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Dufour and thermal radiation effects of Kuvshinski fluid on double diffusive and convective MHD heat and mass transfer flow past a porousvertical plate in the presence of radiation absorption, viscous dissipation and chemical reaction

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Abstract.

In this paper an analysis is presented to investigate the influence of diffusion thermo, thermal radiation, radiation absorption, chemical reaction and viscous dissipation on hydro magnetic free convective heat and mass transfer flow of Kuvshinski fluid past a porous vertical plate. A uniform magnetic field of is applied in the direction of the flow field. Analytical solutions for velocity, temperature and concentration are obtained by using a Perturbation technique. Skin friction, rate of heat and mass transfer coefficients are also derived. The results have been analyzed graphically and numerically for various values of the flow parameters.

Keywords: Kuvshinski fluid; MHD; Dufour effect; Chemical reaction; Radiation absorption; Thermal radiation; Viscous dissipation.

1. Introduction

The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear reactor, chemical process industries and many engineering applications in which the fluid is a working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convective flows.

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. Free convective flow past a vertical plate has been studied extensively by Ostrach[1]. Siegel [2] investigated the transient free convection from a vertical flat plate. Hossain and Begum [3] have discussed unsteady free convective mass transfer flow past a vertical porous plate.

Unsteady effect on MHD free convective and mass transfer flow through porous medium with constant suction and constant heat flux in rotating system was studied by Sharma [4]. Agrawal et al.[5] have discussed the effect of stratified viscous Kuvshinski fluid on MHD free convective flow with heat and mass transfer past a vertical porous plate. Sharma and Varshney **[**6**]** have extended the problem of Agrawal et al.[5] and investigatedthe effect of stratified Kuvshinski fluid onMHD free convective flow past a vertical porous plate with heat and mass transfer neglecting induced magnetic field in comparison to applied magnetic field.

Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and various propulsion devices for aircrafts, missiles, satellites and space vehicles are examples of such engineering areas. Takhar et al. [7] studied the radiation effects on MHD free convective flow for nongray-gas past a semi-infinite vertical plate. Effect of mass transfer on radiation and free convective flow of Kuvshinski fluid through a porous medium was studied by Harish Kumar and Kaanodia [8].Salam [9]examined a coupled heat and mass transfer flow in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in fluid saturated porous medium under the influence of radiation and viscous dissipation. Cookey et al. [10] investigated the influence of viscous dissipation and radiation on steady MHD free convective flow past an infinite heated vertical plate in a porous medium with time dependent suction. Aravind Kumar Sharma et.al [11] have studied the effect of Kuvshinski fluid on double-diffusive unsteady convective heat and mass transfer flow past a porous vertical moving plate with heat source and soret effect. Kumar [12] investigated the radiative heat transfer with the viscous dissipation effect in the presence of transverse magnetic field.Prasad and Reddy [13] investigated radiation and mass transfer effects on an unsteady MHD free convective flow past a semi-infinite vertical permeable moving plate with viscous dissipation. Shvets and Vishevskiy [14] discussed the effect of dissipation on convective heat transfer in the flow of non-Newtonian fluids. Combined effects of magnetic field and viscous dissipation on a power law fluid over a plate with variable surface heat flux embedded in a porous medium was studied by Amin [15].

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler heat and the mass transfer occur simultaneously. Possible application of this type of flow can be found in many industries such as polymer production, manufacturing of ceramics or glassware and food processing. For example in the power industry, among the methods of generation, electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary.

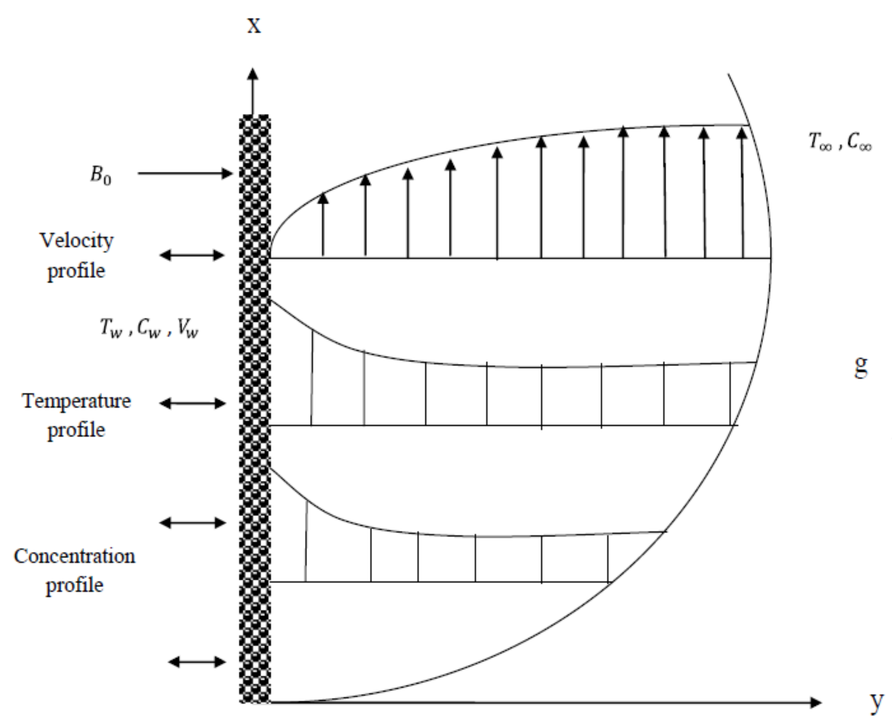
The chemical reactive species in a laminar boundary layerflow over a flat plate was demonstrated by Chambre and Young [16].Rajesh [17] discussed the chemical reaction and radiation effects on the transient MHD free convective flow of dissipative fluid past an infinite vertical porous plate with ramped wall temperature. Gireesh Kumar et. al. [18] discussed the effects of chemical reaction and mass transfer on radiation and MHD free convection flow of Kuvshiski fluid through a porous medium. Devasena and LeelaRatnam[19] analyzed the combined effects of chemical reaction, thermo diffusion and thermal radiation and dissipation on connective heat and mass transfer flow of a Kuvenski fluid past a vertical plate embedded in a porous medium. Manjulatha et al. [20] investigated the radiation and chemical reaction effects on the unsteady MHD oscillatory flow in a channel filled with saturated porous medium in an aligned magnetic field.

Sreekantha Reddy et al.[21] have discussed the effects of chemical reaction and thermo-diffusion on non-Darcy convective heat and mass transfer flow in a vertical channel with heat sources. Diffusion thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite vertical plate in the presence of a chemical reaction of first order have been studied by Raveendra Babu et al. [22].
Kinyanjui et al. [23] considered magneto hydrodynamic free convective heat and mass transfer of a heat generating fluid past an impulsively saturated infinite vertical porous plate with Hall current and radiation absorption. Ibrahim et al. [24] analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convective flow past a semi infinite vertical permeable moving plate with source and suction. Manjulatha et.al. [25] have discussed the effects of radiation absorption and mass transfer on the steady free convective flow of a viscous, incompressible and electrically conducting fluid past an infinite vertical flat plate through a porous medium with an aligned magnetic field. Unsteady magneto hydrodynamic (MHD) free convective flow of a viscous, incompressible and electrically conducting, well known non-Newtonian fluid named as Kuvshinski fluid past an infinite vertical

porous plate in the presence of homogeneous chemical reaction, radiation absorption and heat source/sink was studied analytically by Reddy et al. [26]. An unsteady MHD two dimensional free convective flow of a viscous, incompressible, radiating, chemically reacting and radiation absorbing Kuvshinski fluid through a porous medium past a semi-infinite vertical plate was investigated by Vidya Sagar et al. [27]. Sudershan Reddy et al.[28] have discussed the chemical reaction and radiation absorption effects on MHD convective heat and mass transfer flow past a semi-infinite vertical moving porous plate with time dependent suction.

The objective of the present problem is to study Dufour and thermal radiation effects of Kuvshinski fluid on double-diffusive convective MHD heat and mass transfer flow past a porous vertical plate in the presence of radiation absorption, viscous dissipation and chemical reaction.

Physical model



2. Formulation of the problem

Two dimensional unsteady flow of a laminar conducting fluid past a moving semi-infinite vertical plate in the presence of uniform porous medium is considered. A transverse magnetic field is applied in the direction of the flow fled. The effects of diffusion thermo, thermal radiation, chemical reaction, viscous dissipation and radiation absorption are taken in to the consideration. According to the coordinate system the x′-axis is taken along the porous plate in the upward direction and y′-axis normal to it. The radiative heat flux in x′-direction is considered negligible in comparison with that in the y′-direction.

The following assumptions were made:

1. The porous plate is of infinite length in x′-direction.

2. The plate velocity and suction velocity both are assumed in the form of
$u^{'} = v_0\left(1+\epsilon e^{-t^{'}n^{'}}\right)$.
3. Induced magnetic field is neglected.
4. Electric field is neglected.
5. Joule's dissipation is neglected.
6. Since the flow of fluid is assumed to be in the direction of x′direction, the physical quantities are functions of y′and t′ only.

Taking into consideration the assumptions made above, the equations describing the flow can be written in the Cartesian frame of reference, as follows:

$$\frac{\partial v^{'}}{\partial y^{'}} = 0 \tag{1}$$

$$\left(1+\lambda^{'}\frac{\partial}{\partial t^{'}}\right)\frac{\partial u^{'}}{\partial t^{'}} + v^{'}\frac{\partial u^{'}}{\partial y^{'}} = g\beta\left(T^{'}-T_{\infty}^{'}\right) + g\beta^{*}\left(C^{'}-C_{\infty}^{'}\right) + \vartheta\frac{\partial^2 u^{'}}{\partial y^{'2}} - \left(\frac{\sigma B_0^{2}}{\rho} + \frac{\vartheta}{K^{'}}\right)\left(1+\lambda^{'}\frac{\partial}{\partial t^{'}}\right)u^{'} \tag{2}$$

$$\left(1+\lambda^{'}\frac{\partial}{\partial t^{'}}\right)\frac{\partial T^{'}}{\partial t^{'}} + v^{'}\frac{\partial T^{'}}{\partial y^{'}} = \frac{K}{\rho C_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} + \frac{\vartheta}{C_p}\left(\frac{\partial u^{'}}{\partial y^{'}}\right)^2 - \frac{1}{\rho C_p}\frac{\partial q_r^{'}}{\partial y^{'}} + \frac{R_1}{\rho C_p}\left(C^{'}-C_{\infty}^{'}\right) + \frac{D_m K_T}{\rho C_s C_p}\frac{\partial^2 C^{'}}{\partial y^{'2}} \tag{3}$$

$$\left(1+\lambda^{'}\frac{\partial}{\partial t^{'}}\right)\frac{\partial C^{'}}{\partial t^{'}} + v^{'}\frac{\partial C^{'}}{\partial y^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}} - K_1\left(C^{'}-C_{\infty}\right) \tag{4}$$

where $u^{'}, v^{'}$are the velocity components, $T^{'}$, $C^{'}$ are the temperature and concentration components, ϑ is the Kinematic viscosity, ρ is the density, σ is the electrical conductivity, $D_m$is the coefficient of chemical molecular diffusivity, K_T is the thermal diffusion ratio, C_s is the concentration susceptibility and K is the thermal conductivity.

The equation (1) gives

$$v^{'} = -v_0 \tag{5}$$

wherev_0 is the constant suction velocity.

The boundary conditions at the wall and in the free stream are

$$u^{'} = v_0\left(1+\epsilon e^{-t^{'}n^{'}}\right),\ T^{'} = T_w^{'},\ C^{'} = C_w^{'} at y^{'} = 0 \tag{6}$$

$$u' \to 0\,,\;\; T' \to T_\infty{}',\;\;\; C' \to C_\infty{}' at y' \to \infty \tag{7}$$

The radiative heat flux $q_r{}'$ using the Rosseland diffusion model for radiation heat transferis expressed as

$$q' = \frac{4\sigma^*}{3K^*}\frac{\partial T'^4}{\partial y'} \tag{8}$$

where$\sigma^*$and $K^*$are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference with in the flow is sufficiently small and T'^4may be expressed as a linear function of the temperature. This is accomplished by expanding in Taylor series about $T_\infty{}'$ and neglecting higher order terms, thus

$$T'^4 \cong 4T_\infty{}'^3 T' - 3T_\infty{}'^4 \tag{9}$$

In view of (8) and (9) the equation (3) reduces to

$$\left(1+\lambda'\frac{\partial}{\partial t'}\right)\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\vartheta}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{16\sigma^* T_\infty{}'^3}{3\rho c_p K^*}\frac{\partial^2 T'}{\partial y'^2} + \frac{R_1}{\rho C_p}\left(C' - C_\infty{}'\right) + \frac{D_m K_T}{\rho C_s C_p}\frac{\partial^2 C'}{\partial y'^2} \tag{10}$$

Introducing the following dimensionless variables

$$u = \frac{u'}{V_0}\,, y = \frac{y' V_0}{\vartheta}, t = \frac{t' V_0{}^2}{\vartheta}\,, n = \frac{n'\vartheta}{V_0{}^2}\,, \theta = \frac{T' - T_\infty{}'}{T_w{}' - T_\infty{}'}\,, \emptyset = \frac{C' - C_\infty{}'}{C_w{}' - C_\infty{}'},$$

$$Gr = \frac{g\beta\vartheta\,(T_w{}' - T_\infty{}')}{V_0{}^3}, Gm = \frac{g\beta^*\vartheta(C_w{}' - C_\infty{}')}{V_0{}^3}\;, Pr = \frac{\mu C_p}{k}, \mathrm{Sc} = \frac{\vartheta}{D}, \mathrm{K} = \frac{K' V_0{}^2}{\vartheta^2},$$

$$\mathrm{M} = \frac{\sigma\beta_0{}^2\vartheta}{\rho V_0{}^2}\;, \mathrm{R} = \frac{4\sigma^* T_\infty{}'^3}{K^* K}, \lambda = \frac{V_0{}^2\lambda'}{\vartheta}, E = \frac{V_0{}^2}{C_p(T_w{}' - T_\infty{}')}, K_r = \frac{K_1\vartheta}{V_0{}^2}$$

$$R_a = \frac{R_1\vartheta(C_w{}' - C_\infty{}')}{kV_0{}^2(T_w{}' - T_\infty{}')}\;, D_f = \frac{D_m K_T(C_w{}' - C_\infty{}')}{C_s K(T_w{}' - T_\infty{}')} \tag{11}$$

and using the equations (11) and (5) into the equations (2), (10) and (4), we obtain

$$\alpha_1\frac{\partial u}{\partial t} + \lambda\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - M_1 u + Gr\,\theta + Gm\,\emptyset \tag{12}$$

$$Pr\frac{\partial\theta}{\partial t} + Pr\lambda\frac{\partial^2\theta}{\partial t^2} = N_1\frac{\partial^2\theta}{\partial y^2} + Pr\frac{\partial\theta}{\partial y} + PrE\left(\frac{\partial u}{\partial y}\right)^2 + R_a\emptyset + D_f\frac{\partial^2\emptyset}{\partial y^2} \tag{13}$$

$$Sc\frac{\partial\emptyset}{\partial t}+Sc\lambda\frac{\partial^2\emptyset}{\partial t^2}=\frac{\partial^2\emptyset}{\partial y^2}+Sc\frac{\partial\emptyset}{\partial y}-KrSc\emptyset \qquad (14)$$

The corresponding boundary conditions in non-dimensional form are

$$u=1+\varepsilon e^{-nt},\ \ \theta=1,\ \ \emptyset=1\ \ at y=0 \qquad (15)$$

$$u\to 0,\ \theta\to 0,\ \emptyset\to 0\qquad as y\to\infty \qquad (16)$$

3. Solution of the problem

The governing equations (12), (13) and (14) of the flow, temperature and concentration respectively are coupled non-linear differential equations. Assuming ε to be very small, the perturbation parameter, we write

$$u=u_0(y)+\varepsilon u_1(y)e^{-nt}+o(\varepsilon^2) \qquad (17)$$

$$\theta=\theta_0(y)+\varepsilon\theta_1(y)e^{-nt}+o(\varepsilon^2) \qquad (18)$$

$$\emptyset=\emptyset_0(y)+\varepsilon\emptyset_1(y)e^{-nt}+o(\varepsilon^2) \qquad (19)$$

By substituting the above equations (17)-(19) into equations (12) to (14), equating the harmonic terms and neglecting the higher order terms of $o(\varepsilon^2)$, we obtain the following pairs of equations for $(u_0,\ \theta_0,\emptyset_0)$ and $(u_1,\ \theta_1,\emptyset_1)$

$$u_0{''}+u_0{'}-M_1u_0=-Gr\theta_0-Gm\emptyset_0 \qquad (20)$$

$$u_1{''}+u_1{'}-M_2u_1=-Gr\theta_1-Gm\emptyset_1 \qquad (21)$$

$$N_1\theta_0{''}+\Pr\theta_0{'}=-PrEu_0{'^2}-R_a\emptyset_0-D_f\emptyset_0{''} \qquad (22)$$

$$N_1\theta_1{''}+\Pr\theta_1{'}+N_2\theta_1=-2PrEu_0{'}u_1{'}-R_a\emptyset_1-D_f\emptyset_1{''} \qquad (23)$$

$$\emptyset_0{''}+Sc\emptyset_0{'}-ScKr\emptyset_0=0 \qquad (24)$$

$$\emptyset_1{''}+Sc\emptyset_1{'}-L_1\emptyset_1=0 \qquad (25)$$

where$M_2=M_1-\alpha_1n+\lambda n^2$,$N_2=nPr-n^2Pr\lambda$,$L_1=nSc-n^2Sc\lambda-KrSc$

The corresponding boundary conditions are

$$u_0=1,u_1=1,\theta_0=1,\theta_1=0,\emptyset_0=1,\emptyset_1=0\quad at\ y=0 \qquad (26)$$

$$u_0\to 0,\ u_1\to 0,\ \theta_0\to 0,\theta_1\to 0,\ \emptyset_0\to 0,\ \emptyset_1\to 0\ as\ y\to\infty \qquad (27)$$

Solving the equations (24) and (25) subject to the corresponding boundary conditions, we obtain

$$\emptyset_0 = e^{-a_2 y} \quad (28)$$

$$\emptyset_1 = 0 \quad (29)$$

The equations (20), (21), (22) and (23) are still coupled non-linear ordinary differential equations, the exact solutions of which are not possible. To solve these equations, assuming the Eckert number E to be very small, we write

$$u_0 = u_{01} + Eu_{02} + o(\varepsilon^2) \quad (30)$$

$$u_1 = u_{11} + Eu_{12} + o(\varepsilon^2) \quad (31)$$

$$\theta_0 = \theta_{01} + E\theta_{02} + o(\varepsilon^2) \quad (32)$$

$$\theta_1 = \theta_{11} + E\theta_{12} + o(\varepsilon^2) \quad (33)$$

Substituting the equations (30) to (33) into equations (20) to (23), equating the coefficients of like powers of E and neglecting the higher order terms of $o(\varepsilon^2)$, we obtain

$$u_{01}'' + u_{01}' - M_1 u_{01} = -Gr\theta_{01} - Gm\emptyset_0 \quad (34)$$

$$u_{11}'' + u_{11}' - M_2 u_{11} = -Gr\theta_{11} - Gm\emptyset_1 \quad (35)$$

$$N_1\theta_{01}'' + Pr\theta_{01}' = -R_a\emptyset_0 - D_f\emptyset_0'' \quad (36)$$

$$N_1\theta_{11}'' + Pr\theta_{11}' + N_2\theta_{11} = -R_a\emptyset_1 - D_f\emptyset_1'' \quad (37)$$

$$u_{02}'' + u_{02}' - M_1 u_{02} = -Gr\theta_{02} \quad (38)$$

$$u_{12}'' + u_{12}' - M_2 u_{12} = -Gr\theta_{12} \quad (39)$$

$$N_1\theta_{02}'' + Pr\theta_{02}' = -Pru_{01}'^2 \quad (40)$$

$$N_1\theta_{12}'' + Pr\theta_{12}' + N_2\theta_{12} = -2Pru_{01}'u_{11}' \quad (41)$$

The corresponding boundary conditions are

$$u_{01} = 1, u_{02} = 0, u_{11} = 1, u_{12} = 0$$

$$\theta_{01} = 1, \theta_{02} = 0, \theta_{11} = 0, \theta_{12} = 0 \quad at y = 0 \quad (42)$$

$$u_{01} \to 0, u_{02} \to 0\,, u_{11} \to 0\,, u_{12} \to 0$$

$$\theta_{01} \to 0\,, \theta_{02} \to 0\,, \theta_{11} \to 0\,, \theta_{12} \to 0 \quad as\, y \to \infty \tag{43}$$

The analytical solutions of equations (34) to (41) under the boundary conditions (42) and (43) are given by

$$\theta_{01} = I_1 e^{-a_2 y} + I_2 e^{-a_6 y} \tag{44}$$

$$u_{01} = I_3 e^{-a_2 y} + I_4 e^{-a_6 y} + I_5 e^{-a_8 y} \tag{45}$$

$$\theta_{02} = I_6 e^{-a_6 y} + I_7 e^{-m_1 y} + I_8 e^{-m_2 y} + I_9 e^{-m_3 y} + I_{10} e^{-m_4 y} + I_{11} e^{-m_5 y} + I_{12} e^{-m_6 y} \tag{46}$$

$$u_{02} = I_{13} e^{-a_8 y} + I_{14} e^{-a_6 y} + I_{15} e^{-m_1 y} + I_{16} e^{-m_2 y} + I_{17} e^{-m_3 y} + I_{18} e^{-m_4 y} + I_{19} e^{-m_5 y} + I_{20} e^{-m_6 y} \tag{47}$$

$$\theta_{11} = 0 \tag{48}$$

$$u_{11} = e^{-a_{12} y} \tag{49}$$

$$\theta_{12} = I_{21} e^{-a_{14} y} + I_{22} e^{-m_7 y} + I_{23} e^{-m_8 y} + I_{24} e^{-m_9 y} \tag{50}$$

$$u_{12} = I_{25} e^{-a_{12} y} + I_{26} e^{-a_{14} y} + I_{27} e^{-m_7 y} + I_{28} e^{-m_8 y} + I_{29} e^{-m_9 y} \tag{51}$$

where the constants are given in Appendix

In view of the solutions (44) to (51), (28) and (29) and the equations (17) to (19) and (30) to (33), the velocity, temperature and concentration distribution in the boundary layer become

$$u(y,t) = [(I_3 e^{-a_2 y} + I_4 e^{-a_6 y} + I_5 e^{-a_8 y}) + E(I_{13} e^{-a_8 y} + I_{14} e^{-a_6 y} + I_{15} e^{-m_1 y} + I_{16} e^{-m_2 y} + I_{17} e^{-m_3 y} + I_{18} e^{-m_4 y} + I_{19} e^{-m_5 y} + I_{20} e^{-m_6 y})] + \varepsilon[e^{-a_{12} y} + E(I_{25} e^{-a_{12} y} + I_{26} e^{-a_{14} y} + I_{27} e^{-m_7 y} + I_{28} e^{-m_8 y} + I_{29} e^{-m_9 y}) e^{-nt})] \tag{52}$$

$$\theta(y,t) = [(I_1 e^{-a_2 y} + I_2 e^{-a_6 y}) + E(I_6 e^{-a_6 y} + I_7 e^{-m_1 y} + I_8 e^{-m_2 y} + I_9 e^{-m_3 y} + I_{10} e^{-m_4 y} + I_{11} e^{-m_5 y} + I_{12} e^{-m_6 y})] + \varepsilon[E(I_{21} e^{-a_3 y} + I_{22} e^{-m_7 y} + I_{23} e^{-m_8 y} + I_{24} e^{-m_9 y}) e^{-nt}] \tag{53}$$

$$\emptyset = e^{-a_2 y} \quad (54)$$

3.1 Skin friction

The skin-friction coefficient at the surface y=0 is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= -[(-a_2 I_3 - a_6 I_4 - a_8 I_5) + E(-a_8 I_{13} - a_6 I_{14} - m_1 I_{15} - m_2 I_{16} - m_3 I_{17} - m_4 I_{18} - m_5 I_{19} - m_6 I_{20})] + \varepsilon[-a_{12} + E(-a_{12} I_{25} - a_{14} I_{26} - m_7 I_{27} - m_8 I_{28} - m_9 I_{29})]e^{-nt}$$

3.2 Rate of heat transfer

Heat transfer coefficient at the surface y=0 is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$
$$= -[(-a_2 I_1 - a_6 I_2) + E(-a_6 I_6 - m_1 I_7 - m_2 I_8 - m_3 I_9 - m_4 I_{10} - m_5 I_{11} - m_6 I_{12})] + \varepsilon[E(-a_{14} I_{21} - m_7 I_{22} - m_8 I_{23} - m_8 I_{24})]e^{-nt}$$

3.3 Rate of mass transfer

Mass transfer coefficient at the surface y=0 is given by

$$Sh = -\left(\frac{\partial \emptyset}{\partial y}\right)_{y=0} = -(-a_2) = a_2$$

4. Results and discussion

Numerical calculations are carried out for different values of dimensionless parameters and a representative set of results is reported graphically in Figures 1-7, These results are obtained to illustrate the influence of the chemical reaction parameter K, Dufour effect D_f, the magnetic field parameter M, radiation absorption parameter R_a as well as other physical parameters namely Gr, Gc, w, Sc, Pr and $t$on the concentration, temperature and the velocity profiles.

The effect of Schmidt number (Sc) on the concentration profiles is shown in Fig. 1.It is observed that the species concentration increases as the Schmidt number (Sc) increases. Fig. 2 displays the effect of chemical reaction parameter on concentration profiles. It is observed that the concentration increases as chemical reaction parameter (K_r) increases. Fig. 3 presents the effect of radiation absorption parameter on temperature profiles. It is observed that the temperature increases as radiation absorption parameter (R_a) increases. Fig. 4 shows the effect of Dufour number (D_f)on temperature profiles. It is observed that temperature increases as Dufour number increases.

Fig. 5 displays the effect of chemical reaction parameter on the temperature profiles. It is observed that temperature increases for $y \leq 1$ near the plate and decreases for $y>1$ as chemical reaction parameter increases. Fig. 6 shows the effect of Schmidt number on the temperature profiles. It is observed that the temperature increases as Schmidt number increases. For different values of magnetic parameter (M), the velocity profile (u) is plotted in Fig. 7.It is noticed that the velocity decreases as the magnetic parameter increases.

Fig. 8 shows the effect of Dufour number on the velocity profiles. It is observed that the velocity increases as Dufour number increases. Fig. 9 shows the effect of radiation absorption parameter on the velocity profiles. It is observed that the velocity increases as radiation absorption parameter increases.
Fig. 10 displays the effect of Grashof number on velocity profiles. It is observed that velocity increases as Grashof number increases. For different values of Eckert number(E), the velocity profile (u) is plotted in Fig. 11. It is observed that the velocity increases as the Eckert number increases.

Table1 shows the numerical values of the Skin friction coefficient τ for various values of Dufour number, Eckert number, chemical reaction parameter, radiation parameter (R), Radiation absorption parameter. From table 1.1, we observed that, an increase in the D_f, R_a, E and λ, increases the value of the skinfriction coefficient, while an increase in K_r and R, the value of the skinfriction coefficient decreases.

Table2 shows the numerical values of heat transfer coefficient in terms of Nusselt number (Nu) for various values of Prandtl number, Dufour number and radiation absorption parameter. It is observed that, as the Prandtl number increases, Nusselt number decreases and as radiation absorption parameter increases, the value of heat transfer coefficient increases.

Table 3 shows the numerical values of mass transfer coefficient in terms of Sherwood number (Sh) for various values of chemical reaction parameter and Schmidt number. It is observed that, an increase in the chemical reaction parameter increases the value of mass transfer coefficient and an increase in the Schmidt number increases the value of mass transfer coefficient.

5. Conclusions

Based on the results and discussions above, the following conclusions have been made:

- The velocity increases for increasing values of radiation absorption parameter or Dufour number or thermal Grashof number or Eckert number where as a reverse trend is noticed in the case of magnetic parameter.
- The temperature increases with an increase in radiation absorption parameter or Dufour number.
- The concentration and Sherwood number increases with increasing Schmidt number or chemical reaction parameter.
- The Nusselt number increases with an increase in radiation absorption parameter but the trend is just reversed with respect to the prandtl number.
- The skin friction increases with an increase in radiation absorption parameter or Dufour number or Eckert number and it decreases in the case of chemical reaction parameter or radiation parameter.

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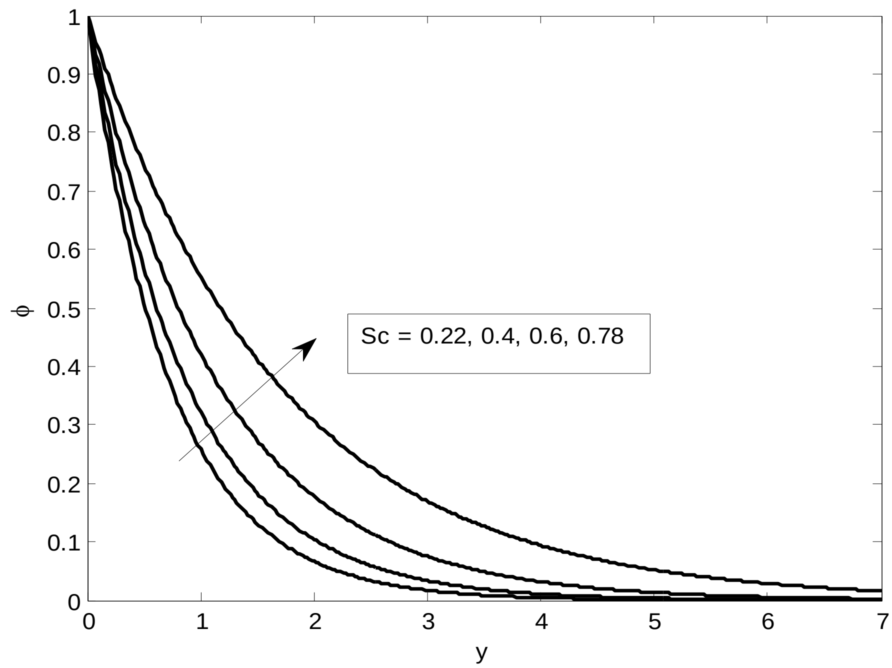

Fig. 1: Effect of Schmidt number on concentration profiles with K_r=1

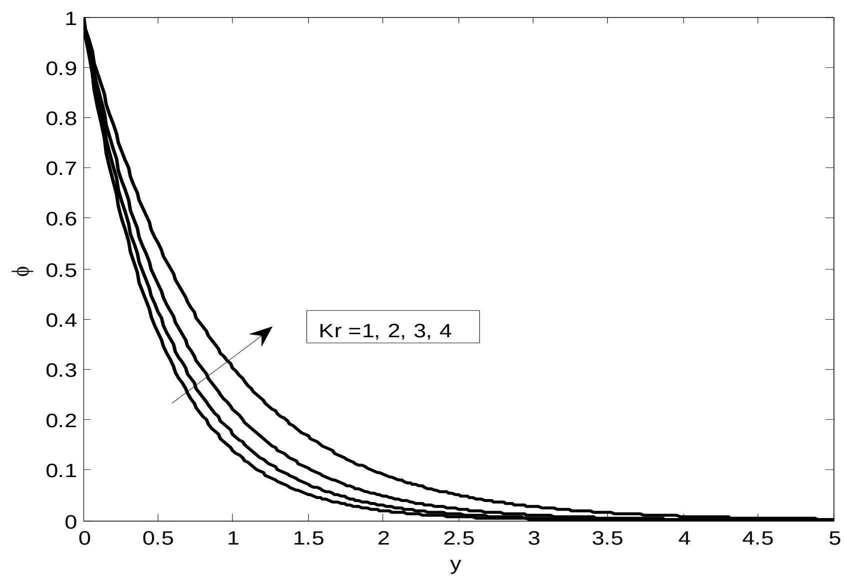

Fig. 2: Effect of chemical reaction on concentration profiles with Sc=0.65

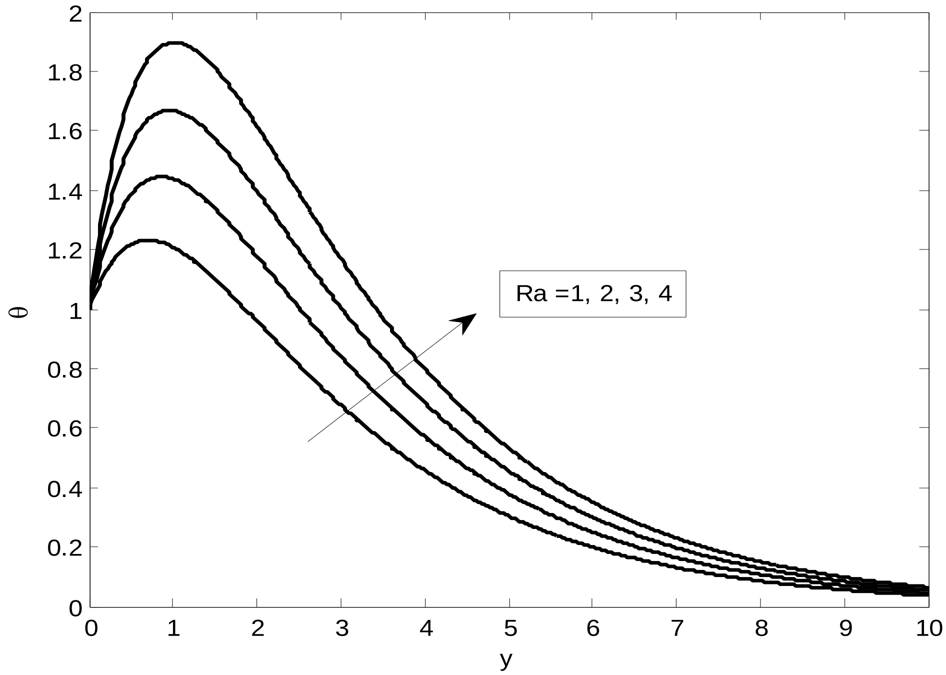

Fig. 3: Effect of radiation absorption parameter on temperature profiles with Sc=0.65, K_r =1,R=0.5,Pr=0.71,K=0.5,Gr=2,M=0.5,E=0.01,n=0.5,t=0.5, $\varepsilon = 0.02,\ \lambda = 0.5$

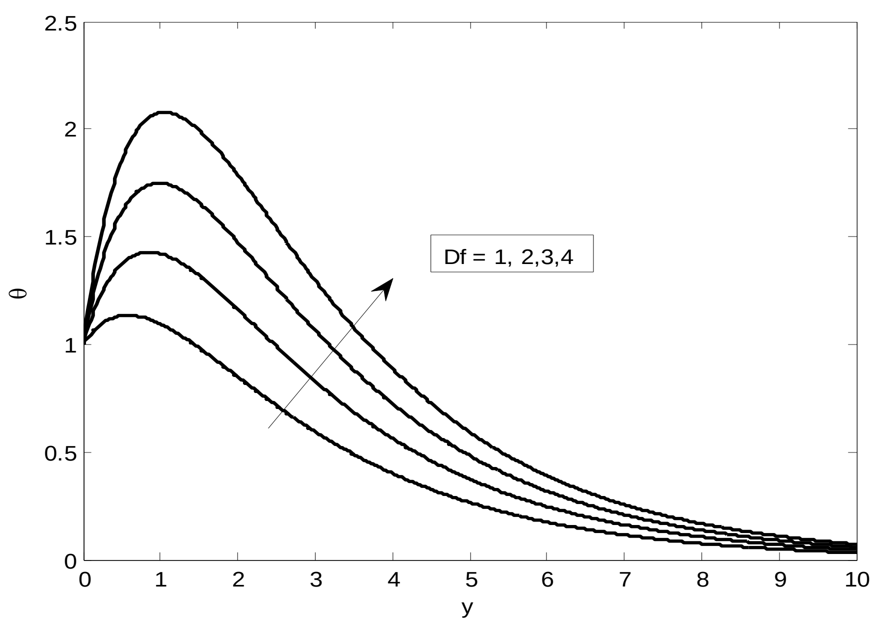

Fig. 4: Effect of Dufour number on temperature profiles with Sc=0.65,K_r=1,R=0.5,R_a =0.5, Pr=0.71,K=0.5,Gr=2,M=0.5,E=0.01,n=0.5,t=0.5, $\varepsilon = 0.02,\ \lambda = 0.5$

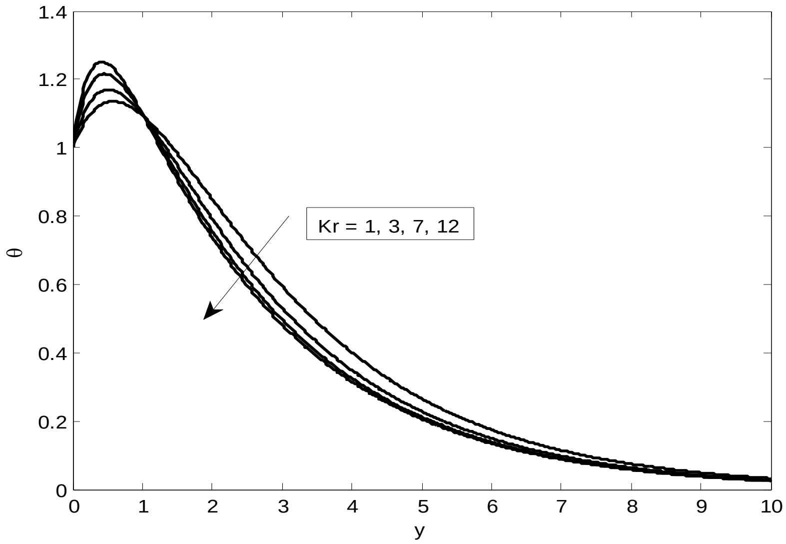

Fig. 5: Effect of Chemical reaction on temperature profiles with Sc=0.65,R=0.5, R_a=0.5, Pr=0.71,K=0.5,Gr=2,M=0.5,E=0.01,n=0.5,t=0.5, $\varepsilon = 0.02,\ \lambda = 0.5$.

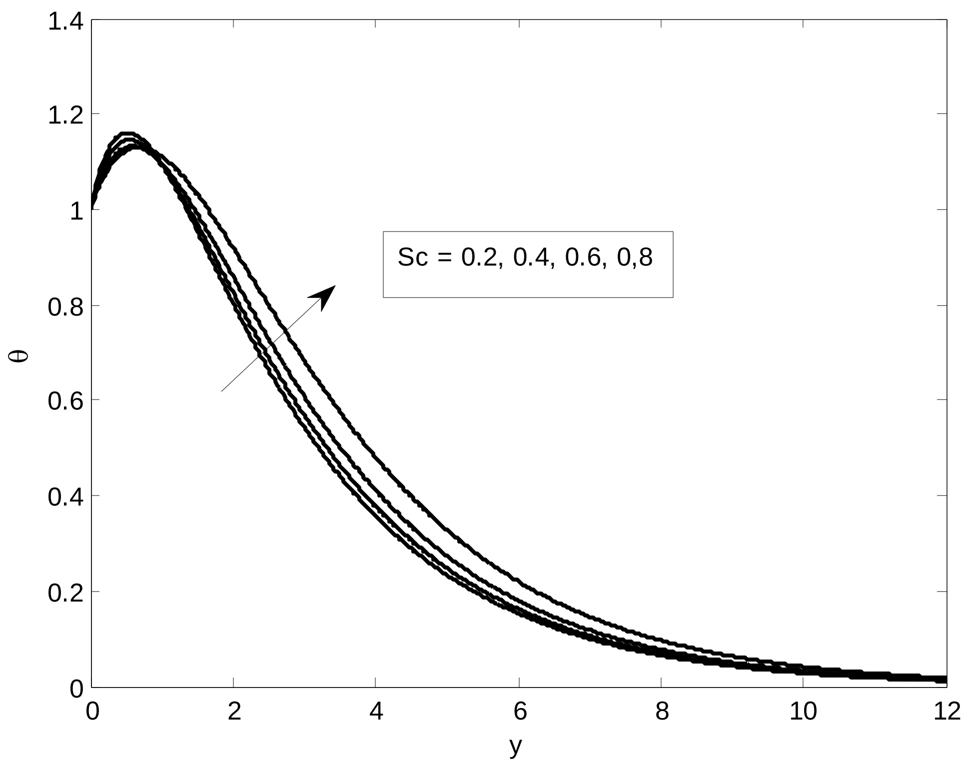

Fig. 6: Effect of Schmidt number on temperature profiles with K_r=1,R = 0.5,R_a=0.5, Pr=0.71,K=0.5,Gr=2,M=0.5,E=0.01,n=0.5,t=0.5, $\varepsilon = 0.02,\ \lambda = 0.5$

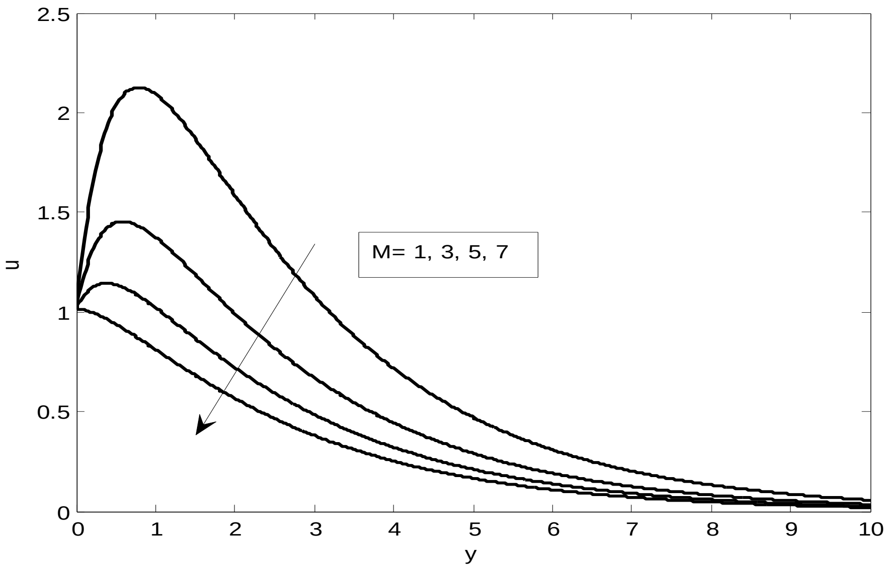

Fig. 7:Effect of Magnetic field parameter M on velocity profiles with Sc=0.65,Pr=0.71,Gr=5,Gm=5, D_f=1,E=0.002,K=0.5,R=0.5, λ=0.5,R_a =1,n=0.5,K_r =1,ε=0.02,t=0.5,ω=0.5.

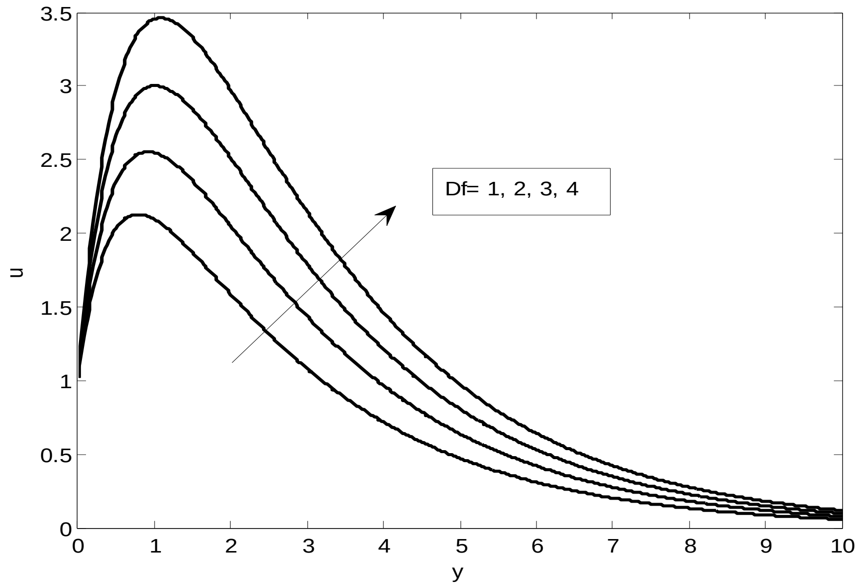

Fig. 8: Effect of Dufour number on velocity profiles with Sc=0.65,Pr=0.71,Gr=5,Gm=5, M=1,E=0.02,K=0.5,R=0.5, λ=0.5,R_a=1,n=0.5,K_r =1, ε=0.02,t=0.5, ω=0.5

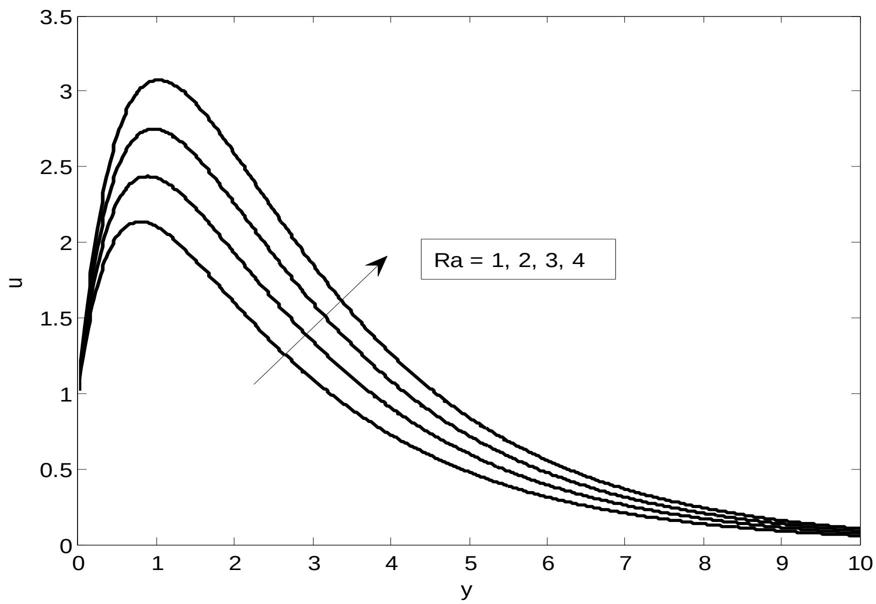

Fig. 9: Effect of Radiation absorption parameter on velocity profiles with Sc=0.65,Pr=0.71,Gr=5,Gm=5,M=1,E=0.02,K=0.5,R=0.5, λ=0.5,n=0.5,K_r =1, ε=0.02, t=0.5, ω=0.5.

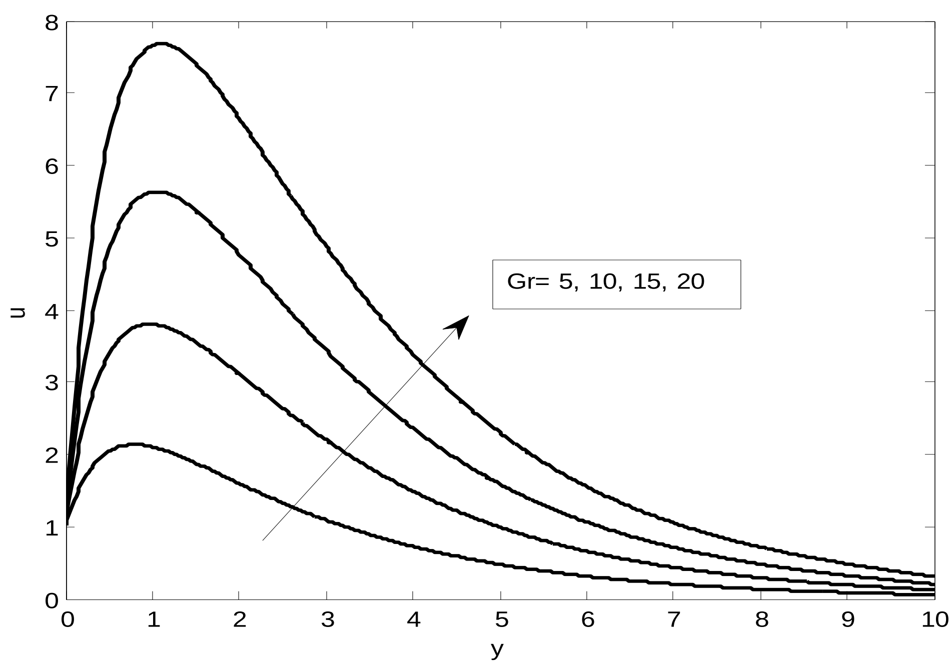

Fig. 10: Effect of Grashof number on velocity profiles with Sc=0.65,K_r=1,R=0.5, R_a =0.5, Pr=0.71,K=0.5,M=0.5,E=0.01,n=0.5,t=0.5, $\varepsilon = 0.02,\ \lambda = 0.5$

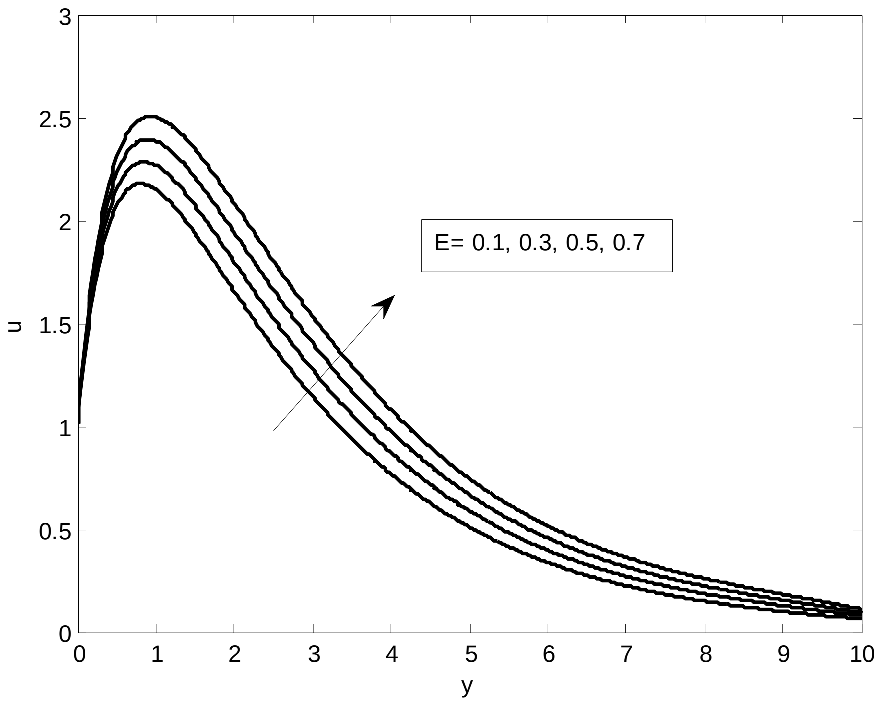

Fig. 11: Effect of Eckert number on velocity profiles with Sc=0.65,K_r=1,R=0.5, R_a =0.5, Pr=0.71,K=0.5,Gr=2,M=0.5,n=0.5,t=0.5, $\varepsilon = 0.02,\ \lambda = 0.5$

Table 1. Skin friction for different values of D_f, K_r, R, R_a, E, λ with Sc=0.65, Pr=0.71, Gr=5, Gm=5, M = 1, K=0.5,$\varepsilon = 0.02$, $\omega = 0.5$, T=0.5

D_f	K_r	R	R_a	E	λ	τ
0.5	1	1	1	0.02	0.5	3.8224
1	1	1	1	0.02	0.5	4.2177
1.5	1	1	1	0.02	0.5	4.4851
1	1	1	1	0.02	0.5	4.2177
1	2	1	1	0.02	0.5	3.9560
1	3	1	1	0.02	0.5	3.9249
1	1	1	1	0.02	0.5	4.2177
1	1	2	1	0.02	0.5	3.8566
1	1	3	1	0.02	0.5	3.7243
1	1	1	0.5	0.02	0.5	3.9519
1	1	1	1	0.02	0.5	4.2177
1	1	1	1.5	0.02	0.5	4.4207
1	1	1	1	0.02	0.5	4.0414
1	1	1	1	0.04	0.5	4.2177
1	1	1	1	0.06	0.5	4.3939
1	1	1	1	0.02	0.5	4.2177
1	1	1	1	0.02	1	4.2213
1	1	1	1	0.02	1.5	4.2259

Table.2. Nusselt number for different values of *Pr*, R_a, D_f with *Sc=0.65, Gr =5, Gm=5, M=1, E=0.04, K=0.5, R=1,* $\lambda = 0.5, \varepsilon = 0.02, \omega = 0.5$

Pr	R_a	D_f	*Nu*
0.71	1	1	0.5928
1	1	1	0.4791
2	1	1	-0.4438
0.71	1	1	0.5928
0.71	2	1	0.9572
0.71	3	1	1.3150
0.71	1	1	0.5928
0.71	1	2	1.1107
0.71	1	3	1.6125

Table.3. Sherwood number for different values of K_r and *Sc*

K_r	*Sc*	*Sh*
1	0.65	1.1949
2	0.65	1.5106
3	0.65	1.7587
1	0.22	0.5918
1	0.40	0.8633
1	0.60	1.1307

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Appendix

$M_1 = M + \frac{1}{K}$, $\alpha_1 = 1 + \lambda M_1$, $N_1 = 1 + \frac{4R}{3}$, $M_2 = M_1 - \alpha_1 n + \lambda n^2$,

$N_2 = nPr - n^2 Pr\lambda$, $L_1 = nSc - n^2 Sc\lambda - KrSc$, $a_2 = \frac{[Sc+\sqrt{Sc^2+4KrSc}]}{2}$, $a_4 = \frac{[1+\sqrt{1+4L_1}]}{2}$

, $a_4 = \frac{[1+\sqrt{1+4L_1}]}{2}$, $a_6 = \frac{Pr}{N_1}$

, $a_8 = \frac{[1+\sqrt{1+4M_1}]}{2}$, $I_1 = \frac{-R_a - {a_2}^2 D_f}{N_1 {a_1}^2 - \mathrm{Pr}\, a_1}$, $I_2 = 1 - I_1$, $I_3 = \frac{-(Gr I_1 + Gm)}{{a_1}^2 - a_1 - M_1}$,

$I_4 = \frac{-Gr\,I_2}{a_6{}^2 - a_6 - M_1}$, , $I_5 = 1 - I_3 - I_4$, $H_1 = -Pra_2{}^2 I_3{}^2$,$H_2 = -Pra_6{}^2 I_4{}^2$

$H_3 = -Pra_8{}^2 I_5{}^2$, $H_4 = -2Pra_2 a_6 I_3 I_4$, $H_5 = -2Pra_2 a_8 I_4 I_5$,

$H_6 = -2Pra_2 a_8 I_3 I_5$, $m_1 = 2a_2$, $m_2 = 2a_6$, $m_3 = 2a_8$,$m_4 = a_2 + a_6$,

$m_5 = a_6 + a_8$, $m_6 = a_2 + a_8$,$I_7 = \frac{H_1}{N_1 m_1{}^2 - \mathrm{Pr} m_1}$, $I_8 = \frac{H_2}{N_1 m_2{}^2 - \mathrm{Pr} m_2}$, $I_9 = \frac{H_3}{N_1 m_3{}^2 - \mathrm{Pr} m_3}$, $I_{10} = \frac{H_4}{N_1 m_4{}^2 - \mathrm{Pr} m_4}$,$I_{11} = \frac{H_5}{N_1 m_5{}^2 - \mathrm{Pr} m_5}$,$I_{12} = \frac{H_6}{N_1 m_6{}^2 - \mathrm{Pr} m_6}$

$I_6 = -(I_7 + I_8 + I_9 + I_{10} + I_{11} + I_{12})$, $I_{14} = \frac{-Gr\,I_6}{a_6{}^2 - a_6 - M_1}$,$I_{15} = \frac{-Gr\,I_7}{m_1{}^2 - m_1 - M_1}$

$I_{16} = \frac{-Gr\,I_8}{m_2{}^2 - m_2 - M_1}$,$I_{17} = \frac{-Gr\,I_9}{m_3{}^2 - m_3 - M_1}$,$I_{18} = \frac{-Gr\,I_{10}}{m_4{}^2 - m_4 - M_1}$, $I_{19} = \frac{-Gr\,I_{11}}{m_5{}^2 - m_5 - M_1}$, $I_{20} = \frac{-Gr\,I_{12}}{m_6{}^2 - m_6 - M_1}$, $I_{13} = -(I_{14} + I_{15} + I_{16} + I_{17} + I_{18} + I_{19} + I_{20})$,

$a_{12} = \frac{\left[1 + \sqrt{1 + 4M_2}\right]}{2}$,$a_{13} = \frac{\left[Pr + \sqrt{Pr^2 - 4N_1 N_2}\right]}{2}$,$H = -2Pra_{12}$,$H_7 = Ha_2 I_3$, $H_8 = Ha_6 I_4$, $H_9 = Ha_8 I_5$, $m_7 = a_2 + a_{12}$, $m_8 = a_6 + a_{12}$,$m_9 = a_8 + a_{12}$

, $I_{22} = \frac{H_7}{N_1 m_7{}^2 - Prm_7 + N_2}$,$I_{23} = \frac{H_8}{N_1 m_8{}^2 - Prm_8 + N_2}$, $I_{24} = \frac{H_9}{N_1 m_9{}^2 - Prm_9 + N_2}$,

$I_{21} = -(I_{22} + I_{23} + I_{24})$,$I_{26} = \frac{-Gr\,I_{21}}{a_{14}{}^2 - a_{14} - M_2}$, $I_{27} = \frac{-Gr\,I_{22}}{m_7{}^2 - m_7 - M_2}$, $I_{28} = \frac{-Gr\,I_{23}}{m_8{}^2 - m_8 - M_2}$

$I_{29} = \frac{-Gr\,I_{24}}{m_9{}^2 - m_9 - M_2}$, $I_{25} = -(I_{26} + I_{27} + I_{28} + I_{29})$