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Speculative investment, heavy-tailed distribution and risk management of Bitcoin exchange rate returns

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Abstract:

Since its launch in 2008, Bitcoin becomes one of the most successful and fast-growing alternative currencies. As of 2017, the market capitalization is around $46 billion and arguably expected to continue growing. The Bitcoin to the US dollar exchange rate has been very volatile and fluctuating significantly. Although Bitcoin was designed as a medium of exchange, it is now more as an investment tool and thus the development of effective quantitative risk management tools becomes quite urgent for all the market participants. In this paper, we investigate empirical distribution of the Bitcoin exchange rate returns by using four types of widely-used heavy-tailed distribution and show that the Skewed *t* distribution has the best empirical performance. We further calculate the VaR based risk measures and found the Skewed *t* distribution generates the VaR values, which are closest to historical VaR values. Our results could be directly used in the industry's stress testing practice, and help financial institutions fulfill the regulatory requirements.

Keywords: Skewed *t* distribution; goodness of fit; Value at Risk; risk management; Bitcoin

JEL classification: C46; C58; G10

1. Introduction

Bitcoin is a digital currency invented in 2009. It follows the ideas initiated in a white paper under the name Satoshi Nakamoto, whose true identity has yet to be verified. The Bitcoin system is peer-to-peer, and transactions take place between users directly, without an intermediary. These transactions are identified by network nodes and recorded in a public distributed warehouse called a block chain. Since the system works without a central repository or single administrator, Bitcoin is called the first decentralized digital currency.

The exchange rate of Bitcoin to USD is currently above $2700, starting from almost nothing in 2010 as in Figure 1. The market capitalization is around $46 billion and arguably expected to continue growing as in Figure 2. The number of active Bitcoin users has increased significantly since its introduction to the market. According to research produced by Hileman and Rauchs (2017) from Cambridge University, there are between 2.9 million and 5.8 million unique users actively using a cryptocurrency wallet as of April 2017, most of them using Bitcoin. The number of active users has grown significantly since 2013, when there were only 0.3 to 1.3 million unique users. As investors and speculators are drawing to the Bitcoin market, it is more and more urgent for market participants and regulators to setup a rigorous framework to manage the risk.

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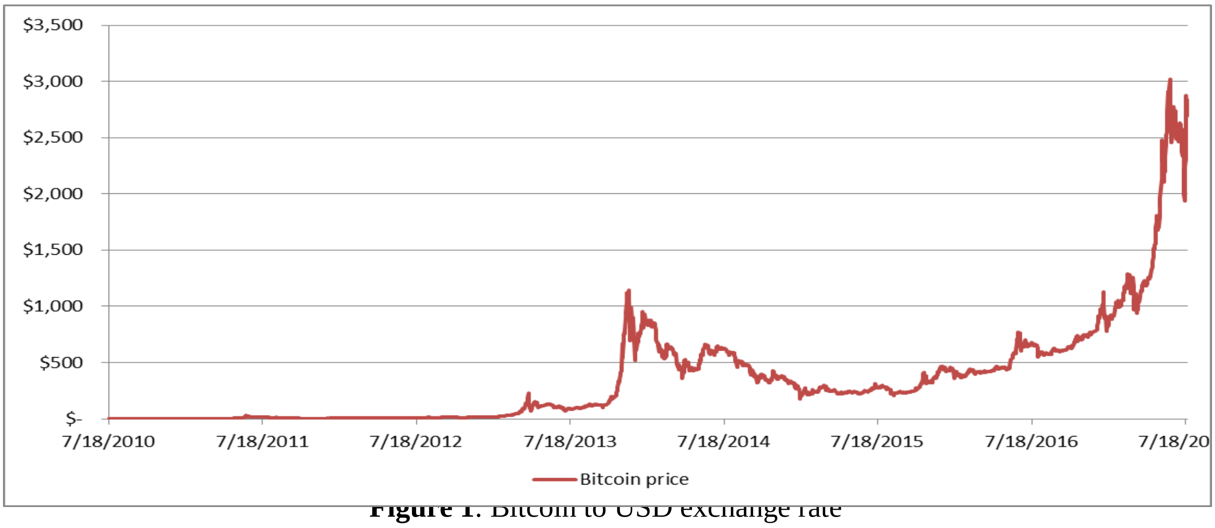

Figure 1: Bitcoin to USD exchange rate

Figure 2: Bitcoin market capitalization

Although Bitcoin was designed as a medium of exchange, it is now more as an investment tool since many speculative investors buy Bitcoin for its investment value rather than as a currency. But their lack of guaranteed value and digital nature means the purchase and use of Bitcoin carries several inherent risks. Moreover, the regulators do not have efficient tools to regulate the market as a lack of sufficient experience. Thus, there are a suite of risks for the Bitcoin market. First, Bitcoin is a rival to government currency and may be used for black market transactions, money laundering, illegal activities or tax evasion, and there is considerable regulatory risk. Second, investors face internet security risk, as Bitcoin exchanges are entirely digital and, as with any virtual system, are at risk from hackers, malware and operational glitches. Third, Bitcoin accounts are not insured by any type of federal or government program, so the insurance risk is substantial for small investors. Finally, as shown in Figure 1, Bitcoin values have increased more rapidly than most of the other assets, and thus investment in Bitcoin might face more serious market risk.

In this paper, we take advantage of the widely-used quantitative risk management tool, Value at Risk (VaR), to investigate risk management of Bitcoin exchange rate returns. As in Guo (2017a), we introduce four different types of heavy-tailed distribution, the Student's *t*, the Skewed *t*, the normal inverse Gaussian (NIG) and the generalized hyperbolic (GH) distributions. Our results indicate the Skewed *t* distribution has the best empirical performance in fitting the Bitcoin exchange rate returns and predicting suitable risk measures of VaR.

Literature Review

Although Bitcoin has only existed for a quite short history, there are still several economic and financial studies. Hencic and Gourieroux (2014) introduced a non-causal autoregressive process with Cauchy errors to fit the Bitcoin exchange rates and showed the model could model speculative bubbles of the exchange rates. Katsiampa (2017) explored the optimal conditional heteroskedasticity model with regards to goodness-of-fit to Bitcoin price data and found the AR-CGARCH model has the best performance. Bouri, et al. (2017) examined whether Bitcoin can hedge global uncertainty using wavelet-based quantile-in-quantile regression techniques, and showed hedging is observed at shorter investment horizons, but not significantly at longer investment horizons. Balcilar, et al. (2017) employed a non-parametric causality-in-quantiles test to analyze the causal relation between trading volume and Bitcoin returns and demonstrated that volume cannot help predict the volatility of Bitcoin returns. Bouri, et al. (2017) investigated the asymmetric GARCH models in fitting the Bitcoin exchange rate returns and showed the leverage effects are only observed for the period before the price crash of 2013.

All the above studies either focus on speculative bubbles of the Bitcoin exchange rates or using GARCH models to fit the volatility processes. In this paper, we investigate risk management of Bitcoin exchange rate returns. Our research is closely related to the work in Osterrieder and Lorenz (2017). Osterrieder and Lorenz employed an extreme value analysis of the returns of Bitcoin and focused on the tail risk characteristics, the authors found that the Bitcoin return distribution not only exhibits higher volatility than traditional G10 currencies, but also stronger non-normal characteristics and heavier tails. Our research differs from Osterrieder and Lorenz (2017) in two aspects. First, we follow the approach in Guo (2017a) and are interested in several widely-used heavy-tailed distributions in empirically fitting the Bitcoin exchange rate returns. Second, we apply the concept of Value at Risk and calculate the values of risk measure and our results could be directly used in the industry's stress testing practice.

The four types of heavy-tailed distribution in Guo (2017a) include: the Student's *t*, Skewed *t*, normal inverse Gaussian (NIG) and generalized hyperbolic (GH) distributions. The Skewed *t* distribution was introduced in Hansen (1994), and the GH distribution was developed by Barndorff-Nielsen (1977). We are interested in the NIG distribution as it is one of the most popular subclasses of the GH distribution in the finance literature (see Figueroa-Lopez, et al., 2011, for a survey). In this paper, we reconsider these four distributions but focus on the Bitcoin exchange rate returns. In Section 2, we summarize the heavy-tailed distributions. Section 3 discusses the data. The estimation results are in Section 4. Finally, we conclude in Section 5.

2. Heavy-tailed Distributions

Similar as in Guo (2017a), four types of widely-used heavy-tailed distribution in addition to the normal distribution are studied: (i) the Student's *t* distribution; (ii) the Skewed *t* distribution; (iii) the normal inverse Gaussian distribution (NIG); and (iv) the generalized hyperbolic distribution (GH). All the distributions have been standardized to ensure mean and standard deviation equal to zero and one respectively. Their probability density functions are given as follows.

(i) Student's *t* distribution:

$$f(e_t \mid \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}} \left(1 + \frac{e_t^2}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \tag{1}$$

Where ν indicates degrees of freedom and e_t is daily equity market index return.

(ii) Skewed *t* distribution:

$$f(e_t \mid \nu, \beta) = \begin{cases} bc\left(1 + \frac{1}{\nu-2}\left(\frac{be_t + a}{1-\beta}\right)^2\right)^{-(\nu+1)/2} & e_t < -a/b \\ bc\left(1 + \frac{1}{\nu-2}\left(\frac{be_t + a}{1+\beta}\right)^2\right)^{-(\nu+1)/2} & e_t \geq -a/b \end{cases}, \tag{2}$$

Where e_t is the standardized log return, and the constants a, b and c are given by $a = 4\beta c\left(\frac{\nu-2}{\nu-1}\right)$, $b^2 = 1+3\beta^2 - a^2$, and $c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The density function has a mode of $-a/b$, a mean of zero, and a unit variance. The density function is skewed to the right when $\beta > 0$, and vice-versa when $\beta < 0$. The Skewed *t* distribution specializes to the standard Student's *t* distribution by setting the parameter $\beta = 0$.

(iii) Normal inverse Gaussian distribution (NIG):

$$f(e_t \mid \mu,\alpha,\beta,\delta) = \frac{\alpha\delta K_1(\alpha\sqrt{\delta^2+(e_t-\mu)^2})}{\pi\sqrt{\delta^2+(e_t-\mu)^2}}\exp(\delta\sqrt{\alpha^2-\beta^2}+\beta(e_t-\mu)), \quad (3)$$

Where $K_\lambda(\cdot)$ is the modified Bessel function of the third kind and index $\lambda = 0$ and $\alpha > 0$. The NIG distribution is specified as in Prause (1997). The NIG distribution is normalized by setting $\mu = -\frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}$ and $\delta = \frac{\left(\sqrt{\alpha^2-\beta^2}\right)^3}{\alpha^2}$, which implies $E(e_t) = 0$ and $Var(e_t) = 1$.

(iv) Generalized hyperbolic distribution:

$$f(e_t \mid p,b,g) = \frac{g^p}{\sqrt{2\pi}\left(b^2+g^2\right)^{\frac{1}{2}\left(p-\frac{1}{2}\right)} d(p,b,g) K_p(g)} q\left(\frac{e_t - m(p,b,g)}{d(p,b,g)};\ p,b,g\right), \quad (4)$$

Where $\tilde{R}_n \square \frac{K_{n+p}(g)}{g^n K_p(g)}$, $d(p,b,g) \square \left[\tilde{R}_1 + b^2\left\{\tilde{R}_2 - \tilde{R}_1^2\right\}\right]^{-\frac{1}{2}} \geq 0$ and $m(p,b,g) \square -b\, d(p,b,g)\tilde{R}_1$. p, b and g Are parameters. The generalized hyperbolic distribution is a standardized version of Prause (1997).

3. Data

The heavy tailed distributions are applied to estimate the normalized returns of the Bitcoin exchange rate returns. We collected the standardized daily Bitcoin exchange rate returns from Coindesk (www.coindesk.com) for the period from July 19, 2010 to July 23, 2017, covering all the available data in Coindesk. There are in total 2562 observations. Figure 3 illustrates the dynamics of daily Bitcoin exchange rate returns. The daily exchange rate returns are very volatile and exhibits significant volatility clustering.

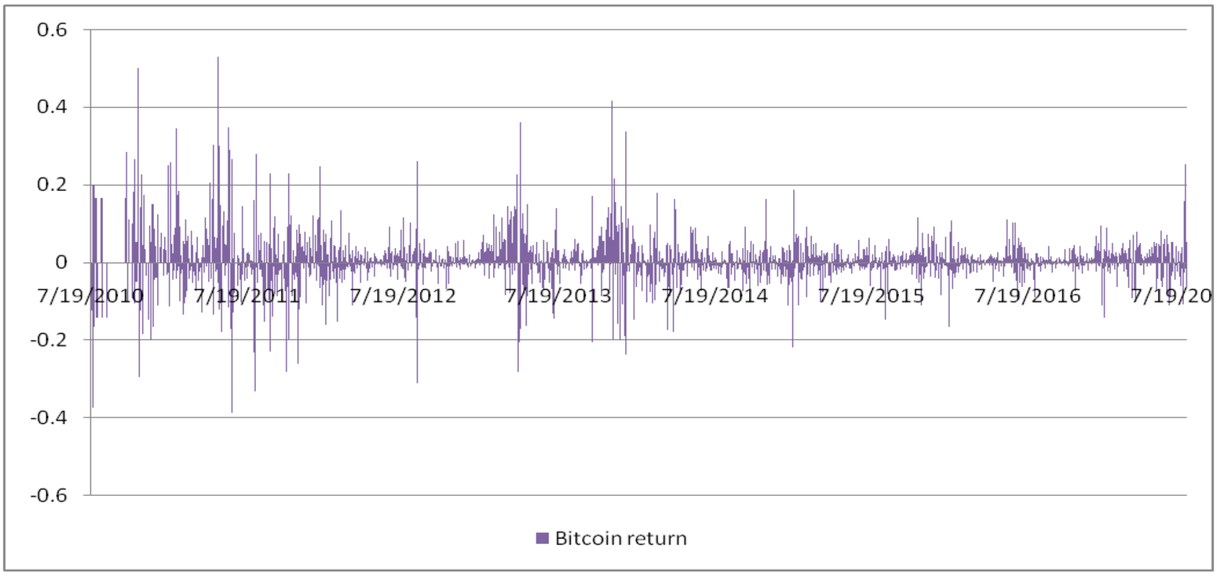

Figure 3: Bitcoin exchange rate returns dynamics

Table 1 presents basic statistics of the daily Bitcoin exchange rate returns. The results show the daily returns are leptokurtotic and positively skewed. The extreme downside move is slightly less than the extreme upside move, which is at odds with most of the major financial assets over the world.

min	max	mean	std	skewness	kurtosis
-38.83%	52.89%	0.58%	5.95%	0.86	12.73

Table 1: Descriptive statistics

Figure 4 is the histogram of the raw data. The distribution of the daily Bitcoin exchange rate returns is compared with the Gaussian distribution and the figure clearly exhibits heavy tails.

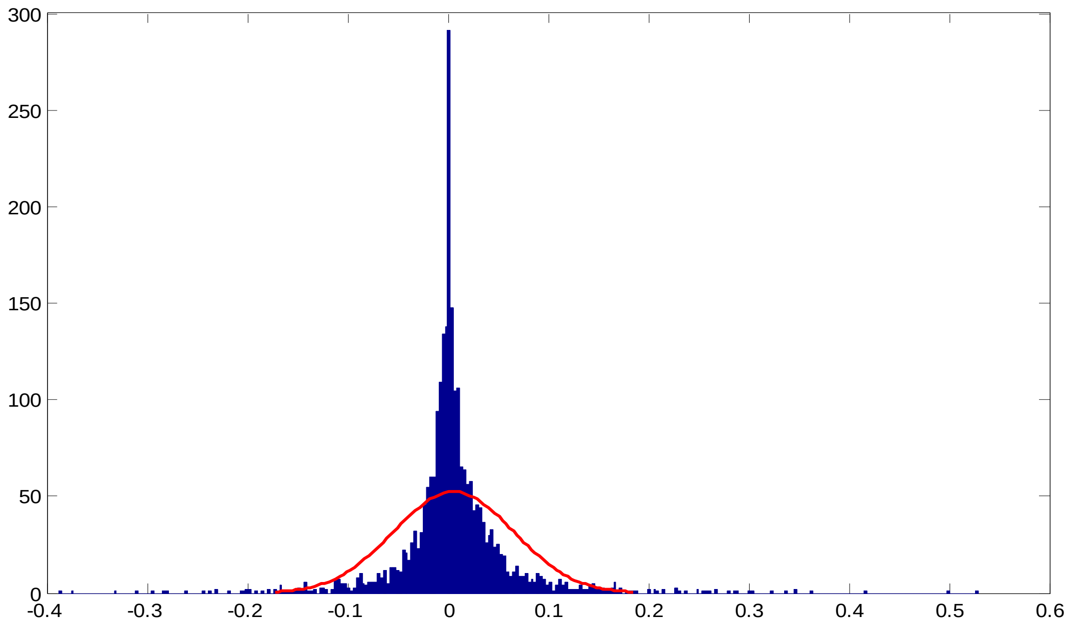

Figure 2: Histogram of Bitcoin exchange rate returns

4. Empirical Results

4.1 Parameters Estimation

We normalize the raw return series to have zero mean and unit standard deviation. The maximum likelihood estimation (MLE) method is adopted and the estimation results of the key parameters are given in Table 2. All the parameters are significantly different from zero at 5% significance level.

	Normal	Student's *t*	Skewed *t*	NIG	Generalized Hyperbolic
Symmetric	Y	Y	N	N	N
Fat-tailed	N	Y	Y	Y	Y
Estimated Parameters		Nu=2.47	Nu=2.53; beta=0.072	alpha=1.18; beta=0.103	p=-1.09; b=.068; g=0.04

Table 2: Estimated values of key parameters

4.2 Goodness of Fit

As discussed in Huber-Carol, et al. (2002) and Taeger and Kuhnt (2014), we compare the four heavy-tailed distributions and the benchmark normal distribution in fitting the daily Bitcoin exchange rate returns by four different criteria: (i) Kolmogorov-Smirnov statistic; (ii) Cramer-von Mises criterion; (iii) Anderson-Darling test; and (iv) Akaike information criterion (AIC).

(i) Kolmogorov-Smirnov statistic is defined as the maximum deviation between empirical CDF (cumulative distribution function) $F_n(x)$ and tested CDF $F(x)$:

$$D_n = \sup_x | F_n(x) - F(x) |, \tag{5}$$

Where $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{[-\infty,x]}(X_i)$.

(ii) Cramer-von Mises criterion is defined as the average squared deviation between empirical CDF and tested CDF:

$$T = n\int_{-\infty}^{\infty} [F_n(x) - F(x)]^2 dF(x) = \frac{1}{12n} + \sum_{i=1}^{n}\left[\frac{2i-1}{2n} - F_n(x_i)\right]^2, \tag{6}$$

(iii) Anderson-Darling test is defined as the weighted-average squared deviation between empirical CDF and tested CDF:

$$A = n\int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1-F(x))} dF(x),$$

And the formula for the test statistic A to assess if data comes from a tested distribution is given by:

$$A^2 = -n - \sum_{i=1}^{n} \frac{2i-1}{n}\left[\ln(F(x_i)) + \ln(1-F(x_i))\right]. \tag{7}$$

(iv) Akaike information criterion (AIC) is defined as:

$$AIC = -2k - 2\ln(L), \tag{8}$$

Where L the maximum value of the likelihood is function for the model, and k is the number of estimated parameters in the model.

The comparison results are showed in Table 3, indicating the Skewed *t* distribution has the best goodness of fit compared with other selected types of distribution, followed by the generalized hyperbolic distribution, and the Student's *t* distribution.

	Normal	Student's *t*	Skewed *t*	NIG	Generalized Hyperbolic
K-S Test	0.054	0.047	0.046	0.048	0.047
Cv-M Test	0.135	0.130	0.128	0.131	0.129
A-D Test	2.33	2.16	2.10	2.14	2.11
AIC	10689	10320	10184	10392	10264

Table 3: Comparison of selected types of distribution

4.3 Value at Risk

In this subsection, we take advantage of the widely-used quantitative risk management tool, Value at Risk, to calculate risk measures for the daily Bitcoin exchange rate returns. VaR is originally developed by JP Morgan in 1994. In quantitative risk management, VaR is defined as: for a given position, time horizon, and probability *p*, the *p* VaR is defined as a threshold loss value, such that the probability that the loss on the position over the given time horizon exceeds this value is *p*.with the estimated parameters in Section 4.1, we calculate VaRs for different confidence levels:

$$VaR_{\alpha}(e_t) = \inf\{e \in \mathbb{R} : P(e_t > e) \leq 1-\alpha\}, \qquad (9)$$

Where $\alpha \in (0,1)$ is the confidence level. We select the following levels for downside moves: {99.99%, 99.95%, 99.9%, 99.5%}, and for upside moves: {0.01%, 0.05%, 0.1%, 0.5%}. From Equation (9), the VaR levels are given as in Table 4. Table 4 indicates that the Skewed *t* distribution has the closest VaRs to the nonparametric historical VaRs compared with other types of distributions.

Left Tail					
Confidence	99.99%	99.95%	99.90%	99.50%	99.00%
Empirical	-40.46%	-38.02%	-32.77%	-28.14%	-25.38%
Normal	-28.71%	-26.89%	-25.96%	-24.45%	-23.51%
T	-39.27%	-36.46%	-34.38%	-30.38%	-27.57%
Skewed T	-40.15%	-38.18%	-31.99%	-28.45%	-25.85%
NIG	-36.25%	-34.48%	-31.62%	-30.01%	-27.41%
GH	-39.37%	-36.88%	-33.39%	-29.28%	-26.37%
Right Tail					
Confidence	0.01%	0.05%	0.10%	0.50%	1.00%
Empirical	47.75%	41.36%	39.09%	34.26%	31.42%
Normal	28.71%	26.89%	25.96%	24.45%	23.51%
T	39.27%	36.46%	34.38%	30.38%	27.57%
Skewed T	48.96%	42.35%	39.94%	34.69%	31.85%
NIG	50.80%	47.04%	42.28%	38.95%	35.82%
GH	53.29%	47.68%	43.13%	38.66%	34.05%

Table 4: Scenarios for daily Bitcoin exchange rate shocks

5. Limitation of the Study

This study only investigated the issue of heavy tails of financial data and did not consider the volatility clustering effect as in Guo (2017b, 2017c).

6. Conclusions

As Bitcoin reaches a market cap exceeding $7 billion and has the potential to become an investable asset class, the development of effective quantitative risk management tool become even more urgent. In this paper, we apply several popular types of heavy-tailed distribution, the Student's *t*, Skewed *t*, normal inverse Gaussian distribution (NIG) and generalized hyperbolic distribution (GH) in fitting daily Bitcoin exchange rate returns. Our results indicate that the Skewed *t* distribution has the best empirical performance. We further calculate the VaR based risk measures and found the Skewed *t* distribution generates VaR values mostly closest to historical VaR values. Our results could be directly used in the industry's stress testing practice, and help financial institutions fulfill the regulatory requirements.

7. Future Research

There are two possible extensions for future research. First, one can compare our finds with the results in Osterrieder and Lorenz (2017), which are based on the extreme value theory. Second, if we incorporate the heavy-tailed distributions into the GARCH framework as in Guo (2017b, 2017c), it would further contribute to the literature.

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