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Fuzzy Inventory Model with Single Item Under Constant Demand and Time Dependent Holding Cost.

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Abstract

The objective of this model is to discuss the inventory model for constant demand and time dependent holding cost. Mathematical model has been developed for determining the optimal order quantity, the optimal cycle time and optimal total inventory cost in fuzzy environment. For defuzzification, graded unit preference integration method is used. Numerical examples are given to validate the proposed model. Sensitivity analysis is carried out to analyze the effect of changes in the optimal solution with respect to change in various parameters.

Keywords: Fuzzy Inventory system; constant demand; time dependent holding cost.

AMS Classification No: $90B05$.

1.0 INTRODUCTION

Various models have been proposed for constant demand rate with constant holding cost. Teng et al. (2005)[17] developed an EOQ model on optimal pricing and ordering policy under permissible delay in payments by assuming that the selling price is necessarily higher than the purchase cost. They established an appropriate model for a retailer to found its optimal price and lot size, simultaneously, when the supplier offered a permissible delay in payment. Muhlemann and Valtis-Spanopoulos (1980)[14] investigated the constant rate EOQ model but with variable holding cost expressed as a percentage of the average value of capital investigated in stock. Vander Veen (1967)[19] presented an EOQ inventory system with the holding cost as a nonlinear function of inventory. Weiss (1982)[21] investigated traditional EOQ model with the holding cost per unit modified as a nonlinear function of the length of time an item was held in stock. Goh (1994)[6] presented an EOQ model with general demand and holding cost function and demand rate for an item was considered as a function of existing inventory level and carrying cost per unit was allowed to change.

Fuzzy set theory has been applied to inventory problems to handle the uncertainties related to the demand or cost coefficients. An extended review of the application of the fuzzy set theory in inventory management can be found in [7]. The advantage of using the fuzzy set theory in modeling the inventory problems is its ability to quantify vagueness and imprecision.

In certain situations, uncertainties are due to fuzziness, primarily introduced by Zadeh[26], is applicable. In 1970, Zadeh et al[27] proposed some strategies for decision making in fuzzy environment. Jain[11] worked on decision making in the presence of fuzzy variables. Kacpryzk et al[12] discussed some long-term inventory policy-making through fuzzy-decision making models. Wide applications of fuzzy set theory can be found in Zimmerman[28], and Park[15]. In basic EOQ model we identify the order size that minimizes the sum of annual costs of inventory holding and fixed setup to place orders.

Thus, EOQ model serves as a useful approximation to many real life problems. In literature, there are many papers on fuzzified problems of EOQ model. Urgeletti [18] treated EOQ model in fuzzy sense, and used triangular fuzzy number. Chen and Wang[3] used trapezoidal fuzzy number to fuzzify the order cost, inventory cost, and backorder cost in the total cost of inventory model without backorder. Then, they found the estimate of the total cost in the fuzzy sense by functional principle.

Vujosevic et al[20] used trapezoidal fuzzy number to fuzzify the order cost in the total cost of inventory model with backorder. Then, they got fuzzy total cost. They obtained the estimate of the total fuzzy cost through centroid to defuzzify.

Further, in a series of papers, Yao et al.[24,23,25], considered the fuzzified problems for the inventory with or without backorder models. In [24], they applied the extension principle to obtain the fuzzy total cost, and then, they defuzzified the fuzzy total cost by centroid. In [23], they considered the fuzzified problems for the inventory with or without backorder models using trapezoidal fuzzy number. In [25], they considered the fuzzified problems for the inventory without backorder models and they fuzzified the order quantity as the triangular fuzzy number. For defuzzification, the study shows that the signed distance method is better than centroid method Yao & Lee [25].

Kao and Hsu [13] considered a single-period inventory model with fuzzy demand. Hsieh[10] analyzed some production inventory models in fuzzy sense and he proposed some optimal strategies. Syed & Aziz [16] used trapezoidal fuzzy number. De and Rawat [5], proposed an EOQ model without shortage cost by using triangular fuzzy number. The total cost has been computed by using signed distance method.

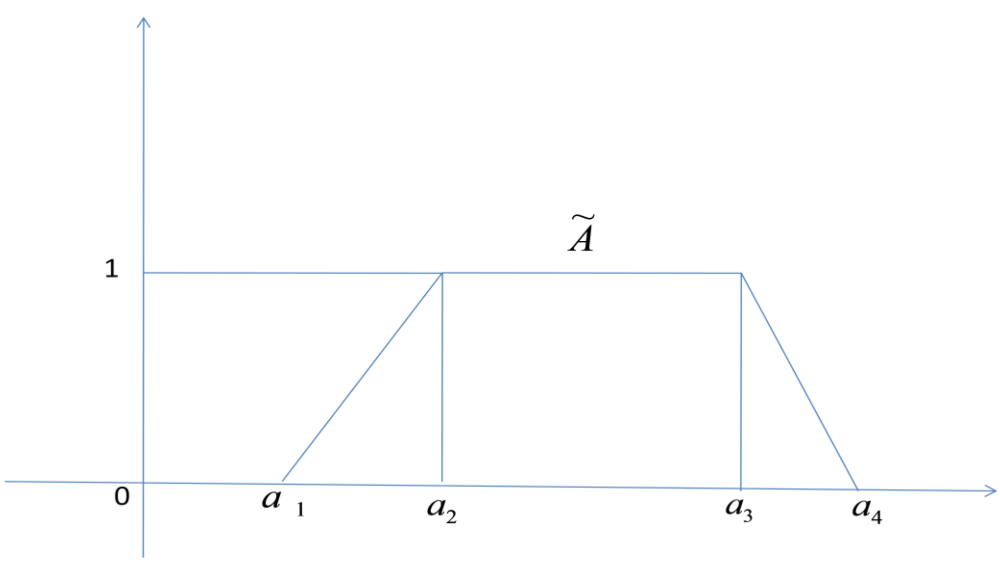


Figure 1 : The Trapezoidal Fuzzy Number $\tilde{A}$

In the proposed study, a fuzzy inventory model has been developed where, we consider the demand rate is constant and holding cost is time dependent. The main objective of this paper is to obtain minimum total inventory cost, order quantity and corresponding order cycle. An algorithm that minimizes the total inventory cost is developed. Numerical examples are discussed to illustrate the procedure of solving the model. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, the related inventory parameters i.e. the inventory holding cost and ordering cost are fuzzified as the trapezoidal fuzzy numbers and then apply the Graded Mean Integration Representation method for defuzzification. The objective is to obtain fuzzy optimal solution to minimize the total cost per time unit of an inventory control system based on the fuzzy arithmetical operations under Function Principle.

1.1. DEFINITION AND PRINCIPLES

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Figure 1, and is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

(1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1],

(2) $\mu_{\tilde{A}}(x) = 0, -\infty < x \le a_1$,

(3) $\mu_{\tilde{A}}(x) = L(x),$ is strictly increasing on $[a_1, a_2]$,

(4) $\mu_{\tilde{A}}(x) = 1, a_2 \le x \le a_3$,

(5) $\mu_{\tilde{A}}(x) = R(x),$ is strictly increasing on $[a_3, a_4]$,

(6) $\mu_{\tilde{A}}(x) = 0,\ a_4 \le x < \infty$,

where a_1, a_2, a_3 *and* a_4 are real numbers.

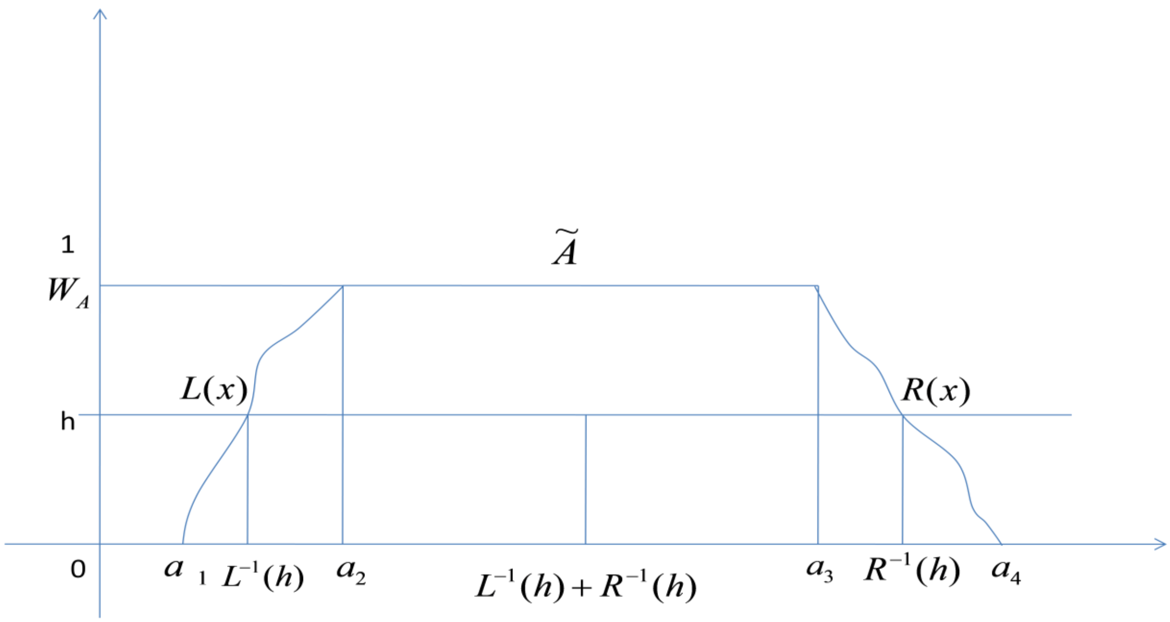


Figure 2 : Generalized Fuzzy Number

In 1998, Chen and Hsieh [1,2,4] propose graded mean integration representation for representing generalized fuzzy number. Now we describe graded mean integration representation (GMIR) as follows.

Suppose L^{-1} and R^{-1} are inverse functions of functions L and R, respectively, and the graded mean h-level value of generalized fuzzy number $A = (c, a, b, d : w)_{LR}$ is $h[L^{-1}(h) + R^{-1}(h)]/2$ as Figure:2. Then the graded mean integration representation of generalized fuzzy number based on the integral value of graded mean h-level is

(1.1) $$P(A)=\int_0^w h\left(L^{-1}(h)+R^{-1}(h)/2\right)dh/\int_0^w h\,dh,$$

where h is between 0 and w and $0<w\le 1$.Generalized trapezoidal fuzzy number and generalized triangular fuzzy number are denoted as $(c,a,b,d:w)$ and $(c,a,d:w)$ respectively. Chen and Hsieh [1,2,4] already find the general formulae of the representation of generalized trapezoidal fuzzy number, or generalized triangular fuzzy number as follows.

Suppose $A=(c,a,b,d:w)$ is a trapezoidal fuzzy number. Since,

$L(x)=w\left(\dfrac{x-c}{a-c}\right),\ c\le x\le a,$ and $R(x)=w\left(\dfrac{x-d}{b-d}\right),\ b\le x\le d,$

then $L^{-1}(h)=c+(a-c)h/w, 0\le h\le w$, $R^{-1}(h)=d-(d-b)h/w, 0\le h\le w$,

and $\dfrac{L^{-1}(h)+R^{-1}(h)}{2}=\dfrac{c+d+(a-c-d+b)h/w}{2}$.

By formula (1.1), the graded mean integration representation of A is

(1.2) $$P(A)=\int_0^w h(c+d+(a-c-d+b)h/w/2)dh/\int_0^w h\,dh=\frac{c+2a+2b+d}{6}.$$

The fuzzy arithmetical operations under Function Principle

Here, we describe some fuzzy arithmetical operations under Function Principle as follows.

Suppose $\tilde{A}=(a_1,a_2,a_3,a_4)$ and $\tilde{B}=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers. Then,

1. The addition of $\tilde{A}$ and $\tilde{B}$ is

   $\tilde{A}\oplus\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$

   where $a_1,b_1,a_2,b_2,a_3,b_3,a_4$ *and* b_4 are any real numbers.

2. The multiplication of $\tilde{A}$ and $\tilde{B}$ is

   $\tilde{A}\otimes\tilde{B}=(c_1,c_2,c_3,c_4)$,

   Where $T=\{a_1b_1,a_1b_4,a_4b_1,a_4b_4\}, T_1=\{a_2b_2,a_2b_3,a_3b_2,a_3b_3\}$,

   $c_1=\min T, c_2=\min T_1, c_3=\max T_1, c_4=\max T$.

   Also, if $a_1,b_1,a_2,b_2,a_3,b_3,a_4$ *and* b_4 are non zero positive real numbers, then

   $\tilde{A}\otimes\tilde{B}=(a_1b_1,a_2b_2,a_3b_3,a_4b_4)$, Where $\tilde{A}\otimes\tilde{B}$ is a trapezoidal fuzzy number.

3. $-\tilde{B}=(-b_4,-b_3,-b_2,-b_1)$, then the subtraction of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A}\theta\,\tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$

where $a_1, b_1, a_2, b_2, a_3, b_3, a_4$ *and* b_4 are any real numbers.

4. $1/\tilde{B} = \tilde{B}^{-1} = (1/b_4, 1/b_3, 1/b_2, 1/b_1)$ *where* b_1, b_2, b_3 *and* b_4, are all positive real numbers.

   If $a_1, b_1, a_2, b_2, a_3, b_3, a_4$ *and* b_4 are all positive real numbers, then the division of $\tilde{A}$ *and* $\tilde{B}$ is $\tilde{A}$ $\varnothing\,\tilde{B} = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1)$.

5. Let $\alpha \in R$, then

$$\begin{cases} (i)\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha\, a_1, \alpha\, a_2, \alpha\, a_3, \alpha\, a_4), \\ (ii)\alpha < 0, \alpha \otimes \tilde{A} = (\alpha\, a_4, \alpha\, a_3, \alpha\, a_2, \alpha\, a_1). \end{cases}$$

Example:

Suppose $\tilde{A} = (1,2,3,4)$ and $\tilde{B} = (1,3,4,6)$ are two trapezoidal fuzzy numbers and $\alpha = 2.5$. Then,

i. $\tilde{A} \oplus \tilde{B} = (2,5,7,10)$,

ii. $\tilde{A} \otimes \tilde{B} = (1,6,12,24)$,

iii. $\tilde{A}\theta\,\tilde{B} = (-5,-2,0,3)$,

iv. $\tilde{A}\ \varnothing\,\tilde{B} = (0.2,0.5,1,4)$,

v. $\alpha \otimes \tilde{B} = (2.5,7.5,10,15)$.

2.0. ASSUMPTIONS AND NOTATIONS:

Following assumptions are made for the proposed model:

i. Single inventory will be used.
ii. Lead time is zero.
iii. The model is studied when shortages are not allowed.
iv. The demand rate λ is assumed to be constant.
v. The holding cost is time dependent and holding cost parameter h i.e. $h(t) = h.t$.

Following notations are made for the given model:

$I(t)$ = On hand inventory level at any time t, $t \geq 0$.

T =The length of cycle time.

A = The ordering cost per unit time

λ = The constant annual demand rate.

$h(t)$ = The time dependent holding cost .

U = Total inventory cost per cycle

Q = Ordering quantity

3.0. FORMULATION:

Let $I(t)$ be the on-hand inventory level at any time $t \geq 0$. The demand rate is assumed to be constant in its entire domain. The amount of stock depletes in the period $[0, T]$ due to the effect of demand. By this process, the stock reaches zero at time T. Hence, the inventory level at any instant of time t is described as follows.

At time $t + \Delta t$, the on-hand inventory in the interval $[0, T]$ will be

$$I(t + \Delta t) = I(t) - d(t)\,\Delta t$$

Dividing by Δt and then taking as $\Delta t \to 0$ we get

(3.1) $$\frac{dI(t)}{dt} = -\lambda \;; \quad 0 \leq t \leq T$$

With the condition

(3.2) $$I(T) = 0.$$

The solution of the differential equation **(3.1)** is given by,

(3.3) $$I(t) = \lambda(T - t)$$

Now Q is the ordering quantity of stock which is given by

(3.4) $$Q = I(0) = \lambda T \quad .$$

From (3.4), we obtain

(3.5) $$T = \frac{Q}{\lambda} \quad .$$

Now the average total cost per cycle is given by

(3.6) $$U(Q) = \frac{1}{T}\text{[Ordering Cost + Holding Cost]}$$

$$= \frac{1}{T}\left[A + h\int_0^T t \,.\, I(t)\,dt\right]$$

$$= \frac{A}{T} + \frac{h\lambda}{T}\int_0^T t(T - t)\,dt = \frac{A}{T} + \frac{h\lambda T^2}{6}$$

Using (3.5), we obtain

(3.7) $$U(Q) = \frac{A}{T} + \frac{h\lambda T^2}{6} = \frac{A\lambda}{Q} + \frac{hQ^2}{6\lambda} \quad .$$

The necessary condition for minimization of $U(Q)$ is,

(3.8) $$\frac{\partial U(Q)}{\partial Q} = 0 \ .$$

The sufficient condition for minimization of $U(Q)$ is for $Q > 0$,

(3.9) $$\frac{\partial^2 U(Q)}{\partial Q^2} > 0 \ .$$

(3.10) $$\frac{\partial U(Q)}{\partial Q} = \frac{-A\lambda}{Q^2} + \frac{hQ}{3\lambda} \ .$$

(3.11) $$\frac{\partial^2 U(Q)}{\partial Q^2} = \frac{2A\lambda}{Q^3} + \frac{h}{3\lambda} \ .$$

Now the function $U(Q)$ will be maximum if

(3.12) $$\frac{\partial^2 U(Q)}{\partial Q^2} > 0$$ which is obvious from (3.11).

Now from **(3.8)** we have

(3.13) $$\frac{-A\lambda}{Q^2} + \frac{hQ}{3\lambda} = 0 \ .$$

Now on solving **(3.13)**, implies minimizing total cost to determine optimal Q^* given by $Q^* = \left[\frac{3A\lambda^2}{h}\right]^{1/3}$ and hence the optimal cost $U(Q^*)$ can be evaluated.

4.0 FUZZY MODEL AND SOLUTION PROCEDURE

We consider the model in fuzzy environment. Due to fuzziness, it is not easy to define all the parameters precisely. We use the following variables

$\tilde{A}$: fuzzy Ordering cost,

$\tilde{h}_c$: fuzzy carrying cost,

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{h}_c = (h_1, h_2, h_3, h_4)$, are nonnegative trapezoidal fuzzy numbers.

The total average cost per unit time is given by

(4.1) $$\tilde{C}_{avg}(Q) = (\alpha \otimes \tilde{A}) \oplus (\theta \otimes \tilde{h}_c)$$

$$where\ \ \alpha = \frac{\lambda}{Q}\ ,\ \theta = \frac{Q^2}{6\,\lambda}\ \ .$$

(4.2) Now $\tilde{C}_{avg}(Q) = (\tilde{C}_{avg\,1}(Q), \tilde{C}_{avg\,2}(Q), \tilde{C}_{avg\,3}(Q), \tilde{C}_{avg\,4}(Q),)$

$$\tilde{C}_{avg\,1}(Q) = (\alpha\ A_1 + \theta\ h_1\ )$$

$$\tilde{C}_{avg\,2}(Q) = (\alpha\ A_2 + \theta\ h_2\ )$$

$$\tilde{C}_{avg\,3}(Q) = (\alpha\ A_3 + \theta\ h_3\ )$$

$$\tilde{C}_{avg\,4}(Q) = (\alpha\ A_4 + \theta\ h_4\ )$$

Now on defuzzifying the fuzzy total average cost $\tilde{C}_{avg}(Q)$ *we have*

(4.3)
$$P(\tilde{C}_{avg}(Q)) = \frac{1}{6}(\alpha\ A_1 + 2\alpha\ A_2 + 2\alpha\ A_3 + \alpha\ A_4) + \frac{1}{6}(\theta\ h_1 + 2\theta\ h_2 + 2\theta\ h_3 + \theta\ h_4)$$

$$= \frac{\alpha}{6}(\ A_1 + 2\ A_2 + 2\ A_3 + A_4) + \frac{\theta}{6}(\ h_1 + 2\ h_2 + 2\ h_3 + h_4)$$

To minimize the average total cost per unit time, the optimal value of Q can be obtained by solving the following equation

(4.4)
$$\frac{d(P(\tilde{C}_{avg}(Q)))}{dQ} = \frac{\alpha'}{6}(\ A_1 + 2\ A_2 + 2\ A_3 + A_4) + \frac{\theta'}{6}(\ h_1 + 2\ h_2 + 2\ h_3 + h_4)$$

$$where\ \ \alpha' = \frac{-\lambda}{Q^2}\ \ ,\ \theta' = \frac{Q}{3\,\lambda}\ \ .$$

Thus minimum value of the total cost $C_{avg}(Q)$ denoted by $C^{*}_{avg}(Q)$

(4.5)
$$C^{*}_{avg}(Q) = \frac{1}{6}\left[(\alpha\ A_1 + \theta\ h_1\right] + \frac{1}{3}\left[\alpha\ A_2 + \theta\ h_2\right]$$

$$+ \frac{1}{3}\left[\alpha\ A_3 + \theta\ h_3\right] + \frac{1}{6}\left[(\alpha\ A_4 + \theta\ h_4\right]$$

5.0. COMPUTATIONAL ALGORITHM:

Step-1: Start.

Step-2: Initialize the value of the variables $A, \lambda, h.$

Step-3: Evaluate $U(Q)$.

Step-4: Evaluate $\dfrac{\partial U(Q)}{\partial Q}$.

Step-5: Solve the equation $\frac{\partial U(Q)}{\partial Q} = 0$.

Step-6: Choose the solution from Step-5.

Step-7: Evaluate $\frac{\partial^2 U(Q)}{\partial Q^2}$.

Step-8: If the value of Step-7 is greater than zero then this solution is optimal (minimum) and go to Step-10.

Step-9: Otherwise go to Step-6.

Step-10: End.

6.0 NUMERICAL EXAMPLES

To illustrate the proposed method, let us consider the following input data:

Crisp Model:

The values of the parameters in proper units are considered as follows:

$\lambda = 500,\ A = 400, h = 40$

Optimal $Q^* = 195.7434, U^* = 1532.6189, T^* = 0.3915$.

Fuzzy Model:

We can apply the fuzzy inventory model with fuzzy order quantity to find the optimal fuzzy total average cost. First, we represent the case of vague value as the type of trapezoidal fuzzy number.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4) = (200,400,400,600)$

$\tilde{h}_c = (h_1, h_2, h_3, h_4) = (20,40,40,60)$, $\lambda = 500$.

Equation (4.5) can be minimized by using**,** Matlab Software to determine optimal $Q^*, U^* \ \& T^*$.

The optimal ordering quantity, average cost and time are found to be $Q^* = 191.108, U^* = 1510.743, T^* = 0.3327$.

7.0. SENSITIVITY ANALYSIS:

TABLE 1

h	Q^*	U^*	T^*
40	195.7434	1532.6189	0.3915
45	188.2072	1593.9879	0.3764
50	181.7121	1650.9636	0.3634
55	176.0298	1704.2569	0.3521
60	170.9976	1754.4106	0.342

TABLE 2			
A	Q^*	U^*	T^*
400	195.7434	1532.6189	0.3915
420	198.9529	1583.2896	0.3979
440	202.062	1633.1621	0.4041
460	205.0783	1682.2843	0.4102
480	208.0084	1730.6995	0.416

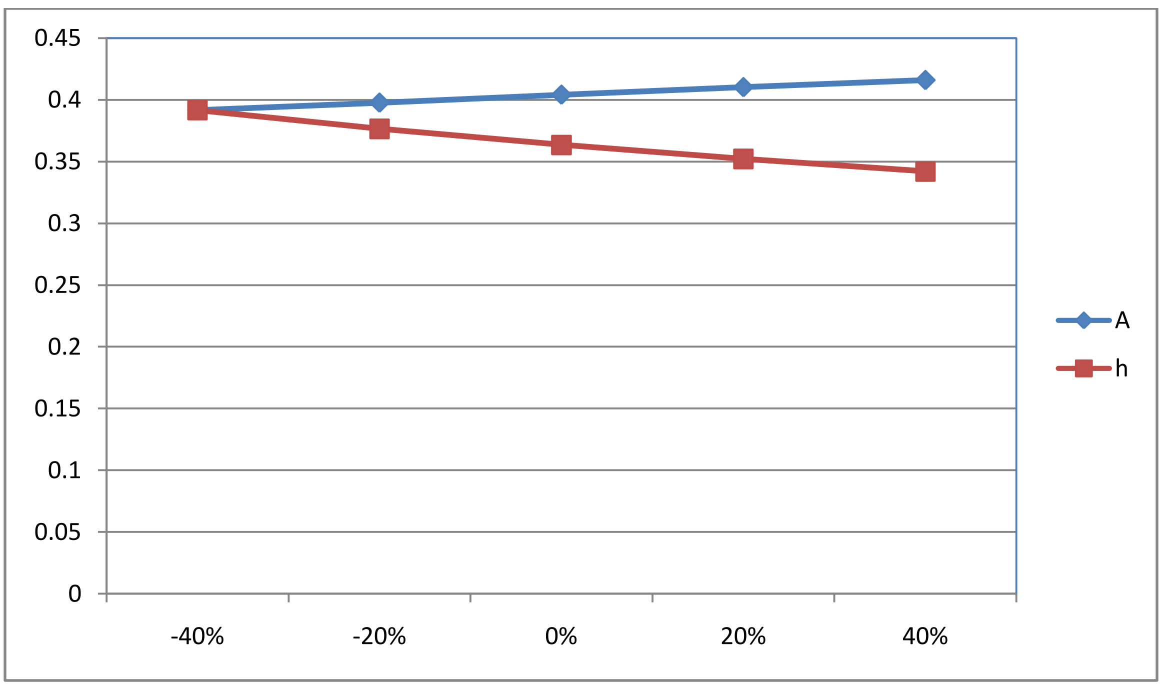
Important points from the table

- The effect of optimality due to change of values of different parameters associated in this model is discussed below.

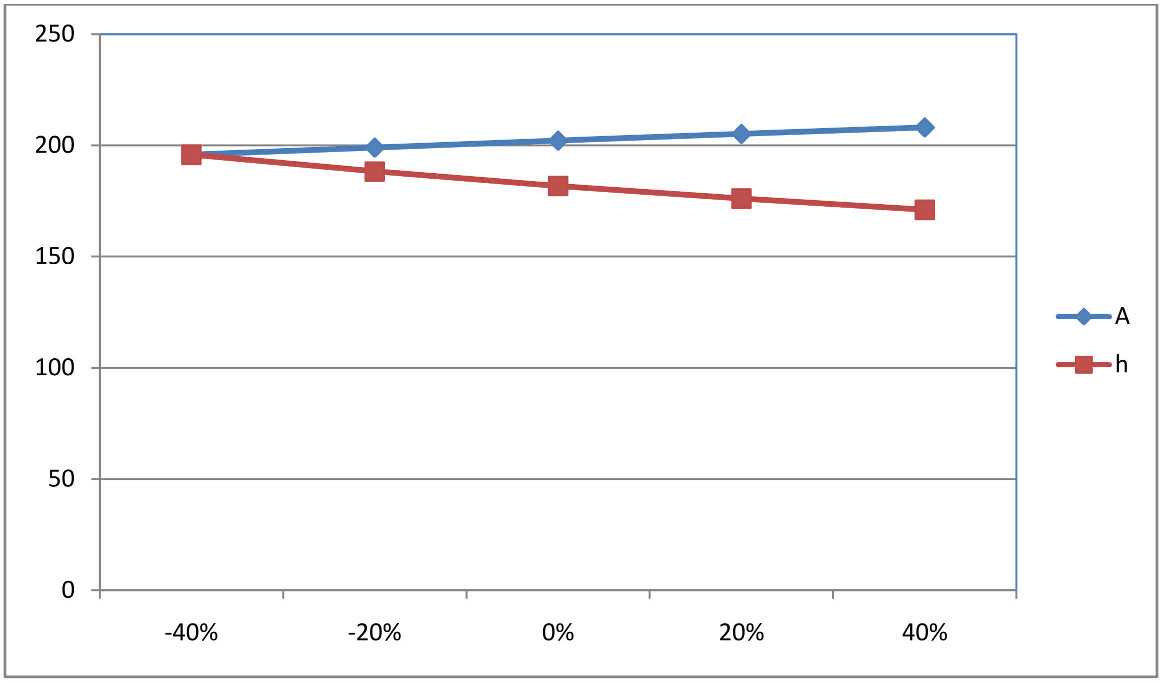
1. U increases while Q & T decrease with increase in value of the parameter h .
2. Q, U & T increase with increase in value of the parameter A .

Variation of Time duration, Ordering Quantity and Total cost w.r.t. different parameters.

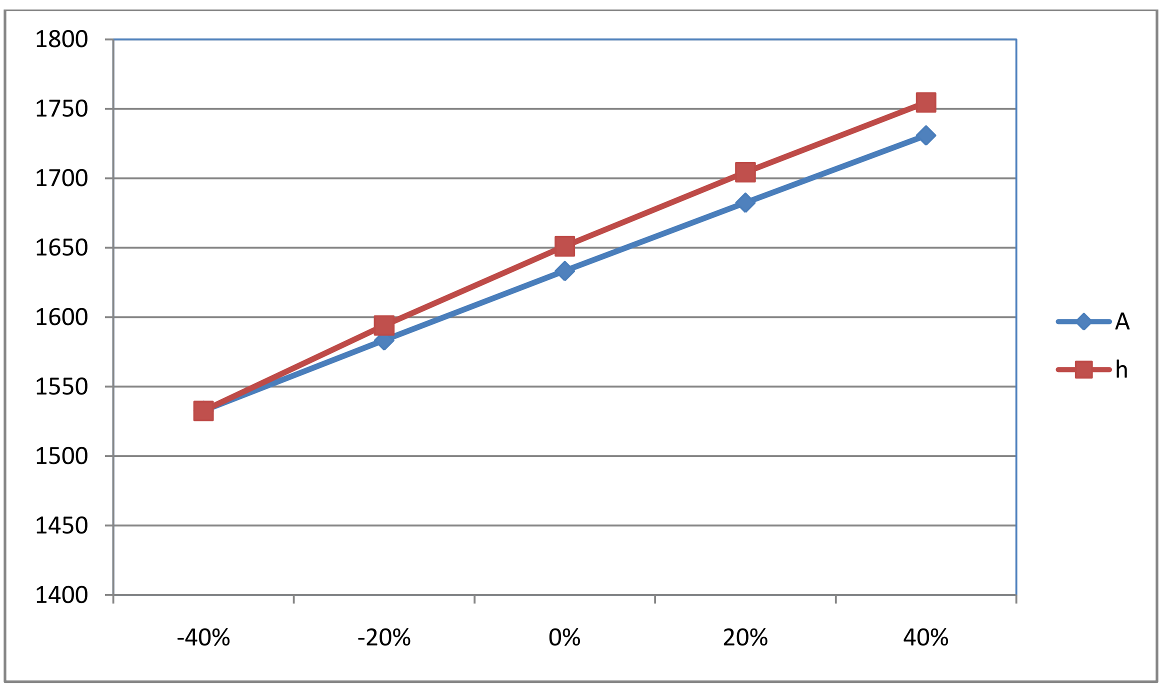
1.**Variation of Time duration**



2.**Variation of Ordering Quantity**



3.**Variation of Total cost**



8.0 CONCLUSION

This paper presented a fuzzy inventory control model for constant demand and time dependent holding cost respectively. The proposed model is developed in both the crisp and fuzzy environments. In fuzzy environment, all related inventory parameters were assumed to be trapezoidal fuzzy numbers. The optimum results of fuzzy model are defuzzified using graded mean level integration representation method.

The model can further be studied for shortage state and for multiple items under identical conditions. This can also be extended for deterioration conditions and also for discounted cash flow approach.

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