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Parametric Non-linear programming approach for N-policy queues with infinite capacity

V. Ashok Kumar**
***Department of Mathematics, Shinas College of Technology, Oman.*

ABSTRACT

This paper proposes a procedure to construct the membership function of N-policy queue with infinite capacity. By using mathematical programming we construct the membership function of the system performance measure in which arrival rate and service rate are fuzzy numbers. Based on α-cut approach and Zadeh's extension principle, the fuzzy queues are converted into a family of crisp queues. Suitable real world example is exemplified to analyze N-policy fuzzy queues. Extending this model to fuzzy environment it would have further more wider applications.

Keywords: α-cut; N-policy; Infinite capacity; Membership function; Nonlinear Programming.

1. Introduction

One of the most important concept in queuing theory is to control the queuing system. The control of queuing system has been widely used in fields like manufacturing / production systems, communication networks, computer systems etc. One of the most popular controllable queues is the N-policy queue introduced by Yadin and Naor [22]. In the N-policy, the idle server is turned on whenever the queue length reaches the threshold $N(N \geq 1)$, and the server is turned off when the system becomes empty. Extensive literature exist on the N-policy queues Bell [2], Kimura [10], Takagi [18], Gakies [7], Wang and Huang [20], Wang and Ke [21], Zhang and Xu [24] and others.

In most of the early literature, inter arrival times and service times are determined by probability distribution. However, in many real-world applications, it is more adequate to describe the arrival and service patters by linguistic terms such as "frequently arrivals" (or) "fast, slow or moderate service", rather than probability distributions. That is to say the inter arrival times and service times are more possibilistic than probabilistic in many real world situations. That is fuzzy queues are more realistic and applicable than crisp queues. By extending the usual crisp N-policy queues to fuzzy N-policy queues, the queuing models will have further wider applications.

Through the use of Zadeh's extension principle, possibility concept and fuzzy Markov chain [see Stanford [17], the problem of fuzzy queues have been investigated by Li and Lee [14], Buckley

[3], Negi and Lee [15]. Kao et al [8] utilized parametric programming approach to construct the membership functions for four simple queues FM/FM/1, F/M/1/1, F/F/1/1 and FM/FM/1/1, where F represents fuzzy time and FM represents fuzzified exponential distributions. Using the same approach, Ke and Lin [11,12] and Lin et al. [13] analyzed the FM/FM/9FM, FM)/1, $FM^{[x]}$/FM/1/FV and $FM^{[x]}$/FM/1/FSET fuzzy systems, respectively, where FV represents the fuzzified exponential vacation rate and FSET represents the fuzzified exponential setup rate. Also, Chen[4,5] analyzed the FM/$FM^{[k]}$/1 and $FM^{[k]}$/FM/1 with varying batch sizes, respectively. Recently, Aydin and Apaydin [1] considered the multi channel fuzzy queuing systems and computed fuzzy queuing characteristics via different membership functions. The results in [1] were also compared with the simulations based on simulating the fuzzy parameter of the model.

The aim of this paper is to provide a mathematical programming approach to derive the membership functions of system characteristic of fuzzy N-policy queue. By using α-cut s and Zadeh's extension principle we transform the fuzzy queue into a family of crisp queue. As α varies, we use parametric nonlinear programming (NLP) to solve the family of crisp queues. Due to the consideration of two fuzzy variables [fuzzified exponential arrival and service rates), it would formulate two pairs of mathematical programs to compute lower and upper bounds of the α-cuts for the fuzzy system performances, including the expected waiting time in the system and expected number of customers in queue. The explicit close form expression for the membership function of the system performances could be obtained. By using inverse function we invert the interval limits and derive the explicit closed-form expression for the membership function of the system performances.

The structure of the paper is as follows. In the next section, we perform the system performances of the classical and fuzzy N-policy queues with Infinite capacity. Section 3 develops a mathematical programming approach to calculate the α-cuts of the membership functions of the system performances. In section 4, a real-world example is exemplified to illustrate the proposed approach. Conclusions are given in the last section.

2. Fuzzy N-policy queues with infinite capacity

2.1. N-policy M/M/1 queue

We consider an N-policy M/M/1 queue with infinite capacity. It is assumed that arrivals of customers follow a Poisson process with parameter λ, service time are according to exponential distribution with a service rate μ. The server can serve only one customer at a time. Arrived customers form a single waiting time at a server based on the order of their arrivals; that is, in a first-come, first-served (FCFS) discipline. If the server is busy, the arriving customers must wail until the server is busy, the arriving customers must wait until the server is available.

Let Ws and Nq denote the expected waiting time in the system and the expected number of customers in queue, respectively. One can easily derive W_s and N_q by a Markov chain approach. It follows that

$$W_s = \frac{N-1}{2\lambda} + \frac{1}{\mu - \lambda} \tag{1}$$

$$N_q = \frac{N-1}{2} + \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{2}$$

It should be noted that $0 < \lambda / \mu < 1$ under the steady-state condition.

2.2. N-policy FM/FM/1 queue

We consider an N-policy queuing model with Infinite capacity in which arriving customers follows a Poisson process with a fuzzy arrival rate $\bar{\lambda}$, and the service times are exponential with a fuzzy service rate $\bar{\mu}$. We will represent $\bar{\lambda}$ and $\bar{\mu}$ by the convex fuzzy sets. Let $\eta_{\bar{\lambda}}(x), \eta_{\bar{\mu}}(y)$ denote the membership functions of $\bar{\lambda}$ and $\bar{\mu}$ respectively. This model will be referred to as the N-policy FM/FM/1 queue. We have

$$\bar{\lambda} = \{(x, \eta_{\bar{\lambda}}(x)) / x \in X\} \tag{3}$$

and $$\bar{\mu} = \{(y, \eta_{\bar{\mu}}(y)) / y \in Y\} \tag{4}$$

where X and Y are the crisp universal sets of the arrival and service rates, respectively. Let us define the system performances of interest by f(x, y). Since $\bar{\lambda}$ and $\bar{\mu}$ are fuzzy numbers, f($\bar{\lambda}$, $\bar{\mu}$) is also a fuzzy numbers. With Zadeh's extension principle (Zadeh [23] and Zimmerman [25], one can obtain the membership function of the system performance f($\bar{\lambda}$, $\bar{\mu}$) in the following:

$$\eta_{f(\bar{\lambda},\bar{\mu})}(z) = \sup_{\Omega} \min\left\{\eta_{\bar{\lambda}}(x), \eta_{\bar{\lambda}}(y) / z = f(x, y)\right\} \tag{5}$$

Where $\Omega = \left\{x \in X, y \in Y / x > 0, y > 0\right\}$

Consider the two fuzzy system performance $\widetilde{W}_s$ and $\widetilde{N}_q$ the expected waiting time the system and the expected number of customers in queue, respectively. Based on the result of the N-policy M/M/1 queue, where the arrival rate x and the service rate y are constants, the expected waiting time in the system and the expected number of customers in queue, respectively are

$$f(x, y) = \frac{N-1}{2x} + \frac{1}{y-x} \tag{6}$$

and

$$f(x, y) = \frac{N-1}{2} + \frac{x^2}{y(y-x)} \tag{7}$$

As a result the membership functions of $\widetilde{W}_s$ and $\widetilde{N}_q$ respectively, become

$$\eta_{\widetilde{W}_s}(z) = \sup_{\Omega} \min \left\{ \eta_{\tilde{\lambda}}(x), \eta_{\tilde{\lambda}}(y), / z = \frac{N-1}{2x} + \frac{1}{y-x} \right\} \tag{8}$$

$$\eta_{\widetilde{N}_q}(z) = \sup_{\Omega} \min \left\{ \eta_{\tilde{\lambda}}(x), \eta_{\tilde{\mu}}(y), / z = \frac{N-1}{2x} + \frac{x^2}{y(y-x)} \right\} \tag{9}$$

For practical use, the membership functions in equations (8) and (9) are not expressed in the usual forms. Furthermore, it is difficult to infer the shapes of the membership function associated with $\widetilde{W}_s$ and $\widetilde{N}_q$. To this end, a mathematical programming technique is applied to solve this complicated problem.

3. The Parametric nonlinear programming (NLP) approach

Our approach is to construct the membership function $\mu_{p(\tilde{\lambda},\mu)}(z)$ is on the basis of deriving the α-cuts of $\mu_{p(\tilde{\lambda},\tilde{\mu})}(z)$. Denote the α-cuts of $\tilde{\lambda}$ and $\tilde{\mu}$ as

$$\lambda_\alpha = [x_\alpha^L, x_\alpha^U] = \left[\min_{x \in X} \left\{ x / \mu_{\tilde{\lambda}}(x) \geq \alpha \right\}, \max_{x \in X} \left\{ x / \mu_{\tilde{\lambda}}(x) \geq \alpha \right\} \right] \tag{10}$$

$$\mu_\alpha = [y_\alpha^L, y_\alpha^U] = \left[\min_{y \in Y} \left\{ y / \mu_{\tilde{\mu}}(y) \geq \alpha \right\}, \max_{x \in X} \left\{ y / \mu_{\tilde{\mu}}(y) \geq \alpha \right\} \right] \tag{11}$$

From equation (10) and (11), it indicates that $\tilde{\lambda}$ and $\tilde{\mu}$ are lying the range of $[x_\alpha^L, x_\alpha^U]$ and $[y_\alpha^L, y_\alpha^U]$ respectively, at a possibility level α. It is evident that the N-policy FM/FM/1 queue reduces to a family of crisp N-policy D/D/1 queue with different α-level sets {λ(α) / 0 < α ≤ 1} and {μ(α) / 0 / α ≤ 1}. By the fundamental property of convexity of fuzzy numbers (see Zimmermann [25], the upper and lower bounds of $\tilde{\lambda}$ and $\tilde{\mu}$ can be represented as functions of α as

$x_\alpha^L = \min \eta_\lambda^{-1}(\alpha), x_\alpha^U = \max \eta_\lambda^{-1}(\alpha), y_\alpha^L = \min \eta_\mu^{-1}(\alpha)$ and $y_\alpha^U = \max \eta_\mu^{-1}(\alpha)$. Therefore, both the membership functions of $\widetilde{W}_s$ and $\widetilde{N}_q$ are also parametrized by α. We can use the α-cut approach to construct the membership functions of $\widetilde{W}_s$ and $\widetilde{N}_q$.

We first derive the membership functions of the expected waiting time in the system. According to (8), $\eta_{\widetilde{W}_s}(z)$ is the minimum of $\mu_{\tilde{\lambda}}(x)$ and $\mu_{\tilde{\mu}}(y)$. To deal with the membership value, w need one of the two cases in the following such that $f(x,y) = \left\{ \frac{N-1}{2x} + \frac{1}{y-x} \right\}$ to satisfy $\eta_{\widetilde{W}_s}(z) = \alpha$.

Case (i) : $\eta_{\tilde{\lambda}}(x) = \alpha$ and $\eta_{\tilde{\mu}}(y) \geq \alpha$

Case (ii) : $\eta_{\tilde{\lambda}}(x) \geq \alpha$ and $\eta_{\tilde{\mu}}(y) = \alpha$.

This can be carried out by using the parametric nonlinear programming (NLP) technique. For case (i), the lower and upper bounds of α-cuts of $\widetilde{W}_s$ can be obtained via solving the corresponding parametric nonlinear program;

$$(w_s)_\alpha^{L1} = \min_{x,y \in R^+} \left\{ \frac{N-1}{2x} + \frac{1}{y-x} \right\} \tag{12}$$

and

$$(w_s)_\alpha^{U1} = \max_{x,y \in R^+} \left\{ \frac{N-1}{2x} + \frac{1}{y-x} \right\} \tag{13}$$

and

$$(w_s)_\alpha^{L2} = \min_{x,y \in R^+} \left\{ \frac{N-1}{2x} + \frac{1}{y-x} \right\} \tag{14}$$

and

$$(w_s)_\alpha^{U2} = \max_{x,y \in R^+} \left\{ \frac{N-1}{2x} + \frac{1}{y-x} \right\} \tag{15}$$

By considering case(i) and case(ii) simultaneously, we set the lower bound $(w_s)_\alpha^L$ and the upper bound $(w_s)_\alpha^U$ of the α-cuts of $\widetilde{W}_s$ which can be written as

$$(w_s)_\alpha^L = \min_{x,y \in R^+} \left\{ \frac{N-1}{2x} + \frac{1}{y-x} \right\} \tag{16}$$

Such that $x_\alpha^L \leq x \leq x_\alpha^U$ and $y_\alpha^L \leq y \leq y_\alpha^U$

and

$$(w_s)_\alpha^U = \min_{x,y\in R^+}\left\{\frac{N-1}{2x}+\frac{1}{y-x}\right\} \quad (17)$$

Such that $x_\alpha^L \le x \le x_\alpha^U$ and $y_\alpha^L \le y \le y_\alpha^U$

This pair of mathematical programs involves the systematic study of how the optimal solutions change when $x_\alpha^L, x_\alpha^U, y_\alpha^L, y_\alpha^U$ vary over the interval α ∈ [0, 1], thus they fall into the category of parametric NLP (see Gal [6]).

From equations (10) and (11), one can replace x ∈ λ(α) and y ∈ μ(α) by x ∈ $[x_\alpha^L, x_\alpha^U]$ and y ∈ $[y_\alpha^L, y_\alpha^U]$ respectively. It is noted that α-cuts of x and y forms a nested structure with respect to α (see Kaufman [9] and Zimmerman [25]). Considering the two possibility levels α_1 and α_2, we have $\left[x_{\alpha_1}^L, x_{\alpha_1}^U\right] \subseteq \left[x_{\alpha_2}^L, x_{\alpha_2}^U\right]$ and $\left[y_{\alpha_1}^L, y_{\alpha_1}^U\right] \subseteq \left[y_{\alpha_2}^L, y_{\alpha_2}^U\right]$ where $0 < \alpha_2 < \alpha_1 \le 1$. Thus, for $(w_s)_{\alpha_1}^L \ge (w_s)_{\alpha_2}^L$ and $(w_s)_{\alpha_1}^U \ge (w_s)_{\alpha_2}^U$; in other words, $(w_s)_\alpha^L$ increases and $(w_s)_\alpha^U$ decreases as α increases. Therefore, the membership function $\eta_{\widetilde{W}_s}(z)$ can be found.

In order to construct the membership function $\eta_{\widetilde{W}_s}(z)$, let us define an increasing function $(w_s)_\alpha^L : \alpha \to (w_s)_\alpha^L$ and $(w_s)_\alpha^U : \alpha \to (w_s)_\alpha^U$. If both $(w_s)^L$ and $(w_s)^U$ are invertible with respect to α, the membership function $\eta_{\widetilde{W}_s}(z)$ can be obtained as follows.

$$\eta_{\widetilde{W}_s}(z) = \begin{cases} L(z), & (w_s)_{\alpha=0}^L \le z \le (w_s)_{\alpha=1}^L \\ 1, & (w_s)_{\alpha=1}^L \le z \le (w_s)_{\alpha=1}^U \\ R(z), & (w_s)_{\alpha=1}^U \le z \le (w_s)_{\alpha=0}^U \end{cases}$$

Where the left shape function L(z) and the right shape function R(z) are $\left[(w_s)^L\right]^{-1}$ and $\left[(w_s)^U\right]^{-1}$ respectively. If both $(w_s)_\alpha^L$ and $(w_s)_\alpha^U$ cannot be derived analytically, they can still be constructed numerically by enumerating different α levels.

At the end of this section, one may notice that the membership function of the expected number of customer $\eta_{\widetilde{N}_q}(z)$ can be derived in the similar manners to $\eta_{\widetilde{W}_s}(z)$.

4. Application example

In order to illustrate how the proposed method can be applied to the fuzzy n-policy queue, let us consider a case study of parcel service system. For cost saving purpose, the clearance of parcel service begins whenever the number of parcel reaches a critical value N. Parcel arrives at a system in accordance with Poisson process and the service time of the system is exponential. The management of the system would like to know the system performance including the expected waiting time of parcel in the system and the expected number of parcels in queue. Clearly, we can model this system as N-policy FM/FM/1 queue, and construction of membership function of the system performances based on the proposed approach in section 3.

4.1. Expected waiting time in the system for the fuzzy N-policy queue

A trapezoidal fuzzy number $\widetilde{A}$ = [a, b, c, d] is a fuzzy set which has a membership function denoted for all x ∈ R by

$$\eta_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a}, & a\le x\le b\\ 1, & b\le x\le c\\ \dfrac{d-x}{d-c}, & c\le x\le d\end{cases}$$

Suppose that both the inter-arrival rate and the service rate are trapezoidal fuzzy numbers described by $\tilde{\lambda}$ = [1, 2, 3, 4] and $\tilde{\mu}$ = [11, 12, 13, 14] per hour respectively. For the threshold value, we choose N = 4 and thereby the reflow machine operates when four parcels are accumulated. It is simple to determine

$$\left[x_\alpha^L, x_\alpha^U\right]=\left[\min_{x\in X}\left\{x/\mu_{\tilde{\lambda}}(x)\right\}\ge\alpha, \max_{x\in X}\left\{x/\mu_{\tilde{\lambda}}(x)\right\}\ge\alpha\right]=[1+\alpha, 4-\alpha]$$

and

$$\left[y_\alpha^L, y_\alpha^U\right]=\left[\min_{y\in Y}\left\{y/\mu_{\tilde{\mu}}(y)\right\}\ge\alpha, \max_{x\in X}\left\{y/\mu_{\tilde{\mu}}(y)\right\}\ge\alpha\right]=[11+\alpha, 14-\alpha]$$

Obviously, the expected waiting time in the system attains its minimum value when $x=x_\alpha^U$ and $y=y_\alpha^U$ and on the contrary it attains its maximum value when $x=x_\alpha^L$ and $y=y_\alpha^L$. According to the equations (16) and (17), the lower and upper bounds of the α-cut of w_s, respectively, are given by

$$(w_s)_\alpha^L=\frac{N-1}{2(4-\alpha)}+\frac{1}{(14-\alpha)-(4-\alpha)}$$

and

$$(w_s)^u_\alpha = \frac{N-1}{2(1+\alpha)} + \frac{1}{(11+\alpha)-(1+\alpha)}$$

$$(w_s)^L_\alpha = \frac{3}{2(4-\alpha)} + \frac{1}{10}$$

and

$$(w_s)^u_\alpha = \frac{3}{2(1+\alpha)} + \frac{1}{12}$$

$(w_s)^L_\alpha$ is invertible

$$\text{Let } z = \frac{3}{2(4-\alpha)} + \frac{1}{10}$$

$$\alpha = \frac{(19-40z)}{(1-10z)}$$

$$(w_s)^L_\alpha = \frac{(19-40z)}{(1-10z)} \qquad \frac{19}{40} \le z \le \frac{18}{30}$$

$(w_s)^u_\alpha$ is invertible

$$\text{Let } z = \frac{3}{2(1+\alpha)} + \frac{1}{12}$$

$$\alpha = \frac{(19-12z)}{(12z-1)}$$

$$(w_s)^u_\alpha = \frac{(19-12z)}{(12z-1)} \qquad \frac{20}{24} \le z \le \frac{19}{12}$$

$$\eta_{\widetilde{W}_s}(z) = \begin{cases} \frac{(19-40z)}{(1-10z)}, & \frac{19}{40} \le z \le \frac{18}{30} \\ 1, & \frac{18}{30} \le z \le \frac{20}{24} \\ \frac{(19-12z)}{(12z-1)}, & \frac{20}{24} \le z \le \frac{19}{12} \end{cases}$$

Table 1: The α-cuts of the arrival rate, service rate and the expected waiting time of parcels in the system at 11 distinct α values

α	x_α^L	x_α^U	y_α^L	y_α^U	$(w_s)_\alpha^L$	$(w_s)_\alpha^U$
0.00	1.00	4.00	11.00	14.00	0.475	1.580
0.01	1.10	3.90	11.10	13.90	0.485	1.447
0.02	1.20	3.80	11.20	13.80	0.495	1.333
0.03	1.30	3.70	11.30	13.70	0.505	1.237
0.04	1.40	3.60	11.40	13.60	0.517	1.155
0.05	1.50	3.50	11.50	13.50	0.529	1.083
0.06	1.60	3.40	11.60	13.40	0.541	1.021
0.07	1.70	3.30	11.70	13.30	0.555	0.966
0.08	1.80	3.20	11.80	13.20	0.569	0.917
0.09	1.90	3.10	11.90	13.10	0.584	0.873
1.00	2.00	3.00	12.00	13.00	0.600	0.833

From the table it is evident that the fuzzy expected waiting time in the system is most likely to fall between 0.6 and 0.833 when 1. At the extreme value, α = 0, the fuzzy expected waiting time in the system is impossible to fall below 0.475 and 1.580. This result will be highly useful for the system practitioner to know the system performances in fuzzy environment.

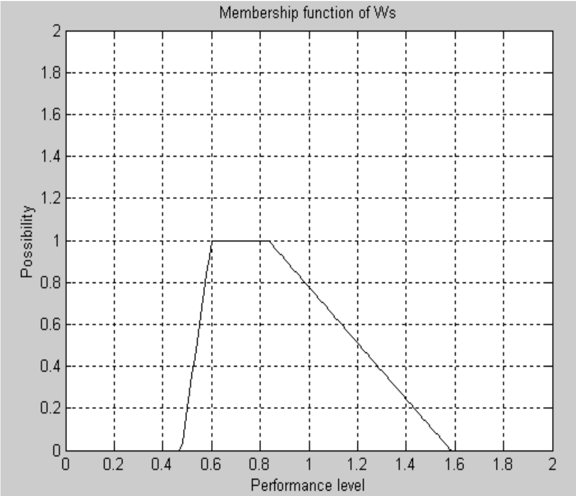

Figure 1: The membership function of the expected waiting time in the system for the fuzzy N-policy queue

4.2. Expected number of customers in the queue for the fuzzy N-policy queue

Similar to α-cuts of waiting time in the system, the α-cuts of number of customers in the queue is described as follows.

$$(N_q)_\alpha^L = \min_{x,y\in R^+}\left\{\frac{N-1}{2}+\frac{x^2}{y(y-x)}\right\} \tag{18}$$

Such that $x_\alpha^L \le x \le x_\alpha^U$ and $y_\alpha^L \le y \le y_\alpha^U$

and

$$(N_q)_\alpha^U = \max_{x,y\in R^+}\left\{\frac{N-1}{2}+\frac{x^2}{y(y-x)}\right\} \tag{19}$$

Such that $x_\alpha^L \le x \le x_\alpha^U$ and $y_\alpha^L \le y \le y_\alpha^U$

Obviously, the expected number of customers in queue attains its minimum value when $x = x_\alpha^L$ and $y = y_\alpha^U$ and on the contrary it attains its maximum value when $x = x_\alpha^U$ and $y = y_\alpha^L$. According to the equations (18) and (19), the lower and upper bounds of the α-cut of N_q, respectively, are given by

$$(N_q)_\alpha^L = \frac{3}{2}+\frac{\alpha^2+2\alpha+1}{(14-\alpha)(13-2\alpha)}$$

and

$$(N_q)_\alpha^U = \frac{3}{2}+\frac{\alpha^2-8\alpha+16}{(11+\alpha)(7+2\alpha)}$$

$(N_q)_\alpha^L$ is invertible

Let z = $\dfrac{8\alpha^2-119\alpha+548}{4\alpha^2-82\alpha+364}$

$$\alpha = \frac{82z-119\pm\sqrt{900z^2+900z-3375}}{8z-16}$$

$$(N_q)_\alpha^L = \frac{82z-119\pm\sqrt{900z^2+900z-3375}}{8z-16} \qquad 1.505 \le z \le 1.546$$

$(N_q)_\alpha^U$ is invertible

Let z = $\dfrac{8\alpha^2+71\alpha+263}{4\alpha^2+58\alpha+154}$

$$\alpha = \frac{58z-71\pm\sqrt{900z^2+900z-3375}}{16-8z}$$

$$(N_q)_\alpha^U = \frac{58z - 71 \pm \sqrt{900z^2 + 900z - 3375}}{16 - 8z} \qquad 1.58 \le z \le 1.707$$

$$\eta_{\tilde{N}_q}(z) = \begin{cases} \dfrac{82z - 119 \pm \sqrt{900z^2 + 900z - 3375}}{8z - 16} & 1.505 \le z \le 1.546 \\ 1 & 1.546 \le z \le 1.58 \\ \dfrac{58z - 71 \pm \sqrt{900z^2 + 900z - 3375}}{16 - 8z} & 1.58 \le z \le 1.707 \end{cases}$$

Table 1: The α-cuts of the arrival rate, service rate and the expected number of parcels in queue at 11 distinct α values

α	x_α^L	x_α^U	y_α^L	y_α^U	$(N)_\alpha^L$	$(N_q)_\alpha^u$
0.00	1.00	4.00	11.00	14.00	1.505	1.707
0.01	1.10	3.90	11.10	13.90	1.533	1.690
0.02	1.20	3.80	11.20	13.80	1.508	1.675
0.03	1.30	3.70	11.30	13.70	1.509	1.659
0.04	1.40	3.60	11.40	13.60	1.512	1.646
0.05	1.50	3.50	11.50	13.50	1.514	1.633
0.06	1.60	3.40	11.60	13.40	1.516	1.622
0.07	1.70	3.30	11.70	13.30	1.519	1.611
0.08	1.80	3.20	11.80	13.20	1.522	1.601
0.09	1.90	3.10	11.90	13.10	1.525	1.592
1.00	2.00	3.00	12.00	13.00	1.527	1.583

From the table it is evident that the fuzzy expected number of parcels in the queue is most likely to fall between 1.527 and 1.583 when α = 1. At the extreme value, α = 0, the fuzzy expected length of queue is impossible to fall below 1.505 or exceed 1.707. Summarizing the above results, the system performances are expressed by membership functions rather than possible values. Accordingly, more information is available on the membership functions. Definitely this information would be highly useful for the system designers and practitioners for designing their desired queuing systems.

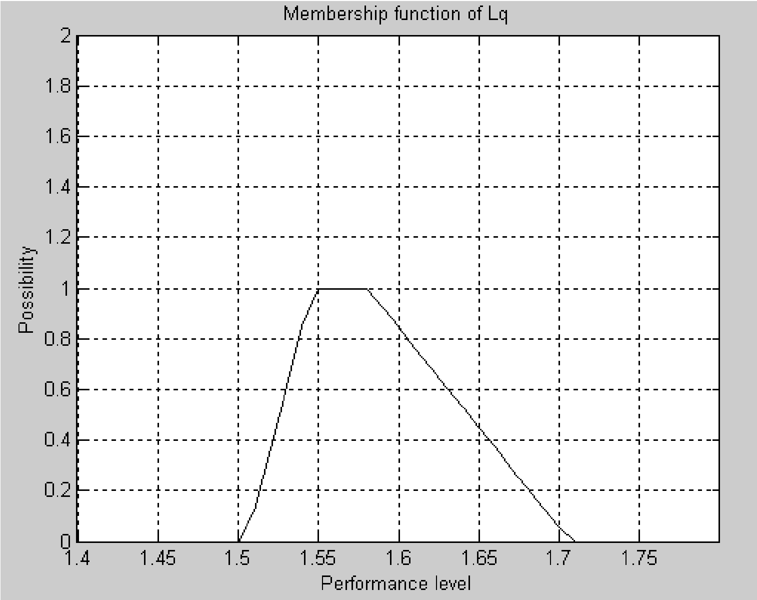

Figure 2: The membership function of the expected queue length for the fuzzy N-policy queue

5. Conclusion

The fuzzy set theory has been applied to some classical queuing systems. In this paper, we investigated the N-policy FM/FM/1 queue. This paper applies the concepts of α-cuts and Zadeh's extension principle to a N-policy FM/FM/1 queuing and construct the membership functions of expected waiting time in the system (w_s) and expected queue length (N_q) using paired NLP approach, α-cuts of the membership functions are found and their interval limits are inverted to attain explicit closed-form expressions for the membership functions of w_s and L_q. It is worth to inform that the system performances are expressed by the membership functions rather than characteristic functions, that it would provide more realistic information to the system practitioners. Finally, a real world example of parcel system is investigated to illustrate the applicability of the proposed approach.

6. Open Problem

Fuzzy set theory has been applied to some classical queuing problems. By using Zadeh's extension principle, the system performances of interest for the expected waiting time of customer in the system (W_s) and the expected number of customers in queue (N_q) are evaluated for N-policy queue which is fuzzy when the arrival rate and service rate are fuzzy. α-cut approach is used to construct the membership functions of Fuzzy system performances $\tilde{W}_s$ and $\tilde{N}_q$ which are modeled by a set of parametric NLP. By inverting α-cuts of the corresponding membership functions the closed form expression is derived for $\tilde{W}_s$ and $\tilde{N}_q$. Here it is worth to mention that the system performances were described by membership functions rather than the characteristic functions. The proposed approach is applied N-policy queue with infinite capacity, nevertheless the procedure can be applied to any N-policy queue like N-policy queue with infinite capacity, nevertheless the procedure can be applied to any N-policy queue like N-policy queue with finite capacity, N-policy queue with vacations (single and multiple) N-policy queue with breakdown etc.,

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