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Solving Problems with The Percentage Bar

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Abstract

At the end of primary school all children more of less know what a percentage is, but yet they often struggle with percentage problems. This article describes a study in which students of 13 and 14 years old were given a written test with percentage problems and a week later were interviewed about the way they solved some of these problems. In a teaching experiment the students were then taught the use of the percentage bar. Although the teaching experiment was very short - just one lesson - the results confirm that the percentage bar is a powerful model that deserves a central place in the teaching of percentages.

Keywords: percentage, model, design research

Abstrak

Pada saat selesai dari sekolah dasar, semua anak kurang lebih tahu apa itu persentase, tetapi mereka masih sering kesulitan dengan masalah-masalah persentase. Artikel ini menggambarkan sebuah studi dimana siswa yang berumur 13 dan 14 tahun diberikan sebuah test tertulis tentang masalah-masalah persentase dan satu minggu kemudian diwawancarai tentang cara mereka menyelesaikan beberapa masalah tersebut. Pada percobaan pengajaran, siswa kemudian diajarkan tentang penggunaan bar persentase. Meskipun percobaan pengajaran tersebut sangat singkat – hanya satu pelajaran – hasilnya menegaskan bahwa bar persentase adalah model yang sangat kuat yang patut dijadikan pusat dalam pengajaran persentase.

Keywords: persentase, model, penelitian desain

Introduction

With a group of Indonesian master students, studying in Utrecht as participants in the Impome project[1], we carried out a study in grade 7 on students' understanding of percentages. To our surprise only four out of the 14 students we tested were able to give a correct answer on the following problem: 'On a bike that normally costs €600 you get a discount of 15%. What do you have to pay?' (See figure 1). The calculations on the scrap paper we supplied in the written test suggest that 8 of the 14 students divided 600 by 15, the original price divided by the discount percentage! The bike problem seems to be a standard percentage problem, so how can it be that children in grade 7 still struggle with such a simple problem? And of course we should also ask what a teacher could do to help them?

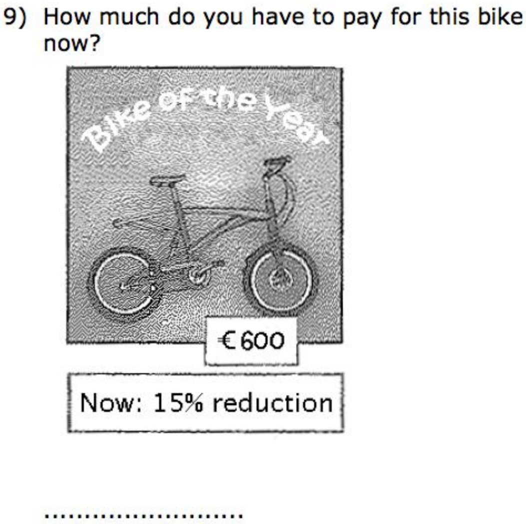

Figure 1. The bike problem.

This article focuses on the last question: How can the teacher support students' understanding of percentages? We think that the percentage bar model can play an important role in building children's understanding of percentages. In our study we saw that 7th grade students who obviously were not familiar with the percentage bar, immediately understood how the bar could be used. It was also clear that drawing a bar was advantageous for them.

In this article we first discuss why it helps if students draw a percentage bar when they solve percentage problems. Then we describe how the students originally tried to solve problems 'in their heads' and the kind of mistakes they made. Finally we discuss a few examples that show how quick students profited from working with the percentage bar.

How Can The Percentage Bar Support Student's Understanding?

When students try to solve percentage problems, drawing a percentage bar has several advantages (van den Heuvel, 2003; van Galen et al., 2008; Rianasari et al., 2012). In the first place it means that students make a representation for themselves of the relations between what is given and what is asked. The bike problem boils down to the question: 'How much is 15% of €600?', which can be represented in a drawing like the first bar of figure 2. Other questions, however, are also possible, like: '€90 of

€600, what percentage is that?' (the second bar), or: '€90 is 15% of the total price; how much is the total price?' (the third bar).

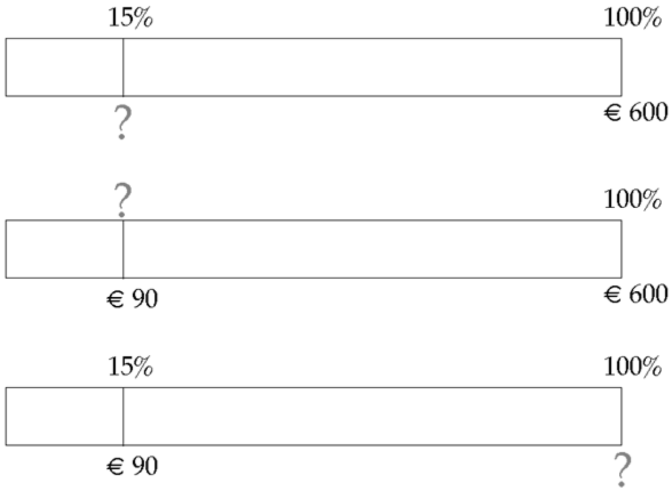

Figure 2. Three different percentage questions.

A second advantage is that the percentage bar offers scrap paper for the intermediate steps in the calculation process. The student can compute, for example, that 50% of €600 equals €300 and that 10% equals €60 (see figure 3). The first step will not lead directly to an answer, but via 10% and 5% the student may find that the answer is €60 + €30, which add up to €90. Thus, the students can keep track of their thinking processes, can see what they are doing, decide what to do next after every step, and make corrections if necessary.

Figure 3. Solving a percentage problem with intermediate steps.

The third advantage is that the percentage bar offers a natural entry to calculating via 1%. When students have learned to solve percentage problems in a flexible way using the percentage bar, it is only a small step to the procedure of working via 1%: first calculate 1% and from there calculate the percentage that is asked for (see figure 4). This is a procedure that is generally applicable and quite often it is also an efficient procedure. Students may discover this procedure by themselves if they are asked to compute, for example, 26%, or 51% of something.

Figure 4. Solving a percentage problem by first computing 1 percent.

The percentage bar is not the only model for working with percentages. Several Dutch textbook series for mathematics promote the use of a percentage table like the one that is shown in figure 5. Another alternative is a double number line, also shown in figure 6. These two models offer to some extend the same advantages as the bar. We prefer the bar because it gives a clear and concrete picture of the relations between the total and its parts.

100%	50%	10%	5%	15%
€600	€300	€60	€30	€90

Figure 5. The percentage table as a tool to solve percentage problems.

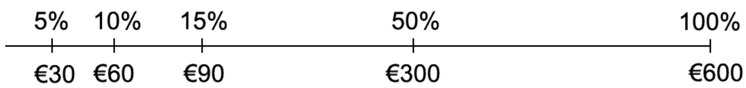

Figure 6. The double number line as a tool to solve percentage problems.

Data from The Written Test and Interviews

The study was conducted by 10 Indonesian master students in mathematics education as part of a course. The study was carried out at an international school where English was spoken. The grade 7 students of this school were given a written test of 15 percentage problems - translated from Janssen et al. 2005 - and were interviewed a few days later about the way they approached such problems. Then two lessons were given to small groups of students.

In the instructions for the written test the students were asked explicitly to use certain areas of their test sheets as scrap paper. Almost all students wrote calculations in these areas. None of them, however, used the scrap paper to draw a picture or model to clarify the relations between the given numbers. Obviously arranging the relations between the given data was done 'in their heads', and often that went wrong.

It was obvious that most of the students in our study did not know a systematic procedure for working with percentages. Only the student with the highest score solved all problems the same way: she translated the percentage into a decimal number and multiplied (for example 0,15 x 600). Other students tried to multiply with fractions - 15/100 x 600 - but they were not consistent and they made calculation errors.

Trying to make proportions equal

Analyzing the written test and interviews, we found that several students used an approach in which they tried to find a proportion that would be equal to the given percentage. Given the bike problem, for example, the student knows that 15% is the same as '15 out of a 100', or 15/100 and then tries to find the same proportion in 'so much out of 600', or 'so many 600ths'. In principle this approach is correct; one may notice for example that 6x100=600, calculate 6x15= and conclude that 15 out of 100 is equal to 90 out of 600. Often, however, the unfriendly proportion of ‘15 out of 100’ confused the students. They thought they had to simplify this proportion and tried, for example, to divide 100 by 15. This led to extensive calculations, but it did not lead to the solution.

Guided by the numbers in the problem

Probably the fact that in the bike problem so many students divided 600 by 15 - the total amount of money divided by the sale percentage - can be explained by the expectation of students that within a test calculations will be kept simple. They see that 100 cannot be divided by 15 easily, but 600 : 15 is doable. We may conclude that we give students too often problems with friendly numbers and this prevents them from trying to understand what is being asked. Problems with 25%, 75% and 20% can easily be solved by finding an equal proportion, as these percentages can be transformed into simple fractions. That is not possible, however, with percentages like 15% and 35%.

A Teaching Experiment

In our opinion a good approach when students start learning about percentages is calculating via 10%. It contributes to students' understanding of percentages, but is only possible if the numbers are easy. Later on when working more complicated numbers a systematic approach is calculating via 1%. This approach is always applicable. Moreover, it does not only work for 'how much is 15% of 600?', but - as can be tried out with the percentage bars in figure 2 - it also works for '90 out of 600, what percentage is that?' and for: '15% of the total equals 90; how much is the total?' This procedure of calculating via 1%, however, should not become a trick, but as long as the students keep the percentage bar in mind, that will not happen.

In the teaching experiment the percentage bar was introduced and the students were given some problems in which they could practice working with the percentage bar. Although the students in this small study were only given one lesson in applying the percentage bar, this lesson showed how effective it can be to draw a bar. The students immediately understood its purpose and most of them could solve problems that they probably could not have done otherwise, as shown by their results on the written test. An example is the problem of figure 7. This student first calculated 50% and 25% of €300, but that did not lead to a discount of €60. As a next step she started with 10% and found €60 by doubling.

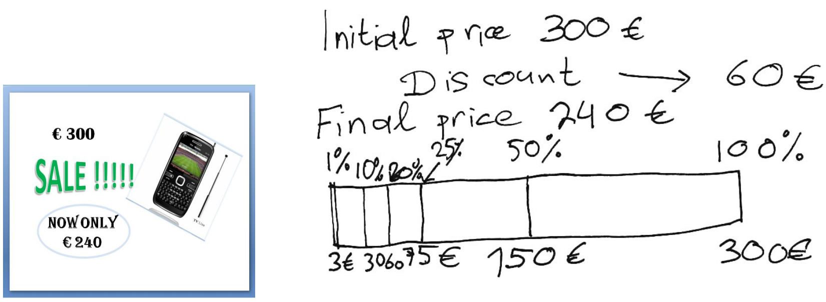

Figure 7. A student finding the discount percentage for an item of €300, sold for €240

Another student was given the problem of figure 8 about downloading a computer file: 11% of 600 Mb has been downloaded already, how many Mb is that? We can see in figure 8 that the student used the percentage bar to try out a few percentages: 50% equals 300, 25% equals 150 and 10% equals 60. Eventually she computed 11% as 10% + 1%. The master student who taught this lesson then went on and asked what 9% of 600 would be. Interestingly the student computed this as 10% - 1% and not as 9 x 6, like the master student expected.

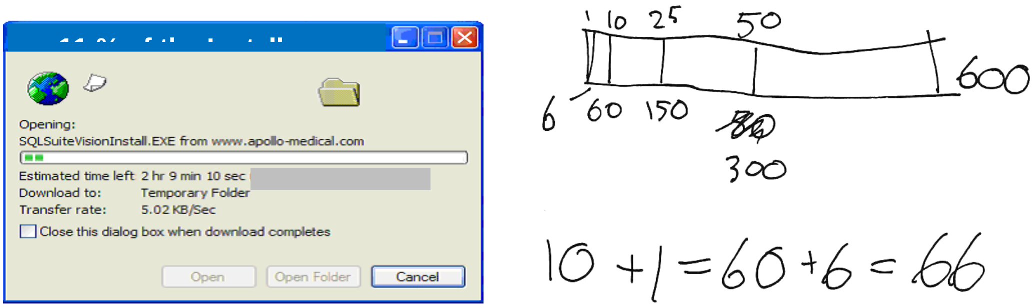

Figure 8. A student finding 11% of 600 Mb

These examples show that students quickly understood how they could use the percentage bar as a kind of scrap paper for flexibly finding intermediate calculation steps. For the next step in the learning process - developing the procedure of working via 1% - more time is needed.

From Flexible Mental Calculations Towards A More Systematic Procedure

Solving percentage problems via 1% seems to be an efficient procedure, so why do we not teach this as a standard procedure from the start? Do we need to first give students the opportunity to solve percentage problems via steps like 50%, 25% and 10%? Our answer to this is that if procedures are insufficiently anchored in understanding, they will quickly become vulnerable tricks.

So we would like to argue for encouraging students to use the percentage bar and for giving them the opportunity to choose intermediate calculation steps, with easy percentages like 50%, 25% and 10%. The percentage bar offers support, because it helps students to oversee the relations between the given numbers. Moreover, the bar relates percentages in a direct way to fractions. In our opinion teachers should promote drawing the percentage bar for quite a long period, because the students need time to learn to appreciate the bar as a mathematical tool that can be applied in all situations. Eventually the percentage bar should come to function as a model for thinking, but that will only happen if the students are thoroughly familiar with it.

We should prevent, at least, students to decide too quickly that they can, or should solve percentage problems in their heads. In that respect the problems in text books and tests with their 'friendly' numbers send out the wrong signal. These problems can indeed be solved without scrap paper, but only if one understands what should be done. For learning to solve all kind of problems, understanding is needed and the percentage bar helps to build that understanding.

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[1] Impome project, reference to http://p4mri.net/new/