Journal of Progressive Research in Mathematics(JPRM)
ISSN: 2395-0218

SCITECH
RESEARCH ORGANISATION

Volume 15, Issue 3
Published online: November 04, 2019|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

Sampling Plans for Generalized Pareto Distribution

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Abstract

In this paper, two sampling plans have been discussed, when the lifetime of the items follows the generalized Pareto distribution, Chain Sampling Plan (ChSP-1) and an improved group acceptance sampling plan (IGASP) using weighted binomial. The minimum number of sample size (n) and minimum number of groups (g) are obtained for two plans respectively. The plans are explain with the help of examples whereas the proposed plan compared with Aslam et al. (2010). The results and comparison are discussed with the help of tables and figure. It observed that sometime proposed plan Improved group acceptance sampling plan (IGASP) using weighted binomial, when the lifetime of the test items follows generalized Pareto distribution showed better results than existing plan, under the same parameter settings.

Keywords: Chain Sampling Plan, Improved Group sampling plan, generalized Pareto distribution.

Introduction

The acceptance sampling plans are concerned with accepting or rejecting a submitted lot of a size of products on the basis of quality of the products inspected in a sample taken from the lot
In most acceptance sampling plans for a truncated life test, major issue is to determine the sample size from the lot under consideration. Sampling inspection in which the criteria for acceptance and non- acceptance of the lot depend in part on the results of the inspection of immediately preceding lots is adopted in Chain Sampling Plan.

The chain sampling plan (ChSP-1) proposed by Dodge (1955), making use of cumulative results of several samples helps to overcome the shortcomings of single sampling plan. It avoids rejection of a lot on the basis of a single nonconforming unit and improves the poor discrimination between good and bad quality that occurs with the c = 0 plan.

The use of cumulative results of several samples is proposed for application to cases where there is repetitive production under the same conditions and where the lots or batches of products to be inspected are submitted for inspection in the order of production. Such situation may arise in receiving inspection of a continuing supply of purchased materials produced within a manufacturing plan. Chain sampling is not suited to intermittent or job lot production, or to occasional purchases. An example situation is a continuing supply of processed material, such as a particular type of copper alloy rod or sheet.

When large samples are not practicable, and the use of c = 0 plan is warranted, for example, when an extremely high quality is essential (extremely low per cent nonconforming units) the use of chain sampling plan is often recommended.

Although with the innovations in the acceptance sampling area the group acceptance sampling plan introduced. When a single item is put on test then it is known as single sampling plan. When we put more than one items on a tester which form a group and the number of items in a group known as group size(r) so, dealing with group is known as group sampling plan (GASP). It showed GASP has advantage over the ordinary sampling plan in terms of time and cost. The key concern is to minimize the number of groups (g) which is as similar as to minimize the sample size (n) when a single item is put on a tester (n = rg).

Aslam et al.(2010) constructed group acceptance sampling plan using binomial distribution when the lifetime of an item follows generalized Pareto distribution also Aslam et al.(2010) Group acceptance sampling plan Pareto distribution of the second kind. Aslam et al.(2011) constructed Improved group acceptance sampling plan for Dagum Distribution under percentiles lifetime. In this paper an improved group acceptance sampling plan using weighted binomial, when the lifetime of an item follows generalized Pareto distribution with known shape parameters, has been discussed. Ramaswamy and Jayasri (2013) worked on time truncated Chain Sampling Plans for Marshall-Olkin Extended Exponential Distributions. Naqvi and Bashir (2016) reported an improved group acceptance sampling plan using weighted binomial on time truncated testing strategy for multiple testers when lifetime of the variate follows exponential distribution.

Generalized Pareto distribution

The probability density function and the cumulative distribution function of generalized Pareto distribution are given below:

$$f(t;\alpha,\beta,\gamma,\delta) = \frac{\delta\alpha}{\beta}\left(\frac{t-\gamma}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\gamma}{\beta}\right)\right]^{-(\alpha+1)} \quad (1)$$

$$F(t;\alpha,\beta,\gamma,\delta) = 1-\left[1+\left(\frac{t-\gamma}{\beta}\right)\right]^{-\alpha} \quad (2)$$

Where $\gamma < t < \infty, \beta > 0, \alpha > 0, \delta > 0$, γ is the location parameter, β is the scale parameter and (α, δ) are the shape parameters. The Generalised Pareto distribution was introduced by AbdElfattah et al. (2007). Aslam et al. (2010a) studied Group Acceptance Sampling Plan based on Generalized Pareto distribution. The mean and variance of Generalized Pareto distribution are

$$\mu = \beta\left[\frac{\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\Gamma(\alpha)}\right]+\gamma \quad (3)$$

$$\sigma^2 = \beta^2\left[\frac{\Gamma\left(1+\frac{2}{\delta}\right)\Gamma\left(\alpha-\frac{2}{\delta}\right)}{\Gamma(\alpha)}-\frac{\Gamma\left(1+\frac{1}{\delta}\right)\Gamma\left(\alpha-\frac{1}{\delta}\right)}{\Gamma(\alpha)}\right]^2 \quad (4)$$

For the existence of mean and variance, $\alpha > {}^{1}/_{\delta}$ and $\alpha > {}^{2}/_{\delta}$ respectively

Methodology

In the forthcoming section we will discussed the proposed methodology for the two sampling(ChSP-1,IGASP) plans when the life time of the variate follows generalized Pareto distribution

Design of the proposed Chain Sampling Plan

The chain sampling plan (ChSP-1) proposed by Dodge (1955), making use of cumulative results of several samples helps to overcome the shortcomings of single sampling plan. It avoids rejection of a lot on the basis of a single nonconforming unit and improves the poor discrimination between good and bad quality that occurs with the c = 0 plan.

The use of cumulative results of several samples is proposed for application to cases where there is repetitive production under the same conditions and where the lots or batches of products to be inspected are submitted for inspection in the order of production. Chain sampling is not suited to intermittent or job lot production, or to occasional purchases.

Chain sampling plan is often recommended when large samples are not practicable, and the use of c = 0 plan is warranted, means extremely high quality is essential. Such situation may arise in receiving inspection of a continuing supply of purchased materials produced within a manufacturing plan. An example situation is a continuing supply of processed material, such as particular type of copper alloy rod or sheet.

The conditions for application and operating procedure of ChSP-1 are as follows:
The cost of destructiveness of testing is such that a relatively small sample size is necessary, although other factors make a large sample desirable.
1) The product to be inspected comprises a series of successive lots produced by a continuing process.
2) Normally lots are expected to be of essentially the same quality.
3) The consumer has faith in the integrity of the producer.

Operating Procedure

Suppose n units are placed in a life test and the experiment is stopped at a predetermined time t. The numbers of failures till the time point t is observed, and suppose it is d. The decision to accept the lot takes place, if and only if the number of failures d at the end of the time point t does not exceed i =1 the acceptance number. The plan is implemented in the following way:

1) For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
2) Accept the lot if d (the observed number of defectives) is zero in the sample of n units, and reject if d > 1.
3) Accept the lot if d is equal to 1 and if no defectives are found in the immediately preceding i samples of size n.
Dodge (1955) has given the operating characteristic function of ChSP-1 as

$$P_a(p) = P_0 + P_1\left(P_0^i\right) \qquad (5)$$

Where

P_a= the probability of acceptance,
P_0= probability of finding no defects in a sample of n units from product of quality p.
P_1 = probability of finding one defect in such a sample.
i = Number of preceding samples.

The Chain sampling Plan is characterized by the parameters n and i. When i = ∞,the OC function of a ChSP -1 plan reduces to the OC function of the Single Sampling Plan with acceptance number zero and when i = 0, the OC function of ChSP -1 plan reduces to the OC function of the Single Sampling Plan with acceptance number 1.

We are interested in determining the sample size required for in the case of generalized Pareto distribution and various values of acceptance number i.

The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability(β) of accepting a bad lot is known as the consumer's risk. Often the consumer risk is expressed by the consumer's confidence level. If the confidence level is p* then the consumer's risk will be β = 1- p*.We will determine the sample size so that the consumer's risk does not exceed a given value β. When the quality is measured in terms of defects, the Poisson approximation is often used. Assuming the Poisson Model, the Chain Sampling Plans (ChSP-1) is designed andthe probability of acceptance in the case of chain sampling plan is given by

$$L(p) = e^{-np} + npe^{-np\,(i+1)} \qquad (6)$$

wherep is the function of $F(t)$ given in (2). It would be convenient to take the termination time as a multiple of the specified number 'a′ , that is, $\mu_o = {t_o}/{a}$. Therefore p is as follows:

$$p = F(t) = 1 - \left[1 + \left\{\frac{a\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\left({\mu}/{\mu_o}\right)\Gamma(\alpha)}\right\}^{\delta}\right]^{-\alpha} \qquad (7)$$

Where p can be evaluated when the shape parameter, the multiplier a and the ratio ${\mu}/{\mu_o}$ are specified. The minimum sample size can be determined such that following inequality is satisfied

$$L(p) \leq p^* \qquad (8)$$

In Table 1 shows minimum values of n, satisfying equation (8) for p* = 0.75, 0.90, 0.95, 0.99 and for ChSP-1 the termination ratio $a =$ 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 4.712, keeping the shape parameter fixed such as $(\alpha, \delta) = 2$.

Table 1:Minimum sample size (n)for Generalized Pareto distribution under proposed plan

p*	i	t/μ_o						
		0.628	0.942	1.257	1.571	2.356	3.927	4.712
0.75	1	7	4	3	2	2	1	1
	2	6	4	2	2	1	1	1
	3	6	4	2	2	1	1	1
	4	6	3	2	2	1	1	1
	5	6	3	2	2	1	1	1
0.90	1	11	6	4	3	2	1	1
	2	10	6	4	3	2	1	1
	3	10	5	4	3	2	1	1
	4	10	5	4	3	2	1	1
	5	10	5	4	3	2	1	1
0.95	1	13	7	5	4	2	2	1
	2	12	7	5	4	2	2	1
	3	12	7	5	4	2	2	1
	4	12	7	5	4	2	2	1
	5	12	7	5	4	2	2	1
0.99	1	18	10	7	5	3	2	2
	2	17	10	7	5	3	2	2
	3	17	10	7	5	3	2	2
	4	17	10	7	5	3	2	2
	5	17	10	7	5	3	2	2

Assume that the life time distribution is generalized Pareto distribution with shape parameter(s) is 2 and that the experimenter is interested in knowing that the true average life is atleast 1000 hours with confidence 0.99. It is assumed that the maximum affordable time is 767 hours and t/γ_0 = 0.942, then the minimum sample required is n = 10 which seems reasonable. Therefore, out of 10 items if not more than 1 item fail and if no defectives are found in the immediately preceding i samples before T = 767 units of time, the lot can be accepted with the assurance that the true average life is at least 1000 with probability 0.99.

For the sampling plan (n = 10, i= 2 and t/γ_0= 0.942) and confidence level p* = 0.99 under generalized Pareto distribution with α = 2 the values of the operating characteristic function are as follows:

Table 2: operating characteristic

μ/μ_o	2	4	6	8	10	12
$L(p)$	0.106638	0.614107	0.864671	0.946361	0.975500	0.987448

The complete Table 2 for the values of the operating characteristic function can be provide on researchers request.

With the modernizations in the acceptance sampling area many more sophisticated acceptance sampling plans introduced the group acceptance sampling plan is one of them. When a single item is put on test then it is known as single sampling plan. When we put more than one items on a tester which form a group and the number of items in a group known as group size(r) so, dealing with group is known as group sampling plan (GASP). It showed GASP has advantage over the ordinary sampling plan in terms of time and cost. The key concern is to minimize the number of groups (g) which is as similar as to minimize the sample size (n) when a single item is put on a tester (n = rg). It showed GASP has advantage over the ordinary sampling plan in terms of time and cost.

Design of the proposed IGASP

Aslam et al.(2011) proposed IGASP as follows:

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be n = rg.
2) Select the acceptance numbers c for a group and the experiment time t_0.
3) Carry out the experiment for the g groups simultaneously and record the number of failures for each group.
4) Accept the lot if at most c failures occurs in each of all groups.
5) Truncate the experiment, if more than c failures occur and reject the lot or, at time t_0.

The stated plan is based on two known plan parameters g and c. The said plan reduces to the ordinary acceptance sampling plan when r = 1.the lot acceptance probability for the IGASP is as follows:

$$L(P) = \sum_{i=1}^{c} \binom{rg-1}{i-1} p^{i-1}(1-p)^{rg-i} \qquad (9)$$

Here p is the probability of failure of an item before termination time t_0.As it's known that p is the function of cumulative distribution function which is, in this paper, generalized Pareto. Also it is convenient to determine the termination time t_0 as a multiple of specified average life μ_0. So we can consider$t_0 = a\mu_0$

for a constant "a" e.g a = 0.5 means that the experiment time is just half of the specified average life (Aslam and June (2009)). So the p is as same as above in equation (7).

The optimal number of groups can be obtained by satisfying the following inequality in (10)

$$\sum_{i=1}^{c}\binom{rg-1}{i-1}\left(1-\left[1+\left\{\frac{a\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\left({}^{\mu}/_{\mu_o}\right)\Gamma(\alpha)}\right\}^{\delta}\right]^{-\alpha}\right)^{i-1}\left(1-\left[1+\left\{\frac{a\Gamma\left(\alpha-\frac{1}{\delta}\right)\Gamma\left(1+\frac{1}{\delta}\right)}{\left({}^{\mu}/_{\mu_o}\right)\Gamma(\alpha)}\right\}^{\delta}\right]^{-\alpha}\right)^{rg-i} \le \beta$$

-------------- (10)

Where β is the consumer risk

Notations

a	-	Test termination time multiplier
c	-	Acceptance number
g	-	Number of groups
n	-	Sample size
p	-	Probability of failure
L(p)	-	Probability of acceptance
r	-	Number of items in a group
t_0	-	Termination time
α	-	Producer's risk
β	-	Consumer's risk
μ	-	True average life
μ_0	-	Specified average life

Table 3 gives the minimal group size of the IGASP for the shape parameters =2 , the termination ratio 0.7, 0.8, 1.0, 1.2, 1.5, 2.0, and the number of tester r =2(1)9, the acceptance number c = 0(1)7 , for consumer's risk 0.25, 0.10, 0.05, 0.01. This parameters setting has used for the comparison purpose.

Table 3: minimal group size (g)for Generalized Pareto distribution under proposed plan

β	r	c	A					
			0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	**2**	**0**	**2**	**2**	**2**	**2**	**1**	**1**
	3	**1**	**3**	**2**	**2**	**2**	**2**	**1**
	4	**2**	**3**	**3**	**2**	**2**	**2**	**1**
	5	**3**	**3**	**3**	**2**	**2**	**2**	**2**
	6	**4**	**3**	**3**	**2**	**2**	**2**	**2**
	7	**5**	**3**	**3**	**2**	**2**	**2**	**2**
	8	**6**	**3**	**3**	**2**	**2**	**2**	**2**
	9	**7**	**3**	**3**	**2**	**2**	**2**	**2**
0.10	**2**	**0**	**3**	**3**	**2**	**2**	**2**	**1**
	3	**1**	**3**	**3**	**2**	**2**	**2**	**2**
	4	**2**	**4**	**3**	**2**	**2**	**2**	**2**
	5	**3**	**4**	**3**	**2**	**2**	**2**	**2**
	6	**4**	**4**	**3**	**2**	**2**	**2**	**2**
	7	**5**	**4**	**3**	**2**	**2**	**2**	**2**

	8	**6**	**4**	**3**	**2**	**2**	**2**	**2**
	9	**7**	**4**	**3**	**2**	**2**	**2**	**2**
0.05	**2**	**0**	**4**	**3**	**3**	**2**	**2**	**2**
	3	**1**	**4**	**3**	**3**	**2**	**2**	**2**
	4	**2**	**4**	**3**	**3**	**2**	**2**	**2**
	5	**3**	**4**	**3**	**3**	**2**	**2**	**2**
	6	**4**	**4**	**3**	**3**	**2**	**2**	**2**
	7	**5**	**4**	**3**	**3**	**2**	**2**	**2**
	8	**6**	**4**	**3**	**3**	**2**	**2**	**2**
	9	**7**	**4**	**3**	**3**	**2**	**2**	**2**
0.01	**2**	**0**	**5**	**4**	**3**	**3**	**2**	**2**
	3	**1**	**5**	**4**	**3**	**3**	**2**	**2**
	4	**2**	**5**	**4**	**3**	**3**	**2**	**2**
	5	**3**	**5**	**4**	**3**	**3**	**2**	**2**
	6	**4**	**5**	**4**	**3**	**3**	**2**	**2**
	7	**5**	**5**	**4**	**3**	**3**	**2**	**2**
	8	**6**	**4**	**4**	**3**	**2**	**2**	**2**
	9	**7**	**4**	**4**	**3**	**2**	**2**	**2**

The table 3 indicates that the required minimal group size decreases as the termination ratio increases. It can also be observed that group size in a life test increases as the number of testers increases, under the prescribed parameter setting.

Comparison:

In this section the comparison of proposed plan for generalized Pareto Distribution with existing plan will be discussed with the help of table and figure.

Table 4: Comparison between proposed and *existing plan for optimal groups (g)

Multiplier "a"	**Existing(GASP)Generalized Pareto Distribution**						**Proposed(IGASP) Generalized Pareto Distribution**					
	β						β					
	c	r	0.25	0.10	0.05	0.01	c	r	0.25	0.10	0.05	0.01
a = 0.7	0	2	2	3	3	5	0	2	**2**	**3**	**4**	**5**
	1	3	4	6	7	11	1	3	**3**	**3**	**4**	**5**
	2	4	7	11	15	22	2	4	**3**	**4**	**4**	**5**
	3	5	14	24	30	46	3	5	**3**	**4**	**4**	**5**
	4	6	30	49	64	98	4	6	**3**	**4**	**4**	**5**
	5	7	64	106	137	211	5	7	**3**	**4**	**4**	**5**
	6	8	138	228	297	456	6	8	**3**	**4**	**4**	**4**
	7	9	301	500	651	999	7	9	**3**	**4**	**4**	**4**
a = 0.8	0	2	2	2	3	4	0	2	**2**	**3**	**3**	**4**
	1	3	3	3	5	8	1	3	**2**	**3**	**3**	**4**
	2	4	5	7	9	14	2	4	**3**	**3**	**3**	**4**
	3	5	8	13	17	25	3	5	**3**	**3**	**3**	**4**
	4	6	14	23	30	46	4	6	**3**	**3**	**3**	**4**
	5	7	26	42	55	84	5	7	**3**	**3**	**3**	**4**
	6	8	47	78	101	154	6	8	**3**	**3**	**3**	**4**

	7	9	87	144	187	287	7	9	**3**	**3**	**3**	**4**
a = 1.0	0	2	1	2	2	3	0	2	**2**	**2**	**3**	**3**
	1	3	2	3	3	5	1	3	**2**	**2**	**3**	**3**
	2	4	2	4	5	7	2	4	**2**	**2**	**3**	**3**
	3	5	4	6	7	11	3	5	**2**	**2**	**3**	**3**
	4	6	5	8	10	16	4	6	**2**	**2**	**3**	**3**
	5	7	7	12	15	23	5	7	**2**	**2**	**3**	**3**
	6	8	11	18	23	35	6	8	**2**	**2**	**3**	**3**
	7	9	16	26	34	52	7	9	**2**	**2**	**3**	**3**

*extracted the part of table 1 from Aslam at el. (2010).

Let a researcher wants to test the quality of an electronic item for 1000h with the following sampling plan parameters:

β =0.05, r=5,c=3, a=0.7 then from table3 he required 4 electrical items for proposed plan whereas for the same parameter setting he requires 30 such electrical items, as much more of existing plan(Aslam at el (2010),Table1) to conduct his experiment. Hence proposed plan seems economical in this case.

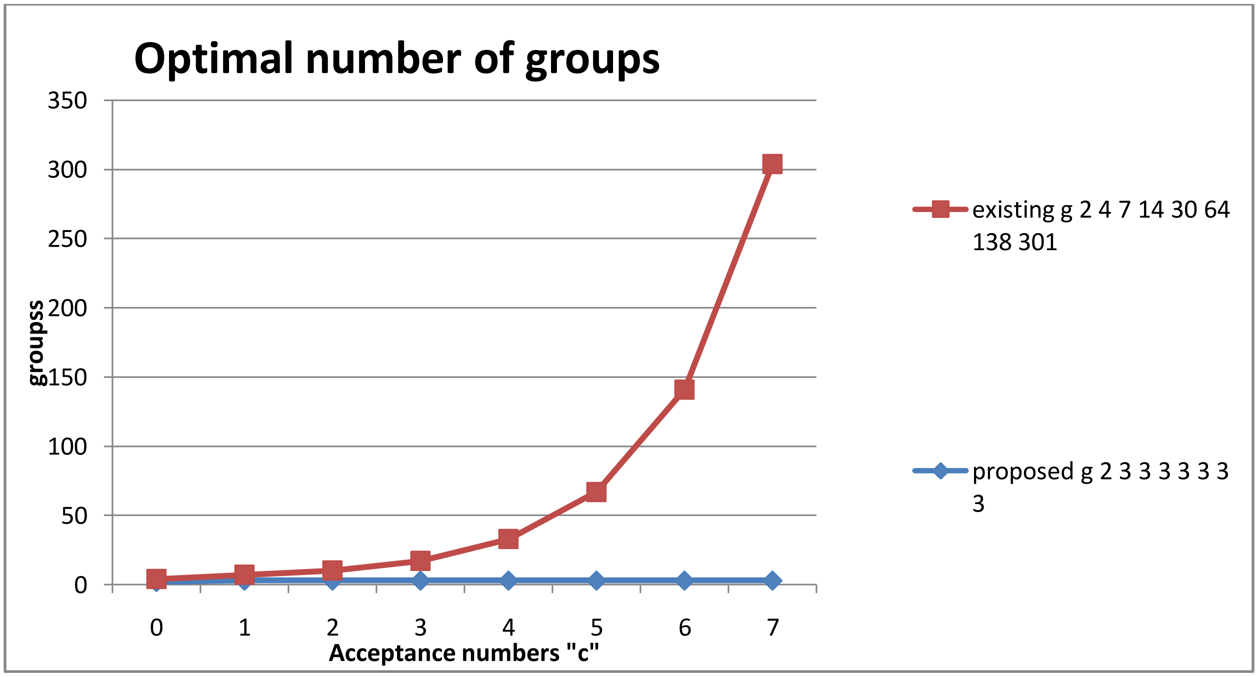

Figure1: Comparison of groups

Figure1 is the comparison of group “g” between the proposed and existing plan. When the acceptance numbers are increasing the proposed plan seems more efficient than the existing plan under the same parameter setting otherwise seems as efficient as the existing one. The above figure is the representation for β=0.25, r=2(1)7,c= 0(1)9, a=0.7

7. Conclusion

In this paper the two sampling plans chain and improved grouped have been discussed when the lifetime variate follows generalized Pareto distribution. The chain sampling plan has an advantage over single sampling plan, as it avoid the rejection of a lot on the basis of a single nonconforming unit and improves the poor discrimination between good and bad quality that occurs with the c = 0 plan. It is appropriate and usually recommended when large samples are not practicable and extremely high quality is essential.

The group sampling plans are seems more generic and cost effective so we presented an improved group acceptance sampling plan by using weighted binomial too, when the lifetime variates follows generalized Pareto distribution Under pre-defined parameter setting the optimal number of groups obtained. It has been observed that proposed plan, sometime for same parameter setting, seems efficient and cost effective than the existing plan Aslam at el.(2010).

Although both plans (chain and group) are seems to be practically applicable according to the researcher requirements and conditions.

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