit only axial force because they are slender, while in the current study, the strands are assumed multilinear isotropic.

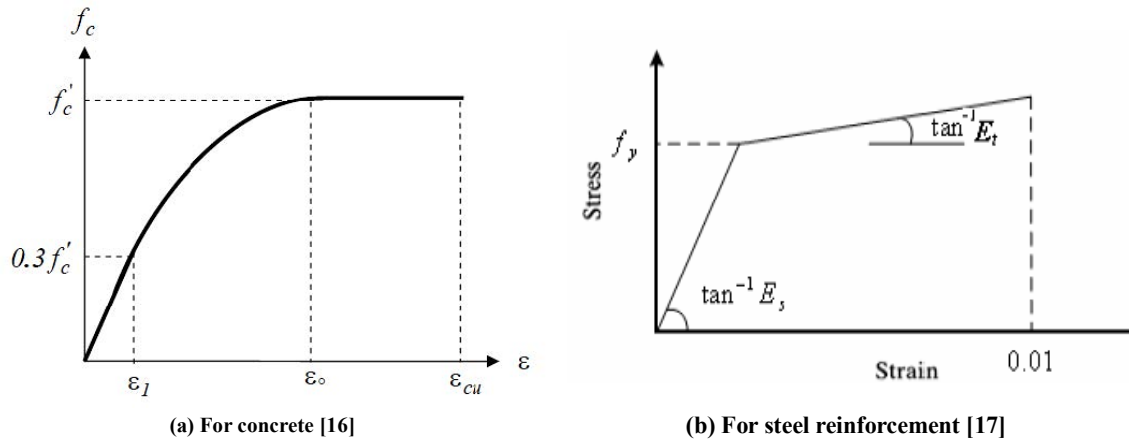


Figure 1. Stress- strain curve

3. Analysis of Post-Tensioned Two-Way Slab

The numerical results were verified by comparison with the experimental results. In order to validate the developed nonlinear models, one of the experimentally simply supported post tensioned concrete two-way slab experimentally tested by Kim and Lee [2] was chosen. One of their slabs was named as the slab FP-PT-xy and tested under static load up to failure. FP-PT-xy had five tendons in both directions with seven wire steel strand 12.7 mm, with a 13 mm diameter non-prestressed bars which was used in both directions at bottom of slab for each 280 mm spacing, with overall slab geometry dimensions equal to 3000 mm, 3000 mm and 250 mm length, width and thickness respectively as shown in Figure 2. The non-prestressed reinforcement was made of high tensile steel deformed bars with a 400 MPa yield stress and 200 GPa modulus of elasticity. The tendon nominal ultimate tensile strength was 1860 MPa. The tendon layout applied similarly to the bending moment induced from the concentrated loads. The tendons were bonded and grouted with 1488 MPa prestressing force. Figure 2. demonstrates the dimensions and section details of the prestressed slab specimen. Figure 3. shows the concrete and tendons mesh.

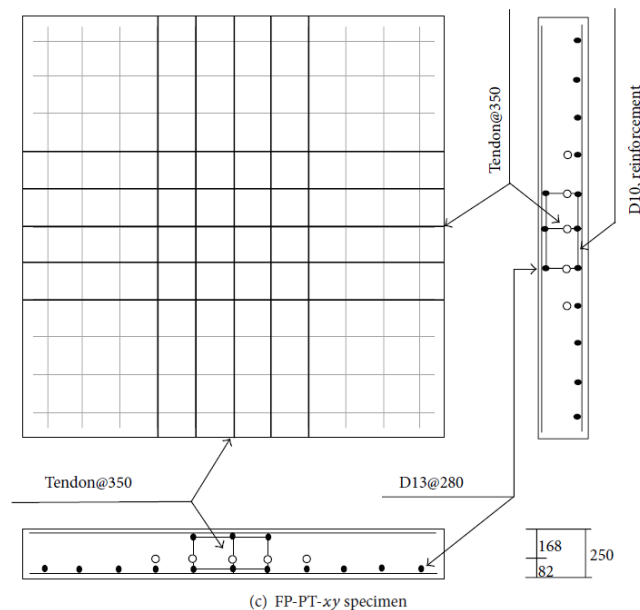


Figure 2. Reinforcement and dimension details of Post tensioned slab FP-PT-xy by Kim and Lee [2]

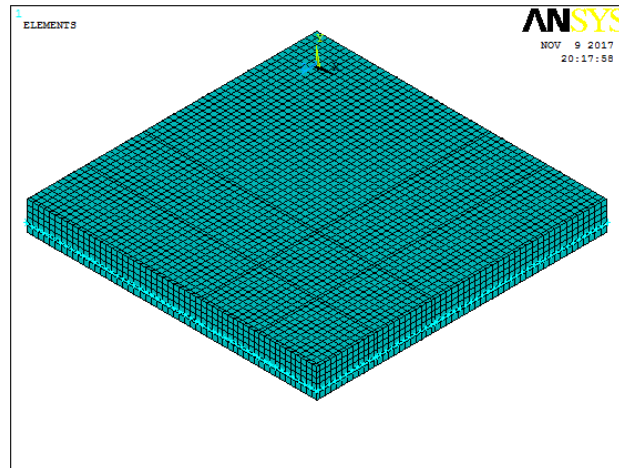


Figure 3. Finite element mesh of 1/4 of the concrete two-way slab FP-PT-xy

After modeling and analyzing the experimental model slab FP-PT-xy by ANSYS program, the results for load-displacement curves were validated and showed good agreement with experimental results as shown in Figure 4.

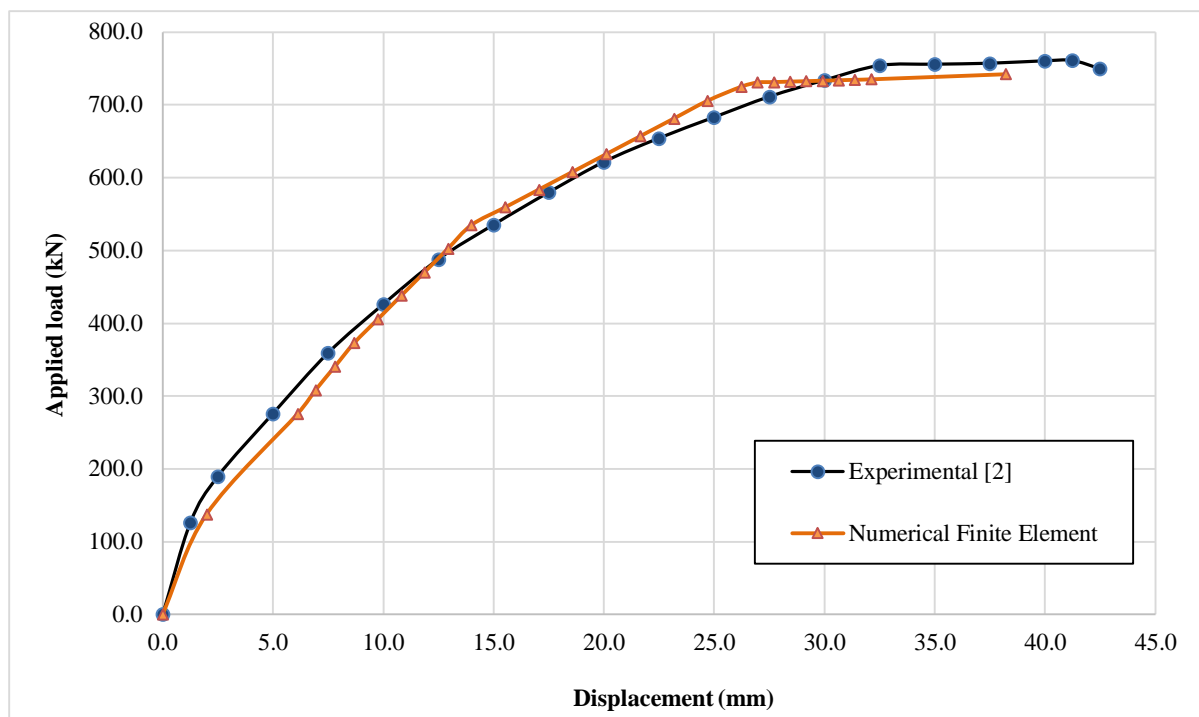


Figure 4. Load vs. deflection curves for the slab FP-PT-xy

4. Parametric Study

When applying external load on the bonded PT slabs, the variation in tendons strain at any section is equal to the variation in strain in adjacent section of concrete. The layout of tendons affects the strain and failure loads of the slab members. In PT concrete slabs, tendons are classically placed continuously through the supports. Tendon layouts with different numbers greatly affect the ultimate capacity. Some parameters were studied to clarify the effect of tendon layout on the general behavior of post-tensioned two-way concrete slab.

The dimensions, loading, and boundary conditions considered for the modeling slab are shown in Figures 5 and 6. The slab was subjected to concentrated point load at center and simply supported at all ends. In the current study, using 3D solid elements, the bonded PT concrete slab was modeled. Only 1/4 of the slab was modeled with 16100 elements because of symmetry. The concrete material properties, reinforcements, and strands for bonded PT two-way slabs considered in this study are shown in Table 1.

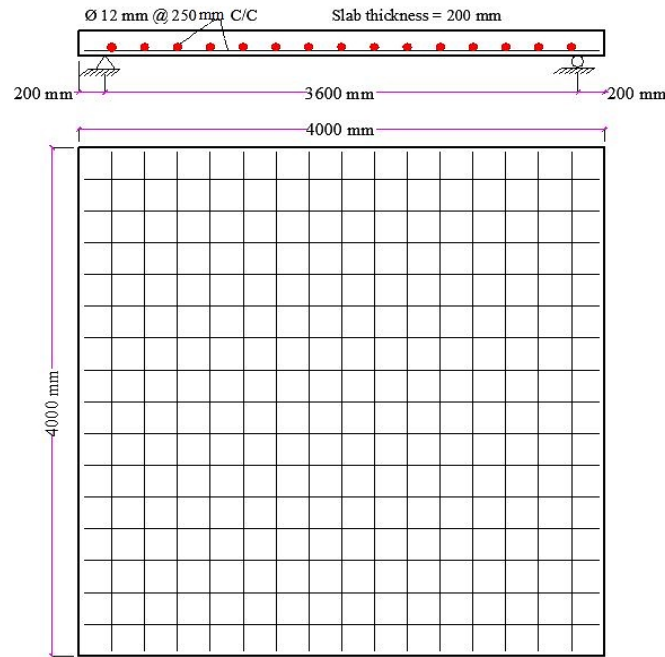


Figure 5. Concrete two-way slab dimension and cross section for parametric study

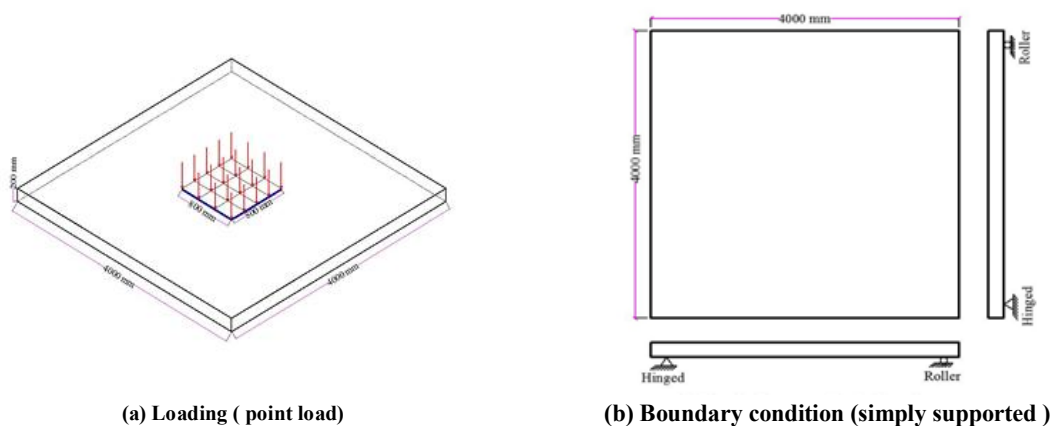


Figure 6. Loading and boundary condition

Table 1. Concretes material properties, reinforcements, and strands

Descriptions	Concrete	Steel plate	Strand
Compressive strength (MPa) (f'_c)	30.00	—	—
Tensile strength (MPa) (f_t)	3.80	—	—
Elasticity modulus of (MPa)	30000.00	200000.00	206290.00
Poisson's ratio (ν)	0.20	0.30	0.30
Yielding strength (MPa)	—	400.00	1741.00

4.1. Tendon Layouts

Prestressing is a way of counteracting the effect of external loads on structures by imposing stresses conflicting to the load effect. The best way to achieve this is by using tendons, which stressed prior to the final loading of structures. Tendons are typically located near the bottom which positive moments occur and near the top which negative moments occur with the intent to install the cable with the maximum total drape. Different tendon layout are considered which are without tendon, PT tendons in one direction with different number and PT tendons in two directions with different number as shown in Figures 7-8. For all cases tendon profiles are straight at the bottom, and spacing between tendons equal to 400 mm. For the slab without tendon, the tendon is not used, it is a control specimen

and the slab only reinforced by deformed steel reinforcement, all other cases had a PT tendon and will be capered with this two-way slab.

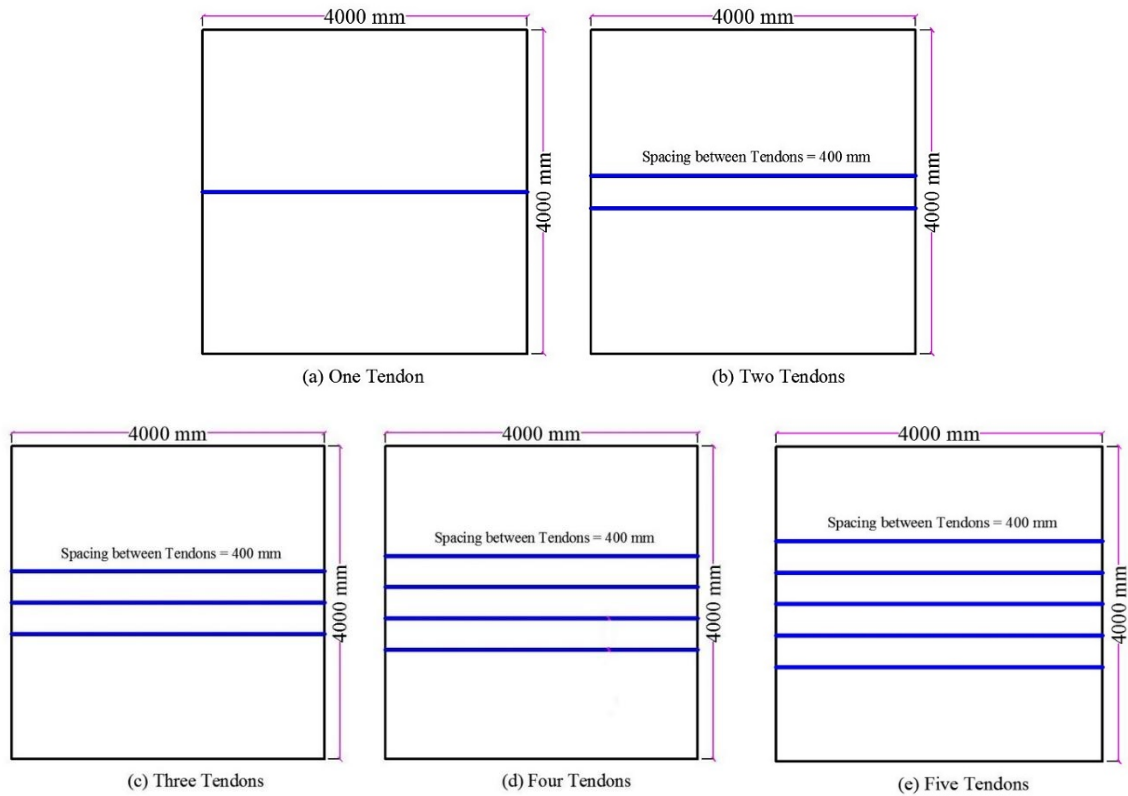


Figure 7. Tendon Layouts in one-direction with different numbers

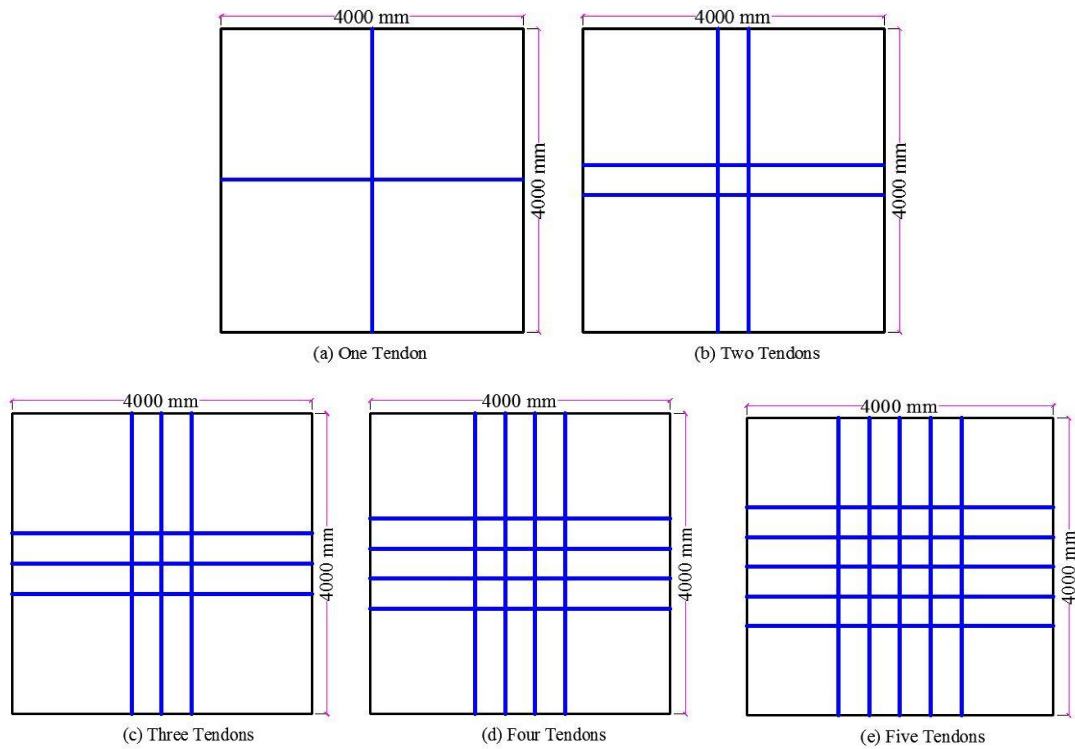


Figure 8. Tendon Layouts in two-direction with different numbers

5. Results and Discussion

The most important factor affected PT concrete two-way slab behavior was tendon layout. The nonlinear FE analysis was achieved to find the optimum layout of the tendon.

From the FE analysis, it can be noted that the ultimate load capacity increased by using tendons in both directions. The slabs with five tendons in both directions showed stiffer response and highest ultimate load capacity compared to the slabs with less than five tendons in addition to that tendons in one direction. Using tendon in both directions has the eccentricities specified values. Eccentricity is defined as the distance between the neutral axis and tendons, which creates an internal moment act in the opposite direction of moments caused by the external load. The Increase in eccentricities will cause an increment in tendon stresses and decrease the tensile stress in concretes. The use of tendons in two directions creates a transverse effect to increase capacity against counteract external load, with the effect of axial, bending and shear. In addition, straight bottom tendons in two directions of the slab give maximum failure load; it is stronger as compared to other slabs. Slab with tendons less than five in two directions or just tendons in one direction shows lesser failure loads and it was weak. Using tendons in two directions makes a resistance moment in both directions with smaller cracks while using tendons in one direction slab has a resistance moment only in one direction and in the other direction causes failure under smaller external load. The tendon is positioned with eccentricity to neutralize the sagging bending moment because of transverse load. Accordingly, the pre-stressed slabs deflect upward in the application of pre-stress. The tendons profile will present the bending moment diagram shape, since the bending moment is the result of the pre-stressing forces and eccentricities.

Figure 9. shows the load-displacement curves at the center of the slab that was obtained from FE analysis. The slabs with two-direction tendons showed stiffer response and higher failure load capacity in comparison with the slabs with one-direction tendons. From this, it can be noted that using tendon in both directions increased the ultimate load capacity in comparison with other tendon layouts.

PT has a strong effect on failure load of the slab, a various number of tendons used in one direction and also for both directions, at first only one tendon used and after that number of tendon increased, analysis results showed that tendon has a good effect for strengthening slabs and reducing cracks. Figures 10 and 11. illustrate the applied load and displacements for all cases. From Figure 10, it can be seen that increasing number of tendons leads to increase the ultimate load capacity in comparison with slab without tendons. Slab without tendons showed the least nominal strength, no change in stiffness till yielding. All models exhibited sudden failures at the center.

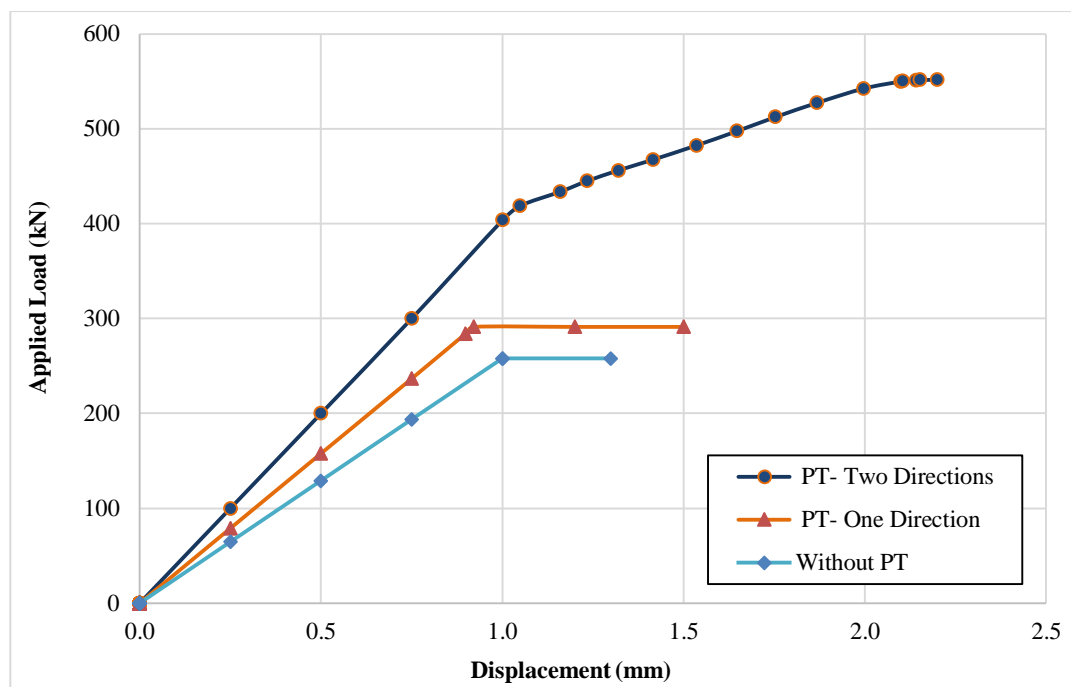


Figure 9. Load- displacement curves at the center of slabs

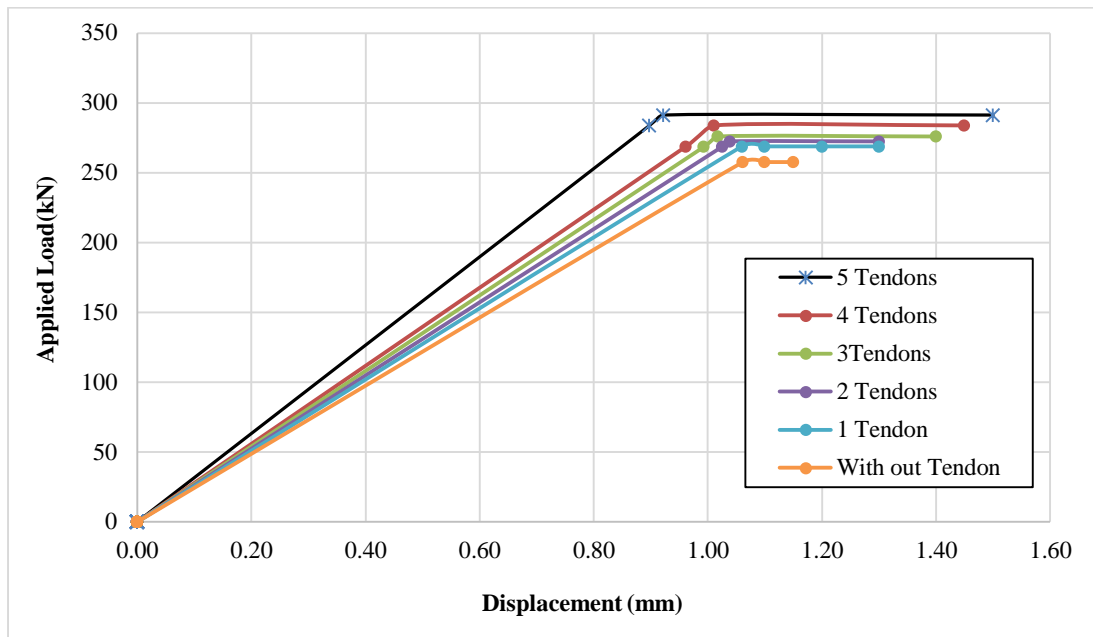


Figure 10. Load- displacement curves at the center of slabs for one-direction tendons

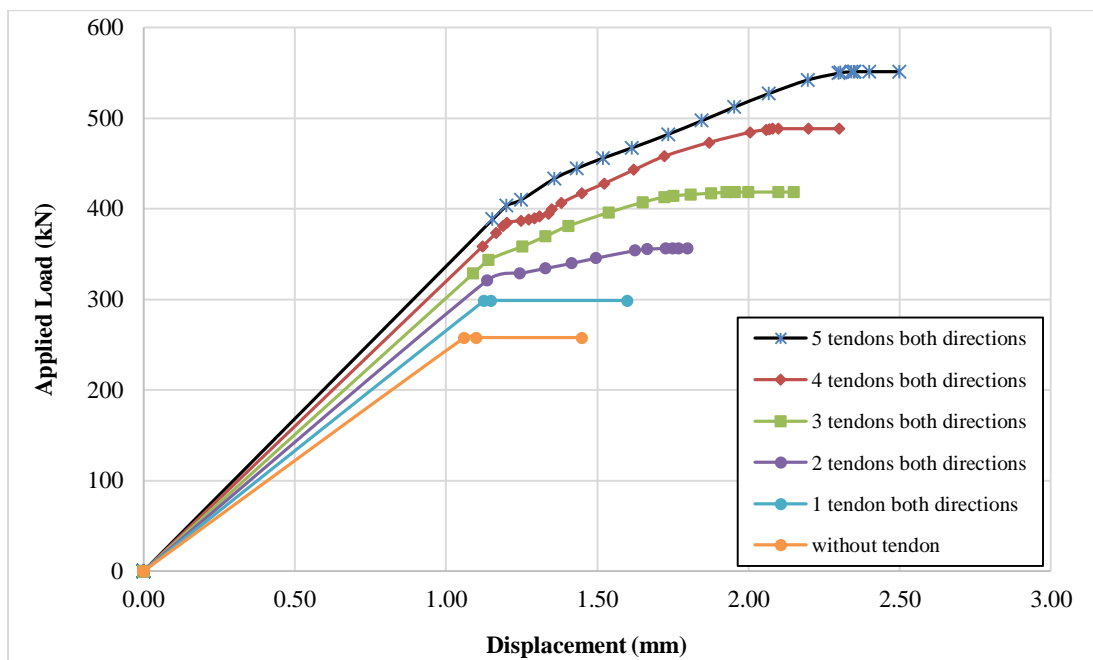


Figure 11. Load- displacement curves at the center of slabs for two-direction tendons.

The different parameters considered in the parametric study and their results are shown in Table 2.

Table 2. The parametric study results

Group	Slab	Tendons Layout	support	Load Type	No. of tendons	Tendons area (mm ²)	Max. deflection (mm)	Failure load (kN)
	S1	Without tendon			0		-1.0	257.7
G1	S2	One-direction	Simply supported	Point	5	150	-0.92	291.2
	S3	Two-directions			5		-2.35	551.6
	S4				5		-0.92	291.2
G2	S5	One-direction	Simply supported	Point	4	150	-1.01	283.8
	S6				3		-1.02	276.2

	S7				2		-1.3	272.5
	S8				1		-1.1	268.8
	S9				0		-1.0	257.7
<hr/>								
	S10				5		-2.35	551.6
	S11				4		-2.07	487.8
	S12				3		-1.96	418.6
G3	S13	Two-directions	Simply supported	Point	2	150	-1.67	355.6
	S14				1		-1.6	298.8
	S15				0		-1.0	257.7
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G4	S16	One-direction	Simply supported	Point	5	150	-0.92	291.2
	S17					175	-0.97	306.2
<hr/>								
G5	S18	Two-directions	Simply supported	Point	5	150	-2.35	551.6
	S19					175	-2.44	595.9

The values presented in the Table 2 are the maximum values from the result analysis of all slabs. From Table 2, it can be shown that the failure loads for the slabs with two-direction tendons are greater than for slabs with one-direction tendons.

In addition, it can be noted that for groups G1-G3 the slab with two-direction tendons (S13) has the highest failure load (551.6 kN), while the highest deflection (2.35 mm) occurs in slab with two-direction tendons (S10).

From Table 2, it can be noted the result shows the ultimate load capacity of slab with two-direction tendons is larger the ultimate load capacity of slab with one-direction tendons when the total number of tendons are constant for two slabs. It can be observed that the failure load in PT in two-directions (S3) about 89 % increases as compared with slab PT in one direction (S2).

Figure 12. shows the stress in concrete (x-direction), stress in concrete (z-direction) and Von Mises stress. Figure 12. shows the compressive stress in a direction parallel of tendons higher the stress perpendicular to tendons, while the inverse for tension stress.

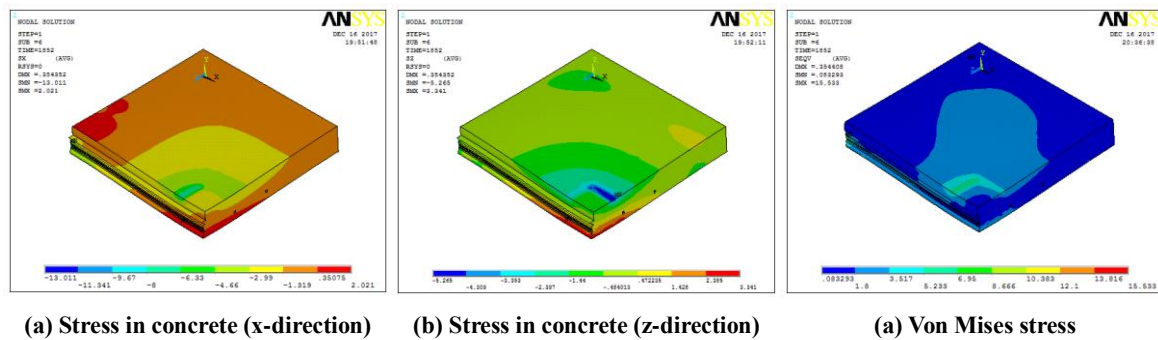


Figure 12. Results for slab S2

6. Conclusion

To enhance cracking loads and decrease deflection at service loads, prestressing was used for flexural strengthening of reinforced concrete slabs. The main objective of this study involves a detailed analytical investigation of the flexural behavior of PT concrete two-way slab with various bonded tendon layout. Finite element package ANSYS program was used to model the PT concrete two-way slab.

The accuracy of the analysis was verified by the comparisons with the experimental results. The results obtained by using FE analysis were close to the experimental results.

To investigate the effect of tendons layout on the overall behavior of post-tensioned two-way concrete slab, a parametric study was conducted. Nineteen models were studied with different tendons layout. Failure loads, deflections for all models and load versus deflection relationships were studied. The prestressing force was constant for all models, that was greatly affected the failure loads and deflections.

The results showed that the ultimate load capacity was highly affected by tendon layouts. More specifically, the ultimate load capacity was increased due to using tendons in both directions of the slabs. The slab models with five tendons in both directions showed a stiffer response and a higher ultimate load capacity compared to the slabs with five tendons in one direction and also those slabs without tendon.

Table 2. showed that the failure load in PT in both directions increased about 114% as compared with slab without tendons, and for PT in one direction increased about 13 % as compared with slab without tendons.

The result shows that the slab ultimate load capacity with two-direction tendons is larger than the ultimate load capacity of slab with one-direction tendons when the total numbers of tendons are constant for two slabs. It can be observed that the failure load in PT in both directions increased about 89 % as compared with slab PT in one direction.

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