中京大学経済学論叢
9号
1997年11月

Illegal Migrants in an Overlapping-Generations Model

Kenji Kondoh*

1. Introduction

The pioneer work concerning migration in an overlapping generations model is Galor (1986). He proved that labour migrated unilaterally from the high (low) to the low (high) time preference country if both countries were under (over) invest relative to the Golden Rule, and that bilateral migration, a theoretical novelty, occurred if the countries were on opposite sides of the Golden Rule. According to his conclusion, under unrestricted unilaterally migration, the entire population of the host country will immigrate to the home country in the long run. (See Lemma 7 and its corollary of Galor (1986).) Karayalcin (1994) analyzed the case of temporary migrants in a similar framework, but his conclusion differed from Galor (1986). Under temporary migration, because the saving of emigrants are invested in the country of origin, thereby raising the capital-labour ratio and wage rate in this country, the entire population will not immigrate.

Galor also concluded that in the case of unilateral migration, the welfare of non-migrants (and their descendants) in the host country would be worsened by immigration, and he only investigated the case of amount restriction policy to prevent the flood of migrants. But in general case, if the authority of the host country knows that immigrants are harmful, immigration will be prohibited and preventing policies to check illegal entrants will be adopted. Now we can remember two policies to prevent illegal immigrants, such as border enforcement and internal enforcement, shown by Ethier (1986). And in this paper, we try to show the effects of border enforcement policy on the factor prices and welfare of the host country in an overlapping generations model which is along the line of Galor (1986).

The model of the world economy is constructed in Section 2. Section 3 is devoted to the main analysis. Conclusion is proposed in Section 4.

2. The Model

Let Galor's framework be adopted. So we consider a world economy which consists of two countries indexed by i, $i=A, B$. In each period two factors of production, capital and labour, and a single good are available. The prices of labour and capital in country i are w^i and r^i, and their quantities are L^i and K^i, respectively; output is the numeraire. For each country the endowment of labour in every period is exogenous, whereas capital is the output produced but not consumed in the proceeding period. Capital depreciates fully after one period.

During each period $\overline{L^i}$ individuals are born in country i. An individual lives two periods. A young individual in country i supplies his endowment of labour inelastically and earns the competitive market wage in the first period and allocates his income between first period consumption, c_1^i, and savings, s^i, so as to maximize his utility function $u^i\,(c_1^i, c_2^i)$, where $c_2^i = r^i s^i$. His optimal behaviour is to solve

$$\underset{c_1^i,\, c_2^i}{Max}\; u\,(c_1^i, c_2^i) \qquad s.t.\, c_1^i + (1/r^i)c_2^i \leq w^i$$
$$c_1^i \geq 0,\; c_2^i \geq 0$$

For all (w^i, r^i), the solution of the above optimization problem exists unique, which satisfies the following first order conditions,

$$u_1^i = \lambda^i$$
$$u_2^i = (1/r^i)\,\lambda^i$$
$$c_1^i + (1/r^i)c_2^i = w^i$$

Production function, $F\,(L^i, K^i)$ is neoclassical, constant return to scale and invariant across countries. The solution for producers' profit-maximization problem is

$$F_L(L^i, K^i) = w^i$$
$$F_K(L^i, K^i) = r^i$$

Now we consider the autarkic equilibria. Then, by using $F(L, K) = Lf(k)$, $k = K/L$, $L^i = \overline{L^i}$ and $K^i = s^i\,\overline{L^i}$, we obtain autarkic steady-state factor prices in country i which are determined as a function of the country's steady-state capital-labour ratio.

$$f(s^i) - f'(s^i)s^i = \hat{w}^i$$
$$f'(s^i) = \hat{r}^i$$

where " ^ " denotes the autarkic equilibria.

Now we proceed our analysis to international migration. Consider the case that the rate of time preference in country A is higher than that in country B. Then according to Galor's Proposition 1, we obtain $\hat{s}^A < \hat{s}^B$, $\hat{w}^A < \hat{w}^B$, and $\hat{r}^A > \hat{r}^B$. We also suppose that the autarkic equilibrium prices in country B, $(\hat{w}^B, \hat{r}^B)$, lies in zone $z_1 = \{(w, r) \mid \tilde{u}^B(w, r) \geq \overline{u}^B_m,\ r < \hat{r}^A\}$ of Fig.1, borrowing from Galor, where $\tilde{u}$ denotes a

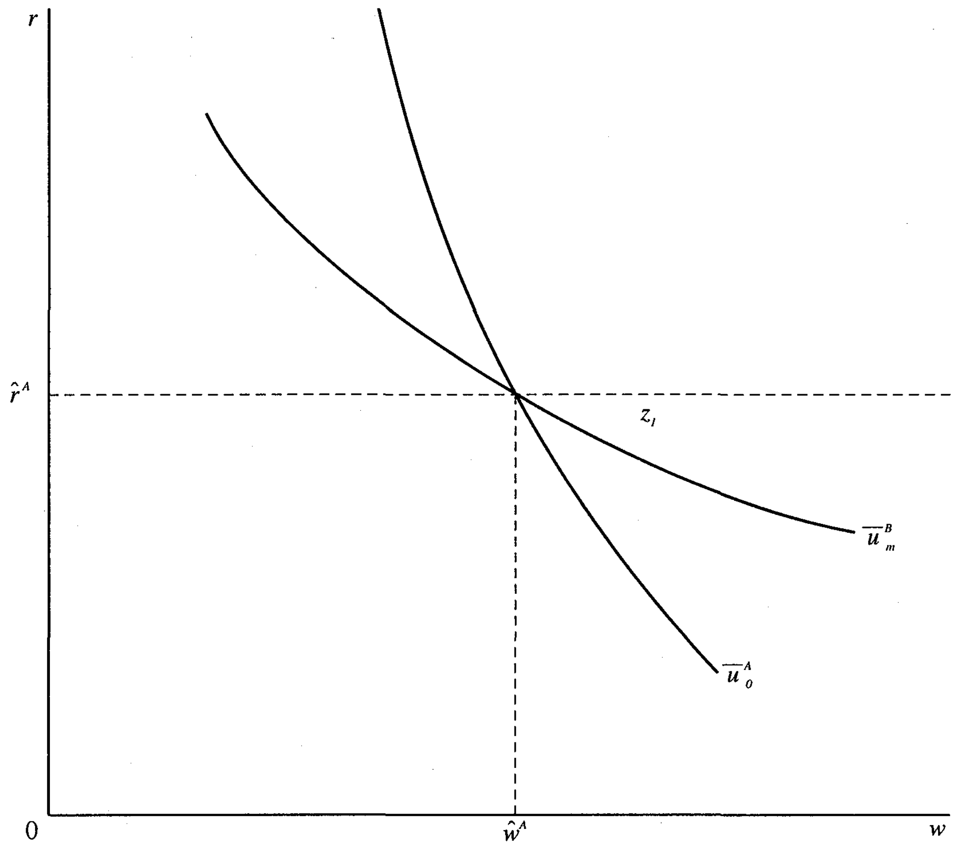

Fig. 1

migrant's utility function, $\overline{u}_0^i$ is the locus of all (w, r) that yield a level of utility $\overline{u}_0^i$ to individual i equal to that attained by him at the autarkic steady-state prices in his country, and $\overline{u}_m^i$ is the locus of all (w, r) that yield a level of utility $\overline{u}_m^i$ to individual i equal to that attained by him if he migrates and faces the autarkic steady-state prices in country j, $i \neq j$. As shown in Fig.1, the curve $\overline{u}_0^A$ is steeper than $\overline{u}_m^B$, as follows from Galor's Lemma 3. $(\hat{w}^B, \hat{r}^B)$ lies in the southeast quadrant relative to $(\hat{w}^A, \hat{r}^A)$ as follows from Galor's proposition 1. And in this case we have unilateral labour migration from country A to country B.

Borrowing the discussion of Galor, we assume that 1) migration is done at the beginning of one's first period, 2) the size and time preference of the population born in each country is unaffected by migration.

Suppose the case that the authority of country B begins the border check to prevent immigrants from country A. This check is not perfect enough to exclude all of immigrants and the possibility of success for each challenger is α $(0 < \alpha < 1)$. Now let us assume α is constant and exogenously given.

Each failed challenger, who should go back to country A, must pay the penalty β to the country B at the border. The authority of country B distributes β to the domestic workers' first period income evenly.

At the equilibrium of period t, we consider a part of young individuals $\mu^t \overline{L}^A$, $0 < \mu^t < 1$, tries to migrate.

The equilibrium conditions in country A are

$$
\begin{aligned}
&k_{t+1}^A = a s_t^A (w_t^A, r_{t+1}^A) + (1-a)\, s_t^{Af} (w_t^A - \beta, r_{t+1}^A), \\
&a = \frac{(1-\mu^t)\overline{L}^A}{(1-\mu^t)\overline{L}^A + \mu^t (1-\alpha)\overline{L}^A} = \frac{1-\mu^t}{1-\alpha\mu^t}, \qquad (1) \\
&r_{t+1}^A = f'(k_{t+1}^A), \\
&w_t^A = f(k_t^A) - f'(k_t^A)\, k_t^A,
\end{aligned}
$$

where s_t^{Af} denotes the savings of the failed challengers at the period t and $s_t^A > s_t^{Af}$.

The equilibrium conditions in country B are

$$\begin{aligned}
&k^B_{t+1}=bs^B_t(w^B_t+X,\ r^B_{t+1})+(1-b)s^{Bm}_t(w^B_t,\ r^B_{t+1}),\\
&b=\frac{\overline{L}^B}{\overline{L}^B+\alpha\mu^t\,\overline{L}^A},\\
&r^B_{t+1}=f'(k^B_{t+1}),\\
&w^B_t=f(k^B_t)-f'(k^B_t)k^B_t,
\end{aligned} \qquad (2)$$

where s^{Bm}_t denotes the saving of a succeeded immigrant at the period t and we may conclude $s^B_t > s^{Bm}_t$. X denotes the distributed income of a domestic worker and is exhibited as

$$X=\frac{\mu^t\,\overline{L}^A(\beta(1-\alpha)}{\overline{L}^B}.$$

3.The Analysis

The steady-state equilibrium factor prices in country i are calculated as

$$\begin{aligned}
&r^i=f'(k^i),\\
&w^i=f(k^i)-f'(k^i)k^i.
\end{aligned}$$

So, in the steady-state, we may rewrite (1) and (2) as

$$\begin{aligned}
&k^A=as^A(w^A,\ r^A)+(1-a)s^{Af}(w^A-\beta,\ r^A),\\
&a=\frac{(1-\mu)\,\overline{L}^A}{(1-\mu)\,\overline{L}^A+\mu(1-\alpha)\,\overline{L}^A}=\frac{1-\mu}{1-\alpha\mu},\\
&r^A=f'(k^A),\\
&w^A=f(k^A)-f'(k^A)k^A,\\
&k^B=bs^B(w^B+X,\ r^B)+(1-b)s^{Bm}(w^B,\ r^B),
\end{aligned} \qquad (1)'$$

$$b = \frac{\overline{L}^B}{\overline{L}^B + \alpha \mu \overline{L}^A}, \tag{2'}$$

$$r^B = f'(k^B),$$

$$w^B = f(k^B) - f'(k^B)k^B.$$

Following Galor, Q is the set of all combinations (w, r) such that $\left.\frac{dr}{dw}\right|_Q = -\frac{1}{k}$. for the given technology $f(k)$.

Since technology is identical across countries, the autarkic steady-state equilibrium in each country lies along Q. Thus, remembering that we assumed the case $(w^B, r^B) \in z_l$, our case is based on Fig.2, borrowing of Galor, and so the larger k^i means the higher utility of the inhabitants in country i because of the shape of indifference curves.

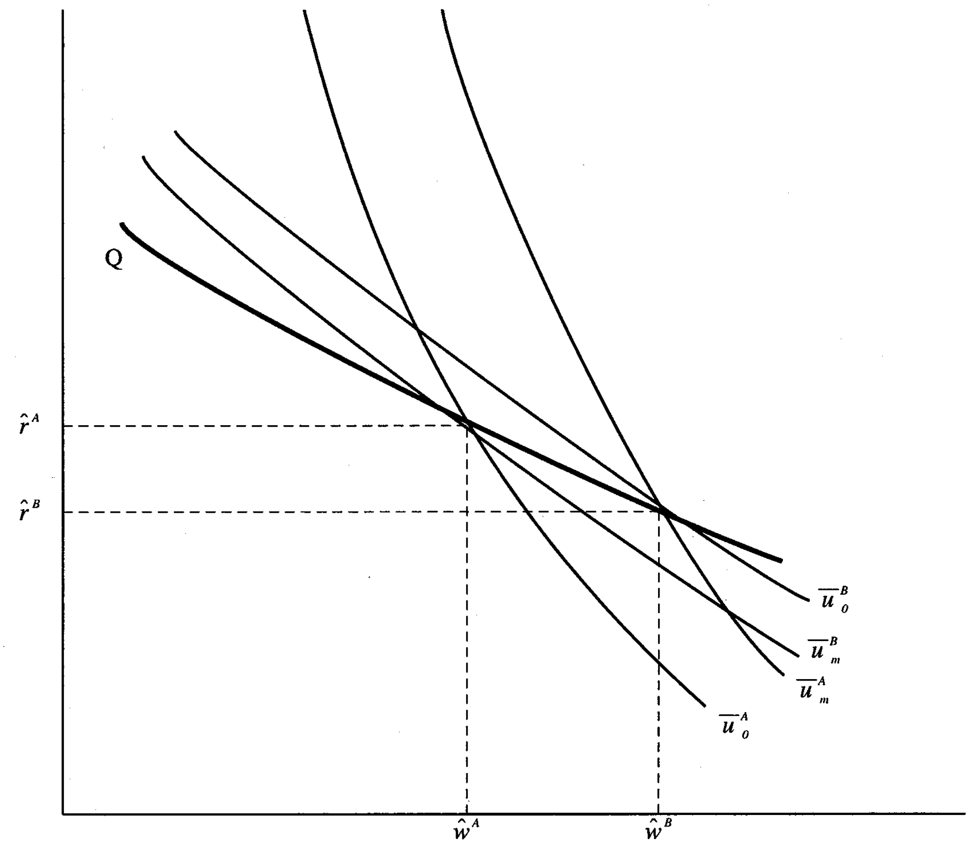

Fig. 2

For a young individual in country A, in the steady-state equilibrium, he is indifferent to migration. So we obtain the following condition,

$$U_1(w^A, r^A) = U_2(\alpha w^B + (1-\alpha)(w^A - \beta),\ \alpha r^B + (1-\alpha) r^A), \qquad (3)$$

where U_1 denotes the utility level of a non-challenger, and U_2 denotes the expected utility level of a challenger. Fig.3 shows this condition graphically.

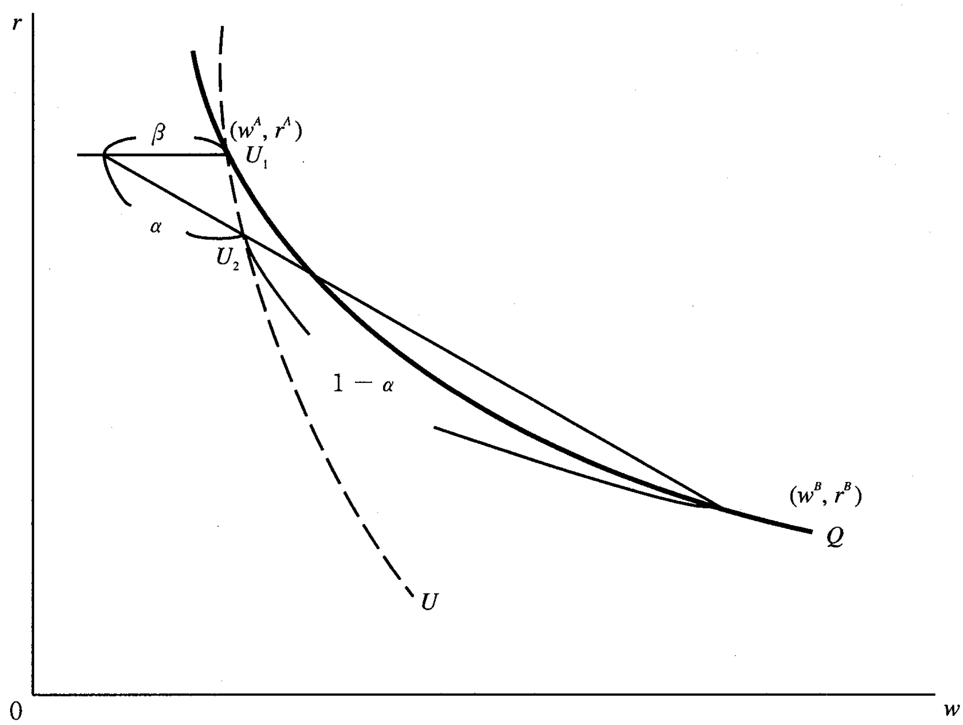

Fig. 3

Now, we assume $\overline{L}^A = \overline{L}^B$ in order to simplify our analysis.

Then by differentiating (1)', (2)' and (3) and taking account of $dr^i = f''(k^i)dk^i$ and $dw^i = k^i f''(k^i)dk^i$, we obtain the following equations.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} dk^A \\ dk^B \\ d\mu \end{bmatrix} = \begin{bmatrix} J \\ K \\ L \end{bmatrix} d\alpha + \begin{bmatrix} M \\ N \\ O \end{bmatrix} d\beta \qquad (4)$$

where

$$A = 1 + k^A f''(k^A) s_A^w - f''(k^A) s_A^r$$
$$B = 0$$
$$C = \frac{1-\alpha}{(1-\alpha\mu)^2}(s^A - s^{Af}) > 0$$
$$D = 0$$
$$E = 1 + \frac{f''(k_B)}{1+\alpha}\ [(s_B^w + \alpha\mu s_{Bm}^w)k_B - (s_B^r + \alpha\mu s_{Bm}^r)]$$
$$F = \frac{\alpha}{(1+\alpha\mu)^2}(s^B - s^{Bm}) - \frac{s_B^w}{1+\alpha\mu}\beta(1-\alpha)$$
$$G = -k^A f''(k^A) U_1^w + f''(k^A) U_1^r + (1-\alpha) k^A f''(k^A) U_2^w - (1-\alpha) f''(k^A) U_2^r$$
$$H = \alpha k^B f''(k^B) U_2^w - \alpha f''(k^B) U_2^r$$
$$I = 0$$
$$J = \frac{(1-\mu)\mu}{(1-\alpha\mu)^2}(s^A - s^{Af}) > 0$$
$$K = \frac{-\mu}{(1+\alpha\mu)^2}(s^B - s^{Bm}) - \frac{1}{(1+\alpha\mu)} s_B^w \beta\mu < 0$$
$$L = U_2^w(w^B - w^A + \beta) + U_2^r(r^B - r^A) > 0$$
$$M = \frac{\mu(1-\alpha)}{1-\alpha\mu} s_{Af}^w < 0$$
$$N = \frac{\mu(1-\alpha)}{1+\alpha\mu} s_B^w > 0$$
$$O = -U_2^w(1-\alpha) < 0$$
$$s_i^j \equiv \partial s^i / \partial j,\ i = A, B, j = w, r \quad \text{and} \quad U_l^j \equiv \partial U_l / \partial j,\ l = 1, 2\ j = w, r$$

Following Galor, we assume all s_i^j and U_l^j are positive in sign.

Now $H < 0$ because of $\partial U_2 / \partial k_B > 0$. We may conclude $G > 0$ because of the stability condition that if k^A increases (decreases) from the equilibrium point, then $U_1 > (<) U_2$. $A > 0$ and $E > 0$ are derived from the stability conditions of the steady-state shown in Lemma 1 of Galor [1)].

Let α be small enough to satisfy $F < 0$ and $\mu > \alpha$. The assumption $F < 0$ implies that we consider the case in which the probability of success for illegal immigration is quite low. (If α is large (small) enough and almost equal to unit (naught), the sign of F is clearly positive (negative).) Then the welfare reduction caused by the increasing amount of challengers is small enough not to overwhelm the welfare gain by distributed β.

Under the above conditions, we obtain the determinant of the LHS matrix in (4) is negative.

Now we proceed our analysis to the comparative statics. The effects of a change in α and β on k^B are as follows.

$$\frac{dk^B}{d\alpha}<0, \frac{dk^B}{d\beta}>0. \qquad (5)$$

[2]

So we obtain the following propositions.

Proposition 1. Consider the probability of success for illegal immigration is quite low. An increase in the possibility of success to illegal immigration lowers the wage rate, raises the rental price of capital and gives rise to a welfare loss in the host country.

Proposition 2. Consider the probability of success for illegal immigration is quite low. An increase in the penalty cost which should be paid by the failed challengers raises the wage rate, lowers the rental price of capital and gives rise to a welfare gain in the host country.

4. Concluding Remarks

According to the above results, in order to prevent the harmful immigrants and to raise the welfare level of the inhabitants in the host country, the useful policies are to make the penalty higher and to enhance the disclosure at the border. Those conclusions look quite normal but we should remark that they are valid only under the conditions $F<0$ and $\mu > \alpha$. In other words, we have no sure policy to raise the welfare level of the inhabitants in the host country if $F<0$ is large enough to satisfy $F\geq 0$. Our conclusions suggest it dangerous to rely on the results of the above preventing policies.

Appendix

$\triangle\dfrac{dk^B}{d\alpha}=G(JF-CK)-AFL$. As we have AFL is negative, G is positive, and both $K<F<0$ and $C>J>0$ if $\mu>\alpha$, we may coclude $\triangle\dfrac{dk^B}{d\alpha}>0$, and so $\dfrac{dk^B}{d\alpha}<0$.

$\triangle\dfrac{dk^B}{d\beta}=G(MF-CN)-AFO$. We have both AFO and G are positive.

We can conclude that $MF-CN<\dfrac{\mu(1-\alpha)^2}{(1-\alpha\mu)(1+\alpha\mu)}[s^w_B(\beta s^w_{Af}-\dfrac{1}{1-\alpha\mu}(s^A-s^{Af}))]$ <0, since $s^{ww}=\partial^2 s/\partial w^2>0$ in general. So we have $\triangle\dfrac{dk^B}{d\beta}<0$, and so $\dfrac{dk^B}{d\beta}>0$.

Footnotes

*) I am very grateful to Professor Makoto Tawada, Nagoya City University, for his advice and meaningful comments.

1) The autarkic steady-state equilibrium is locally stable if $1-f''(\hat{s}^i)[(\partial\hat{s}^i/\partial r^i)-(\partial\hat{s}^i/\partial w^i)\hat{s}^i]>0$.

2) See Appendix for derivation.

Reference

Ethier, W.J., 1986, Illegal Immigration-The Host Country Problem, *The American Economic Review*, 76, 56-71.

Galor, O., 1986, Time Preference and International Labor Migration, *Journal of Economic Theory*, 38, 1-20.

Karayalcin,C., 1994, Temporary and permanent migration with and without an immobile factor, *Journal of Development Economics*, 43, 197-215.

Abstract

By extending the framework of Galor (1986), this paper shows that making the penalty of illegal migrants higher or enhancing the disclosure at the border will surely raise the wage rate, lower the rental price of capital, and give rise to a welfare gain in the host country under the condition of the low possibility of success for illegal immigration. In other words, if the above condition is not satisfied, the validity of those policies are not assured.