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Effect of Slip velocity on Blood Flow in Catheterized Tapered Artery

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Abstract: Effect of slip velocity on blood flow through an arterial stenosis which is developed along a tapering wall is studied here. A uniform catheter is inserted in a stenosed tube. Blood is assumed to behave like Newtonian fluid. No slip as well as slip condition is taken in the present paper, at the arterial wall a velocity slip condition is employed and a no slip at the catheter boundary. Analytic expressions are obtained for different flow parameters and their behavior discussed through graphs. For the numerical solution of the problem, which is described by Navier-Stokes equations with appropriate boundary conditions, the Perturbation method is adopted. It is found that due to the introduction of an axial slip velocity and flow rate increases but wall shear stress decreases. The effect of tapering is also seen in the present model.

Keywords: Newtonian fluid; Blood; Catheterization; Tapered artery; Slip velocity.

1. Introduction

One major type of arterial disease is atherosclerosis in which localized deposits and accumulation of cholesterol and lipid substances, as well as proliferation of connective tissues cause a partial reduction in the arterial cross-sectional area (stenosis) and a considerable increase in the wall stiffness. Tu and Deville (1992) gave an idea that the assumption of Newtonian behavior of blood is acceptable for high shear rate flow, e.g. in the case of flow through large arteries. It has also been pointed out that in some diseased conditions, e.g. patients with severe myocardial infarction, cerebrovascular diseases and hypertension, blood exhibits remarkable non-Newtonian properties.

It is true that the Casson fluid model can be used for moderate shear rates $\gamma < 10$ s−1 in smaller diameter tubes whereas, the Herschel–Bulkley fluid model can be used at still lower shear rate of flow in very narrow arteries where the yield stress is high. Also Herschel–Bulkley fluid model can be reduced to that of power law, Bingham and Newtonian fluid models by suitable choice of the parameters. The same model can be used for larger arteries where the effect of yield stress can be ignored by Maruti et al. (2008).Sankar and Hemalatha (2007) have mentioned that, for tube diameter 0.095 mm, blood behaves like H-B fluid rather than power law and Bingham fluids.Mandal (2005)has considered an unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis.

In recent years, many researchers have investigated the tapered angles in the clinical study of human arterial blood flow.How and Black (1987) investigated that the study of blood flow through tapered tube is important not only for an understanding of the behavior of the marvelous body fluid in arteries, but also for the design of prosthetic blood vessels.Guyton (1970) has found in human systems, there prevail different geometries in blood vessels such as, circular, branched, bifurcated, tapered, inclined etc.Biswas and Paul (2012)have been investigated blood flow through an inclined tapered artery for Newtonian fluid. Some of them have divided the stenosed arteries into three types of non-tapered angle, divergent tapered angle and convergent tapered angle to explore the relationship between the arterial stenosis and tapered angles.

Mu et. al (2013) have seen the effect of tapered angles in an artery on distribution of blood flow pressure with gravity.Arora (2011) has designed an ANN model to study the effect of blood flow and cross sectional area through tapered artery with mild stenosis , considering blood flow as a two-fluid model with the suspension of all the erythrocytes in the core region as Herschel-Bulkley fluid and the plasma in the peripheral layer as Newtonian fluid.

Pressure-flow relationship alters the blood flow in a stenosed artery. Sometimes, for some clinical purposes, catheters are inserted in arteries. The pressure-flow relationship changes appreciably when a catheter is inserted in a stenosed artery. Blood flow models through catheterized stenosed artery have been proposed by Jayaraman (1995). Biswas et al. (2003, 2011) described blood flow in annular region of a catheterized stenosed artery.Mekheimer et al. (2008) described the micropolar fluid model for blood flow through a tapered artery with a stenosis. Verma and Parihar (2010) have studied the effects of stenosis and hematocrit on the flow rate, wall shear stress, and resistance parameter for Newtonian fluid through the tapered artery.The problem of blood flow through a stenosesd segment of an artery where the rheology of the blood is described by the Herschel Bulkley model has been explained by Kumar et al. (2015).Misra et al. (2018) examined the effects of catheterization on various physiological flow variables when blood flows in a bifurcated artery in the presence of stenosis on the endothelium, which is located in the neighborhood of the bend.

In the present analysis, we have considered the pulsatile flow of blood to study the effects of velocity slip (at the stenotic vessel wall), tapering tube, catheterization of the artery on the flow variables for annular blood flow through a catheterized tapering artery with the formation of an axially asymmetric mild stenosis, by considering blood to behave as a Newtonian fluid.

2. Formulation of the problem

Consider a pulsatile flow of blood through a tapered, catheterized artery in the presence of slip velocity at the stenotic wall. We consider a steady, laminar and fully developed and incompressible flow of blood through the annular region between a constricted tapered tube of normal radius R_0 and a co-axial rigid catheter of radius R_1. The flow geometry of the catheterized tapering vessel is shown in Figure 1.

Mathematical expression for the geometry of stenosed tapered artery is given by eq. (1)Liu et. al (2004),

$$\bar{R}(\bar{z}) = \begin{cases} \bar{R}_0 - \bar{m}(\bar{z} + \bar{d}) - \dfrac{\bar{\delta}_s \cos\alpha}{2}\left(1 + \cos\dfrac{\pi \bar{z}}{\bar{z}_0}\right) & |\bar{z}| \le \bar{z}_0 \\ \bar{R}_0 - \bar{m}(\bar{z} + \bar{d}) & |\bar{z}| > \bar{z}_0 \end{cases} \tag{1}$$

where $\bar{R}(\bar{z})$ is the radius of the tapered arterial segment in the stenotic region, $\bar{R}_0$ is the radius of the normal straight artery, α is the angle of tapering, $\bar{\delta}_s \cos\alpha$ is the length of the stenosis at the location d for the tapered artery, $\bar{z}_0$ is the half length of the stenosis and $m = \tan\alpha$ represents the slope of the tapered vessel. Cylindrical polar coordinate system (r, θ, z) has been used to analyze the flow field, where z-axis is taken along the axis of the artery and r, θ are along the radial and circumferential directions respectively.
The geometry of tapered stenosed artery is shown in Fig. 1.

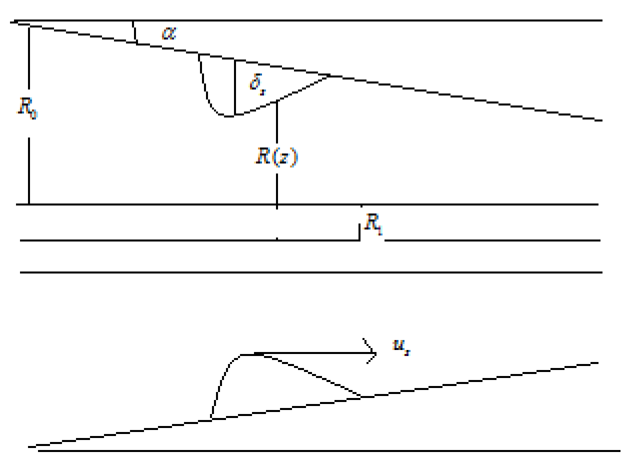

Fig. 1 Schematic diagram of a catheterized tapered artery with stenosis

It is found that the radial velocity being negligibly small can be neglected for low Reynolds number in case of mild stenosis. The momentum equations governing the fluid flow are given by Schlichting and Gersten (2004).

$$\bar{\rho}\left(\partial \bar{u} / \partial \bar{t}\right) = -\left(\partial \bar{p} / \partial \bar{z}\right) - (1/\bar{r})\frac{\partial}{\partial \bar{r}}\left(\bar{r}\bar{\tau}\right), (2)$$

$$\partial \bar{p} / \partial \bar{r} = 0, (3)$$

where $\bar{u}$ represent the axial velocity along z-direction, $\bar{p}$ is the pressure, $\bar{\rho}$ is the density, $\bar{t}$ the time, $\bar{\tau}$ the shear stress.

Blood has been considered as Newtonian fluid described in equation (4)

$$\bar{\tau} = -\bar{\mu}\left(\partial \bar{u} / \partial \bar{r}\right), (4)$$

where $\bar{\mu}$ is the shear viscosity of blood.

The boundary conditions are

$$\bar{u} = \bar{u_s} \ at \ \bar{r} = \bar{R}\left(\bar{z}\right), \quad (5)$$

$$\bar{u} = 0 \ at \ \bar{r} = \bar{R_1}, \quad (6)$$

where $\bar{u_s}$ is the slip velocity at the stenotic wall, $\bar{R_1}$ is the radius of the catheter.
Since the pressure gradient is the function of $\bar{z}$ and $\bar{t}$, we take

$$\frac{-\partial \bar{p}}{\partial \bar{z}}\left(\bar{z}, \bar{t}\right) = A_0 + A_1 \cos\left(\bar{\omega}\bar{t}\right), t \geq 0, \quad (7)$$

where A_0 is the steady state pressure gradient, A_1 is the amplitude of the fluctuating component, $\bar{\omega} = 2\pi \bar{f}$, where $\bar{f_p}$ is the pulse rate frequency.

We introduce the following non-dimensional variables

$$u=\frac{\bar{u}}{\frac{A_0 R_0^2}{4\bar{\mu}}},\ z=\frac{\bar{z}}{R_0},\ R(z)=\frac{\bar{R}(\bar{z})}{R_0},\ r=\frac{\bar{r}}{R_0},\ t=\bar{t}\bar{\omega},$$

$$\delta_s=\frac{\bar{\delta}_s}{R_0},\ u_s=\frac{\bar{u}_s}{\frac{A_0 R_0^2}{4\bar{\mu}}},\ \tau=\frac{\bar{\tau}}{\frac{A_0 R_0}{2}},\ \alpha^2=\bar{R}_0^2\bar{\omega}\bar{\rho}/\bar{\mu},\ e=\frac{A_1}{A_0},\ R_1=\frac{\bar{R}_1}{R_0} \tag{8}$$

The non-dimensional momentum equation (2) can be written as

$$\alpha^2(\partial u/\partial t)=4(1+eCos\ t)-(2/r)\frac{\partial}{\partial r}(r\tau), \tag{9}$$

where $\alpha^2=\omega_p R_o^2/(\mu/\rho)$, is called Womersley frequency parameter.

Equation (4) can be written as

$$\tau=-\frac{1}{2}\frac{\partial u}{\partial r}, \tag{10}$$

On substituting the value of τ in Equation (9) we have

$$\alpha^2(\partial u/\partial t)=4(1+eCos\ t)+(1/r)\frac{\partial}{\partial r}\left(r\,\partial u/\partial r\right). \tag{11}$$

The boundary conditions (5) and (6) reduces to

$$u=u_s\ at\ r=R(z), \tag{12}$$

$$u=0\ at\ r=R_1, \tag{13}$$

The geometry of an arterial stenosis in dimensionless form is given by

$$R(z)=\begin{cases} R_0-m(z+d)-\dfrac{\delta_s\cos\alpha}{2}\left(1+\cos\dfrac{\pi z}{z_0}\right) & |z|\le z_0 \\ R_0-m(z+d) & |z|>z_0 \end{cases} \tag{14}$$

The non-dimensional volumetric flow rate and effective viscosity is defined by equation (15) and (16) respectively.

$$Q(z,t) = 4\int_0^{R(z)} r\, u(z,r,t)\,dr \tag{15}$$

Where $Q(z,t) = \overline{Q}(\overline{z},\overline{t}) \Big/ \dfrac{\pi A_0 \left(\overline{R_0}\right)^4}{8\overline{\mu}}$

Effective viscosity $\overline{\mu_e}$ defined as

$$\overline{\mu_e} = \pi\left(-\frac{\partial \overline{p}}{\partial \overline{z}}\right)\left(\overline{R}\left(\overline{z}\right)\right)^4 \Big/ \overline{Q}\left(\overline{z},\overline{t}\right)$$

$$\mu_e = R^4\left(1 + e\cos t\right) \Big/ Q(z,t) \tag{16}$$

3. Method of Solution:

Let the velocity u can be expressed in the following form

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + \ldots \tag{17}$$

Substituting Eq.(17) in Eq. (11), (12), (13) respectively and equating constant term and α^2 term we get

$$\frac{\partial}{\partial r}\left(r\,\partial u_0 \big/ \partial r\right) = -4r(1 + e\cos t) \tag{18}$$

$$\partial u_0 \big/ \partial t = \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\partial u_1 \big/ \partial r\right)\right) \tag{19}$$

$$u_0 = u_s \ \ and \ \ u_1 = 0 \ \ at \ \ r = R(z) \tag{20}$$

$$u_0 = 0 \ \ and \ \ u_1 = 0 \ \ at \ \ r = R_1 \tag{21}$$

On solving Eq. (18) and Eq. (19) using boundary conditions we get the expression for velocity as,

$$u(z,r,t) = f(t)\left(R_1^2 - r^2\right) + \frac{\ln\left(r/R_1\right)}{\ln\left(R/R_1\right)}\left(u_s + f(t)\left(R^2 - R_1^2\right)\right)$$

$$+\alpha^2 D\left[\begin{array}{l}\left(R_1^2 r^2/4 - (r^4/16) - 3(R_1^4/16)\right) + \frac{\left(R^2 - R_1^2\right)}{4\ln\left(R/R_1\right)}\left(r^2 \ln\ln\left(r/R_1\right) - r^2 + R_1^2\right) \\ + \frac{\ln\left(R_1/r\right)}{\ln\left(R/R_1\right)}\left(\begin{array}{l}\left(\left(R_1^2 R^2/4\right) - \left(R^4/16\right) - 3(R_1^4/16)\right) \\ + \frac{\left(R^2 - R_1^2\right)}{4\ln\left(R/R_1\right)}\left(\ln\ln\left(R/R_1\right) R^2 - R^2 + R_1^2\right)\end{array}\right)\end{array}\right] \tag{22}$$

The wall shear stress τ_w can be obtained as

$$\tau_w = \left[\begin{array}{l} Rf(t) - \frac{1}{2R\ln\left(R/R_1\right)}\left(u_s + f(t)\left(R^2 - R_1^2\right)\right) - \\ \frac{\alpha^2 D}{2}\left(\begin{array}{l}\left(\left(R_1^2 R/2\right) - \left(R^3/4\right)\right) + \frac{\left(R^2 - R_1^2\right)}{4\ln\left(R/R_1\right)}\left(2R\ \ln\left(R/R_1\right) - R\right) \\ - \frac{1}{R\ln\left(R/R_1\right)}\left(\begin{array}{l}\left((R_1^2 R^2/4) - (R^4/16) - \left(3R_1^4/16\right)\right) \\ + \frac{\left(R^2 - R_1^2\right)}{4\ln\left(R/R_1\right)}\left(\ln\ln\left(R/R_1\right) R^2 - R^2 + R_1^2\right)\end{array}\right)\end{array}\right)\end{array}\right] \tag{23}$$

Volumetric flow rate *Q(z ,t)* can be obtained as

$$\begin{aligned} Q(z,t) = {} & f(t)\left(2R_1^2 R^2 - R^4 - R_1^4\right) + \frac{1}{\ln\left(R/R_1\right)}\left(u_s + f(t)\left(R^2 - R_1^2\right)\right) \\ & \times (2\ln\left(R/R_1\right) R^2 - R^2 + R_1^2) + \alpha^2 D\left(B_1 + \frac{\left(R^2 - R_1^2\right)}{\ln\left(R/R_1\right)} B_2 B_3 B_4\right), \end{aligned} \tag{24}$$

where $B_1 = \left((R_1^2 R^4/4) - (R^6/24) - (3R_1^4 R^2/8) - (5R_1^6/24) + (3R_1^6/8) \right),$

$$B_2 = \left(\ln(R/R_1)(R^4/4) - (5R^4/16) + (R_1^2 R^2/2) - (3R_1^4/16) \right),$$

$$B_3 = \left(\frac{1}{\ln(R/R_1)} (R_1^2 R^2 - (R^4/4) - (3R_1^4/4) \right) + \frac{(R^2 - R_1^2)}{\ln(R/R_1)} (\ln(R/R_1) R^2 - R^2 + R_1^2)),$$

$$B_4 = \ln(R_1/R)(R^2/2) + (R^2/4) - (R_1^2/4).$$

The expression for effective viscosity μ_e can be obtained from equation (16) and (24).

4. Results and Discussion

In the present analysis our aim is to study the effect of slip velocity and tapering of an artery in a catheterized stenosed artery assuming blood as a Newtonian fluid. The tapered effect is one of the important factors leading to nonlinearity in such hemodynamic parameters as blood flow pressure. The expressions for different flow parameters are obtained by solving the governing equation of flow using perturbation method. Computer codes are developed for the numerical evaluation of analytical results using MATLAB.
The following values of different parameters are used in the present model for the quantitative analysis. $R_1 = 0.2, \alpha = 0.2, e = 1, \delta_s = 0.4$.

Fig.2 shows the variation of axial velocity with the radial distance showing the effect of different tapering angles and slip velocities. Also a comparison of velocity profile for different time values has been shown in Fig. 3. Velocity profile in fig. 2 indicates that the axial velocity of blood decreases with the increase in the tapering angle of the artery. It also shows that velocity is higher for flows with slip than that of no slip case. Fig. 3 shows the parabolic profile for axial velocity for different time values. As time increases velocity becomes low.

Fig. (4-6) shows the variation of wall shear stress with the catheter radius and axial distance z for different slip velocity and time. Fig. 4 reveals that the increase of catheter radius, increases the wall shear stress.Wall shear stress is reduced with the application of slip velocity. It further decreases with the increase in slip velocity.

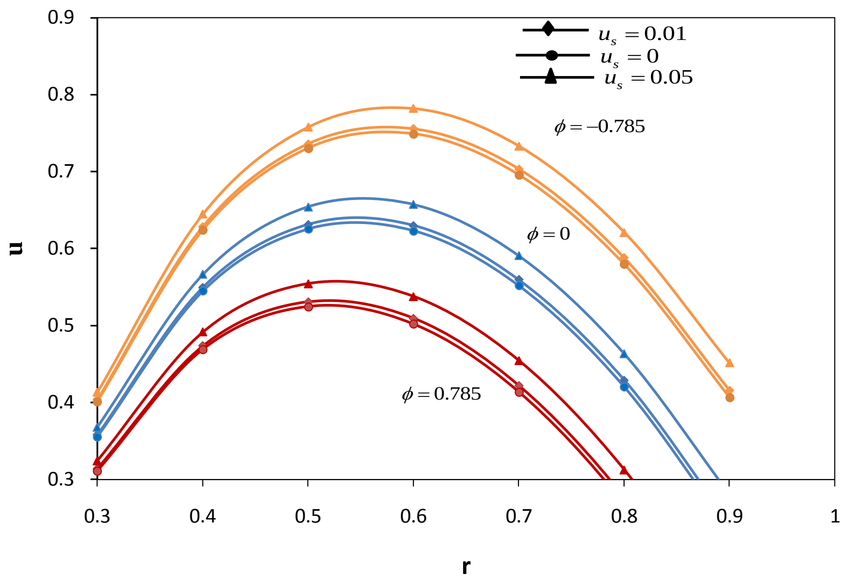

Fig. 2 Variation of axial velocity with the radial distance for different values of Tapered angle and slip velocity

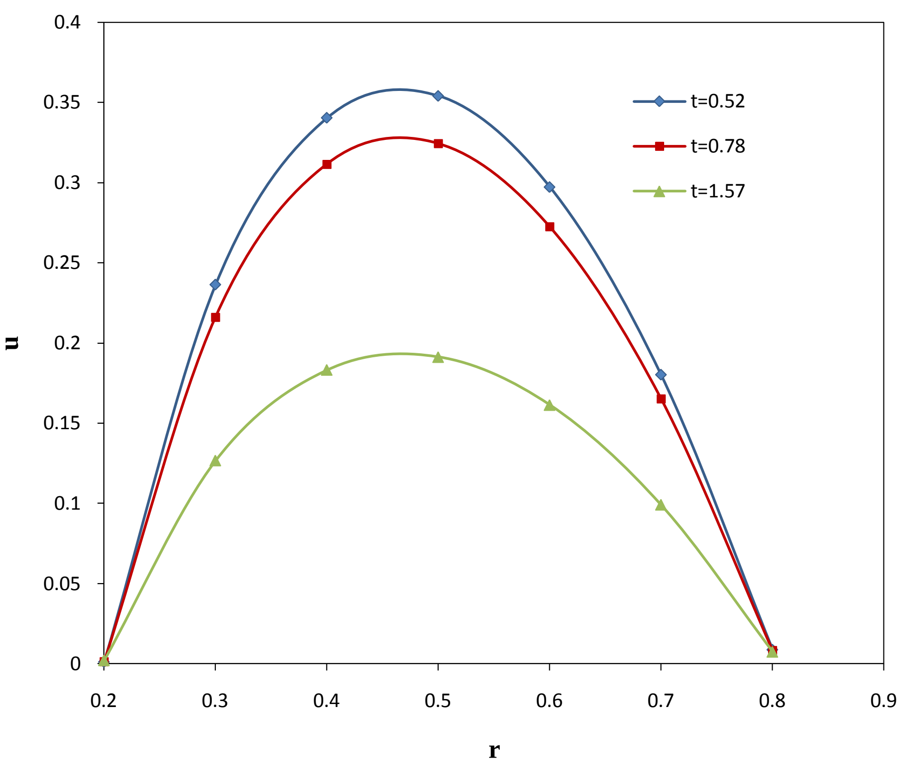

Fig. 3 Variation of axial velocity with the radial distance for different values of Time

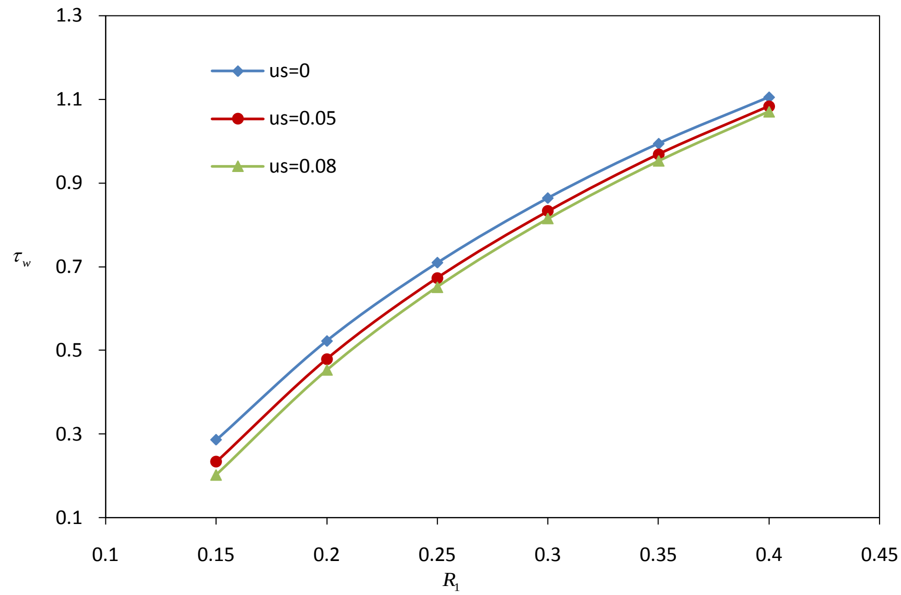

Fig. 4 Variation of wall shear stress with the catheter radius for different values of slip velocity.

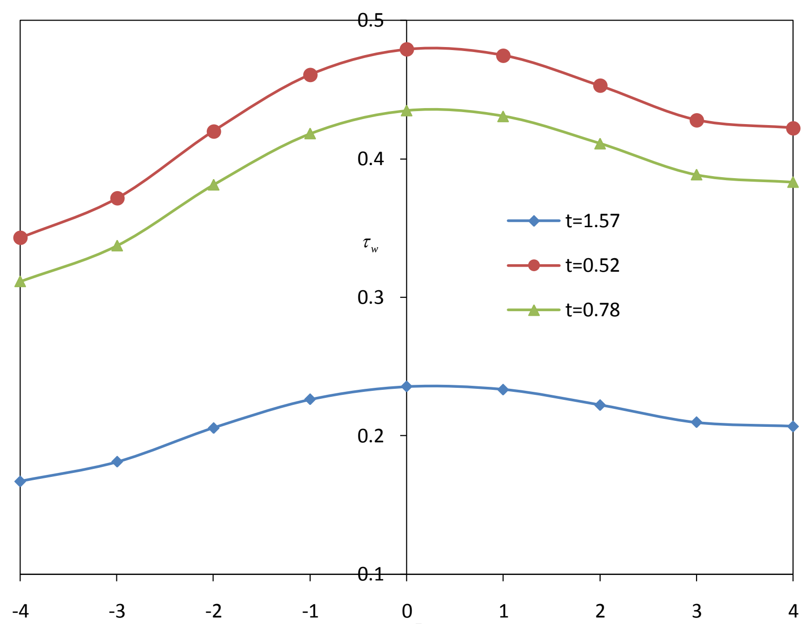

Fig. 5 Variation of wall shear stress with the axial distance for different values of time.

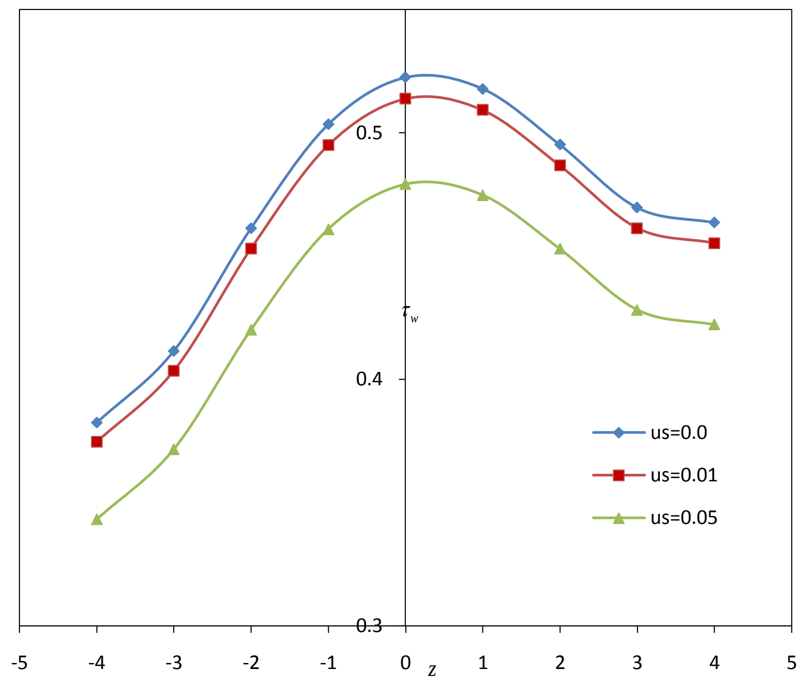

Fig. 6 Variation of wall shear stress with the axial distance for different values of slip velocities

Fig. (5-6)depicts that as axial distance z increases from $z = -z_0$ to $z = 0$, wall shear stress increases from the minimum value at the mouth of the constricted annular region to the maximumone at the throat of the annulus. After that it decreases from the maximum value (at z=0) to the lower one at the other end of the annular stenotic region $z = z_0$. However throughout this variation, wall shear stress is low with the application of slip velocity at the flow boundary and also decreses with the increase in time.

From Fig. (7-9) it is found that flow rate changes with the axial direction in non-uniform co-axial region. It is minimum at the throat of the stenosis, in the two equal regions $-z_0 \le z \le 0$ and $0 \le z \le z_0$ flow rate increases from the smallest value to the greater values at the initiation or the termination of the stenosis. Fig. 7 shows the variation of volumetric flow rate with the axial distance z for different values of tapered angles. It is found that flow rate in artery with tapering is low than without tapering also increase in tapering angle further reduces the flow rate.

Fig. 8 shows that flow rate obtained with the wall slip is more than that obtained with no slip at the boundary. As the magnitude of slip velocity is increases flow rate becomes higher and higher. Flow rate also depends on time is shown in fig.9. it decreases with the increase in time.

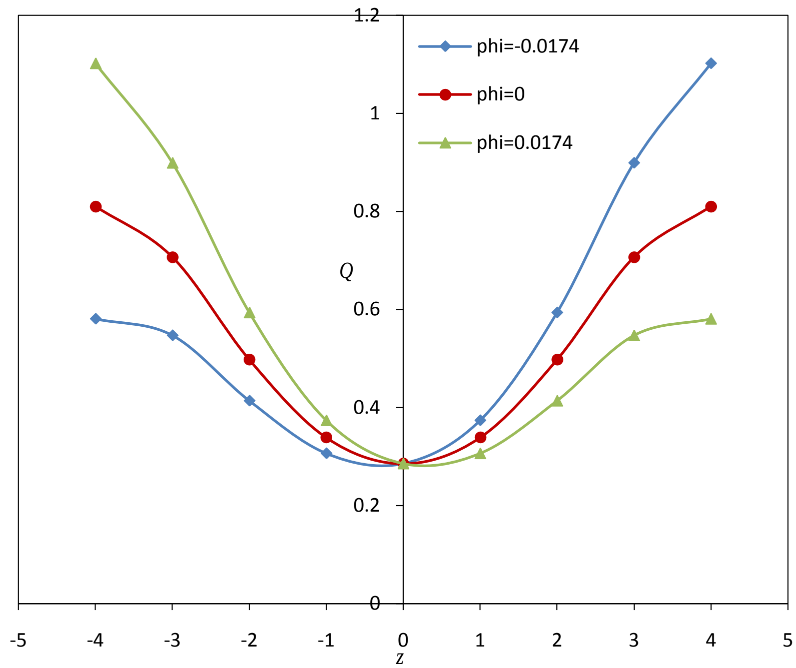

Fig. 7 Variation of flow rate with the axial distance
for different values of tapered angles.

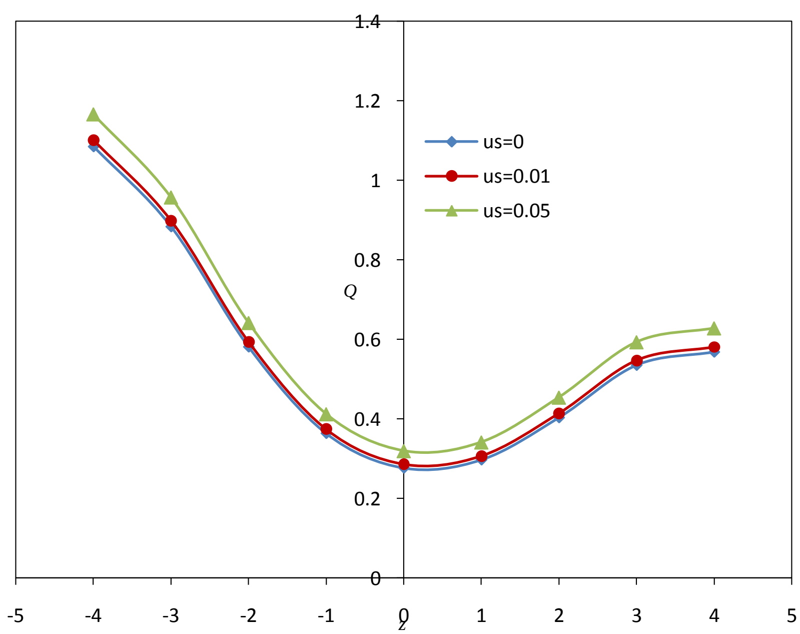

Fig. 8 Variation of flow rate with the axial
distance for different values of slip velocities.

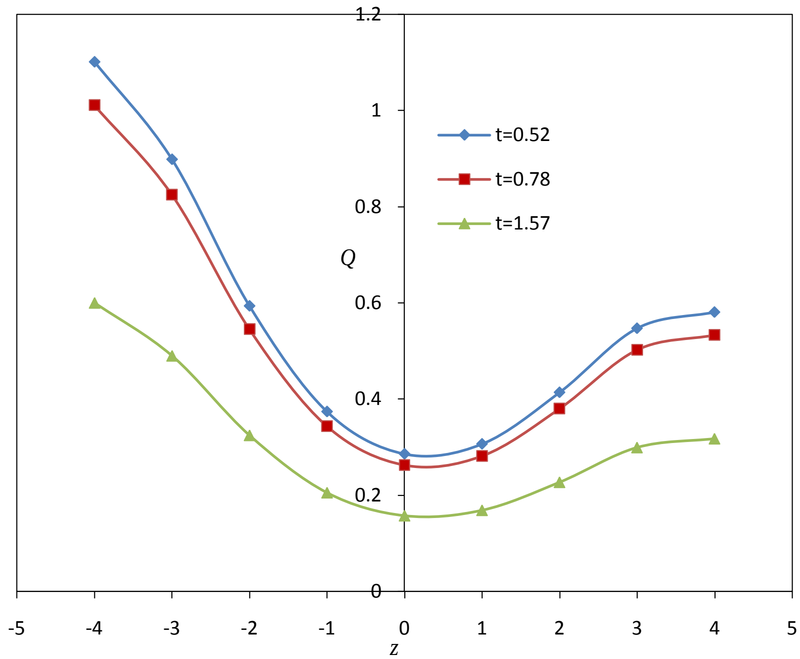

Fig. 9 Variation of flow rate with the axial distance
for different values of time.

4. Conclusion

In the present analysis combined influence of several flow parameters like slip velocity, catheter radius, tapering geometry, on assuming that blood is represented by Newtonian fluid, has been considered. The annular region is spaced within tapering, constricted (-asymmetric stenosis) wall and a co-axial catheter. Analytical expressions of the important flow characteristics such as, velocity, flow rate, wall shear stress and effective viscosity variables are obtained and the variations have been shown and discussed through graphs.

In this analysis, it is observed that flow rate accelerated on one hand but wall shear stress retarded on the other hand while applying a velocity slip condition at the constricted tapering vessel wall.Therefore, bore of the blood vessel could be increased and damages to the diseased vessel wall could be reduced by the application of appropriate slip velocity. It is therefore necessary to determine an appropriate velocity slip in accordance with the stenosis size, artery radius and other physiological situations. It is also found that the magnitudes of velocity and flow rate are found to be smaller in the present model than those in the non tapering catheterized artery with stenosis. Therefore it is concluded that tapering of an artery does not alter the flow pattern. Such models could be used as a device in the initiation of atherosclerosis and also in the treatment modalities of cardiovascular complications, stroke, thrombosis, renal and sickle cell diseases and other arterial disorders.

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