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# Production Lot Sizing Problem with the Lead Time

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# Abstract

The issue in lot sizing problem is to plan production processes, so that mean the production quantities must be equal to customer demand quantities such that the inventory cost and setup production cost is minimized. In this work, we use the model Multi Level Capacitated Lot Sizing problem with consideration the Lead times, which means that the problem of finding a feasible solution is complex. For this, we propose a new formula in comparison with the classic model. The efficiency of the new formula is demonstrated and infeasible solutions are solved by a heuristic method that's based on Lagrangian relaxation. Computational tests conducted in 1000 instances with up to 40 components and 16 periods have shown that optimal solutions were obtained on average 96.43% of the large instances. For the improvement of the best solution, the heuristic is able to find the efficiency with 97.62% on average. The solution quality is evaluated through initial iterations, the average solution time provided by Lagrangian relaxation is less than 0.43s.

Keywords: Lot Sizing; Multi Level Capacitated Problem; Lagrangian Relaxation.

## 1. Introduction

A methodology of research mostly used in production systems in the planning of material requirements (MRP). An important task in such systems consists of the determination of the lot size of products in each period of a finite horizon, so that forecast demand is met. If we take into account the production or purchasing lead time of each product and the product structure (Bill-Of-Materials or BOM), the MRP provides a synchronized production plan of the products Vollmann, Berry and Whybark (1988). Most MRP systems generate production plans that ignore capacity constraints. In addition, MRP plans rarely consider setup production and inventory costs. As a consequence, the production plans generated are mostly infeasible.

This paper focuses on problems in a Multi Level capacitated lot Sizing Problem (MLCLSP), in order to minimize setup and inventory costs and obtain feasible production plans. The problem addressed here include lead times and no setup times. When lead time is incorporated, the model and the solution methods become more complex due to the need of synchronizing the production planning periods of the products. Different model formulations and solution methods for MLCLSP can be found in Buschkühl et al. (2010) points out that it is necessary to consider lead times. Berretta et al. (2005) assumes that some products have no lead times and others have positive lead times of one period or more.

Lot sizing problems have been studied by several ways. The formulations of the Capacitated Lot Sizing Problem (CLSP) are considered as a reference model for addressing the plan generation of problematic production manager in a single site environment Comelli et al. (2008), the Multi Level Capacitated Lot Sizing Problem (MLCLSP), is recognized as a reference model and deals with Manufacturing Resources Planning (MRP and MRPII) issues (Almeder et a, 2011; Berretta et al, 2005; Chen & Chu, 2003) and as pointed out by Nascimento et al. (2010), the lack of a reference model for multi site issues. This model only determines the production quantities and periods, regardless of the actual production sequence of commands, this type of modeling has the advantage that it allows flexible sequencing orders in a period. The problem Multi Level Capacitated Lot Sizing Problem (MLCLSP) is one of the most difficult optimization problems known in the production Bel (1998) demonstrated that to find feasible solutions for MLCLSP is complex, and when there are considered setup times. The approach was based on the control of quantities compared on demand, compliance with

the BOM structure and the level of stock.

Most of the models and algorithms proposed for MLCLSP Almeder, C (2010) rely on one of the following two assumptions: either lead times are neglected, thus allowing predecessors and successors to be produced in the same period; or lead times account for at least one period for each component, forcing the throughput time (in number of periods) of the finished products to be at least equal to the number of levels of the BOM. According to studies, the zero lead time assumption leads to plans that are not implementable, as the lower level scheduling problem is likely to be infeasible Almeder, C (2010). On the other hand, positive lead time usually results in extensive amounts of work-in process, which tends to increase with a larger number of levels in the BOM. This problem has been studied by different researchers Buschkühl et al. (2010).

A recent review about a different model formulations and solution methods for MLCLSP can be found in Almeder et al. (2014), the authors proposed two models: The batching formulation considers that products produced within each lot can be processed as raw materials on the following BOM level once the whole batch is completed. And considering that products can be transformed they are released. They show that the solution of quality is specified by the quality of the approximation of the sub problem by imposing a new developed penalty variant to stabilize the solutions of the problem.

The majority of the solutions published for MLCLSP suffer from the assumption of lead time considered. For this the present work is motivated by the zero lead time assumption which leads to plans that are infeasible. On the other hand, positive lead time usually results in extensive amounts of work-in-process. For this we will propose a new formulation of the model taking in consideration of the lead time. In order to solve large instances, we resort to a well known heuristic approach, Lagrangian relaxation a solution for certain optimization problems.

In Section 2 we introduce MLCLSP and the major difficulties of its classical formulation proposed in the literature. In Section 3 novel multi-level formulation was developed for the capacitated lot-sizing and scheduling problem, based on the assumption of one period of lead time. In Section 4 we present the proposed heuristic method that's based on Lagrangian relaxation. Section 5 is devoted to computational test results for small and large instances and we then compare the proposed formulation with the classical formulation of MLCLSP.

#### 2. Mathematical Formulation

In this section, we study the classical MLCLSP model which is formulated according to Billington et al. (1983). Given i products, t time periods and m machines, each product has a manufacturing lead time and deterministic demand in each time period. The problem is to generate a production plan that minimizes the sum of setup costs and inventory costs. Consider the following mathematical notation:

#### **Parameters:**

- i : Product
- t : Period
- m : Machine

P<sub>mi</sub> : Time to produce a unit of product i on machine m

- C<sub>i</sub> : Cost of setup of product i
- D<sub>it</sub> : Demand for the product i at time t (external)
- G : Arbitrarily larger number (e.g., total demand or maximum capacity)
- $H_i$  : The cost of stock of product i
- $C_{mt}$  : The available capacity (time) at time t
- $S_{mi}\;\;$  : Time for setting up machine m for the production of product i
- l<sub>i</sub> : Lead time of product i (non negative integer corresponding to the number of periods)
- $a_{ii}$ : Amount of product i to produce a unit of product j (go into-factor).

(1)

Variables:

- $I_{it}$  : Stock level of product at the end of period t
- $X_{it}$  : Quantity of product i in period t
  - $\int 1$  if the product and manufactured in the period
- <sup>Y</sup>it  $\begin{bmatrix} 0 \text{ no production} \end{bmatrix}$

The mathematical model is:

$$\mathbf{Min} \sum_{i=1}^{N} \sum_{t=1}^{T} (C_i \cdot Y_{it} + H_i \cdot I_{it})$$

Under the constraints:

$$I = I + X_{i(t-Li)} - \sum_{j=1}^{N} a^* X - D_{j=1} ij jt it$$
 i,t (2)

$$\sum_{i=1}^{N} (P_{mi} * X_{it} + S_{mi} * Y_{it}) \le C_{mt}$$
 i,t (3)

$$X_{it} \le G^* Y_{it} \qquad \qquad i,t \qquad (4)$$

$$I_{it} \ge 0$$
,  $X_{it} \ge 0$  i,t (5)

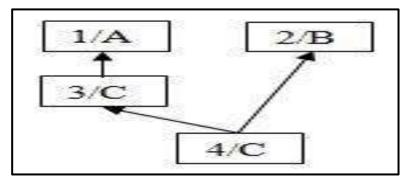
$$Y_{it} \{0,1\}$$
 i,t (6)

The objective function (1) captures the fixed setup cost and the underlying holding cost. Constraints (2) are standard lotsizing flow requirements capturing BOM and lead times. Constraint (3) expresses the fact that the plan that we would compute to be finite capacity. Indeed, for the realization of a plan, we have an amount of resources that will be consumed by the production of one or more references. Total consumption should be less than the available capacity. Constraint (4) model the following condition: if the setup of production, while the quantity produced must not exceed an upper bound of the output G. This represents the minimum between the maximum amount of the reference can be produced and the total demand on the horizon [t, .., T]. Constraint (5) means that  $X_{it}$  and  $I_{it}$  variables are continuous no negative for any product i, for each period t. The last constraint (6) expresses the fact that  $Y_{it}$  is a binary variable for any product i in each period t.

Many researchers dealing with this model assume that the lead time is negligible, to the effect that the predecessors and successors could be produced in the same period ( $l_i = 0$  for every i). The MLCLSP is a big bucket model and the periods are supposed to cover the long time intervals with a number of production batches, so it would result in significant amounts of work in production and the possibility of using overtime to compensate a potential lack of capacity. If we assume that the lead time is positive for at least one period ( $l_i = 1$  for every i), we deliver the requested quantity. But positive lead times contradict the big bucket assumption, because the objective function (1) does not account for additional work-in-process.

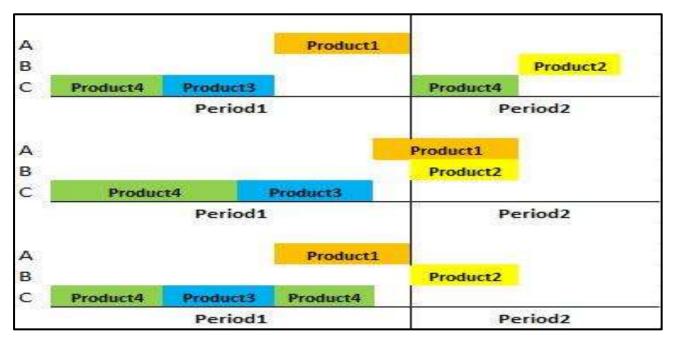
To verify the model, the data in table 1 presents a lot sizing problem in two periods, 4 products, three machines and one way to overcome the feasibility problem of the MLCLSP is to consider a minimum lead time of one period. As aforementioned this lead time may cause a substantial increase of stock and work. Figure 2 shows the structure of the nomenclature Almeder et al. (2014).

TABLE 1. DATA OF THE EXAMPLE							
Product	Machine	<b>D</b> 11	<b>D</b> <sub>12</sub>	H <sub>i</sub>	Ci	P mi	
1	А	3	0	3	5	0.1	
2	В	0	2	3	5	0.1	
3	С	0	0	3	5	0.1	
4	С	0	0	3	5	0.1	



#### FIG 1: BOM OF THE EXAMPLE

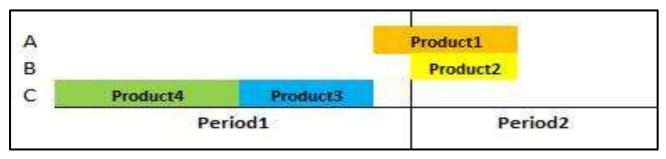
Figure 2 presents the different scenarios of the production plan. If we take the nomenclature presented in Figure1 and to meet the need for the customer, the product 1 must occur in Period 1 and product 2 in Period 2, therefore to respond to this request, he must produce 3 and product 4 after it manufactures product 1, but with this configuration, there will not be an optimization of the setup of the product 4. If we keep the setup of the product 4 in Period 1, we will have a delay of deliveries of product 1 and an average of product 4 in period1, these constraints affect the objective function of our classic model. The final simulation figure the manufacture of product 4 in the period 1 in two setup, it resolves the constraint of the delivery of product 1 and product 2 on time but remains a solution that is not optimal because it does not respond to the constraint of the stock and setup of the product 4.



#### FIG 2: DIFFERENT SCENARIOS OF PRODUCTION PLAN

Taking into consideration of the period of lead time, we will have the solution presented in the scenario below, which violates the requirements of the demand; product 1 will be delivered in the period 2 with a stock of product 4 in the period 1.

#### FIG 3: CONSTRAINT OF CLASSICAL MODEL.



Thus, according to the solution above the industrial plan of our classic model is not optimal, with the constraints that characterize the model, it is for this reason we propose on the next chapter a solution that we resolve the problem described previously.

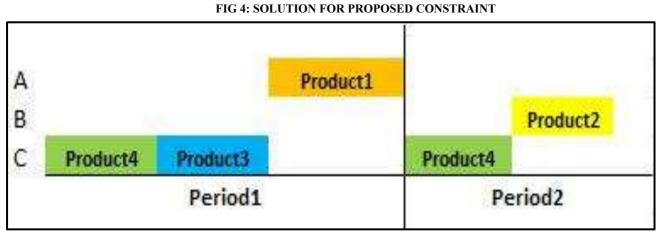
#### 3. Proposed Formulation

According to the previous chapter, the constraint which violates the objective function of the classic model is lot-sizing flow requirements capturing BOM. To resolve this problem we thought to have a stock of the predecessor product in place of the manufacture with consideration of a period of lead time.

We propose to replace the formulation of the constraint (2) of the classical model by the formulation (7) to define the flow of lot sizing with consideration of lead time.

$$I_{it+1} = I - X + a * I + D_{it} + D_{it} + it = it = i, t = i$$

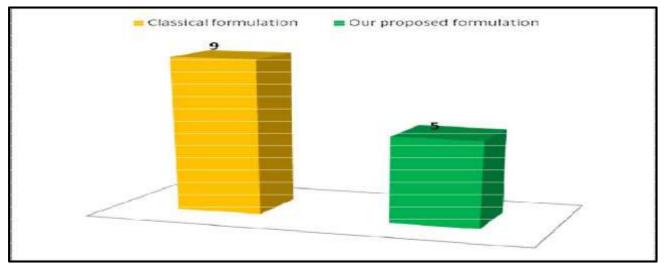
The solution of the model with the new proposed formulation, is presented in the figure below, as presented we can manufacture the successor product after the predecessor product, that means that product 3 will be made only after the manufacture of the product 4, and the same thing for the other products, the objective of the proposed constraint is to have a stock with the successor product with a period of lead time to gain setup time to optimize the stock. In the global manufacture of the need that's requested.



In order to simulate and compare the results of the model MLCLSP with the classic formulation and the proposed formulation, we have adopted the Software CPLEX.

Considering a period of lead time for the two models (classical and proposed) and doing a simulation under CPLEX, we will have an objective of 9 for the classical formulation and an objective of 5 with the proposed formulation. As presented in the Figure 5, it means that we have a gain of 56% of optimization of the costs of inventory and setup with the proposed formulation compared with the classical model.

FIG 5: COMPARISON OF THE OBJECTIVE OF THE SOLUTIONS WITH CPLEX



## 4. The Heuristic Method

In development heuristics, have been widely sought to solve the practical problems. For some production planning problems, it has been found that Lagrangian relaxation is an efficient method for obtaining approximate solutions. This technique is applied to problems of strategic planning J. M. Swaminathan (2000) or operational scheduling, planning (T.-R. M. Chen et T. Hsia.1997; C. A. Kaskavelis et M. C. Caramanis, 1998; D.-Y. Liao, S.-C. Chang, K.-W. Pei et C.-M. Chang, 1996).

The Lagrangian relaxation approach we propose consists of:

- Relax (via Lagrange multipliers) the constraints that make the problem more complex to solve and introduce the penalization cost obtained by relaxation in the objective function of the problem
- Solve the relaxed problem for each Lagrange multiplier in order to obtain a lower bound (LB).
- Find a suitable solution (feasible solution to the problem) to determine an upper bound (UB).
- Solve the dual problem using the sub gradient method.
- The effectiveness of the Lagrangian relaxation method is ensured by:
  - Lower bounds
  - The quality of the admissible solutions of the initial problem

The purpose of the Lagrangian relaxation approach is to decompose the main problem of planning into sub-problems, by relaxing capacity constraints through the use of Lagrange multipliers. The starting dates of each job are then determined by considering the capacity multipliers and the specification of the job process as well as the priority requirements.

The Lagrange relaxation technique has been the subject of diver's studies Fisher, M. (2004) and was raised in the problems of integration of production. This method can be used to approximate solutions, seek a lower bound on the problem or to obtain more optimal solutions. A metaheuristic was proposed in Toledo et al. (2014) to resolve an extension of CLSP with a carry over. Nascimento et al. (2010) proposed a strategy that incorporates a genetic algorithm with a linear program to find approximate solutions to a lot sizing level problem and scheduling.

The approach of the Lagrangian relaxation is to relax a subset of constraints while penalizing their violation in the objective function by associating a Lagrangian multiplier  $\lambda_{it}$ .

As shown in section 3, our new formulation of MLCLSP model allows us to achieve an optimal plan, but design the MLCLSP as a big bucket model and periods are supposed to cover long intervals with several production batches, so it would lead to a significant production capacity, and our equation (3) expresses that our plan must be computed with finite capacity. That is why the approach of the Lagrangian relaxation will be based on the penalization of capacity constraints. Berretta et al. (2005) presented a heuristic based on the Lagrangian relaxation of the capacity constraints of the mathematical formulation. To find a lower bound for the problem.

Our news formulation of the objective function is:

$$\mathbf{Min} \sum_{i=1}^{N} \sum_{t=1}^{T} (C_i \cdot Y_{it} + H_i \cdot I_{it}) + \sum_{i=1}^{N} \sum_{t=1}^{T} \lambda_{it} (C_{mt} - \sum_{i=1}^{N} (P_{mi} * X_{it} + S_{mi} * Y_{it})$$
(8)

Under the constraints:

$$I = I - X + a * I + D$$
  
it it i,t (9)

$$X_{it} \le G^* Y_{it} \qquad \qquad i,t \qquad (10)$$

 $I_{it} \ge 0, X_{it} \ge 0$  i,t (11)

$$Y_{it} \{0,1\}$$
 i,t (12)

Sambasivan et al. (2005) points out the following approaches in his study: the sub-gradient method and the multiplier  $\lambda_{it}$  adjustment method. This according to Fisher, M. (2004) proved to be too costly compared to the sub-gradient method. Although the adjustment method has a high potential, exceeding the sub-gradient method in some case studies, but the sub-gradient is the most used to determine the Lagrange multipliers tool.

So to solve this dual problem, the method chosen is the sub-gradient. The sub-gradient algorithm introduced in (8). That updates iteratively multipliers:

$$\lambda_{it} = \max \{0; \lambda_{it} + TG_i\} \qquad i,t \qquad (13)$$

T is the step of the iteration method, and  $G_i$  is the difference between the time required to produce all units of product i in period t and the capacity limit in period t, calculated in Equation (14):

$$G_{i} = C_{mt} - \sum_{i=1}^{N} (P_{mi} * X_{it} + S_{mi} * Y_{it})$$
 i,t (14)

It is necessary to initialize the values T and  $\lambda_{it}$  for each iteration. The step T is important to optimize our solution.

The choice of the step size T, is of importance for the convergence of the sub-gradient method, for this, the T update is given by the equation below:

$$T = \pi (UB-LB)/G_1^2$$
(15)

The algorithm of the principle of our approach is given below:

Data: Approximate dual solution

Result: Either a heuristic solution for the primal problem or infeasible solution

Repeat

Initialize arrays and variables used in the loop that follows

Initialize  $\pi \in [0; 2]$ 

Initialize  $\lambda_{it}$  values

Executes Lower Bound and Upper Bound model

Compute sub-gradient 
$$G_i = C_{mt} - \sum_{i=1}^{N} (P_{mi} X_{it} + S_{mi} Y_{it})$$

Compute  $T = \pi (UB-LB)/G_i^2$ 

Solve the model to get the Upper Bound

Update  $\lambda_{it}$  to pass it as input data to Lower Bound model in next iteration

End iteration

End loop

To ensure the convergence of the method, the solutions at each iteration step means that the T tends to 0. According to the equation (15), T depend the upper bound and lower bound if no lower bound is found to iterate, so the solution is infeasible. Almeder, C. (2010) most big bucket models provide the best lower bound.

#### 5. Computational Results

We evaluate that the proposed model would also be optimal or feasible for our approach Razki H and Moussa A. (2017). For this purpose, we considered the data in Table 2, the results are obtained by solver integer CPLEX 12.2 (User's Guide Standard Version Including CPLEX Directives, 2010). All tests were implemented in C ++ and run on a PC with 4G HP Core i5 processor.

Table 2. Size of the Instances					
Instance	Period	Product			
1	10	4			
2	10	16			
3	40	4			
4	40	16			

#### 5.2. Comparing the Solutions of the MLCLSP Classical and Proposed Model

We compare the solutions derived from the classic and proposed model with CPLEX. For this purpose, we considered the data in table 2. The small instances with 10 products produced on 3 machines and a planning horizon of 4 periods and 16 periods, for these instances, consider no setup times with one period of lead time. The large instances with 40 products produced on 3 machines and a planning horizon of 4 periods and 16 periods. All instances with a general and assembly product structure, with different demand profiles and different levels of capacity utilization.

Table 3 shows the results of these tests. We clearly observe that the solutions with MLCLSP for small instances are feasible 8.74% on average for classical model, and the solutions are optimal 77% on average for proposed model. For large instances, we obtain infeasible solution for the classical model 96.43% on average for the proposed model.

Table 3: Comparison of the solutions of the MLCLSP classical and proposed model with CPLEX						
	Classical model		Proposed model			
Instance (i*t)	Time (s)	GAP (%)	Time (s)	GAP (%)		
10*4	0.31	1.48	0.23	75		
10*16	3.4	16	0.16	79.17		
40*4	6.33	70	1.05	92.86		
40*16	infeasible	infeasible	7.3	100		

In Figure 6: We show the optimality of the solution for the proposed model compared with the classical model.

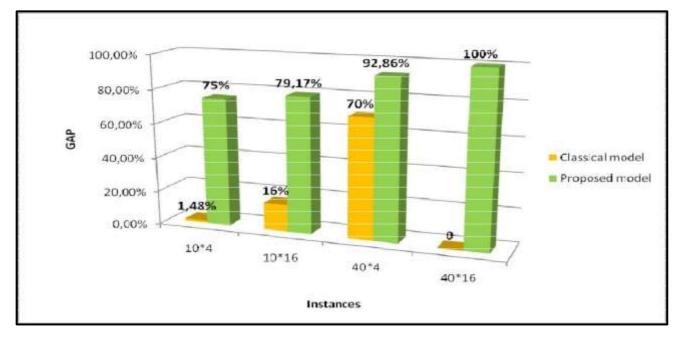


FIG 6: SOLUTIONS OF THE MLCLSP CLASSICAL AND PROPOSED MODEL

## 5.3. Lagrangian Relaxation Approach

Considering that the proposed model incorporates more details, so we evaluate if solutions derived from the proposed model would also be optimal or feasible with our approach. For this purpose we consider the same small and large instances describe in the previous paragraph. For most of these instances the solutions of MLCLSP are infeasible, for this we take the best solutions for these instances by Lagrangian relaxation approach.

Table 4 shows the results of these tests. We set the number of tested solutions for each of testing instances 1000 iterations. The column labeled GAP denotes the average improvement of the objective function of the best solution found compared to the optimal solution of the MLCLSP with the consideration of lead time.

Table 4: Solutions of the MLCLSP model proposed with Lagrangian relaxation approach				
	Lagrangian relaxation			
Instance (i*t)	Time (s)	GAP (%)		
10*4	0.11	60		
10*16	0.28	96.55		
40*4	0.34	95.24		
40*16	0.51	100		

In Figure 7 we show the improvement of the best solution for the proposed model with Lagrangian relaxation, the average of the GAP of small instances is 78% and 97.62% of large instances. We observe clearly that the solutions are made in initial iterations that are configured in solution time, the average of the time 0.20s of the small instances and 0.43s of the large instances.

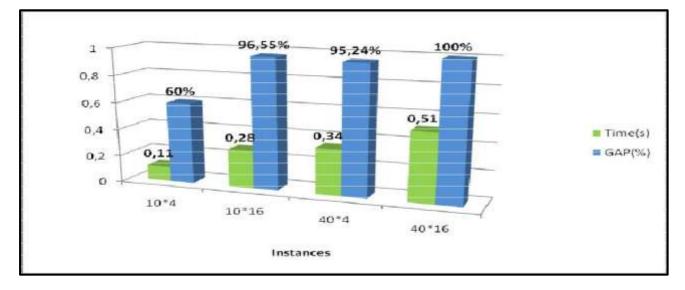


FIG. 7: THE IMPROVEMENT OF THE SOLUTIONS WITH LAGRANGIAN RELAXATION

## 6. Conclusion

This article proposed a new formulation for lot-sizing capacitated multi-level (MLCLSP). The mathematical model includes features, such as lead times that can be found in practical applications and not often considered in the lot-sizing literature. The proposed optimization method of our problem based on Lagrangian relaxation, attempts to obtain an optimal solution by the penalization capacity constraints. Computational experiments involving 40 products for large instances and with 10 products for small instances showed that the proposed method was able to find optimal solutions, 78% for small instances and 97.62% for large instances while reducing the costs of stock and setup. The proposed heuristic method was able to find an optimal solution in initial iterations that are configured in solution time, the average time of 0.20s for the small instances and 0.43s for the large instances.

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## **MOUSSA Ahmed :**

Was born in 1970 in Oujda, Morocco. He received the Licence in Electronics from the University of Oujda, Morocco, in 1994, and the Ph.D. in Automatic Control and Information Theory from the University of Kenitra, Morocco, in 2001. He worked two years as a post-graduate researcher at the University of Sciences and Technology of Lille, France. In 2003 he worked for two years as a researcher, at the Sanofi-Aventis research laboratory in Montpellier France, in the field of data microarray analysis. From 2006 to 2010 he was an assistant professor in the National School of Applied Sciences in the Abdelmalek Essaadi University of Tangier Morocco, and from 2010 to 2016 he worked as an associated professor in the same school. Since 2017 he is full professor in Data Analysis and the Director of the national School of Applied Sciences of tangier.