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A Group Acceptance Sampling Plan for Weighted Binomial on Truncated Life Tests Using Exponential and Weibull Distributions

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Abstract

In this paper, a group acceptance sampling plan using weighted binomial is developed for a truncated life test when the lifetime of an item follows exponential and Weibull distributions. The minimum number of groups required for a given group size and the acceptance number is determined when the consumer's risk and the test termination time are specified. The operating characteristic values according to various quality levels are obtained. The results are explained with examples.

Keywords: Exponential and Weibull distributions; Group acceptance sampling using weighted binomial; consumer's risk; Operating characteristics; Producer's risk; truncated life test.

1. Introduction

Acceptance sampling is concerned with inspection and decision making regarding lots of product and constitutes one of the oldest techniques in quality assurance. In the 1930s and 1940s acceptance sampling was one of the major components in the field of statistical quality control and was used primarily for incoming or receiving inspection. Sampling plans often used to determine the acceptability of lots of items. Although in recent years more emphasis is placed on process control and off-line quality control methods, acceptance sampling remains as a major tool of many practical quality control system. In acceptance sampling, if the quality variable is the lifetime of an item, the problem of acceptance sampling is known as the reliability sampling, and the test is called the life test. We would like to know whether the lifetimes of items reach our standard or not. When the life test shows that the mean lifetime of items exceeds or equals to the specified one, we treat the lot of items as acceptable; conversely, when the life test shows that the mean lifetime of the lot of items is less than our standard, we treat the lot as unacceptable and reject it.

In most acceptance sampling plans for a truncated life test, determining the sample size from a lot under consideration is the major issue. In the usual sampling plan, it is implicitly assumed that only a single item is put in a tester. However testers accommodating a multiple number of items at a time are used in practise because testing those items simultaneously. Items in a tester can be regarded as a group and the number of items in a group is called as group size. The acceptance sampling plan based on this group of items is called group acceptance sampling plan. The quality of the product is tested on the basis of few items taken from an infinite lot. The statistical test can be stated as: Let μ be the true average life and μ0 be the specified average life of a product. Based on the failure data, we want to test the hypothesis H0: $\mu \geq \mu_0$ against H1: $\mu < \mu_0$. A lot is considered as good if $\mu \geq \mu_0$ and bad if $\mu < \mu_0$. This hypothesis is tested using the acceptance sampling scheme as: In a life test experiment, a sample of size n selected from a lot of products is put on the test. The experiment is terminated at a pre – assigned time t0. when we set acceptance number as c, H0 is rejected if more than c failures are recorded before time t0 and H0 is accepted if there are c or fewer failures before t0. Probability of rejection of good lot is called the producer's risk and probability of accepting a bad lot is known as consumer's risk. If the confidence level is p*, then the consumers risk will be β = 1-p*. A well acceptance sampling plan minimizes both the risks.

Many authors have discussed acceptance sampling based on truncated life tests. Aslam M. and Shahbaz M.Q. (2007) have studied economic reliability test plans using the generalized exponential distributions. Epstein B. (1954) discussed truncated life tests in the exponential case. Fertig F.W. and Mann N.R. (1980) have studied life-test sampling plans for two-parameter Weibull populations. Goode H.P. and Kao J.H.K. (1961) have studied sampling plans based on the Weibull distribution. Gupta R.D. and Kundu D. (1999) have studied generalized exponential distributions. Again Gupta R.D. and Kundu D. (2003) studied discriminating between the Weibull and generalized exponential distributions. Jun C.H. Balamurali S. and Lee S.H. (2006) discussed variables sampling plans for Weibull distributed lifetimes under sudden death testing. Radhakrishnan R. and Alagirisamy K. (2011) have studied on construction of group acceptance sampling plan using weighted binomial distribution. Sobel M. and Tischendrof J. A. (1959) have discussed acceptance sampling with new life test objectives.

Here we apply GASP using weighted binomial on the truncated life tests when a lifetime of the product assumed to follow exponential and Weibull distributions. In this type of tests, determining the sample size is equivalent to determining the number of groups.

2. Lifetime Distributions

Weibull Distribution:
The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t,\sigma) = 1 - e^{-(\frac{t}{\sigma})^m} \quad ,t > 0 \qquad (1)$$

where σ is a scale parameter and m is the shape parameter.

Exponential distribution:

The cumulative distribution function (cdf) of the exponential distribution is given by

$$F(t/\sigma) = 1 - e^{-t/\sigma} \quad ,t > 0 \qquad (2)$$

where σ is a scale parameter.

If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/ σ.
The failure probability of an item by time t0 is given by

$$p = F(t_0 : \sigma) \qquad (3)$$

The quality of an item is usually represented by its true mean lifetime although some other options such as median lifetime is sometimes used. Let us assume that the true mean μ can be represented by the scale parameter. Also, it is convenient to specify the test time as a multiple of the specified life so thataμ0 and the quality of an item as a ratio of the true mean to the specified life (μ/ μ0).
Then we can rewrite (3) as a function of 'a' (termination time) and the ratio μ/μ0

$$p = F(a\, \mu_0 : \mu/ \mu0) \qquad (4)$$

When the underlying distributions are the Gamma, exponentiated log – logistic and Marshall – Olkin extended exponential distributions having known shape parameter γ and unknown scale parameter σ. Then the true mean life of a product under the above distributions is given by

$$\mu = \gamma\sigma \qquad (5)$$

Here when the underlying distribution is the Weibull distribution

$$p = 1 - e^{-\left(\frac{ba}{\mu/\mu_0}\right)^m} \quad \text{where } b = (\Gamma(1/m)/m)^m \qquad (6)$$

When the underlying distribution is the Exponential distribution

$$p = 1 - e^{-\frac{1.2279a}{\mu/\mu_0}} \qquad (7)$$

3 Design of the proposed sampling plan

Procedure:

1) Select the number of groups g and allocate predefined r items to each group so that the sample size for a lot will be n = gr.

2) Select the acceptance number c for a group and specify the experiment time t_0.

3) Perform the experiment for the g groups simultaneously and record the number of failures for each group.

4) Accept the lot if at most c failures occur in each of all groups by the experiment time.

5) Terminate the experiment as soon as more than c failures occur in any group and reject the lot.

We are interested in determining the number of groups g, whereas the various values of acceptance number c and the termination time t_0 are assumed to be specified. Since it is convenient to set the termination time as a multiple of the specified life μ_0, we will consider $t_0=a\mu_0$ for a specified constant a (time multiplier).

The lot acceptance probability will be

$$L(p) = \left(\sum_{i=1}^{c} \binom{r-1}{i-1} p^{i-1} (1-p)^{r-1} \right)^g \qquad (8)$$

where p is the probability that an item in a group fails before the termination time t0 = aμ0.

The minimum number of groups required can be determined by considering the consumer's risk when the true median life equals the specified median life ($\mu = \mu_0$) (worst case) by means of the following inequality:

$$L(p_0) \le \beta \qquad (9)$$

where p_0 is the failure probability at $\mu = \mu_0$. Here minimum group size (g) is obtained using (6) and (7) in (8) at worst case.

4 Operation Characteristic Functions

The probability of acceptance can be regarded as a function of the deviation of the specified value μ0 of the median from its true value μ. This function is called Operating Characteristic (OC) function of the sampling plan. Once the minimum sample size is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is sufficiently good. As mentioned earlier, the product is considered to be good if $\mu \ge \mu0$. Te probabilities of acceptance are displayed in Table 3 and 4 for various values of the median ratios μ/μ0, producer's risks β and time multiplier a.

5 Notations

g	-	Number of groups
r	-	Number of items in a group
n	-	Sample size
d	-	Number of defectives
c	-	Acceptance number

t_0	-	Termination time
a	-	Test termination time multiplier
m	-	Shape parameter
σ	-	Scale parameter
β	-	Consumer's risk
p	-	Failure probability
L(p)	-	Probability of acceptance
p*	-	Minimum probability
μ	-	Mean life
μ0	-	Specified life

6 Description of tables and examples

6.1 Example 1

Based on various test values of consumer's risk and the test termination time multiplier, the number of groups of GASP is found using (7) and (8). Suppose that we want to develop an economic reliability group sampling plan to test if the median is greater than 1,000 hours based on a testing time of 700 hours and using testers equipped with 6 items each. It is assumed that c = 2 and $\beta = 0.1$. This gives the termination multiplier $a = 0.7$. If the life time follows Exponential distribution, from Table 1 the design parameters can be written as (g, c) = (3, 2). We will implement the above sampling plan as, draw the first sample of size n = 18 items and put to 6 testers, if not more than 2 failures occur during 700 hours in any one of the groups, we accept the lot, otherwise reject it. If suppose the life time follows Weibull distribution, from Table 2 the design parameters can be written as (g, c) = (4, 2). We will implement the above sampling plan as, draw the first sample of size n = 24 items and put to 6 testers, if not more than 2 failures occur during 700 hours in any one of the groups we accept the lot, otherwise reject it. For this proposed sampling plan if r = 4, β = 0.25 and the life time follows exponential distribution, the probability of acceptance is 0.996096 when the true mean is 10,000 hrs from Table 3 and if the life time follows Weibull distribution the probability of acceptance is 0.996606 when the true mean is 10,000 hrs from Table 4.

When the the probability of acceptance is compared exponential distribution has less probability of acceptance than the Weibull distribution. Thus Weibull distribution is comparatively good for the group acceptance sampling plan with weighted binomial. It is observed that the lot acceptance probability increases as the mean ratio increases and the number of groups tends to increase as the test duration decreases and is shown in Figure 1.

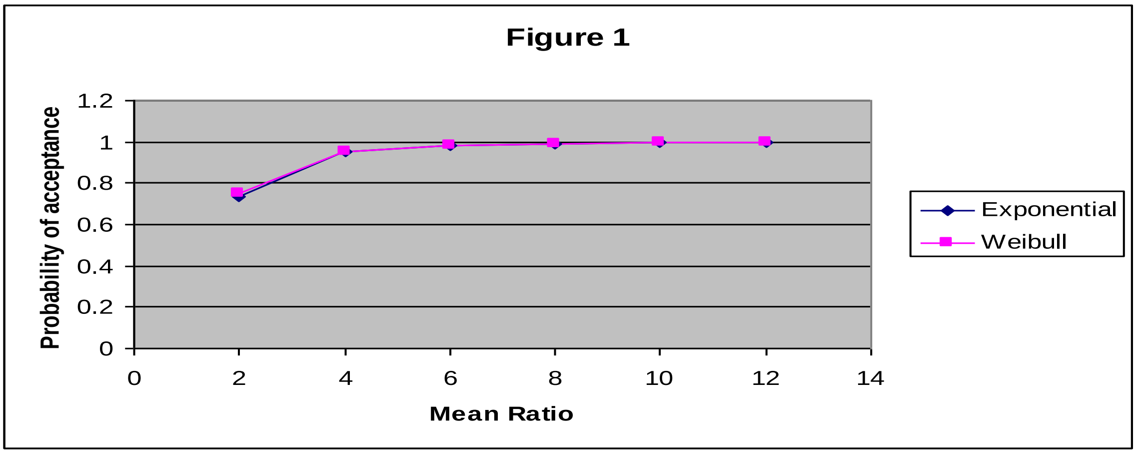

Figure 1: Operating Characteristic curve for the weighted group acceptance sampling plan when the lifetime of the items follows different lifetime distributions with $r = 4$, β = 0.25, $a = 0.7$

Table 1: Minimum number of groups (g) for the weighted acceptance group sampling plan when the lifetime of the items follows the exponential distribution

β	r	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	2	1	1	1
	3	1	4	3	3	2	2	1
	4	2	7	5	4	3	2	1
	5	3	12	9	5	4	2	1
	6	4	22	14	9	3	2	1
	7	5	38	26	11	6	4	2
0.10	4	0	1	1	1	1	1	1
	5	1	2	2	1	1	1	1
	6	2	3	2	2	1	1	1
	7	3	4	3	2	2	1	1
	8	4	5	4	3	2	1	1
	9	5	8	6	3	2	2	1
0.05	5	0	1	1	1	1	1	1
	6	1	2	2	1	1	1	1
	7	2	2	2	2	1	1	1
	8	3	3	3	2	2	1	1
	9	4	4	3	2	2	1	1
	10	5	6	4	3	2	1	1
0.01	7	0	1	1	1	1	1	1
	8	1	2	1	1	1	1	1
	9	2	2	1	1	1	1	1
	10	3	3	2	2	1	1	1
	11	4	3	3	2	2	1	1
	12	5	4	3	2	2	1	1

Table 2: Minimum number of groups (g) for the weighted group acceptance sampling plan when the lifetime of the items follows the Weibull distribution

β	r	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	2	2	1	1
	3	1	5	4	3	3	2	2
	4	2	11	8	5	4	3	2
	5	3	21	15	8	6	4	2
	6	4	43	27	14	8	5	3
	7	5	85	50	22	12	6	3
0.10	4	0	2	1	1	1	1	1
	5	1	2	2	2	1	1	1
	6	2	4	3	2	2	1	1
	7	3	6	4	3	2	2	1
	8	4	9	7	4	3	2	1
	9	5	15	10	5	3	2	1
0.05	5	0	2	1	1	1	1	1
	6	1	2	2	2	1	1	1
	7	2	3	3	2	2	1	1
	8	3	5	4	3	2	1	1
	9	4	7	5	3	2	2	1
	10	5	10	7	4	3	2	1
0.01	7	0	2	1	1	1	1	1
	8	1	2	2	2	1	1	1
	9	2	3	2	2	2	1	1
	10	3	4	3	2	2	1	1
	11	4	5	4	3	2	2	1
	12	5	7	5	3	2	2	1

Table 3: Probability of acceptance for the weighted group acceptance sampling plan with $c = 2$ when the lifetime of the items follows the exponential distribution

β	r	a	g	μ/μ₀					
				2	4	6	8	10	12
0.25	4	0.7	7	0.737143	0.950476	0.983476	0.992623	0.996096	0.997690
	4	0.8	5	0.739979	0.949432	0.982897	0.992310	0.995912	0.997575
	4	1.0	4	0.666157	0.928145	0.974886	0.988527	0.993843	0.996325
	4	1.2	3	0.632318	0.914770	0.969345	0.985788	0.992305	0.995379
	4	1.5	2	0.611518	0.902031	0.963403	0.982681	0.990502	0.994247
	4	2.0	1	0.646467	0.903429	0.962106	0.981531	0.989675	0.993661
0.10	6	0.7	3	0.449204	0.849412	0.943299	0.973177	0.985314	0.991117
	6	0.8	2	0.494124	0.860330	0.946511	0.974410	0.985883	0.991416
	6	1.0	2	0.332834	0.775802	0.908033	0.954511	0.974410	0.984240
	6	1.2	1	0.460058	0.825987	0.927540	0.963602	0.979278	0.987122
	6	1.5	1	0.314249	0.734603	0.880796	0.937649	0.963602	0.976991
	6	2.0	1	0.153824	0.576917	0.786329	0.880796	0.927540	0.952908
0.05	7	0.7	2	0.420450	0.826471	0.931729	0.966916	0.981612	0.988764
	7	0.8	2	0.323780	0.771093	0.905945	0.953451	0.973807	0.983866
	7	1.0	2	0.179632	0.651671	0.843831	0.919323	0.953451	0.970855
	7	1.2	1	0.304660	0.729093	0.878119	0.936208	0.962749	0.976448
	7	1.5	1	0.176653	0.608368	0.807261	0.894055	0.936208	0.958813
	7	2.0	1	0.064982	0.423830	0.675334	0.807261	0.878119	0.918603
0.01	9	0.7	2	0.184380	0.658280	0.847801	0.921653	0.954893	0.971799
	9	0.8	1	0.340787	0.756624	0.893069	0.944752	0.967999	0.979879
	9	1.0	1	0.205387	0.642541	0.828826	0.907344	0.944752	0.964569
	9	1.2	1	0.118310	0.531139	0.756624	0.862230	0.915502	0.944752
	9	1.5	1	0.048672	0.383341	0.642541	0.784347	0.862230	0.907344
	9	2.0	1	0.009941	0.205387	0.461961	0.642541	0.756624	0.828826

Table 4: Probability of acceptance for the weighted group acceptance sampling plan with *c* = 2 when the lifetime of the items follows the Weibull distribution

β	r	a	g	μ/μ₀					
				2	4	6	8	10	12
0.25	4	0.7	11	0.750508	0.955414	0.985409	0.993550	0.996606	0.998000
	4	0.8	8	0.746827	0.953332	0.984545	0.993126	0.996370	0.997856
	4	1.0	5	0.730334	0.947043	0.982040	0.991914	0.995698	0.997447
	4	1.2	4	0.680194	0.932155	0.976387	0.989233	0.994229	0.996557
	4	1.5	3	0.620887	0.911039	0.967881	0.985082	0.991914	0.995141
	4	2.0	2	0.558636	0.881878	0.954963	0.978471	0.988123	0.992777
0.10	6	0.7	4	0.505182	0.877819	0.955775	0.979493	0.988906	0.993344
	6	0.8	3	0.503420	0.872239	0.952793	0.977862	0.987939	0.992729
	6	1.0	2	0.481334	0.854705	0.944082	0.973185	0.985186	0.990984
	6	1.2	2	0.349212	0.7859996	0.912901	0.957083	0.975911	0.985186
	6	1.5	1	0.448293	0.819702	0.924503	0.961964	0.978306	0.986501
	6	2.0	1	0.263564	0.693782	0.858091	0.924503	0.955459	0.971639
0.05	7	0.7	3	0.426863	0.839812	0.939308	0.971212	0.984214	0.990444
	7	0.8	3	0.325065	0.785743	0.915420	0.959114	0.977338	0.986186
	7	1.0	2	0.311208	0.762789	0.901919	0.951310	0.972553	0.983074
	7	1.2	2	0.192824	0.665494	0.851504	0.923663	0.956081	0.972553
	7	1.5	1	0.293470	0.720425	0.873378	0.933488	0.961074	0.975351
	7	2.0	1	0.137811	0.557860	0.774295	0.873378	0.922770	0.949694
0.01	9	0.7	3	0.179119	0.673197	0.860459	0.930059	0.960437	0.975569
	9	0.8	2	0.227745	0.699917	0.870276	0.934199	0.962441	0.976650
	9	1.0	2	0.108367	0.560306	0.789982	0.888033	0.934199	0.958306
	9	1.2	2	0.047568	0.429729	0.699917	0.831775	0.898152	0.934199
	9	1.5	1	0.111225	0.519598	0.748536	0.856978	0.912017	0.942355
	9	2.0	1	0.032679	0.329192	0.593886	0.748536	0.836611	0.888810

7. Conclusion

In this paper, a group acceptance sampling plan with weighted binomial from a truncated life test is proposed in the case of an exponential and Weibull distributions. The number of groups and the acceptance number are determined when the consumer's risk (β) and the other plan parameters are specified. It is observed that the minimum number of groups required decreases as the test termination time multiplier increases. Moreover, the operating characteristic function increases disproportionately when the quality improves. This group acceptance sampling plan with weighted binomial can be used when a multiple number of items are tested simultaneously. Clearly, such a tester would be beneficial in terms of test time and test cost.

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