SCITECH
RESEARCH ORGANISATION

Volume 7, Issue 3

Published online: May 02, 2016|

Journal of Progressive Research in Mathematics
www.scitecresearch.com/journals

Study of Transport of Nanoparticles with Power Law fluid Model for Blood Rheology in Capillaries

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Abstract.

The present paper deals with a mathematical model for blood flow through an axially symmetric blood capillary with peripheral layer and slip at the wall. The longitudinal transport of nanoparticles in blood vessels has been analyzed with blood as a power law fluid in a central core region of suspension of all the erythrocytes and a Newtonian fluid in a peripheral layer of plasma. In present analysis, the capillary walls are impermeable and not absorbent for the nanoparticles. The expressions for velocity profile, flow rate, mean velocity and concentration of the solute have been obtained and results have been discussed through graphs.

Keywords: Nanoparticles; power law fluid; erythrocytes; concentration; longitudinal transport; peripheral layer; slip velocity.

Introduction

In recent years, the study of flow and transport of nanoparticles in small blood vessels has received considerable attention in scientific research and technological development. The understanding of anatomy and physiology of an organic system depends on the knowledge of blood flow through blood vessels. The cause of many arterial diseases are related to the flow characteristics of blood and the behavior of blood vessels walls. The vessel walls may be elastic, movable or permeable. Therefore, to understand the mechanics of the circulation of blood, it would have a clear idea of the basic mechanics of fluids. A large number of theoretical and experimental efforts have been made to explain the behavior of blood flow through the vessel of circulatory system of living beings. Blood can no longer be treated as a single phase homogeneous viscous fluid in small vessels (of diameter $\leq$ 1000 μm). Blood flows through capillaries, the two phase nature of blood occurs, one is Newtonian fluid in cell free layer of plasma and other is non-Newtonian power law fluid in a central core region of suspension of all erythrocytes.

Now a days, nanofluids among researchers are considered an active area of research. In fact, the longitudinal transport of nanoparticles in a fluid flow is of fundamental importance in several fields because these particles are highly specific, efficient and rapidly internalized by the target cells. Through the successful delivery of drug with nanoparticles in small blood vessels such as capillary, kill the diseased cells and finally bind the target region. The efficiency of nanoparticles in capillary depends on its size and concentration in a base fluid.

Many authors have studied the problems of non-Newtonian fluids using nanoparticles in fluid flow under different considerations. Gentile et al. [2008, 2010] studied the longitudinal transport of nanoparticles in blood flow by considering a Casson -like fluid model. They described the effect of vessel permeability and the rheology of blood. Ellahi et al.[2013] discussed the blood flow of nanofluid through an artery with composite stenosis and permeable walls. Several investigators have highlighted different aspects of blood flow analysis in arteries. Mekheimer and El-kot [2012] have studied the different aspects of blood flow in stenosed arteries. In a series of papers (Texon 1957, May et al. 1963,Caro et al. 1971, Lee 1974, Richard et al. 1977 etc.), the effect of cardiovascular system can be understood by studying the blood flow with its characteristics. Generally, blood being a suspension of cells, behaves as a non- Newtonian fluid at low shear rates (Charm and Kurland 1965, Hershey et al. 1964, Shukla et al. 1990). Misra and Shit [2007], Verma et al. [2011], Mallik et al. [2013] have developed mathematical models for blood flow by taking the velocity slip condition at the wall of the artery. V. P. Srivastava [2007], presents a theoretical model for blood flow in small vessels and shown that the effect of hematocrit and the peripheral layer on the flow characteristics. Mostafa A. A. Mahmoud [2011], investigated the slip velocity effect on a non- Newtonian Power- law fluid over a continuously moving surface with heat generation. Verma S. R. et al. [2014] studied the flow of blood as a two-phase model. The aim of the present study is to analyze the effect of peripheral layer and slip condition on the transportation of nanoparticles in capillary. Also we have studied the effect of power law index on the velocity and concentration of nanoparticles in a base fluid.

Formulation

An axially symmetric capillary of radius R_e with length l is considered in a cylindrical co-ordinate system (r, z), whose axis is along the direction of z-axis. When blood flows through the capillary, there is two phase model developed, one is the region of suspension of erythrocytes treated as a non-Newtonian power law fluid model and other is the cell-free layer of plasma treated as a Newtonian fluid. μ_0 and u_0 be the c_o be the constant concentration of blood in a capillary, π_i is interstitial fluid pressure, λ be the thickness of peripheral layer of plasma. The transport of nanoparticles of concentration c in capillary is studied under the presence of peripheral layer of plasma and a small slip u_s at the wall of the capillary.

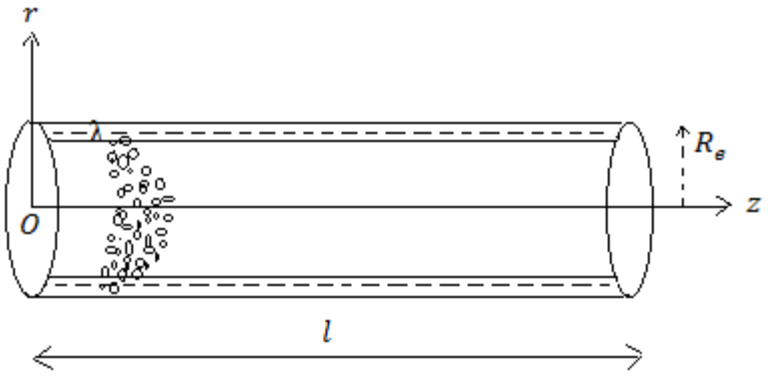

Fig. 1 Longitudinal transport of nanoparticles in blood capillary

Governing Equations

(i) For the central region, the constitutive equation expressed by the Power law fluid

$$\frac{du_1}{dr} = -\left(\frac{1}{2}\frac{P}{\mu_1}\right)^{1/n} r^{1/n} \tag{1}$$

(ii) For the cell-free layer, the constitutive equation is given by

$$-\frac{\partial p}{\partial z} + \frac{\mu_2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_2}{\partial r}\right) = 0 \tag{2}$$

$$-\frac{\partial p}{\partial r} = 0 \tag{3}$$

(iii) The transport equation along the capillary can be given as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) = \frac{u}{D_m}\frac{\partial c}{\partial z} \tag{4}$$

The non dimensional scheme are

$$z' = \frac{z}{l},\ p' = \frac{p}{\pi_i},\ u_1' = \frac{u_1}{u_0},\ u_2' = \frac{u_2}{u_0},\ \mu_1' = \frac{\mu_1}{\mu_0},\ \mu_2' = \frac{\mu_2}{\mu_0},\ r' = \frac{r}{R_e},\ c' = \frac{c}{c_0}. \tag{5}$$

Boundary Conditions

(i) u' is finite at $r' = 0$.

(ii) $u_2' = u_s'$ at $r' = 1$.

(iii) $u_1' = u_2'$ at $r' = 1 - \lambda'$.

(iv) $\mu_1' \frac{\partial u_1'}{\partial r'} = \mu_2' \frac{\partial u_2'}{\partial r'}$ at $r' = 1 - \lambda'$.

(v) $c^{'}$ is finite at $r^{'} = 0$.

(vi) $c^{'} = 0$ at $r^{'} = 1$.

(vii) $\frac{\partial c^{'}}{\partial r^{'}} = 0$ at $r^{'} = 0$.

using above boundary conditions, the velocity profile in central region is given by the expression:

$$u_1{}^{'} = \psi \left(\frac{P^{'}}{2\mu^{'}}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[(1-\lambda^{'})^{\frac{1}{n}+1} - \ r^{'\frac{1}{n}+1}\right] + B_1 + B_2 \log\ (1-\lambda^{'}) + u_s{}^{'} \tag{6}$$

and the velocity profile in the cell free layer is given by

$$u_2{}^{'} = \frac{1}{\mu_2{}^{'} A}\frac{P^{'}}{4}\left(1 - \ r^{'2}\right) + B_2 \log\ r^{'} + u_s{}^{'} \tag{7}$$

where $B_1 = \frac{1}{\mu_2{}^{'} A}\frac{P^{'}}{4}(1-(1-\lambda^{'})^2)$;

$$B_2 = \frac{(1-\lambda^{'})}{\mu_2{}^{'}}\left[\frac{1}{A}P^{'}\frac{(1-\lambda^{'})}{2} - \mu_1{}^{'}\psi\left(\frac{P^{'}}{2\mu^{'}}\right)^{\frac{1}{n}}(1-\lambda^{'})^{\frac{1}{n}+1}\right] ;$$

$$\psi = \frac{R_e}{u_o}\left(\frac{\pi_i}{l}\frac{R_e}{\mu_0}\right)^{1/n}; \qquad P^{'} = \frac{dp^{'}}{dz^{'}} ; \qquad A = \frac{l\mu_0 u_0}{\pi_i {R_e}^2}.$$

The volumetric flow rate is calculated as

$$Q^{'} = Q_1{}^{'} + Q_2{}^{'} \tag{8}$$

where $Q_1{}^{'} = 2\pi\int_0^{1-\lambda^{'}} u_1{}^{'}\ r^{'} d\, r^{'}$; $\qquad Q_2{}^{'} = 2\pi\int_{1-\lambda^{'}}^{1} u_2{}^{'}\ r^{'} d\, r^{'}$

$$Q_1{}^{'} = \pi(1-\lambda^{'})^2\left[\psi\left(\frac{P^{'}}{2\mu^{'}}\right)^{\frac{1}{n}}\frac{2n}{3n+1}(1-\lambda^{'})^{\frac{1}{n}+1} + B_1 + B_2\log\ (1-\lambda^{'}) + u_s{}^{'}\right] \tag{9}$$

$$Q_2{}^{'} = \pi\left[\frac{1}{\mu_2{}^{'} A}\frac{P^{'}}{4}\left(\frac{1}{2} - (1-\lambda^{'})^2 + \frac{1}{2}(1-\lambda^{'})^4\right) - B_2\left((1-\lambda^{'})^2\log\ (1-\lambda^{'}) + \frac{1}{2}(1 - (1-\lambda^{'})^2)\right) + u_s{}^{'}(1-(1-\lambda^{'})^2)\right] \tag{10}$$

The mean velocity $U^{'}$ is derived as

$$U^{'} = \frac{Q^{'}}{\pi} \tag{11}$$

$$= (1-\lambda')^2\left[\psi\left(\frac{P'}{2\mu_1'}\right)^{\frac{1}{n}}\frac{2n}{3n+1}(1-\lambda')^{\frac{1}{n}+1} + B_1 + B_2\log\,(1-\lambda') + u_s'\right] + \left[\frac{1}{\mu_2' A}\frac{P'}{4}\left(\frac{1}{2} - (1-\lambda')^2 + \frac{1}{2}(1-\lambda')^4\right) - B_2\left((1-\lambda')^2\log\,(1-\lambda') + \frac{1}{2}(1-(1-\lambda')^2)\right) + u_s'(1-(1-\lambda')^2)\right] \quad (12)$$

The expression of the concentration c' for $r' \neq 0$ derived as a solution of equation (4) using the boundary conditions (6) and (7), are given on taking $D_m' = \frac{D_m}{u_0 {R_e}^2/l}$

$$c'(r') = \frac{1}{D_m'}\frac{\partial c'}{\partial z'}\left[\left\{\psi\left(\frac{P'}{2\mu_1'}\right)^{\frac{1}{n}}\frac{n}{2(3n+1)}(1-\lambda')^{\frac{1}{n}+3} + B_1\frac{(1-\lambda')^2}{2} + B_2\frac{(1-\lambda')^2}{2}\log(1-\lambda') + u_s'\frac{(1-\lambda')^2}{2}\right\}\log r' + \left\{\frac{1}{\mu_2' A}\frac{P'}{4}\left(\frac{r'^2}{4} - \frac{r'^4}{16} - \frac{(1-\lambda')^2}{2}\log r' + \frac{(1-\lambda')^4}{4}\log r' - \frac{3}{16}\right) + B_2\left(\frac{r'^2}{4}\log r' - \frac{r'^2}{8} - \frac{(1-\lambda')^2}{2}\log r'.\log(1-\lambda') + \frac{(1-\lambda')^2}{4}\log r' + \frac{1}{8}\right) + u_s'\left(\frac{r'^2}{4} - \frac{(1-\lambda')^2}{2}\log r' - \frac{1}{4}\right)\right\}\right] \quad (13)$$

Result and Discussion

In this part of the paper, we have studied the graphical features of pertinent parameters on the velocity profile and concentration profiles. The effect of parameters u_s', λ' and the flow behavior index n are given in fig. 2 - 6. The graph of velocity profiles are drawn with the radius of the capillary for different thickness of peripheral layer, slip velocity and the flow index. Fig. 2, 3 & 4 depicts that the velocity profile with capillary radius for different values of slip velocity, width of peripheral layer and flow index. Fig. 5 and 6 depicts that the concentration profiles with capillary radius for different values of slip velocity, width of peripheral layer and flow index.

Fig. 2 shows that velocity profiles decreases as the peripheral layer increases while velocity profile decreases on increase the radius of capillary. Velocity profile is exactly parabolic in nature up to the point of the boundary of peripheral layer region. For $\lambda' = 0.1$, the velocity decreases exactly parabolically up to the point 0.9 of the radius of capillary. After this, velocity profile shows that a straight line in the peripheral layer region. On increase the peripheral layer thickness, the width of straight line also increases, and velocity decreases gradually.

Fig. 3 shows that the velocity decreases parabolically on increase the radius of capillary while velocity increases for the greater change in slip velocity. the profile profiles breaks at the certain value of slip velocity. This graph also shows that at a fixed slip $u_s' = 1$, velocity takes the values in parabolic nature and breaks after reach the value 1 of the velocity in capillary. Similarly the same features occurs for different values of slip velocity.

Fig. 4 shows that the variation of velocity against the radius of capillary. These graphs are drawn with the fixed slip $u_s^{'} = 0.1$ and fixed peripheral layer thickness $\lambda^{'} = 0.1$, for different values of flow index n. At the fixed $n = 0.7$, velocity profile decreases slowly in parabolic nature up to the point $r^{'} = 0.9$ in the region of suspension of erythrocytes, and thereafter decreases rapidly at $r^{'} = 1$. That is to say that velocity decrease and shown in straight line in the peripheral layer region.

Fig. 5 and 6 shows that the variation of concentration of nanoparticles with capillary radius for different value of peripheral layer width and flow index. In figure 5, the concentration of nanoparticles decreases with the radius of capillary and with the width of peripheral layer and concentration becomes zero at the wall of capillary. Fig. 6 represent the graphs of concentration with the radius of capillary for fixed values of $u_s^{'} = 0.1$ and fixed $\lambda^{'} = 0.1$ for different n. From here, it has been shown that the concentration increases as the flow index increases and gives the same behavior with the radius of capillary as shown in fig. 5.

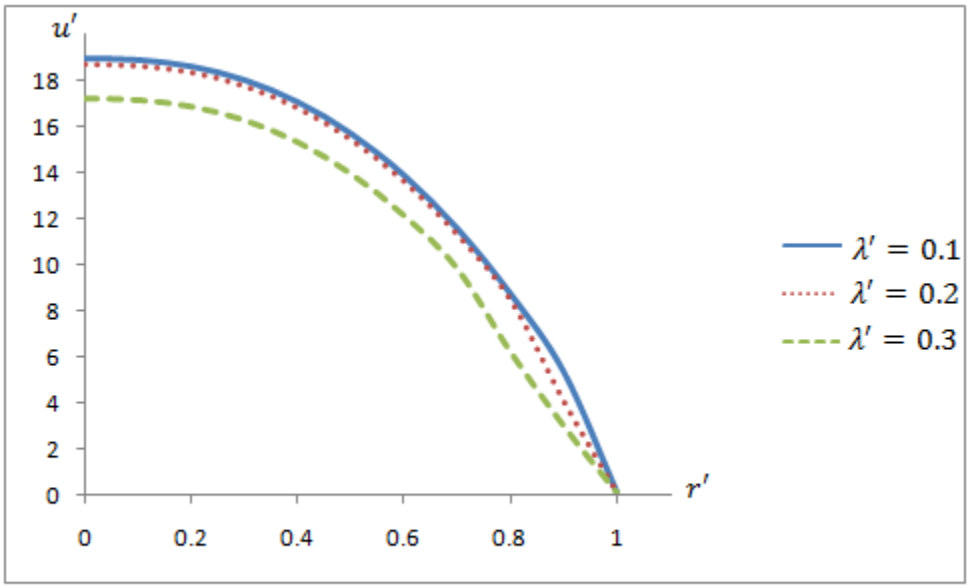

Fig. 2: Variation of velocity u' against r' for $\lambda' = 0.1,\ 0.2, 0.3.$

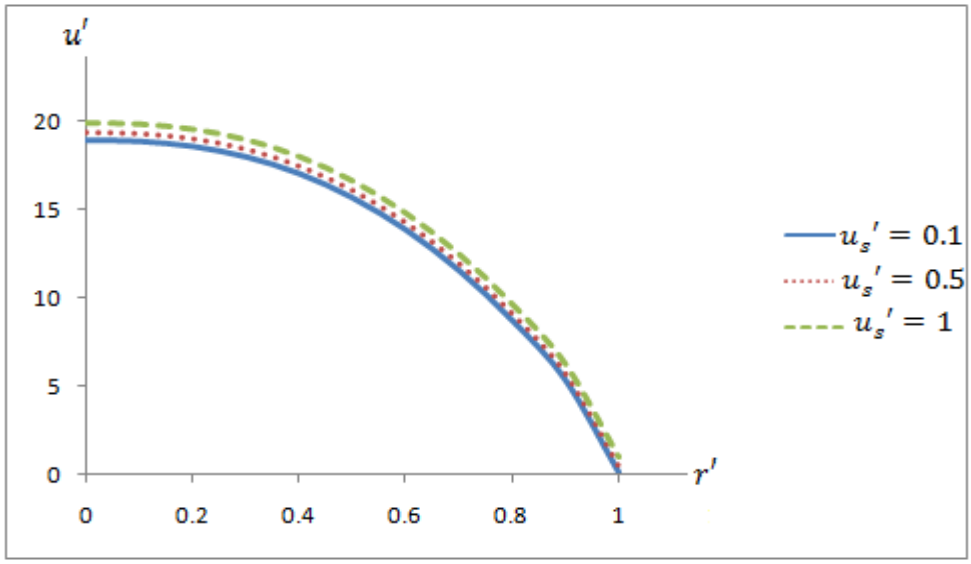

Fig. 3: Variation of velocity u' against r' for $u_s' = 0.1,\ 0.5,\ 1.$

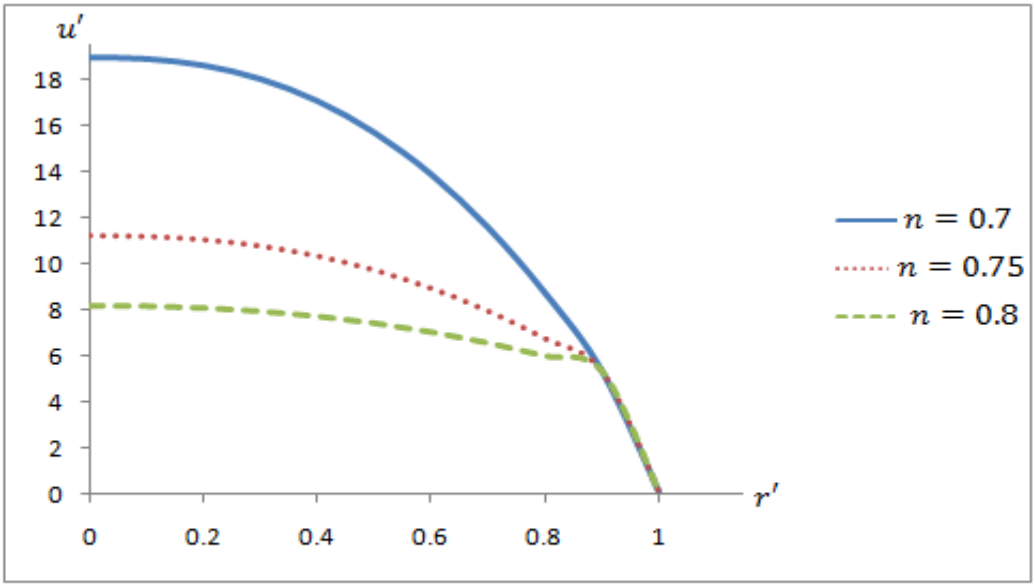

Fig. 4: Variation of velocity u' against r' for $n = 0.7,\ 0.75,\ 0.8$

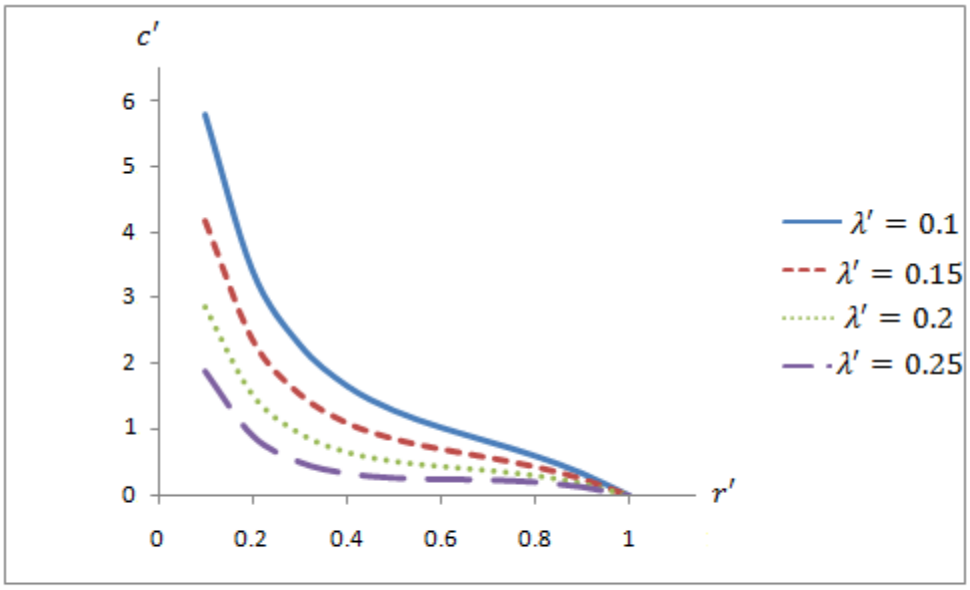

Fig. 5: Variation of concentration c' against r' for $\lambda' = 0.1,\ 0.15,\ 0.2,\ 0.25$

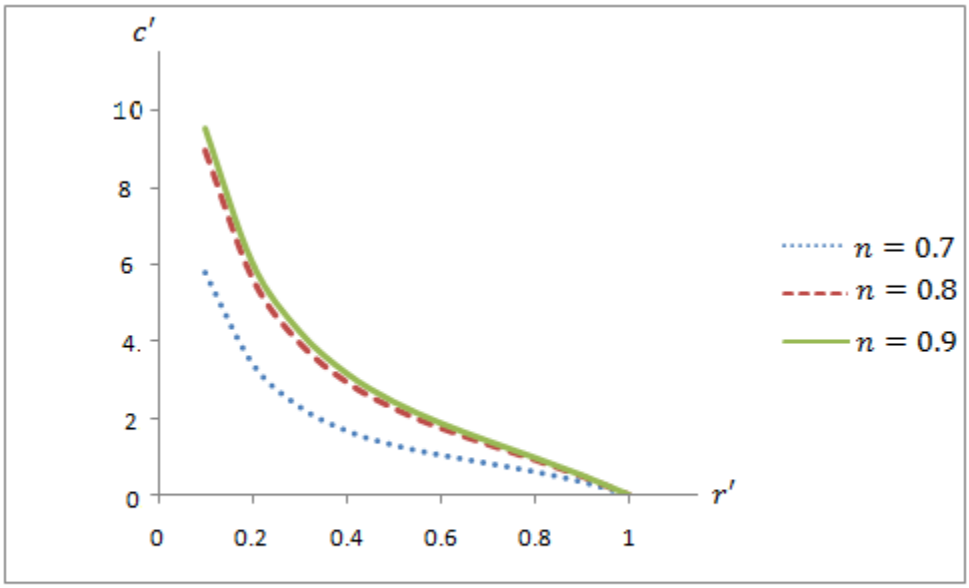

Fig. 6: Variation of concentration c' against r' for $n = 0.7,\ 0.8,\ 0.9$

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