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A PROOF OF BEAL'S CONJECTURE

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Abstract. Beal's Conjecture : The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z with μ, ξ and ν odd primes at least 3. A proof of this longstanding conjecture is given.

Beal's Conjecture: The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z with ξ, μ and ν odd primes at least 3. A history of this problem can be found in [1].

Suppose $z^\xi = x^\mu + y^\nu$ is true for any relatively prime positive integers x, y, z and odd primes ξ, μ and ν with ξ, μ, ν at least 3. When x, y and z are relatively prime, $(z^\xi), (x^\xi)$ and (y^ξ) are also relatively prime. Then $(z^\xi)^\xi = (x^\xi)^\mu + (y^\xi)^\nu$. That is, suppose $(z^\xi)^\xi = (x^\mu)^\xi + (y^\nu)^\xi$.

The Proof.

We claim the following:

$$x^\mu + y^\nu - z^\xi \equiv 0 \ (mod\ \xi),$$

and

$$(x^\mu + y^\nu)^\xi - (z^\xi)^\xi \equiv 0 \ (mod\ \xi^2)$$

To prove the above claims:

Note that by expanding $(x^\mu + y^\nu - z^\xi)^\xi$ using binomial expansion,

$$(x^\mu+y^\nu-z^\xi)^\xi-((x^\mu+y^\nu)^\xi-(z^\xi)^\xi) = \sum_{k=1}^{\xi-1} C(\xi,k)(x^\mu + y^\nu)^{\xi-k}(-z^\xi)^k, \qquad (1)$$

Again, using binomial expansions for $(x^\mu + y^\nu)^\xi$ and $((x^\mu + y^\nu - z^\xi) + z^\xi)^\xi$, we have,

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$$(x^\mu + y^\nu)^\xi - (z^\xi)^\xi - (x^\mu + y^\nu - z^\xi)^\xi \equiv 0 \ (mod \ \xi). \qquad (2)$$

The right hand side of equation (2) is divisible by ξ and hence the left hand side is divisible by ξ. The expansion of $(x^\mu + y^\nu)^\xi - (z^\xi)^\xi$ shows that $(x^\mu + y^\nu)^\xi - (z^\xi)^\xi$ is divisible by ξ and hence $(x^\mu + y^\nu - z^\xi)^\xi$ is divisible by ξ. Thus

$$x^\mu + y^\nu - z^\xi \equiv 0 \ (mod \ \xi). \qquad (3)$$

So,

$$(x^\mu + y^\nu - z^\xi)^\xi \equiv 0 \ (mod \ \xi^\xi).$$

Also from equations (2) and (3), and since

$$(x^\mu + y^\nu)^\xi - (z^\xi)^\xi - (x^\mu + y^\nu - z^\xi)^\xi = \xi S,$$

where ξS represents a sum of terms with $(x^\mu + y^\nu - z^\xi)$ as a factor and a multiple of ξ as coeficient, we have

$$(x^\mu + y^\nu)^\xi - (z^\xi)^\xi \equiv 0 \ (mod \ \xi^2). \qquad (4)$$

In view of equations (3) and (4), equation (1) gives that

$$z^\xi \equiv 0 \ (mod \ \xi) \qquad (5)$$

and

$$x^\mu + y^\nu \equiv 0 \ (mod \ \xi). \qquad (6)$$

Hence, in view of equation (3),

$$(z^\xi)^\xi - (x^\mu)^\xi - (y^\nu)^\xi = (x^\mu + y^\nu)^\xi - (x^\mu)^\xi - (y^\nu)^\xi$$

$$= \sum_{k=1}^{\xi-1} C(\xi, k)(x^\mu)^{\xi-k}(y^\nu)^k \equiv 0 \ (mod \ \xi^\xi). \qquad (7)$$

So,

$$y^\nu \equiv 0 \ (mod \ \xi) \qquad (8)$$

and

$$x^\mu \equiv 0 \ (mod \ \xi) \qquad (9).$$

Thus we get $x \equiv 0 \ (mod \ \xi), y \equiv 0 \ (mod \ \xi)$ and $z \equiv 0 \ (mod \ \xi)$. Hence x, y, z are not relatively prime and thus proves Beal's Conjecture.

REFERENCES

[1] https://www.bealconjecture.com/

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