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The Color, Power Spectrum, and Hurst Exponent Associated with the Linear and chaotic Nature of Changes in Pitch of Selections of Music By Philip Glass

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ABSTRACT

White noise is what we call random or white colored noise. It is a simple measure of the frequency at which the system changes over time. The color of noise is a measure of the instability (white noise) or probabilistic nature of a system's dynamics (i.e. white noise is the most unstable random colored noise), whereas, red noise is the most stable, non-random colored noise. In all types of music, changes in the pitch of the music is a proxy for measuring change in the color of noise, and hence, the stability of noise over time. In the following paper, I hypothesize that particular pieces of Philip Glass's music can be used to measure (qualitatively and quantitatively), linear and chaotic or fractal (unstable) dynamic change in pitch, which can best be qualitatively and quantitatively explored by determining the color and chaotic (or fractal) nature of a particular measure within a composition. I use statistical models and analysis of those models using computer software to test my hypothesis. The models presented here are analogs, which do not explicitly model the pitch patterns ingrained in a measure from a particular Glass composition. Rather, these models are meant to generate the types of patterns one expects Glass's music to generate. Surprisingly, I found that one of the analog models predicted that a measure of one of Glass's compositions consisted of both linear and chaotic pitch components, resulting in the generation of a measure with an indefinite pitch (blue color) state, one, whose dynamics are in-between a stable and unstable state. Analysis of each individual note of a measure of a particular composition is not the intent of this paper at this moment in time. Future research the explicitly models pitch dynamics is needed. Lastly, I opine about further application of this approach to quantify the color of noise and associated power spectrum in many other disciplines.

INTRODUCTION

Philip glass is a contemporary composer, who has produced profound works, ranging from piano etudes, music for plays, and symphonies with piano accompaniment [1, 2]. I will focus on the latter to analyze the color of noise that his music generates.

Music Theory

Presently, contemporary classical musical compositions such as those produced by Philip Glass, particularly in his Glassworks, utilize pitch and chord structure, such that his compositions induce a hierarchy of relations, stabilities, instabilities, and attractions [3]. Pitch is the lowness or highness of a tone, for example the difference between middle D and a higher D. Frequencies of sound waves producing a pitch can be measured with a high degree of precision. Tonality and pitch are very similar musical properties, each a complex combination of frequencies, and therefore a mix of pitches [3].

The frequency of vibration of the pitch generated from a single measure of music enables the listener to assign musical tones to relative positions on a musical scale based primarily on the frequency of vibration [4, 5].

Thus, pitch is closely related to frequency, but the two are not equivalent. Sound waves do not have pitch, and their oscillations can be measured to obtain a frequency. Pitches are usually quantified as frequencies in cycles per second, or hertz (Hz), by comparing sounds with pure tones, which have periodic, sinusoidal waveforms. Complex and a periodic sound waves can often be assigned a pitch by this method[5, 6].

MATERIALS AND METHODS

Each measure of music corresponds to a discrete set of notes, each consisting of its own pitch. Hence, music is discrete if measured over successive measures of a musical composition, and continuous if the musical composition is listened to in its entirety, which can be thought of as listening to an entire composition at once and not focusing explicitly on each measure of music. Hence, discrete time analog models were developed in this paper to generate the color and respective power spectrum associated with color produced from a discrete component of a section of a measure from a small number of Philip Glass compositions.

The Linear and Fractal Structure of Glass musical compositions

A general tool for exploring discrete time series is discrete Fourier analysis. The underlying idea of spectral analysis is that a simulation of a specific type of spectral density *S(f)* will give rise to fractional Brownian motion (fBm) with an exponent $0<H<1$. If a random function *X(t)* contains equal power for all frequencies *f*, the process is called white noise[6]. Thus, a white noise process is analogous to the white light made up of radiations of all wavelengths, and is the most random or unstable noise. Extending this analogy, a time-series signal is considered less variable and is called red if $1/2<H\leq 1$ [7], and a time-series signal is considered a signal that is in-between a stable and unstable state and is called blue if $0\leq H<1/2$[7]. If *S(f)* is proportional to $\frac{1}{f^2}$, it is true that $S(f) \propto \frac{1}{f}\ \beta \sim fBm$, with $H = \frac{\beta - 1}{2}$. Sincethe mean-square increments for fBM with exponent H are proportional to Δt^{2H}, β and H are directly related to the autocorrelation function of X, which in turn defines the spectral density by means of a Fourier transformation.

In what follows, I discuss the linear and fractal compositional structure of a single measure from two musical compositions from Philip Glass's Glass Works. The particular musical compositions that I focus on epitomizes Glass's use of linear and fractal structure in pitch for a single measure. For example, figure's 1 and 2 represent examples of the structure of a measure derived from 2 different compositions, each composed of notes from Glasswork's composition 1 ("Opening") and composition 4 ("Rubric").

(a)

(b)

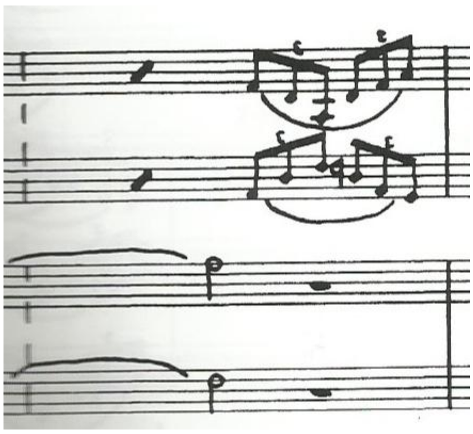


Figure 1. An example of change in pitch for measures in each of 2 compositions from Glassworks. Pitch is the lowness or highness of a tone, for example the difference between middle D and a higher D. Figure (a) illustrates the compositional structure of a measure that exhibit linear change in pitch (composition 1 from Glassworks). Figure (b) illustrates the compositional structure of a measure that exhibits both linear and chaotic change in pitch (composition 2 from Glassworks). In (b), the flute is playing the non-linear component

of the measure while the alto saxophone and tenor saxophone are playing the non-linear component of the measure.

For (a), the musical composition Opening uses triplet eighth notes, over duple eighth notes, over whole notes in 4/4. For (b), the musical composition Rubric begins with open fifths in the horns while the other members of the ensemble enter with oscillating arpeggio figures. There are two formulaically identical sections to the movement. Although rhythmically driven, the melodic implications of Rubric occur somewhat coincidentally by orchestration. There is no modulation, but the harmonic progression simply repeats over and over again, with a linear and nonlinear component [8].

RESULTS: THE COLOR OF NOISE IN A GLASS COMPOSITION

For Glassworks compositions 1 and 4, the linear and fractal structure of each measure from each composition was measured separately. Thus, I was able to analyze the music as it occurred in discrete time by using discrete time statistical models. The first analog or reference model (linear model(1)) is a discrete-time one-dimensional sinusoidal model for change in pitch (for 1 measure) in composition 1, and can be described mathematically as

$$X_{t+1} = C + \alpha \sin(\omega X_t + \theta) \tag{1}$$

Where $\boldsymbol{C}$ is a constant defining a mean pitch level (415 Hz as defined by "Baroque pitch", set in the 20th century), $\boldsymbol{\alpha}$ is an amplitude for the sine wave, where the amplitude varies from -1(low peak amplitude) to 1 (peak amplitude), $\boldsymbol{\omega}$ is the frequency, where values are consistent with music with frequencies ranging from 400 to 2000 Hz [9]. Thus, frequency values are based on the assumption that the frequency component of measure 1 in composition 1 is based on an equal-tempered scale. In model 1, $\boldsymbol{X_{t+1}}$ is a time variable that measures the pitch of the measure that follows measure $\boldsymbol{X_t}$, where $\boldsymbol{X_t}$ is the initial pitch frequency (range 400–3480 Hz), and $\boldsymbol{\theta}$ is the phase, where the values of the phase are scaled from -3π to 3π. Model (1) is the linear analog that will be analyzed to determine the color of noise and power spectrum generated by a measure from Glass's composition 1.

The second model (non-linear model(2)) is a discrete-time one-dimensional white noise/blue noise sinusoidal model [10] for change in pitch (for 1 measure) in compositions 4, and can be described mathematically as

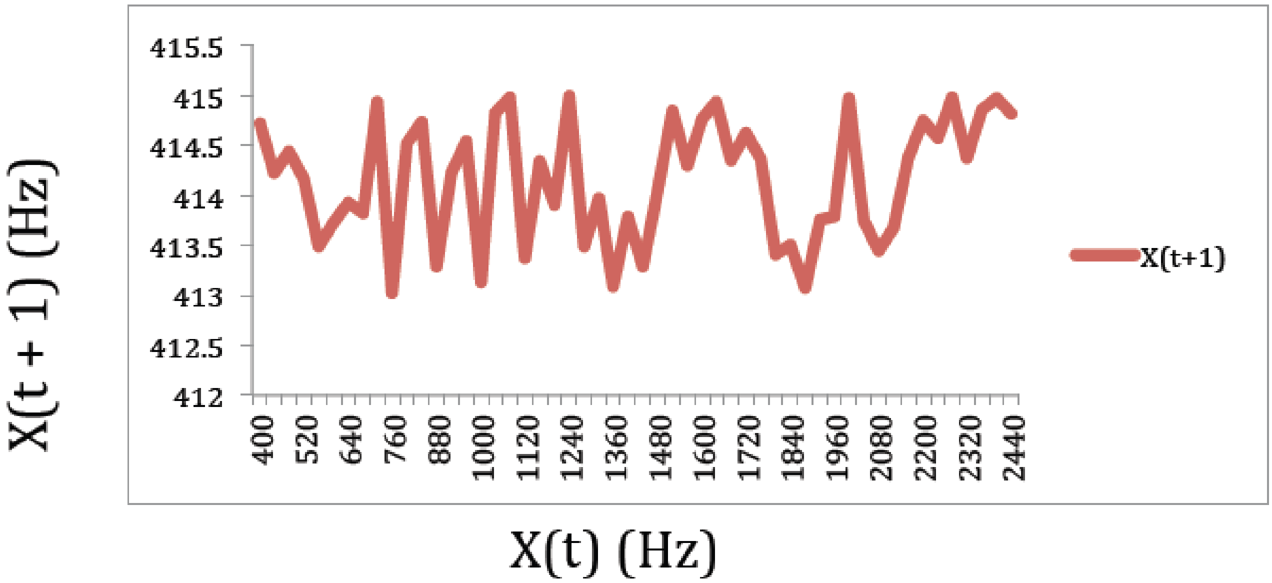
$$Y_{t+1} = C + \alpha \sin(\omega Y_t + \theta) + \varepsilon_t \tag{2}$$

Where $\boldsymbol{C}$ is a constant defining a mean pitch levels with model (1), $\boldsymbol{\alpha}$ is an amplitude for the sine wave (with values equivalent to those used in model (1)), $\boldsymbol{\omega}$ is the frequency (with values equivalent to those used in model (1)), $\boldsymbol{Y_{t+1}}$ is a time variable that measures the pitch of the measure that follows measure $\boldsymbol{Y_t}$, with values equivalent to those used in model (1), $\boldsymbol{\theta}$ is the phase (with values equivalent to those used in model (1)), and $\boldsymbol{\varepsilon_t}$ is the error sequence in approximating the sequence Y_t by the model (error was simply a deviate sampled from a normal distribution). Model (2) is the linear/non-linear analog that will be analyzed to determine the color of noise and power spectrum generated by a measure from Glass's composition 4.

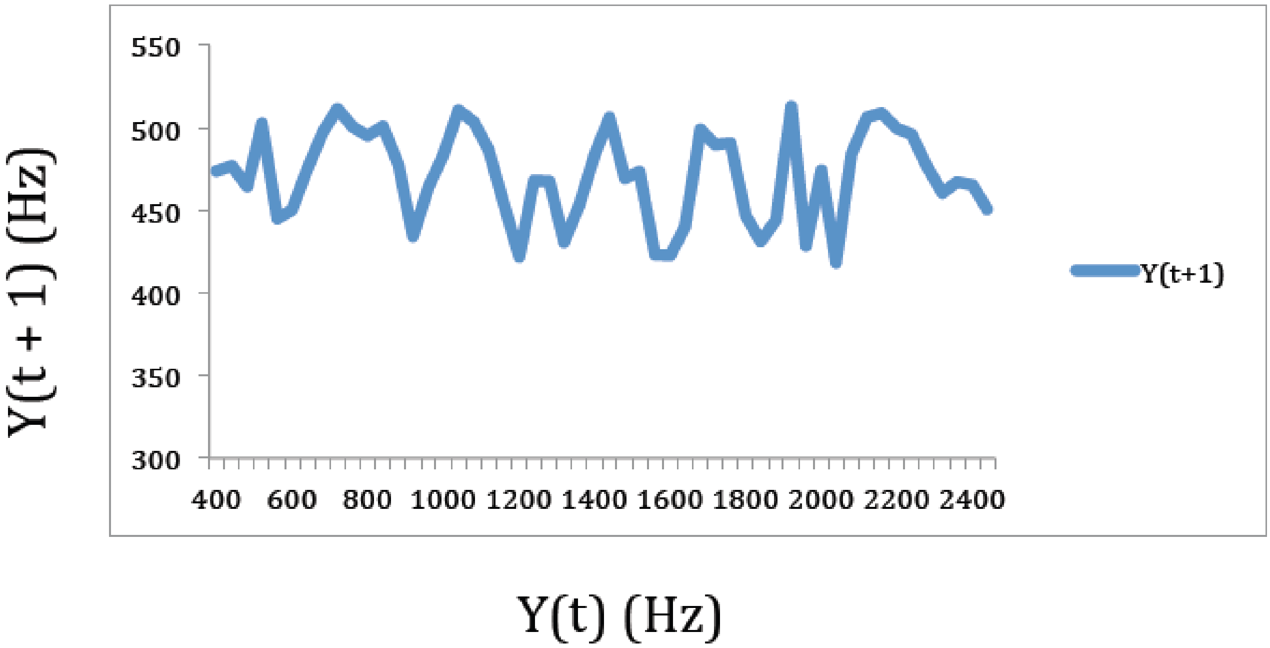
Simulations of models(1) and (2)generate two time-series vectors (X_{t+1} and Y_{t+1}), each of which contains the value of the pitch at time t +1. Discrete Fourier analysis of the simulated data was performed with the Benoit Truesoft© package. Average linear trends in the time series were removed and a taper function applied [10].Tapering was done to avoid edge effects and to minimize spectral leaking [14]. The algorithm multiplied the input series by a taper function that smoothly approached zero at the extremities of the power spectrum. Signal noise was smoothed by averaging period grams obtained in equal logarithmic intervals of the data set. The Hurst exponent *H* was then calculated for all simulated time series.

Figure 2(a-d) show the numerical dynamics and associated power spectrums of reference model's (1-2), their power spectral densities, and associated Hurst exponents. Table 1 summarizes the results and describes the color of noise generated from each simulation.

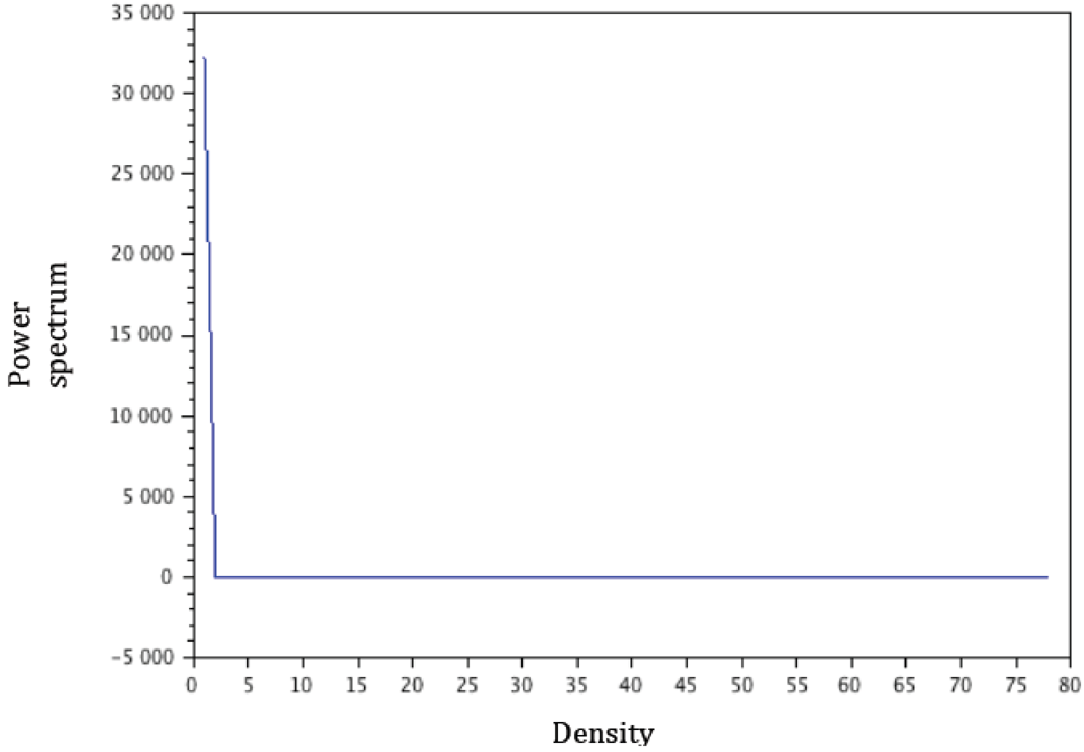
(a)


415.5
415
414.5
414
413.5
413
412.5
412
400
520
640
760
880
1000
1120
1240
1360
1480
1600
1720
1840
1960
2080
2200
2320
2440
X(t+1)
X(t + 1) (Hz)
X(t) (Hz)

(b)


550
500
450
400
350
300
400 600 800 1000 1200 1400 1600 1800 2000 2200 2400
Y(t+1)
Y(t + 1) (Hz)
Y(t) (Hz)

(c)


35 000
30 000
25 000
20 000
15 000
10 000
5 000
0
-5 000
Power spectrum
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80
Density

(d)

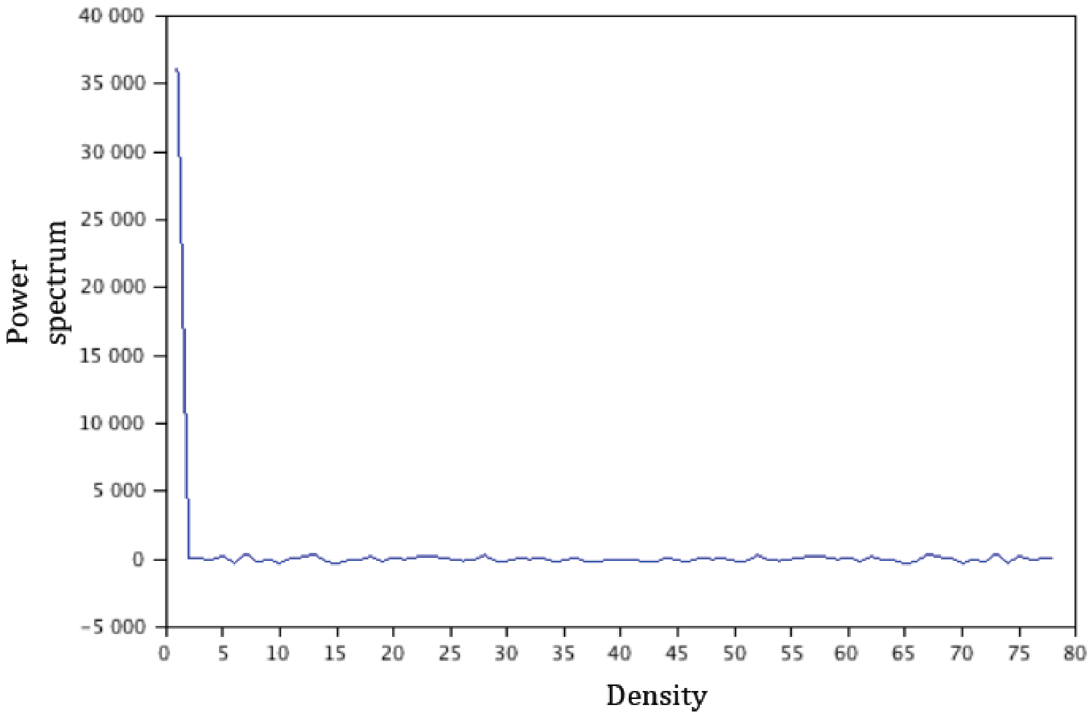

40 000
35 000
30 000
25 000
20 000
15 000
10 000
5 000
0
-5 000
Power spectrum
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80
Density

Figure 1(a)-(d) show results from simulation of model (1) and (2). (a): Change in pitch dynamics (X_{t+1}) measured in Hz as a function of X_t.(b): Change in pitch dynamics (Y_{t+1}) measured in Hz as a function of Y_t. (c): The power spectrum generated by (a) and (d) is the power spectrum generated by (b). For parameter values C = 400 Hz, α is the amplitude for the sine wave (-1 to 1), ω is the frequency, and varies from 400 to 2000 Hz.

Table 1. Model dynamics, calculated *H*, and associated color for each power spectrum

Composition	Dynamics	***H***	Color
(1) Glass's composition Opening	Stable	0.8000	Red
(2) Glass's composition Rubric	Stable and unstable	0.1830	Blue

APPLICATIONS AND SUMMARY

In summary, 2 simple discrete –time model analogs ((1) and (2)) have been introduced. These models are used as references for determining the meaning of color in the time-series signal of the dynamics of changes in pitch in 1 measure of music taken from 2 of Philip Glass's compositions. The dynamics of model (1) and (2) are explored through simulation of the models. As expected, the linear sinusoidal model (model (1) generates linear dynamics with red colored noise. Unexpectedly, the non-chaotic/chaotic (i.e. linear/non-linear) sinusoidal model (model (2)) generates a blue colored noise signal, which is indicative of indefinite pitch dynamics. The power spectrum followed what one would expect from the dynamics exhibited by model (1) but not (2). The power spectrum generated by model(2) is indicative of a linear spectrum that tapers off towards zero in a smooth pattern, however, unlike the entirely smooth spectrum obtained by model (1), there is variation (the chaotic or unstable component) about zero, which is indicative of a system that is concurrently stable and unstable. This makes sense, since composition 4 is composed of both linear and non-linear pitch components.

There are several caveats that should be mentioned in this summary. First, quantifying pitch in any musical composition is a very difficult task. Hence, the models presented here to investigate the dynamics (time-series color and power spectrum) of change in pitch from selected work from Glassworks were used as analogs, rather than exact models for measuring pitch. Thus, these analog models can be used as a template for future studies of dynamical change in many aspects of musicality, including pitch, tonality, modality, and harmony, in addition to many other musical metrics.

Analog models like those presented in this paper can be used by research from such field as physics, biology (cellular processes and biochemical reactions), and large ecosystems (e.g. ecosystems with interacting populations of predators and prey). Examples of analog models that could apply to the above mentioned fields of study for analyzing the linear and chaotic dynamics of the studied system, and to determine their color, include, but are not limited to:

(1) The investigation of Jump Markov processes, in which the differential equations used to predict behavior generated by the dynamics of the system have no memory of past events [11].

(2) The cell cycle, where several differential equation models have been developed including a generic eukaryotic cell cycle model which can represent a specific eukaryote. However, the qualitative dynamics generated from simulations of the model show sensitivity to initial conditions as a function of slight differences in the parameters in the model. Often, these models are simple analogs of equations like those presented in this paper [12, 13], where their predictions match those of models designed to predict the existence of stable and unstable dynamics.

(3) The application of mathematical models similar to those described by equations (1)-(2) to predict the qualitative dynamics of swarming behavior in insects [12]

(4) The application of mathematical models similar to those described by equation's (1) and (2) to predict the dynamics of the spatial distribution of organisms in large ecosystems [13]

(5) The stability of natural communities, including their response to environmental perturbations rests to a large degree on interactions and components of biotic communities [15–16]. Thus, studying the affects of environmental noise (analogous to $\boldsymbol{\varepsilon}$in model (2) to extreme conditions in the dynamics of single populations without including inter specific competition is biologically unrealistic. Using the approach set forth in this paper may enable researchers to analyze the affects of environmental noise on the color of noise and power spectrum generated by competing populations for different extreme conditions (i.e. different values of $\boldsymbol{\varepsilon_t}$).

Lastly, I would conclude that systems in-between stable and unstable states that exhibit blue colored noise and power spectrums that are concurrently smooth and rough are probably quite prevalent in natural systems, because these systems (particularly, those mentioned above), can simultaneously exhibit linear and nonlinear dynamics. For example, the coexistence of predator and prey populations, where one population is stable and the other is unstable.

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