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# Experimentally Comparative Study on Different Strengthening Methods of Reinforced Concrete Deep Beams

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#### Abstract

The aim of this study is to investigate the effect of strengthening reinforced concrete deep beams. An experimental study was done using six reinforced concrete deep beams have the same dimensions of  $1150 \times 800 \times 150$  mm, and subjected to mid-span concentrated load up to failure. Beams were different in the type, Location of strengthening and the ratio of reinforcement. Beams were divided into three groups. The first group included beams strengthened internally by single strut and either vertical or horizontal additional reinforcement. The second group included beams strengthened using double embedded strut or using CFRP as external strengthening. The third group included one beam strengthened using inclined stirrups. One of the specimens was tested without any strengthening and one specimen was strengthened by external CFRP sheets for comparison purposes the results of the experimental study shown remarkable improvement for using each type of strengthening. Results shown that using the mechanism of increasing stirrups by double rate and using single strut reinforcing is the optimum choice. This is due to the fact that this type of strengthening provides significant increase in the beam capacity in additional to the enhanced behavior of the beam. By this study comparison between each type of strengthening was done and the optimum type to be used in accordance with parameters of gained load capacity of tested deep beams.

Keywords: Strut and Tie; Inclined Stirrups; CFRP; Single Embedded Strut; Double Embedded Strut.

# 1. Introduction

Deep Beam is one of the most important structural elements used in the construction industry which has the ability to carry heavy loads in long spans. It has many applications such as high rise buildings and transfer girders. Nowadays strengthening of structural elements became an important issue. An Experimental study was done to investigate the effect of strengthening reinforced concrete deep beams internally and externally on the capacity of the beam section. The main parameters considered in this study were the type of strengthening whether adding single strut, double strut or inclined stirrups; and type of strengthening material whether steel or CFRP. All beams were subjected to mid-span concentrated load up to failure. Different ratios of steel bars and stirrups were used in the horizontal, vertical, and inclined directions to act as embedded strengthening struts. The results showed great improvement in the ultimate capacity of tested deep beams due to proposed strengthening methods and confirmed the ability to use CFRP sheets as external strengthening in case of retrofitting deep beams. A cost analysis was done in this research to calculate accurately

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cost of each type of strengthening and relation between gained section capacity. Research of deep beams starts from 1962 till present and many researchers have proposed various methods to strengthen deep beams.

Khattab et al. (2017) studied reinforced concrete deep beams and concluded that increasing of strips lead to an increase in beam capacity [1]. Using of FRP sheets in the strengthening of deep beams, increase beam capacity [2-7], also using of FRP bars in the strengthening of deep beams led to increasing beam capacity [8-10]. Another researcher studies other parameters that shall be considered in the design of deep beams [11-13]. Chen et al. (2018) concluded that using simplified strut and tie model without iterative procedure can predict accurately shear strength of the beam. Also, Hussein et al. (2018) concluded that size effect action of the deep beam shall be considered in beams less than 1m size [14]. Increasing of fiber in a concrete mix of the deep beam can control cracks and increase the load capacity of the beam [15-17].

Also, Biao and Yu-Fei (2018) studied span to depth ratio effect on shear strength of reinforced concrete deep beams. They used 11 beams in their study using the parameter of a/d (span-depth ratio) [18]. They concluded that shear carried by concrete in small a/d governed shear strength of the beam. On the other hand, in large a/d less than 2.5, shear strength was governed by shearcarried by reinforcement. Albidah et al. studied strengthening deep beam using near surface mounted (NSM) carbon fiber reinforced polymer (CFRP) U-wrap strips and externally bonded horizontal CFRP strips, and welded wire mesh and its effect on properties of deep beam specially shear strength. By this study, researcher concluded that using CFRP strips and welded wire mesh improving shear strength of the beam than that in steel web reinforcement. Also for beams strengthened using CFRP deformation capacity enhancement was not significant [19-23]. Chen C. et al. studied the use of steel beam embedded in reinforced concrete deep beam as internal strengthening technique and concluded that using of wide flange steel beams increased the shear strength by 7% up to 23% and the wide flange shape affect shear strength but it was hard to evaluate the shear increase accurately [24].

Amin and Foster (2016) tested ten simply supported reinforced concrete deep beams having the same dimensions but different ratio of fiber reinforcement as external strengthening. Results of the study showed that by using a sufficient ratio of steel fibers, can replace transverse shear reinforcement. Also using of large ratio of fiber led to the dispersed crack pattern [25].

## 2. Experimental Program

#### 2.1. Specimens Details

Six simply supported RC deep beams were constructed for doing this experimental study with the same concrete dimensions of 1150 mm for longitudinal direction, 800 mm for depth and 150 mm for width with a rectangular shape. Concrete strength was 30 kN/mm<sup>2</sup> and all beams was simply supported and subjected to concentrated load in mid-span, two strain gauge with factor 2.12 in each beam, Steel rebar all six beams had different shape and ratio of reinforcement or CFRP strengthening sheets, Figure 1 shows general Details of Beams and Table 1 shows Reinforcement Details of Beams.

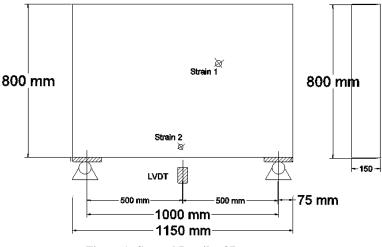


Figure 1. General Details of Beams

The specimens include one control specimen (B1-CS) and the other specimens were classified into three groups. The first group includes the specimens with duplication of either vertical or horizontal specimen with single strut reinforcement, Figures 2 (b and c), respectively. The second group includes the internally reinforced specimen using embedded double strut and the strengthened specimen externally using CFRP sheets, Figures 2 (d and e), respectively. The third group includes the specimen with inclined stirrups as shown in Figure 2 (f).

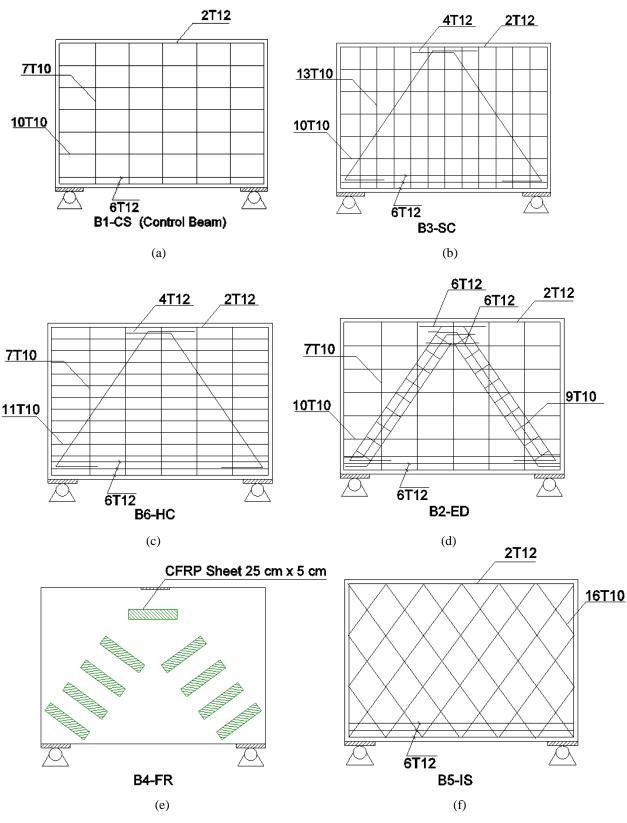


Figure 2. Reinforcement Details of Beams

## 2.2. Strengthening of Beam B4-FR

For Beam B-4-FR external strengthening with CFRP sheets was done as external strut strengthening in both sides of the beam according to the following stages. Figure 8 shows the scratching of the beam by chisel and hummer, Figure 9 shows CFRP sheets and Epoxy Resin, Figure 3 shows steps of strengthening using CFRP.

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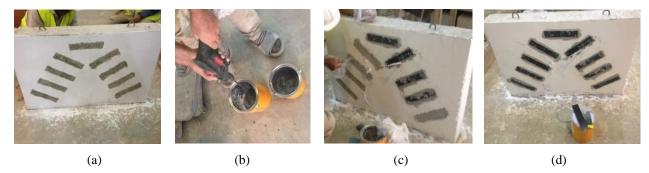


Figure 3. Steps of Strengthening using CFRP

#### 2.3. Measuring System

In this study, each beams which lifted for testing on compression machine using manual crane, during this stage, all precautions were done to ensure leveling, and placing on supports and load aligning plate, LVDT (Linear Voltage Differential Transducers) was placed at mid-span of the beam and connected to a data logger and their coefficients were defined to the computer program. All devices were calibrated by Cairo University lab and accuracy of all devices were between 95 to 98%.

## **3. Results and Discussion**

### 3.1. Group I

For the control specimen, the cracks were propagated and other cracks start to appear. Then the beam was collapsed due to shear cracking which occurred at the mid portion of the beam after the tension reinforcement yielding as shown in Figure 4 (a). The specimens with duplication of vertical stirrups (B3-SC) and horizontal reinforcement (B6-HC) had experienced a similar failure as the control specimen as shown in Figures 4 (b and c).

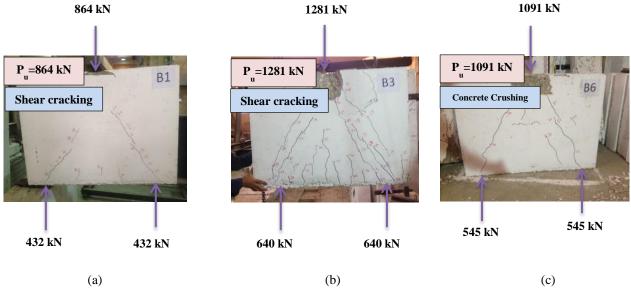


Figure 4. Failure Mode for Group I

Beams (B3-SC) and (B6-HC) had a similar cracking load at 730 kN while the control specimen had a cracking load of 630 kN. The ultimate loads were 864, 1281 and 1091 kN for the control specimen, the beam with duplicated vertical stirrups and single strut reinforcement (B3-SC), and the beam with duplicated horizontal reinforcement and single strut reinforcement (B6-HC), respectively . In comparison with the control specimen, beam (B3-SC) and beam (B6-HC) increased the toughness by 84% and 3%, respectively also these beams increased the initial stiffness by 26.7% and 31.3%, respectively. Table 1 shows the results of the first group.

|       | -   |             |              | oup -                  |                            |                                    |
|-------|---|-------------|--------------|------------------------|----------------------------|------------------------------------|
| Beam  | Strengthening Technique   | Pcr<br>(kN) | Pult<br>(kN) | Max Deflection<br>(mm) | Increase in<br>toughness % | Increase in Initial<br>Stiffness % |
| B1-CS | Unstrengthen Beam   | 630         | 864          | 2.14                   |                            |                                    |
| B3-SC | Duplicated vertical stirrups and single strut reinforcement           | 730         | 1281         | 2.26                   | 84 %                       | 26.7 %                             |
| B6-HC | Duplicated horizontal reinforcement and<br>single strut reinforcement | 730         | 1091         | 1.91                   | 3 %                        | 31.3 %                             |

Table 1. Results of Group I

The corresponding deflections at the ultimate load were 1.85, 2.35 and 1.79 mm for the control specimen, B3-SC, and B6-HC, respectively. The max deflections were 2.14, 2.26 and 1.91 mm, respectively as shown in Figure 5.

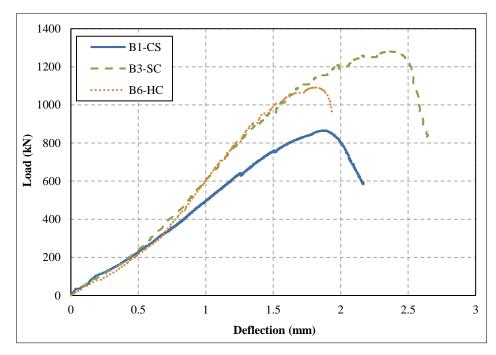


Figure 5. Deflection of Group I

## 3.2. Group II

Group II included two beams (B2-EB) and (B4-FR). Beam (B2-EB) was strengthened using embedded diagonal double strut, beam (B6-FR) was strengthened with CFRP strips. Beam (B1-CS) was included in the group for comparison purpose only. Fig (6) showed Failure mode for group II.

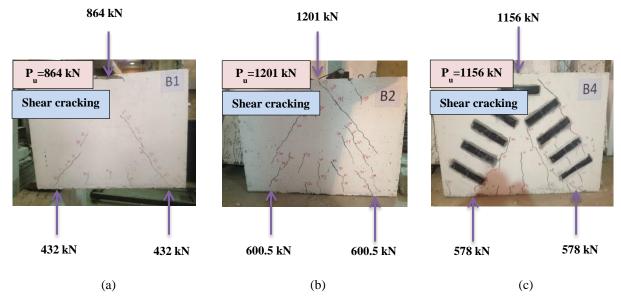


Figure 6. Failure Mode for Group II

Beams (B2-EB) and (B4-FR) had a cracking load at 1201 and 1156 kN, respectively. The beam with an embedded diagonal double strut (B2-EB), and the beam with CFRP strips (B4-FR), respectively. In comparison with the control specimen, beam (B2-EB) and beam (B4-FR) increased the toughness by 83% and 31%, respectively also these beams increased the initial stiffness by 17.3% and 9.2%, respectively. Table 2 shows the results of the second group.

| Table 2. Results of Group II |                                |             |              |                        |                         |                                    |  |  |
|------------------------------|--------------------------------|-------------|--------------|------------------------|-------------------------|------------------------------------|--|--|
| Beam                         | Strengthening Technique        | Pcr<br>(kN) | Pult<br>(kN) | Max Deflection<br>(mm) | Increase in toughness % | Increase in Initial<br>Stiffness % |  |  |
| B1-CS                        | Unstrengthen Beam              | 630         | 864          | 2.14                   |                         |                                    |  |  |
| B2-EB                        | Embedded Diagonal Double Strut | 850         | 1201         | 2.65                   | 83%                     | 17.3%                              |  |  |
| B4-FR                        | CFRP strips                    | 770         | 1156         | 1.93                   | 31%                     | 9.2%                               |  |  |

#### Table 2. Results of Group II

The corresponding deflections at the ultimate load were 1.85, 2.28 and 2.28 mm for the control specimen, B2-EB, and B4-FR, respectively. The max deflections were 2.14, 2.65 and 1.93 mm, respectively as shown in Figure 7.

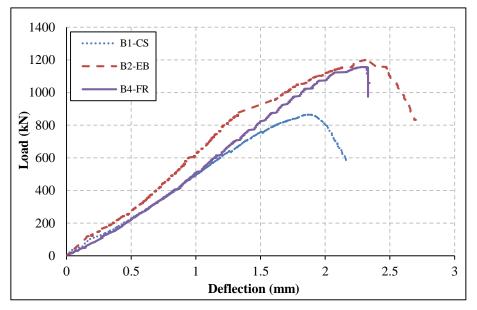


Figure 7. Deflection of Group II

## 3.3. Group II

Group III included beam (B5-IS). Beam (B5-IS) was strengthened using inclined stirrups. Beam (B1-CS) was included in the group for comparison purpose only. Figure 8 showed Failure mode for group III.

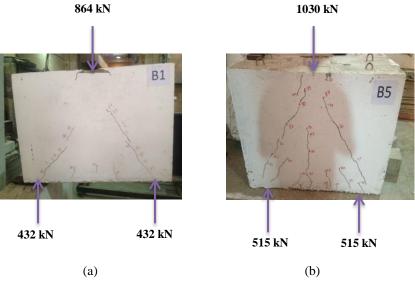


Figure 8. Failure Mode for Group III

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Beams (B5-IS) with inclined stirrups, had a cracking load at 1015 kN. In comparison with the control specimen, beam (B5-IS) increased the toughness by 22% also the beam increased the initial stiffness by 7%. Table 3 shows the results of the third group.

| Beam  | Strengthening<br>Technique | Pcr<br>(kN) | Pult<br>(kN) | Max Deflection<br>(mm) | Increase in toughness<br>(%) | Increase in Initial<br>Stiffness (%) |  |  |  |
|-------|----------------------------|-------------|--------------|------------------------|------------------------------|--------------------------------------|--|--|--|
| B1-CS | Unstrengthen Beam          | 630         | 864          | 2.14                   |                              |                                      |  |  |  |
| B5-IS | inclined stirrups          | 700         | 1015         | 2.27                   | 22%                          | 7%                                   |  |  |  |

| Table 3. Results of Group 1 | Ш |
|-----------------------------|---|
|-----------------------------|---|

The corresponding deflections at the ultimate load were 1.85 and 2.03 mm for the control specimen and B5-IS, respectively. The max deflections were 2.14 and 2.27 mm, respectively as shown in Figure 6.

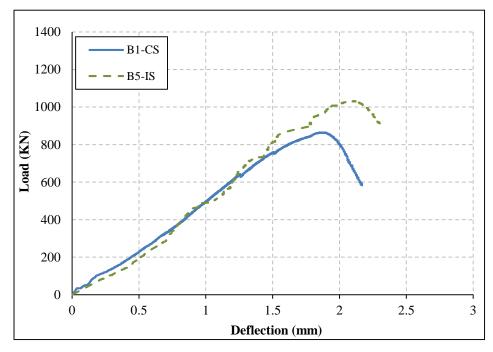


Figure 9. Deflection of Group III

## 4. Results Discussion

### 4.1. Load and Max Deflection of Tested Specimens

The results of the six tested deep beams are compared in terms of ultimate load capacity and max deflection. Figure 10 shows Load Capacity and Deflection of all Tested Beams. It can be seen from this figure that beam (B3-SC) with an embedded single strut and high ratio of vertical stirrups had the highest value for ultimate capacity and the associated maximum deflection. The dense stirrups and inclined struts helped to delay the shear failure. Beam (B2-EB) with double struts came second highest ultimate load. The CFRP strengthening method used in the beam (B4-FR) showed a 34% increase in the ultimate load capacity compared to the control beam without strengthening. The results show that using inclined stirrups (Beam B5-IS) presented the lowest increase in ultimate capacity by 19% compared to the control beam.

### 4.2. Absorbed Energy Capacity

The absorbed energy capacity was calculated by integrating the area under the load-deflection curve for all tested specimens. Beam (B3-SC) with duplicated vertical stirrups and single strut reinforcement shows the highest increase in absorbed energy capacity by 84% in comparison with control specimen, Also Beam (B2-ER) with embedded diagonal double strut shows increase in absorbed energy capacity by 83%. For beams (B4-FR), (B5-IS) and (B6-HC) shows increasement of absorbed energy capacity by 31%, 22%, and 3%, respectively.

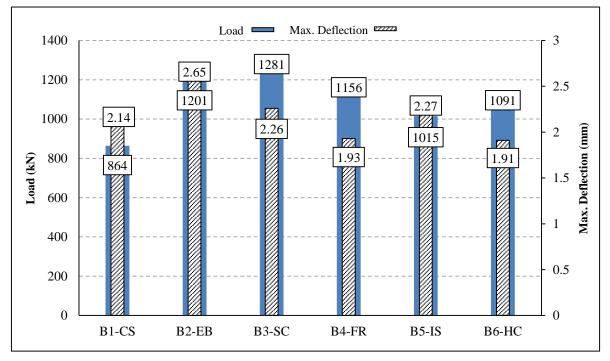


Figure 10. Load Capacity and Deflection of all Tested Beams

## 4.3. Initial Stiffness

The initial stiffness was determined as the slope of the initial straight line of the load-deflection curve for all tested specimens. Beam (B6-HC) with duplicated horizontal reinforcement and single strut reinforcement shows the highest incensement of initial stiffness by 31.3%. Beams (B2-ER), (B3-SC), (B4-FR) and (B5-IS) shows incensement of initial stiffness by 17.3 %, 26.7%, 9.2% and 7%, respectively.

## 4.4. Cost of Tested Beams

The cost of all tested beams were calculated on USD including labor, equipment, materials and strengthening materials cost. The cost effectiveness for each technique of strengthening is evaluated and listed in Table 3.

| Beams | Concrete | Rebar | CFRP | wood | Transportation | Labor | Total |
|-------|----------|-------|------|------|----------------|-------|-------|
| B1-CS | 11       | 31    | 0    | 22   | 9              | 23    | 96    |
| B2-EB | 11       | 39    | 0    | 22   | 9              | 23    | 104   |
| B3-SC | 11       | 39    | 0    | 22   | 9              | 23    | 104   |
| B4-FR | 11       | 28    | 252  | 22   | 9              | 35    | 357   |
| B5-IS | 11       | 33    | 0    | 22   | 9              | 23    | 98    |
| B6-HC | 11       | 41    | 0    | 22   | 9              | 23    | 106   |

| Table 3. Cost Analysis of Beams on US | SD | (\$) |
|---------------------------------------|----|------|
|---------------------------------------|----|------|

The results indicate that the CFRP strengthening is the most expensive technique. The cost of increasing 1% of the capacity using CFRP needs to pay 8.21 USD. Although this technique is very expensive compared to other techniques, it provides convenient solution for existing structures where we need external strengthening. This is in additional to the fact that CFRP gives easy and rapid solution to strengthen a structure. The use of inclined stirrups (Beam B5-IS) provided the cheapest technique for strengthening. According to this analysis, it increase in strength costs only 0.11 USD as per Table 4.

### Table 4: Ratio between increased capacity and cost

| Beam  | Ultimate load<br>(kN) | Increase in<br>Capacity (%) | Cost (\$) | Increase in<br>cost (%) | Cost of 1% increase<br>in strength (\$) |
|-------|-----------------------|-----------------------------|-----------|-------------------------|---|
| B1-CS | 864                   | Control Beam                | 96        | Control                 | Control                                 |
| B2-EB | 1201                  | 39%                         | 104       | 8.33%                   | 0.21                                    |
| B3-SC | 1281                  | 48%                         | 104       | 8.33%                   | 0.21                                    |
| B4-FR | 1156                  | 34%                         | 375       | 290%                    | 8.21                                    |
| B5-IS | 1030                  | 19%                         | 98        | 2.1%                    | 0.11                                    |
| B6-HC | 1091                  | 26%                         | 106       | 10.4%                   | 0.38                                    |

## 5. Conclusions

According to the results obtained from this experimental work, the following conclusions can be made:

- Using embedded inclined struts increase capacity by 39 % compared to the deep beam control specimen. Also, led to increasing the toughness by 83% also increased the initial stiffness by 17.3%. This beam has the smallest deflection at mid span.
- Increasing stirrups by double rate and using single strut reinforcing led to increasing beam capacity by 48.2 % compared to the control beam. Also, led to increasing the toughness by 84% also increased the initial stiffness by 26.7%. This beam has the highest increasing in toughness.
- Using CFRP sheets as external strengthening technique improved shear strength capacity by 33.7% in comparison with control specimen. Also, led to increasing the toughness by 31%, also increased the initial stiffness by 9.2%. The cost of this type of strengthening was the most expensive compared to the other techniques.
- Using of inclined stirrups improved shear strength capacity by 19.2% in comparison with the control specimen. Also, increased the toughness by 22% also the beam increased the initial stiffness by 7%. This type of strengthening was the cheapest among the strengthening techniques investigated in this research. This beam has the smallest increasing in toughness.
- Using of additional horizontal sidebars with bars, single inclined strut led to increasing capacity by 26.2% in comparison with control specimen. And increased the toughness by 3%, also increased the initial stiffness by 31.3%. This beam has the smallest deflection at mid span.

The following points can be recommended from the current tests.

- Using the mechanism of increasing stirrups by double rate and using single strut reinforcing is the optimum choice although it is not the cheapest one. This is due to the fact that this type of strengthening provides significant increase in the beam capacity in additional to the enhanced behavior of the beam.
- Using embedded beams as inclined embedded strut is the second optimum choice of strengthening.
- Using the mechanism of CFRP sheets as external strengthening is a good and convenient choice for existing structural elements despite of the high cost of the materials used.

# 6. Conflicts of Interest

The authors declare no conflict of interest.

# 7. References

- [1] Khatab, M.A.T., A.F. Ashour, T. Sheehan, and D. Lam. "Experimental Investigation on Continuous Reinforced SCC Deep Beams and Comparisons with Code Provisions and Models." Engineering Structures 131 (January 2017): 264–274. doi:10.1016/j.engstruct.2016.11.005.
- [2] Londhe, Poonam A., and C. M. Deshmukh. "Strengthening of Deep Beam Using FRP Wrapping: A Review." SKN International Journal of Scientific and Multidisciplinary Engineering Research (SKNIJSMER): 26, 2017.
- [3] Tetta, Zoi C., Lampros N. Koutas, and Dionysios A. Bournas. "Textile-Reinforced Mortar (TRM) Versus Fiber-Reinforced Polymers (FRP) in Shear Strengthening of Concrete Beams." Composites Part B: Engineering 77 (August 2015): 338–348. doi:10.1016/j.compositesb.2015.03.055.
- [4] Rasheed, M.M. "Retrofit of Reinforced Concrete Deep Beams with Different Shear Reinforcement by Using CFRP." Civil and Environmental Research Vol.8, No.5, (2016).
- [5] El Maaddawy, Tamer, and Sayed Sherif. "FRP Composites for Shear Strengthening of Reinforced Concrete Deep Beams with Openings." Composite Structures 89, no. 1 (June 2009): 60–69. doi:10.1016/j.compstruct.2008.06.022.
- [6] Alsadat Asghari, Azadeh, Zahra Tabrizian, Morteza Hossein Ali Beigy, Gholamreza Ghodrati Amiri, and Bahram Navayineya. "An experimental study on shear strengthening of RC lightweight deep beams using CFRP." Journal of Rehabilitation in Civil Engineering 2, no. 2 (2014): 9-19. doi:10.22075/JRCE.2014.204.
- [7] Lee, H.K., S.H. Cheong, S.K. Ha, and C.G. Lee. "Behavior and Performance of RC T-Section Deep Beams Externally Strengthened in Shear with CFRP Sheets." Composite Structures 93, no. 2 (January 2011): 911–922. doi:10.1016/j.compstruct.2010.07.002.
- [8] Burningham, Clayton A., Chris P. Pantelides, and Lawrence D. Reaveley. "Repair of Reinforced Concrete Deep Beams Using Post-Tensioned CFRP Rods." Composite Structures 125 (July 2015): 256–265. doi:10.1016/j.compstruct.2015.01.054.

- [9] Breveglieri, M., A. Aprile, and J.A.O. Barros. "Embedded Through-Section Shear Strengthening Technique Using Steel and CFRP Bars in RC Beams of Different Percentage of Existing Stirrups." Composite Structures 126 (August 2015): 101–113. doi:10.1016/j.compstruct.2015.02.025.
- [10] Mostofinejad, Davood, and Masoud Shameli. "Performance of EBROG Method under Multilayer FRP Sheets for Flexural Strengthening of Concrete Beams." Procedia Engineering 14 (2011): 3176–3182. doi:10.1016/j.proeng.2011.07.401.
- [11] Chen, Hui, Wei-Jian Yi, and Hyeon-Jong Hwang. "Cracking Strut-and-Tie Model for Shear Strength Evaluation of Reinforced Concrete Deep Beams." Engineering Structures 163 (May 2018): 396–408. doi:10.1016/j.engstruct.2018.02.077.
- [12] Liu, Jian, and Boyan I. Mihaylov. "A Comparative Study of Models for Shear Strength of Reinforced Concrete Deep Beams." Engineering Structures 112 (April 2016): 81–89. doi:10.1016/j.engstruct.2016.01.012.
- [13] Hanoon, Ammar N., M.S. Jaafar, Salah R. Al Zaidee, Farzad Hejazi, and F.N.A. Abd Aziz. "Effectiveness Factor of the Strutand-Tie Model for Reinforced Concrete Deep Beams Strengthened with CFRP Sheet." Journal of Building Engineering 12 (July 2017): 8–16. doi:10.1016/j.jobe.2017.05.001.
- [14] Hussein, G., Sayed H. Sayed, Nasr E. Nasr, and Ahmed M. Mostafa. "Effect of Loading and Supporting Area on Shear Strength and Size Effect of Concrete Deep Beams." Ain Shams Engineering Journal 9, no. 4 (December 2018): 2823–2831. doi:10.1016/j.asej.2017.11.002.
- [15] Yoo, Doo-Yeol, and Jun-Mo Yang. "Effects of Stirrup, Steel Fiber, and Beam Size on Shear Behavior of High-Strength Concrete Beams." Cement and Concrete Composites 87 (March 2018): 137–148. doi:10.1016/j.cemconcomp.2017.12.010.
- [16] Moradi, M., and M. Reza Esfahani. "Application of the Strut-and-Tie Method for Steel Fiber Reinforced Concrete Deep Beams." Construction and Building Materials 131 (January 2017): 423–437. doi:10.1016/j.conbuildmat.2016.11.042.
- [17] Islam, M.R., M.A. Mansur, and M. Maalej. "Shear Strengthening of RC Deep Beams Using Externally Bonded FRP Systems." Cement and Concrete Composites 27, no. 3 (March 2005): 413–420. doi:10.1016/j.cemconcomp.2004.04.002.
- [18] Hu, Biao, and Yu-Fei Wu. "Effect of Shear Span-to-Depth Ratio on Shear Strength Components of RC Beams." Engineering Structures 168 (August 2018): 770–783. doi:10.1016/j.engstruct.2018.05.017.
- [19] Albidah, Abdulrahman, Aref Abadel, Husain Abbas, Tarek Almusallam, and Yousef Al-Salloum. "Experimental and Analytical Study of Strengthening Schemes for Shear Deficient RC Deep Beams." Construction and Building Materials 216 (August 2019): 673–686. doi:10.1016/j.conbuildmat.2019.05.024.
- [20] Zhang, Fasheng, Yining Ding, Jing Xu, Yulin Zhang, Weiqing Zhu, and Yunxing Shi. "Shear Strength Prediction for Steel Fiber Reinforced Concrete Beams Without Stirrups." Engineering Structures 127 (November 2016): 101–116. doi:10.1016/j.engstruct.2016.08.012.
- [21] Colotti, Vincenzo. "Mechanical Shear Strength Model for Reinforced Concrete Beams Strengthened with FRP Materials." Construction and Building Materials 124 (October 2016): 855–865. doi:10.1016/j.conbuildmat.2016.07.146.
- [22]. Dhahir, Mohammed Kareem. "Shear Strength of FRP Reinforced Deep Beams Without Web Reinforcement." Composite Structures 165 (April 2017): 223–232. doi:10.1016/j.compstruct.2017.01.039.
- [23] Elsanadedy, Hussein M., Yousef A. Al-Salloum, Tarek H. Almusallam, Abdulhafiz O. Alshenawy, and Husain Abbas. "Experimental and Numerical Study on FRP-Upgraded RC Beams with Large Rectangular Web Openings in Shear Zones." Construction and Building Materials 194 (January 2019): 322–343. doi:10.1016/j.conbuildmat.2018.10.238.
- [24] Chen, Cheng-Cheng, Keng-Ta Lin, and Yu-Jen Chen. "Behavior and Shear Strength of Steel Shape Reinforced Concrete Deep Beams." Engineering Structures 175 (November 2018): 425–435. doi:10.1016/j.engstruct.2018.08.045.
- [25] Amin, Ali, and Stephen J. Foster. "Shear Strength of Steel Fibre Reinforced Concrete Beams with Stirrups." Engineering Structures 111 (March 2016): 323–332. doi:10.1016/j.engstruct.2015.12.026.