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An Adaptive Preconditioner Matrix on N-P Group AOR Iterative Poisson Solver

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Abstract

Hadjidimos [1], proved that the Accelerated OverRelaxation (AOR) is more powerful compared with the other well-known method called the Successive OverRelaxation (SOR) for solving linear systems of equations. The formulation of group iterative schemes for approximating the solution of the two dimensional elliptic partial differential equations have been the subject of intensive study during the last few years. The recent convergence results of nine-point (N-P) group iterative schemes from the Successive OverRelaxation (SOR) family have been presented by Saeed [2]. In this paper, we extend the work of Saeed [2] with the new application of suitable preconditioning techniques to the N-P Group iterative schemes from the Accelerated OverRelaxation (AOR) for solving Poisson's Equation. The results reveal the significant improvement in number of iterations and execution timings of the proposed preconditioned Group iterative method compared to Preconditioned N-P SOR.

Keywords: Preconditioning Techniques; Nine-Point Group Iterative Method; AOR.

1. Introduction

It has been confirmed that the discretisation of partial differential equations (PDEs) using finite difference schemes normally yield a system of linear equations, which are large and sparse in nature. Iterative methods are usually used to solve these types of systems since these methods need less storage and are capable of preserving the sparsity property of the large system. Many researchers have considered preconditioners which applied to these iterative methods for solving linear systems ([3], [4], [5], [6]). In Saeed [2], the application of the new preconditioner in block formulation for the N-P Group SOR iterative method is presented to accelerate the convergence rate of this group method. The resulted preconditioned system showed improvements in the number of iterations and the execution time. In this research the most efficient preconditioned group AOR iterative method for solving elliptic partial differential equations will be investigated. Furthermore, we will compare the proposed method with the original nine-point group AOR iterative method and the earlier preconditioned group SOR [2] for solving the two dimensional Poisson equation.
Consider the Poisson equation in the form:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y), \quad (x,y) \in \Omega \tag{1.1}$$

with specific Dirichlet boundary conditions

$$U(x,y) = g(x,y), \qquad (x,y) \in \partial\Omega .$$

In SOR method, we have to determine the parameter ω, where a suitable value of w could lead to drastic improvements in convergence. The AOR method involves two parameters, r and ω. We observe that for specific values of these parameters, we can obtain Jacobi, Gauss-Seidel and SOR iterative methods. If $r=0$ and $\omega=1$, we have the Jacobi method. If $r=\omega=1$, Gauss-Seidel method can be obtained, and for the SOR method, we consider $r=\omega$ [1]. It is well known that the AOR method is an iterative method for the numerical solution of the linear systems of equations,

$$A\bar{u} = \overline{f} \tag{1.2}$$

and also, we can consider the AOR method as a generalization method of Jacobi, Gauss-Seidel and SOR iterative methods.

This paper is organised as follows: in Section 2, we present the derivation of the proposed preconditioned N-P AOR method. The numerical results are presented to show the efficiency of the preconditioned N-P AOR method in Section 3. Finally, we report a brief conclusion in Section 4.

2. Derivation of The Proposed Preconditioned N-P AOR Method

Suppose equation (1.1) is discretised using some finite difference scheme, this will normally lead to a large, block and sparse system of equation (1.2).

Equation (1.1) may be approximated at the point (x_i, y_j) in many ways. Assume that a rectangular grid in the (x,y) plane with equal grid spacing h in both directions with $x_i = ih, y = jh(i, j = 0,1,....N)$ are used, where $u_{i,j} = u(x_i, y_j)$ and $h = 1/N$. By neglecting terms of $O(h^2)$, we obtain the simplest approximation for (1.1) which is known as the standard five-point difference formula:

$$u_{i,j+1} + u_{i,j-1} + u_{i+1,j} + u_{i-1,j} - 4u_{ij} = h^2 f_{ij} \tag{2.1}$$

According to Saeed [2], the explicit 9-point group iterative equations are given by:

$$u_{ij} = \frac{1}{224}[67t_1 + 22t_2 + 7t_7 - 14t_0 + 6t_5 + 3t_6], \qquad u_{i+1,j} = \frac{1}{112}[37t_{19} + 11t_8 + 7t_9 - 14t_0 + 5t_{20} + 3t_{10}],$$

$$u_{i+2,j} = \frac{1}{224}[67t_3 + 22t_{13} + 7t_{18} - 14t_0 + 6t_{14} + 3t_4], \qquad u_{i,j+1} = \frac{1}{112}[37t_{21} + 11t_{15} + 7t_{16} - 14t_0 + 5t_{22} + 3t_{17}],$$

$$u_{i+1,j+1} = \frac{1}{16}[2t_{11} - 6t_0 + t_{12}], \qquad u_{i+2,j+1} = \frac{1}{112}[37t_{22} + 11t_{17} + 7t_{16} - 14t_0 + 5t_{21} + 3t_{15}],$$

$$u_{i,j+2} = \frac{1}{224}[67t_4 + 22t_{14} + 7t_{18} - 14t_0 + 6t_{13} + 3t_3], \qquad u_{i+1,j+2} = \frac{1}{112}[37t_{20} + 11t_{10} + 7t_9 - 14t_0 + 5t_{19} + 3t_8],$$

$$u_{i+2,j+2} = \frac{1}{224}[67t_6 + 22t_5 + 7t_7 - 14t_0 + 6t_2 + 3t_1] \tag{2.2}$$

where:

$t_0 = h^2 f_{i+1,j+1}$, $t_1 = u_{i-1,j} + u_{i,j-1} - h^2 f_{i+1,j+1}$, $t_2 = u_{i+1,j-1} + u_{i-1,j+1} - h^2 f_{i+1,j} - h^2 f_{i,j+1}$,
$t_3 = u_{i+2,j-1} + u_{i+3,j} - h^2 f_{i+2,j}$, $t_4 = u_{i-1,j+2} + u_{i,j+3} - h^2 f_{i,j+2}$, $t_5 = u_{i+3,j+1} + u_{i+1,j+3} - h^2 f_{i+2,j+1} - h^2 f_{i+1,j+2}$,
$t_6 = u_{i+3,j+2} + u_{i+2,j+3} - h^2 f_{i+2,j+2}$, $t_7 = t_3 + t_4$, $t_8 = t_1 + t_3$, $t_9 = u_{i+3,j+1} + u_{i-1,j+1} - h^2 f_{i+2,j+1} - h^2 f_{i,j+1}$,
$t_{10} = t_4 + t_6$, $t_{11} = t_2 + t_5$, $t_{12} = t_8 + t_{10}$, $t_{13} = u_{i+1,j-1} + u_{i+3,j+1} - h^2 f_{i+1,j} - h^2 f_{i+2,j+1}$,
$t_{14} = u_{i-1,j+1} + u_{i+1,j+3} - h^2 f_{i,j+1} - h^2 f_{i+1,j+2}$, $t_{15} = t_1 + t_4$, $t_{16} = u_{i+1,j-1} + u_{i+1,j+3} - h^2 f_{i+1,j} - h^2 f_{i+1,j+2}$,
$t_{17} = t_3 + t_6$, $t_{18} = t_1 + t_6$, $t_{19} = u_{i+1,j-1} - h^2 f_{i+1,j}$, $t_{20} = u_{i+1,j+3} - h^2 f_{i+1,j+2}$, $t_{21} = u_{i-1,j+1} - h^2 f_{i,j+1}$,
$t_{22} = u_{i+3,j+1} - h^2 f_{i+2,j+1}$.

and then the nine-point SOR iterative scheme can be written as:

$$u_{ij}^{(k+1)} = \frac{1}{224}[\omega(67t_1 + 22t_2 + 7t_7 - 14t_0 + 6t_5 + 3t_6)] + (1-\omega)u_{ij}^{(k)},$$

$$u_{i+1,j}^{(k+1)} = \frac{1}{112}[\omega(37t_{19} + 11t_8 + 7t_9 - 14t_0 + 5t_{20} + 3t_{10})] + (1-\omega)u_{i+1,j}^{(k)},$$

$$u_{i+2,j}^{(k+1)} = \frac{1}{224}[\omega(67t_3 + 22t_{13} + 7t_{18} - 14t_0 + 6t_{14} + 3t_4)] + (1-\omega)u_{i+2,j}^{(k)},$$

$$u_{i,j+1}^{(k+1)} = \frac{1}{112}[\omega(37t_{21} + 11t_{15} + 7t_{16} - 14t_0 + 5t_{22} + 3t_{17})] + (1-\omega)u_{i,j+1}^{(k)},$$

$$u_{i+1,j+1}^{(k+1)} = \frac{1}{16}[\omega(2t_{11} - 6t_0 + t_{12})] + (1-\omega)u_{i+1,j+1}^{(k)},$$

$$u_{i+2,j+1}^{(k+1)} = \frac{1}{112}[\omega(37t_{22} + 11t_{17} + 7t_{16} - 14t_0 + 5t_{21} + 3t_{15})] + (1-\omega)u_{i+2,j+1}^{(k)},$$

$$u_{i,j+2}^{(k+1)} = \frac{1}{224}[\omega(67t_4 + 22t_{14} + 7t_{18} - 14t_0 + 6t_{13} + 3t_3)] + (1-\omega)u_{i,j+2}^{(k)},$$

$$u_{i+1,j+2}^{(k+1)} = \frac{1}{112}[\omega(37t_{20} + 11t_{10} + 7t_9 - 14t_0 + 5t_{19} + 3t_8)] + (1-\omega)u_{i+1,j+2}^{(k)},$$

$$u_{i+2,j+2}^{(k+1)} = \frac{1}{224}[\omega(67t_6 + 22t_5 + 7t_7 - 14t_0 + 6t_2 + 3t_1)] + (1-\omega)u_{i+2,j+2}^{(k)}, \tag{2.3}$$

where

$t_0 = h^2 f_{i+1,j+1}$, $\quad t_1 = u_{i-1,j}^{(k+1)} + u_{i,j-1}^{(k+1)} - h^2 f_{i,j}$, $\quad t_2 = u_{i+1,j-1}^{(k+1)} + u_{i-1,j+1}^{(k+1)} - h^2 f_{i+1,j} - h^2 f_{i,j+1}$,
$t_3 = u_{i+2,j-1}^{(k+1)} + u_{i+3,j}^{(k)} - h^2 f_{i+3,j} - h^2 f_{i+2,j}$, $\quad t_4 = u_{i-1,j+2}^{(k+1)} + u_{i,j+3}^{(k)} - h^2 f_{i,j+2}$,
$t_5 = u_{i+3,j+1}^{(k)} + u_{i+1,j+3}^{(k)} - h^2 f_{i+2,j+1} - h^2 f_{i+1,j+2}$, $\quad t_6 = u_{i+2,j+2}^{(k)} + u_{i+2,j+3}^{(k)} - h^2 f_{i+2,j+2}$,

$t_7 = t_3 + t_4$, $\quad t_8 = t_1 + t_3$, $\quad t_9 = u_{i+3,j+1}^{(k)} + u_{i-1,j+1}^{(k+1)} - h^2 f_{i+2,j+1} - h^2 f_{i,j+1}$,
$t_{10} = t_4 + t_6$, $\quad t_{11} = t_2 + t_5$, $\quad t_{12} = t_8 + t_{10}$, $\quad t_{13} = u_{i+1,j-1}^{(k+1)} + u_{i+3,j+1}^{(k)} - h^2 f_{i+1,j} - h^2 f_{i+2,j+1}$,
$t_{14} = u_{i-1,j+1}^{(k+1)} + u_{i+1,j+3}^{(k)} - h^2 f_{i,j+1} - h^2 f_{i+1,j+2}$, $\quad t_{15} = t_1 + t_4$, $\quad t_{16} = u_{i+1,j-1}^{(k+1)} + u_{i+1,j+3}^{(k)} - h^2 f_{i+1,j} - h^2 f_{i+1,j+2}$,
$t_{17} = t_3 + t_6$, $\quad t_{18} = t_1 + t_6$, $\quad t_{19} = u_{i+1,j-1}^{(k+1)} - h^2 f_{i+1,j}$,
$t_{20} = u_{i+1,j+3}^{(k)} - h^2 f_{i+1,j+2}$, $\quad t_{21} = u_{i-1,j+1}^{(k+1)} - h^2 f_{i,j+1}$, $\quad t_{22} = u_{i+3,j+1}^{(k)} - h^2 f_{i+2,j+1}$.

Matrix A of (1.2) is also written as

$$A = D - L - U, \tag{2.4}$$

where D is a diagonal matrix and L and U are strictly lower and upper triangular matrices, respectively. The AOR iterative method can be written as:

$$u^{(k+1)} = L_{r,\omega}u^{(k)} + \omega(D - rL)^{-1} f \tag{2.5}$$

where $\quad L_{r,\omega} = (I - rD^{-1}L)^{-1}[(1-\omega)I + (\omega - r)D^{-1}L + \omega D^{-1}U]$.

Equation (2.5) can be rewritten as

$$(D - rL)\underline{u}^{(k+1)} = (\omega - r)L\underline{u}^{(k)} + \omega U\underline{u}^{(k)} + \omega\overline{f} + (1-\omega)D\underline{u}^{(k)} \tag{2.6}$$

We also can write equation (2.6) as

$$D\underline{u}^{(k+1)} = rL(\underline{u}^{(k+1)} - \underline{u}^{(k)}) + \omega L\underline{u}^{(k)} + \omega U\underline{u}^{(k)} + \omega\overline{f} + (1-\omega)D\underline{u}^{(k)}, \tag{2.7}$$

We can observe that the coefficient for expressions $u_{i-1,j}^{(k+1)}$, $u_{i,j-1}^{(k+1)}$, $u_{i+1,j-1}^{(k+1)}$, $u_{i+2,j-1}^{(k+1)}$, $u_{i-1,j+2}^{(k+1)}$ and $u_{i-1,j+1}^{(k+1)}$ contained in L.

In order to construct AOR scheme, we have to change these expressions to $u_{i-1,j}^{(k)}$, $u_{i,j-1}^{(k)}$, $u_{i+1,j-1}^{(k)}$, $u_{i+2,j-1}^{(k)}$, $u_{i-1,j+2}^{(k)}$ and $u_{i-1,j+1}^{(k)}$. After that, add expressions $\alpha r(u_{i-1,j}^{(k+1)} - u_{i-1,j}^{(k)})$, $\alpha r(u_{i,j-1}^{(k+1)} - u_{i,j-1}^{(k)})$, $\alpha r(u_{i+1,j-1}^{(k+1)} - u_{i+1,j-1}^{(k)})$, $\alpha r(u_{i+2,j-1}^{(k+1)} - u_{i+2,j-1}^{(k)})$, $\alpha r(u_{i-1,j+2}^{(k+1)} - u_{i-1,j+2}^{(k)})$ and $\alpha r(u_{i-1,j+1}^{(k+1)} - u_{i-1,j+1}^{(k)})$ to correspond SOR iterative scheme, where α is the coefficient for those expressions.

Hence, nine-point group AOR iterative scheme can be written as:

$$u_{ij}^{(k+1)} = \frac{1}{224}[\omega(67t_1 + 22t_2 + 7t_7 - 14t_0 + 6t_5 + 3t_6) + r(67c_7 + 22c_8 + 7c_9] + (1-\omega)u_{ij}^{(k)},$$

$$u_{i+1,j}^{(k+1)} = \frac{1}{112}[\omega(37t_{19} + 11t_8 + 7t_9 - 14t_0 + 5t_{20} + 3t_{10}) + r(37c_3 + 11c_{10} + 7c_4 + 3c_6] + (1-\omega)u_{i+1,j}^{(k)},$$

$$u_{i+2,j}^{(k+1)} = \frac{1}{224}[\omega(67t_3 + 22t_{13} + 7t_{18} - 14t_0 + 6t_{14} + 3t_4) + r(67c_5 + 22c_3 + 7c_7 + 6c_4 + 3c_6] + (1-\omega)u_{i+2,j}^{(k)},$$

$$u_{i,j+1}^{(k+1)} = \frac{1}{112}[\omega(37t_{21} + 11t_{15} + 7t_{16} - 14t_0 + 5t_{22} + 3t_{17}) + r(37c_4 + 11c_{12} + 7c_3 + 3c_5] + (1-\omega)u_{i,j+1}^{(k)},$$

$$u_{i+1,j+1}^{(k+1)} = \frac{1}{16}[\omega(2t_{11} - 6t_0 + t_{12}) + r(2c_8 + c_{11}] + (1-\omega)u_{i+1,j+1}^{(k)},$$

$$u_{i+2,j+1}^{(k+1)} = \frac{1}{112}[\omega(37t_{22} + 11t_{17} + 7t_{16} - 14t_0 + 5t_{21} + 3t_{15}) + r(11c_5 + 7c_3 + 5c_4 + 3c_{12}] + (1-\omega)u_{i+2,j+1}^{(k)},$$

$$u_{i,j+2}^{(k+1)} = \frac{1}{224}[\omega(67t_4 + 22t_{14} + 7t_{18} - 14t_0 + 6t_{13} + 3t_3) + r(67c_6 + 22c_4 + 7c_7 + 6c_3 + 3c_5] + (1-\omega)u_{i,j+2}^{(k)},$$

$$u_{i+1,j+2}^{(k+1)} = \frac{1}{112}[\omega(37t_{20} + 11t_{10} + 7t_9 - 14t_0 + 5t_{19} + 3t_8) + r(11c_6 + 7c_4 + 5c_3 + 3c_{10}] + (1-\omega)u_{i+1,j+2}^{(k)},$$

$$u_{i+2,j+2}^{(k+1)} = \frac{1}{224}[\omega(67t_6 + 22t_5 + 7t_7 - 14t_0 + 6t_2 + 3t_1) + r(7c_9 + 6c_8 + 3c_7] + (1-\omega)u_{i+2,j+2}^{(k)}$$

where

$$t_0 = h^2 f_{i+1,j+1}, \quad t_1 = u_{i-1,j}^{(k)} + u_{i,j-1}^{(k)} - h^2 f_{i,j}, \quad t_2 = u_{i+1,j-1}^{(k)} + u_{i-1,j+1}^{(k)} - h^2 f_{i+1,j} - h^2 f_{i,j+1},$$

$$t_3 = u_{i+2,j-1}^{(k)} + u_{i+3,j}^{(k)} - h^2 f_{i+3,j} - h^2 f_{i+2,j}, \qquad t_4 = u_{i-1,j+2}^{(k)} + u_{i,j+3}^{(k)} - h^2 f_{i,j+2},$$

$$t_5 = u_{i+3,j+1}^{(k)} + u_{i+1,j+3}^{(k)} - h^2 f_{i+2,j+1} - h^2 f_{i+1,j+2}, \qquad t_6 = u_{i+2,j+2}^{(k)} + u_{i+2,j+3}^{(k)} - h^2 f_{i+2,j+2},$$

$$t_7 = t_3 + t_4, \qquad t_8 = t_1 + t_3, \qquad t_9 = u_{i+3,j+1}^{(k)} + u_{i-1,j+1}^{(k)} - h^2 f_{i+2,j+1} - h^2 f_{i,j+1},$$

$$t_{10} = t_4 + t_6, \quad t_{11} = t_2 + t_5, \quad t_{12} = t_8 + t_{10}, \quad t_{13} = u_{i+1,j-1}^{(k)} + u_{i+3,j+1}^{(k)} - h^2 f_{i+1,j} - h^2 f_{i+2,j+1},$$

$$t_{14} = u_{i-1,j+1}^{(k)} + u_{i+1,j+3}^{(k)} - h^2 f_{i,j+1} - h^2 f_{i+1,j+2}, \qquad t_{15} = t_1 + t_4,$$

$$t_{16} = u_{i+1,j-1}^{(k)} + u_{i+1,j+3}^{(k)} - h^2 f_{i+1,j} - h^2 f_{i+1,j+2}, \qquad t_{17} = t_3 + t_6, \qquad t_{18} = t_1 + t_6,$$

$$t_{19} = u_{i+1,j-1}^{(k)} - h^2 f_{i+1,j}, \quad t_{20} = u_{i+1,j+3}^{(k)} - h^2 f_{i+1,j+2}, \quad t_{21} = u_{i-1,j+1}^{(k)} - h^2 f_{i,j+1}, \quad t_{22} = u_{i+3,j+1}^{(k)} - h^2 f_{i+2,j+1},$$

$$c_1 = u_{i-1,j}^{(k+1)} - u_{i-1,j}^{(k)}, \qquad c_2 = u_{i,j-1}^{(k+1)} - u_{i,j-1}^{(k)}, \qquad c_3 = u_{i+1,j-1}^{(k+1)} - u_{i+1,j-1}^{(k)},$$

$$c_4 = u_{i-1,j+1}^{(k+1)} - u_{i-1,j+1}^{(k)}, \quad c_5 = u_{i+2,j-1}^{(k+1)} - u_{i+2,j-1}^{(k)}, \quad c_6 = u_{i-1,j+2}^{(k+1)} - u_{i-1,j+2}^{(k)},$$

$$c_7 = c_1 + c_2, \qquad c_8 = c_3 + c_4, \qquad c_9 = c_5 + c_6, \qquad c_{10} = c_5 + c_7,$$

$$c_{11} = c_{10} + c_6, \qquad c_{12} = c_6 + c_7.$$

The convergence rates of the system (1.2) depend on the spectral properties of the coefficient matrix *A*. A preconditioner is a matrix that transforms the linear system into one that is equivalent in the sense that it has the same solution, but that has more favorable spectral properties.

For the nine-point group method, the matrix *A*, vectors $\bar{u}$ and $\bar{f}$ are as defined in (1.2)). Therefore the precondetioner P, is obtained in the form: $P = I + k\,L$; $1 \le k \le 2$ and then, we can write the preconditioned system as the following:

$$P(A)\bar{u} = P\,\bar{f} \;\Rightarrow\; (I + k\,L)(A)\bar{u} = (I + k\,L)\;\bar{f} \tag{2.8}$$

It can be seen that the proposed preconditioned system (2.8) have same solution of the original system but these proposed scheme has more favorable convergence properties.

3. Numerical Results and Discussions

For comparison purpose, we will use the model problem of Poisson equation in the form [2]:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)e^{xy}, \tag{3.1}$$

with $\quad u(x,0) = u(0,y) = 1, \quad u(x,1) = e^x, \quad u(1,y) = e^y, \qquad 0 \le x, y \le 1.$

The exact solution for this problem is $u(x,y) = e^{xy}$. In this experimental work, we choose the value of tolerance; $\varepsilon = 10^{-6}$. The computer processing unit is Intel(R) Core(TM) i5 with memory of 4Gb and the software used to implement and generate the results was Developer C++ Version 4.9.9.2. We have computed the average absolute errors and record the number of iterations for convergence for different size of grids 45, 85, 105, 145, 185 and 225.

Table 1 shows the comparison of the results for nine-point group AOR and preconditioned nine-point group AOR iterative methods. The results show the corresponding values of *r* and optimum *w*, number of iterations (k), the CPU time, and the maximum error (e). In addition, Fig.1 shows the comparison of the number of iterations between these two methods. The graph explained that the preconditioned nine-point group AOR method gives the minimum number of iterations and the difference became obvious when the value of N increased.

Table 1. Comparison of number of iterations, execution time N-P AOR and preconditioned N-P AOR iterative methods

N	*r*	*w*	N-P Group AOR			Preconditioned N-P Group AOR		
			k	t	e	k	t	e
45	1.678	1.673-1.681	30	0.003	2.85E-06	28	0.000	2.74E-06
85	1.784	1.704-1.713	35	0.011	2.87E-06	32	0.000	2.76E-06
105	1.871	1.744-1.761	64	0.029	2.89E-06	46	0.012	2.83E-06
145	1.895	1.763-1.779	88	0.034	2.37E-06	72	0.019	2.41E-06
185	1.931	1.855-1.892	102	0.057	1.99E-06	80	0.038	2.36E-06
225	1.934	1.903-1.944	114	0.108	2.36E-06	96	0.088	2.07E-06

Since the convergence of the iteration methods relies on the spectral radius, which is defined as the largest of the moduli of the eigenvalues of the iteration matrix. It is stated and proven that a linear system with smaller value of spectral radius will have better convergence rate [7]. Thus, the spectral radius of the coefficient matrix of the original system and the preconditioned system will be compared in order to justify the performance and suitability of the preconditioner. Since there are no special theoretical formulas that can be used to determine the spectral radiuses of the preconditioned matrices, therefore, we use Matlab software to estimate the values of the spectral radius by the same manner of the work by Saeed [2].

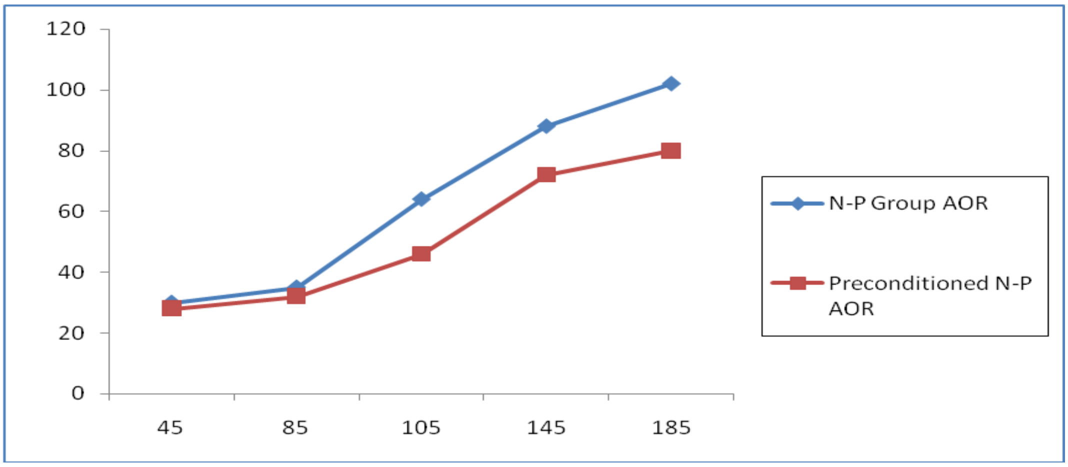

Fig. 1: Comparison of number of iterations (k) for N-P AOR and preconditioned N-P AOR iterative methods

Table 2 and Fig.2 show the comparison of the spectral radius between the original N-P Group AOR and the preconditioned N-P Group AOR systems. Clearly it can be seen that the spectral radius of the preconditioned system is smaller compared to the original system, thus justifying our findings.

Table 2: Comparison of spectral radius between the original and the preconditioned linear systems

N	Original N-P Group AOR system	Preconditioned N-P Group AOR system
45	0.6851	0.4037
85	0.7944	0.4225
105	0.8252	0.4654
145	0.8804	0.5006
185	0.8957	0.5691
225	0.9342	0.6153

Furthermore, we can observe that the results reveal the significant improvement in number of iterations and execution timings of the proposed preconditioned Group iterative method compared to the results obtained in [2].

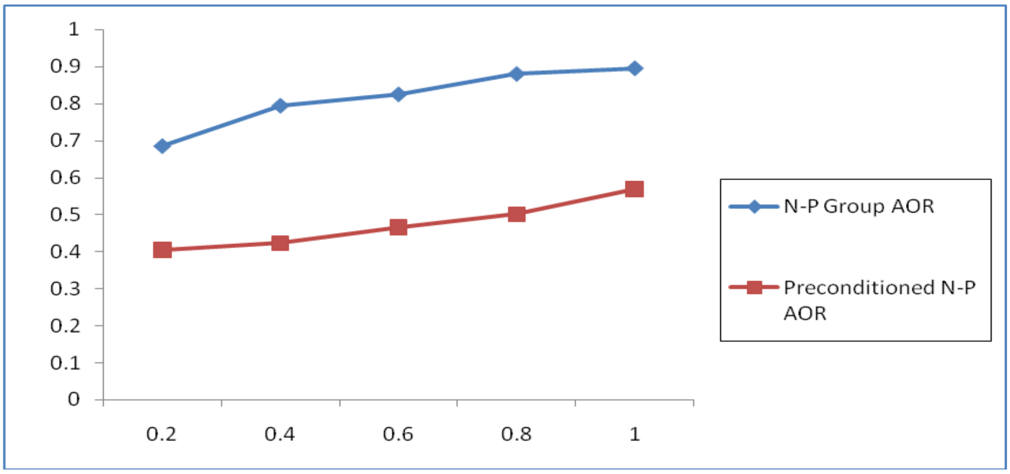

Fig. 2: Comparison of spectral radius for N-P AOR and preconditioned N-P AOR iterative methods

4. Conclusion

In this paper, we proposed a new preconditioner in block formulation for the N-P Group AOR iterative method to accelerate the convergence rate of this group method. From observation of all experimental results by imposing the N-P AOR and Preconditioned N-P AOR iterative methods, the number of iterations and the execution time for Preconditioned N-P AOR iterative method have been declined tremendously as compared with the original N-P AOR iterative method. Furthermore, we can see that our proposed method showed improvements in the number of iterations and the execution time compared to the earlier Preconditioned N-P SOR introduced by Saeed [2]. For future work, it would be worthwhile effort to investigate the application of Preconditioned N-P AOR iterative method for solving other types of equations.

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References

[1] Hadjidimos, A. (1978). Accelerated overrelaxation method. *Mathematics of Computation 32*, 149-157.

[2] Saeed, A. M. (2016). Solving Poisson's Equation Using Preconditioned Nine-Point Group SOR Iterative Method. *International Journal of Mathematics and Statistics Invention (IJMSI) 4 (8)*, 20-26.

[3] Ali, N. H. M. & Saeed A. M. (2013). Preconditioned Modified Explicit Decoupled Group for the Solution of Steady State Navier-Stokes Equation. *Applied Mathematics & Information Sciences.7 (5)*, 1837-1844.

[4] Saeed A. M. and Ali, N. H. M. (2014). Accelerated Solution of Two Dimensional Diffusion Equation, *World Applied Sciences Journal 32 (9)*, 1906-1912.

[5] Saeed A. M. (2014). Fast Iterative Solver For The 2-D Convection Diffusion Equations. *Journal of Advances In Mathematics 9 (6)*, 2773-2782.

[6] Martins M. M, Evans D. J, Yousif W. (2001) Further results on the preconditioned SOR method, *International Journal of Computer Mathematics, 77(4)*, 603-610.

[7] Ali, N. H. M. & Saeed A. M.(2012). Convergence Analysis of the Preconditioned Group Splitting Methods in Boundary Value Problems, *Abstract and Applied Analysis, 2012*, 1-14.