TEACHERS EXPECTATION OF STUDENTS' THINKING PROCESSES IN WRITTEN WORKS: A SURVEY OF TEACHERS' READINESS IN MAKING THINKING VISIBLE

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Abstract

The trends of teaching mathematical thinking and the existence of two thinking skills (critical dan creative thinking) the required by 21st-century skills have created needs for teachers to know their students' thinking processes. This study is intended to portray how mathematics teachers expect their students showing their thinking processes in students' written work. The authors surveyed Whatsapp and Telegram group of mathematics teachers. First, the authors shared the result of the literature review and the governmental regulations about the need to develop thinking skills. Second, the authors stated that the potentials of students' written works as a tool for knowing students' thinking processes. Third, the authors sent a simple mathematical problem with the topic of algebra and asked the mathematics teachers how should their students answer that problem such that they can easily monitor and assess their students' thinking processes. A total of 25 teachers participated voluntarily in this survey. Results of the survey were triangulated with direct trial data in lecture classes at both undergraduate and postgraduate levels. The result indicates that participating mathematics teachers do not expect too much for their students to show their thinking processes in written work. Teacher's focus is mostly on the accuracy and the correctness of their students' mathematics answer.

Keywords: mathematics teachers' expectation, making thinking visible, thinking processes, written works

Abstrak

Kecenderungan tentang pembelajaran berpikir matematis, dan keberadaan dua keterampilan berpikir (kritis dan kreatif) sebagai bagian dari keterampilan abad ke-21, telah menjadikan guru perlu mengetahui proses berpikir para siswanya. Sebagai salah satu alat yang dapat digunakan untuk melihat proses berpikir siswa, yaitu pekerjaan tertulis siswa, penelitian ini dimaksudkan untuk memotret bagaimana guru matematika mengharapkan para siswanya menunjukkan proses berpikirnya pada tugas tertulis. Peneliti mengadakan survey terhadap kelompok guru yang membentuk grup guru dalam Whatsapp dan Telegram. Pertama, peneliti membagikan hasil kajian pustaka dan peraturan menteri tentang perlunya mengembangkan keterampilan berpikir. Kedua, peneliti menyampaikan perlunya memanfaatkan tugas tertulis siswa sebagai wahana untuk mengetahui proses berpikir siswa. Ketiga, peneliti mengirimkan satu soal matematika sederhana dengan topik aljabar, dan meminta guru menuliskan jawaban yang seharusnya ditulis oleh siswa sehingga para guru dengan mudah mengetahui proses berpikir siswanya. Sebanyak 25 orang guru yang bergabung pada kedua grup aplikasi berpartisipasi dalam penelitian ini tanpa paksaan. Hasil dari survey dilakukan triangulasi dengan data ujicoba langsung di dalam kelas perkuliahan baik di tingkat sarjana ataupun pascasarjana. Hasil menunjukkan bahwa guru-guru matematika tidak berkeinginan untuk mengetahui proses berpikir siswanya dari pekerjaan tertulisnya. Guru matematika lebih terfokus kepada ketepatan dan kebenaran konten matematika pada jawaban tertulis siswanya.

Kata kunci: harapan guru matematika, menjadikan berpikir terlihat, proses berpikir, pekerjaan tertulis

Mathematical thinking now becomes the focus of mathematics instruction (Hollebrands & Dick, 2009; Stacey, 2006). Mathematical thinking is useful for students and also for teachers (Stacey, 2006). For students, mathematical thinking is a way of learning mathematics. Through specializing,

generalizing, conjecturing, and convincing, which are the fundamental processes of mathematical thinking, students can learn mathematics effectively. They can understand the mathematical concepts, principles, procedures deeply, and even better solve mathematical problems. For the teachers, Stacey (2006) claimed that mathematical thinking could help mathematics teachers provide suitable lessons for their students. The Royal Society (2014) stated that the ability to think mathematically is useful for decision making either in the classroom context or in real life situations.

Considering the importance of teaching mathematical thinking, The National Research Center (Burns et al., 2006) has created a publication on a comprehensive approach for teaching thinking skills. Even a book illustrating the use of tasks and questions to strengthen the thinking practices and processes is just currently published (Small, 2017). Chinese education, even, evolutionarily has set mathematical thinking as the focus of mathematics instruction (Li, Mok, & Cao, 2019). In addition, several other countries, namely Japan, Singapore, and Australia, also focus on mathematical thinking in every implementation of mathematics learning in class (Katagiri, 2004). So, teaching mathematical thinking now become international trends in mathematics teaching. Consequently, mathematics teaching in Indonesia should also change the focus into teaching mathematical thinking rather than mathematical content acquisition.

Furthermore, in this 21st century, two types of thinking skills (critical thinking and creative thinking skills) are considered as 21st-century skills. Several scholars have set the 4Cs (critical thinking, creative thinking, collaboration, and communication) as the essential skills needed a human being to live in the 21st century (Kay, 2009; National Education Association, 2014; Pacific Policy Research Center, 2010; Vockley & Lang, 2008). In the year 2016, the government of Indonesia through the Ministry of Education and Culture has set these thinking skills as the goals of educational practices in primary and secondary school level (Menteri Pendidikan dan Kebudayaan Republik Indonesia, 2016). Teachers should implement teaching and learning activities, which could lead their students to have critical and creative thinking skills.

Therefore, Indonesian mathematics teachers should also shift their teaching focus from content acquisition into developing thinking skills. This call is truly appropriate since facts indicated that Indonesian students have the lowest score on Programme for International Student Assessment (Argina, Mitra, Ijabah, & Setiawan, 2017) who measured students' reasoning and thinking skills (OECD, 2017; Prenzel, Blum, & Klieme, 2015; Stacey, 2011). Even prospective mathematics teachers do not have excellent thinking skills. They are not critical thinkers yet (As'ari, Mahmudi, & Nuerlaelah, 2017).

To make the teaching thinking skills is carried out optimally, teachers need to continuously know the level and process of their students' thinking skills. They need to know how their students use their thinking skills in understanding the mathematical concepts, principles, procedures, and in solving the problems. Without understanding students' thinking, teachers will not be able to provide

suitable teaching treatments. Therefore, teachers should encourage their students to show their thinking process on every occasion. Teachers need to make their students' thinking is visible.

Many ways can be used by teachers to make students' thinking visible. Teachers may ask their students to think aloud, write reflective journals, involve in focus group discussion, and produce written answers of mathematical tasks. Compare to the other types of tasks; the students' written works is auspicious. The students' written work has some advantages. First, it can be done regularly every day. Second, it can be integrated with the teaching of mathematical content. Third, it is very easy to record and analyze the students' works. Fourth, teachers and students are familiar with this type of assignment.

From the review of literature, many scholars have tried to study students work that mostly related to feedback provision (Gjerde, Padgett, & Skinner, 2017; Fu & Nassaji, 2016; Cohen-Sayag, 2016; Chaqmaqchee, 2015; Fonseca et al., 2015; Osterbur, Hammer, & Hammer, 2016; Costello & Crane, 2013). None of these existing studies are talking about teachers expectation of their students' written work. The only research result available related to teachers' expectation says that having several times involvement in workshops about working on challenging tasks, teachers finally expect students to persist when working on challenging task (Clark, Roche, Cheeseman, & Sulivan, 2014). There is no information available about the type of mathematical thinking processes that should be incorporated into students' work.

The existence of teachers' examples of how to develop written works where thinking processes are shown in it is very important. Examples given by mathematics teachers represent their expectation on students' mathematical thinking processes. From the examples, teachers could guide the students what and how their mathematical thinking processes should be described in their written works. Examples are references for students. If the students follow the examples, teachers will have a chance to understand better their students' mathematical thinking skills. Zhu, Yu, & Cai (2018) stated that understanding students' thinking is helpful for their teaching practice. On the other hands, Neumann (2014) explains that teachers should listen, probe, interpret and responds to students' thinking.

This article is a report of the authors' investigation about teachers expectation in incorporating their students' mathematical thinking through written works. From the study, it can be seen whether the expected thinking processes by the teachers are useful or not. Therefore, the result of this study is significant for the development of teaching thinking. This study provides a portray of the current mathematics education practices which can be used to provide a recommendation of better practices in the future. Mathematics teacher educators can use the result of this study as insights on how to create the best tasks for their pre-service mathematics teacher students to enable the students to have many ideas on how to monitor and evaluate students' thinking processes. In-service mathematics teacher trainers may also use the result of this study to train and up-grade existing mathematics teachers' competencies on how to encourage their students to produce better students works that express their thinking skills.

METHOD

General Background of Research

This research was a qualitative descriptive study, as it was aimed at knowing mathematics teachers expectation in incorporating their students' mathematical thinking through written works.. Stages of this research referred to the stage of a qualitative content analysis method. Written works referred to in this reserach were students' written answers expected by mathematics teachers to determine their mathematical thinking processes.

Participants

Participants in this study were 25 mathematics teachers who joined both WhatsApp and Telegram groups. The conditions for determining the research participants were (1) mathematics teacher who have teaching experience between 10-15 years, (2) mathematics teachers who respond to problems given by writing down the written works, and (3) the mathematics teacher who are willing to be the research participants.

Instruments

To best way to know teachers' expectation about the mathematical thinking processes that their students should incorporate into their written works is through asking the teachers to provide examples of how students should express their thinking processes in their written works. The examples have the potential as the reference for the students to write their responses. The examples could function as the standards criteria for the students to develop their written responses. Therefore, in this study, the authors send a simple mathematics problem to the participating mathematics teachers and they are then expected to pretend as students, They have to write the solution to the problem such that all of the thinking processes required to solve the problem are made visible in it. The simple mathematical problem sent to the group was displayed in Figure 1.

Original Problem	Translation
Harga dua topi dan dua kaos adalah Rp100.000. Harga 3 topi dan 1 kaos adalah Rp80.000. Berapa harga 1 topi?	The price of two hats and two shirts is Rp100.000. The price of three hats and one shirt is Rp80.000. What is the price of one hat?

Figure 1. The Simple Mathematical Problem

This simple problem was used in this study for several reasons. First, the authors wanted the participating teachers focused mostly on their thinking processes. The authors did not want these teachers to reject to response caused by the difficulty of the problem. Second, there are several thinking processes required to solve this problem; (1) the mathematics teachers should use apply horizontal mathematizing to develop mathematical representation suitable to the problem, (2) the mathematics teachers should also employ mathematical things to simplify the mathematical expression, and (3) the mathematics teachers should use reasoning skills to interpret the result correctly and evaluate the appropriateness of the answer.

Data Collection Methods

In a digital era now, Whatsapp and Telegram are two among many mobile phone application that facilitates teachers to communicate and share ideas, research finding, and other things. These applications have been utilized by most teachers, including mathematics teachers, to communicate with each other. Even, 2985 mathematics teachers across the nation have organized themselves as a group in Telegram application where they routinely discuss and share many aspects of mathematics teaching (see Figure 2). Groups of a smaller number of mathematics teachers may use Whatsapp application to communicate and share their ideas. Therefore, the authors decided to use these two mobile phone applications as the way for data collection.
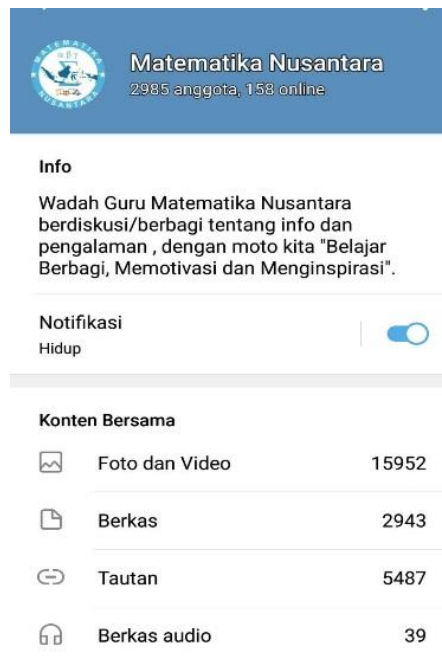
Figure 2. The Screenshoot of Telegram Group

The teachers were asked to write the best-expected of their students' written work so that they can see their students' mathematical thinking processes. The authors assumed that teachers' answer expressed in the written answer is the one that they are expecting from their students.

Data Analysis

Answers from participated respondents were then analyzed qualitatively using content analysis. The stages were (1) providing math problems in both groups namely Whatsapp and Telegram, (2) asking to mathematics teachers who were in the group with 10-15 years of teaching experience to respond to the problems given by writing down the answers and sending photos of the answers in the group, (3) leveling research participants in 3 levels of mathematics knowledge based on teaching experience, namely advanced level, good level, and medium or lover level, (4) analyzing written works that given a mathematics teacher as a research participant, (5) triangulating the data with the answers of prospective mathematics teacher at the undergraduate or postgraduate levels, and (6)

summarizing the expectations of the mathematics teacher to their students by showing their thought processes on written works.

RESULTS AND DISCUSSIONS

A number of 25 mathematics teachers responded to the survey who join both the WhatsApp and Telegram groups. By analyzing their responses, it can be seen that all are related to its mathematical content. The focus of their answer is showing the procedure that should students used to solve the problem. Based on the data analisys, there are no indications that they are expecting their students to show and communicate their thinking processes. Students' thinking processes seem to be implicit and invisible. Students' thinking processes were assumed to be automatic, self-contained, and there is no expectation from the teacher side that their students need to explain the reasons for every step of their answer. It seems that the teachers do not want to know and understand the thinking processes happening in their students' mind when the students are working to solve the problem. Teachers did not take care of students' thinking processes in solving the problem. Teachers seem to assume that students' mathematical thinking is good if they can provide the correct answer to the problems.

To make it clear, following are 25 answers provided by 25 mathematics teachers, consists of 5 teachers with advanced level mathematical knowledge, 15 teachers with good mathematical knowledge, and 5 teachers with medium or lower mathematical knowledge. The only reason for showing teachers' answers by their mathematical competencies is that although they have different mathematical competencies, they have the same expectation. They expect that the correct answer to the mathematical aspect is the most important in their students' written work.

One of the participants for the advanced level mathematics teachers is Participant #1 replied authors' request explained in Figure 3.

Harga dua topi dan dua kaos adalah 100.000 rupiah.
Harga tiga topi dan satu kaos adalah 80.000 rupiah.
Berapa harga datu topi?

Alternatif jawaban:
Misal,
x = harga satu topi
y = harga satu kaos

Dari soal diperoleh
$2x + 2y = 100.000 \Rightarrow x + y = 50.000$ .... (1)
$3x + y = 80.000$ .... (2)

Eliminasi y pada persamaan (1) dan (2)
$x + y = 50.000$
$3x + y = 80.000$ _
$-2x = -30.000 \Rightarrow x = 15.000$

Jadi, harga satu topi adalah 15.000

Translation:
The price of two hats and two t-shirts are IDR100.000. The price of three hats and one t-shirt is IDR80.000. What is the price of one hat?
Alternative solution
Let
x = the price of one hat
y = the price of one t − shirt
From the problem, therefore
$2x + 2y = 100.000 \rightarrow x + y$
$= 50.000 ......$ (1)
$3x + y = 80.000 ....$ (2)
Eliminating y in equation (1) dan (2)
$x + y = 50.000$
$3x + y = 80.000$
$-2x = -30.000 \rightarrow x$
$= 15.000$
So, the price of one hat is IDR15.000

Figure 3. One of Five Advanced Mathematics Teacher's Expectation

He is an outstanding and very famous mathematics teacher in the group, even in Indonesia. He is a very experienced mathematics teacher. He has also written books for national exam preparation, and he also manages his blog for mathematics education. He is known as a very strategic mathematics tutor in helping students and other teachers to solve mathematics exams smartly, efficiently, and effectively.

From his written response (Figure 3), it can be seen that he is very good at mathematics. He knows that x and y are symbols of variable, and therefore, he let x and y correctly as the price of each thing, not the price of things. However, he did not mention the origin of the equation $2x + 2y = 100.000$. He just mentioned "*from the given information*" as its premis for writing the equation. He did not expect his students to write a *bridge* statement, namely: "Since 100.000 is the price of two hats and two shirts, then $2x + 2y = 100$" which will make him better understand his students' thinking processes. It seems that writing this bridge statement, which telling teachers about their students thinking process, is not important for him.

In another chance, he writes a procedure to solve the problem, namely: elimination procedure. He did not expect their students to show the reasons why he/she choose that procedure. There are no indicators that he wants their students to express their thinking process during comparing and contrasting all the procedures that can be used to solve the problem, process of deciding the most appropriate procedure to solve the problem. He did not pay attention to students thinking processes. He cares more on the correctness of its mathematical answer. The mathematical thinking process is allowed to be implicit and invisible.

One of the participants for the good level mathematics teachers is Participant #2 replied authors' request described in Figure 4.
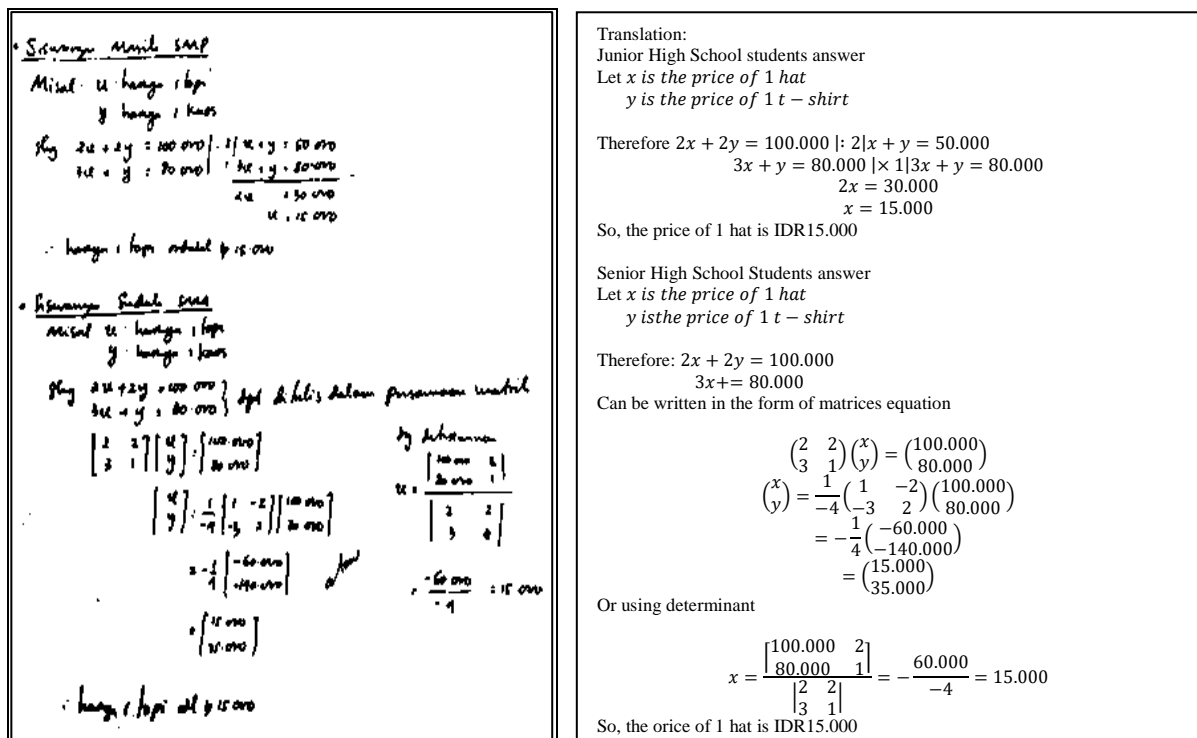

Figure 4. One of Fifteen Good Mathematics Teacher's Expectation

He is also an excellent mathematics teacher. However, he is not as productive as participant 1. He just regular good mathematics teacher who teaches mathematics at a big city in East Java. He is good in mathematics. His mathematical concept about the variable is good. His ability to let x as the price of 1 hat (not the price of hats or the hat itself) shows that he understand the concept variable correctly. He shows that he is also mastery in employing mathematical procedures to solve the problem. Even, he shows that he has good pedagocical content knowledge for the topic. He showed two different approaches that can be used by students from different level to solve the problems.

Unfortunately, his response also indicated that he did not pay attention to the mathematical thinking processes that should be expressed by his students. He also focused on the mathematical aspects of the solution. He did not show the origin of coefficient 2 from expression $2x + 2y = 100.000$. He just writes it down with no explanation.

When he used $\begin{cases} 2x + 2y = 100.000 \mid \div 1 \mid x + y = 50.000 \\ 3x + y = 80.000 \mid \div 2 \mid 3x + y = 80.000 \end{cases}$, which resulting in $2x = 30.000$, there is no information at all about the thinking processes involved in it. The reader will never know the reasons why he chose to use division rather than multiplication in the problem-solving process. The thinking process is invisible and implicit. The students' work did not tell automatically all of their mathematical thinking processes. Students' written works do not function well in inspiring the teachers about the thinking processes done by students. Teachers need additional efforts to know and understand students mathematical thinking processes.

The tendency of the response given by participant 2 has the same tendency as the Graduate Mathematics Education student namely her mathematical concept about the variable is good (see Figure 5). Her ability to let x as the price of 1 hat (not the price of hats or the hat itself) and let y as the price of 1 t-shirt shows that she understand the concept variable correctly. Even, she shows that she has good pedagocical content knowledge for the topic especially the process of elimination. But, her response also indicated that she did not pay attention to the mathematical thinking processes that should be expressed by her students. According to the authors, students only focus on the truth of the solution of the problem given.

Figure 5. Response of Graduate Student

Another the women participant for the good level mathematics teachers is Participant #3 replied authors' request as seen in Figure 6.

Misal t = harga topi	Translation
k = harga kaos	Let t = the price of hats
Berarti dari soal diketahui:	K = the price of t-shirt
(i) 2t + 2k = 100.000 (: 2)	Therefore, from the information given
t + k = 50.000	(1) *2t + 2k = 100.000*
(ii) 3t + 1k = 80.000	*t + k = 50.000*
2t + t + k = 80.000	(2) *3t + k = 80.000*
2t + 50.000 = 80.000	*2t + t + k = 80.000*
2t = 30.000	*2t + 50.000 = 80.000*
t = 15.000	*2t = 30.000*
	t = 15.000

Figure 6. One of Fifteen Good Mathematics Teacher's Expectation

She is a junior mathematics teacher, and she has not many experiences in mathematics teaching. However, right now, she is taking her first semester of master degree in mathematics education. Therefore, in this study, the authors classify her as a good mathematics teacher too.

From her written response, it seems that her mathematical concepts could be incorrect. Instead of writing $t = the\ price\ of\ ONE\ hat$, she wrote $t = the\ price\ of\ hats$. She did not differentiate between "harga 1 topi" which can be translated into *the price of one hat* and "harga topi" which can be translated in,to the *prince of hats*. It seems that she did not realize that what she did may lead to different and wrong mathematical representation. She also relied on *what is known* or *what is given* as to come up with an equation $2t + 2k = 100.000$. There is no information about the thinking processes used to decide that its mathematical representation is $2t + 2k = 100.000$.

Having let $t = the\ price\ of\ hats$, she wrote an expression $2t + 2k = 100.000$ and followed by a sign "($\div 2$)". There was no information on what the meaning of this sign and why she used it. For those who are having difficulty on solving this kind of problem, they will never know why they have to divide the each side of the equation by 2. These students will be having difficulty in answering the question, "Is dividing by 2 applicable to every problem?"

She also did not communicate the origin of $2t + t + k = 80.000$ in the (ii) part of the response. The thinking process that "since $3t = 2t + t,$ and $3t + k = 80.000$, then $2t + t + k = 80.000$ was implicit. No explicit explanation was provided. According to the authors, the good mathematics teacher also did not pay attention to the thinking processes.

One of the participant for the medium or lower level mathematics teachers is Participant #4 replied authors' request as seen in Figure 7.

Translation
Alternative
Let
$$topi = t$$
$$t - shirt = k$$

$$2t + 2k = 100.000 \,|\times 1|2t + 2k = 100.000$$
$$3t + k = 80.000 \,|\times 2|6t + 2k = 160.000$$
$$4t = -60.000$$
$$t = -\frac{60.000}{-4}$$
$$t = 15.000$$

The price of 1 hat is IDR15.000

Figure 7. One of Five Medium or Lower Mathematics Teacher's Expectation

Figure 7 shows that he is not good at mathematics. He did not show us that he has a sense of understanding about the concept of variable in mathematics. He did not realize that the equal sign in his answer was wrongly used in this expression. He did not realize that his mathematical expression is meaningless, that is: the left side (the x) represents a quantity, and the right side represents tthe hing that not a quantity, and those two things are related using the equation sign. However, he showed his capability to execute analytical procedures fluently. He used the procedure clearly and came up to the correct solution.

The tendency of the response given by participant 4 has the same tendency as the Postgraduate Mathematics Education student who is not able to distinguish the given variables, namely for the variable x = price of 1 hat with the variable x = hat (see Figure 8). It seems that he did not realize that what dhe did might lead to different and wrong mathematical representation. He also relied on what is known or what is given as to come up with an equation $2x + 2y = 100,000$. There is no information about the thinking processes used to decide that its mathematical representation is $2x + 2y = 100,000$.

Jawaban :

❖ **Diketahui:** harga 2 buah topi dan 2 kaos adalah Rp.100.000,00
        harga 3 buah topi dan 1 kaos adalah Rp.80.000,00
❖ **Ditanya** : Berapa harga 1 buah topi ?

❖ **Penyelesaian :**
Misalkan
Topi   = x
Kaos   = y
Sehingga diperoleh :
2x + 2y = 100.000,00
3x + y = 80.000,00
Dengan melakukan proses eliminasi diperoleh nilai x = 15.000,00
Karena yang ditanya hanya topi dimana topi merupakan pemisalan dari variable x sehingga harga 1 buah topi adalah Rp.15.000,00

Translation

Given: Price of 2 hats and 2 t-shirts are IDR. 100.000
        Price of 3 hats and 1 t-shirt are IDR. 80.000
Ask: What is the price of 1 hat?
Solution:
Let: Hat = x
    T-shirt = y
So: $2x + 2y = 100.000,00$
            $3x + y = 80.000,00$
With the elimination process, we get the value of $x = 15.000,00$
Because what is asked is only the hat where the hat is a variable x so the price of one hat is IDR. 15.000.00

Figure 8. Response of Postgraduate Student

Furthermore, the student writes "with a process of elimination" without writing down the process of elimination like what so as to get the value of variable x. At the end of the completion process, the student also wrote "Because what is asked is only the hat where the hat is an example of the variable x so the price of 1 hat" and indicates that the student has not been able to distinguish the "price of hat" variable from the "hat". According to the authors, students do not focus on the thought process and only focus on the truth of the solution of the problem given.

So, from the above of mathematics teachers responds, teachers' expectation on students thinking processes in students written work is inexistent. Teachers still assume that thinking skills do not need to be made explicit. Thinking skills are still implicit and informally developed. The researchers then followed up this activity by asking their reasons for focusing on its mathematical content only. Two types of responses were identified.

First, teachers feel that the development of 4Cs is not essential. They refer to the type of national examination test which assesses the mathematical content only. They argue that if the development of 4Cs is essential, the government should develop an assessment approach that measures students 4Cs, and up to now, there is no such examination.

Second, teachers feel that the content to be taught in the curriculum is already very tight, and students readiness are not right. The coverage of mathematical content to be taught in a semester is very much with a limited time allocated. Furthermore, teachers have to spend much time to review and teach the prerequisite materials every time they have to facilitate their students to learn new mathematical materials.

According to the authors, many things can be used to discuss the result of the study. According to the authors, many factors influence the practices of teaching conducted by teachers. Among all of the factors, some of them are teachers understanding about the mathematical thinking itself, their perceptions about changing teaching practices, their perception about teaching mathematical thinking.

First, the discussion about the importance of developing mathematical thinking is a new topic in mathematics education (Royal Society, 2014). Used to be, the focus of discussion in mathematics education is mostly about mathematical content. During their pre-service mathematics teachers education program, especially in Indonesia, there was a minimal discussion related to the development of mathematical thinking. Therefore, not every teacher understand the complete figure mathematical thinking. Some teachers even consider students' mathematical thinking just as their prior knowledge (Celik & Guzel, 2017). They perceived mathematical thinking just as knowledge, not as skills. If their students have shown their prior knowledge correctly, students mathematical thinking are assumed to be already excellent. How the students use their logic to process their existing or prior knowledge into another knowledge is not considered as an essential issue.

Second, teachers' perceptions of changing teaching practices. According to Cho (2014), many factors may hinder teachers from changing their teaching practices. Factors such as curriculum, teacher culture, school environment, the educational policy could lead to teachers' reluctant in

implementing innovation. Based on this Cho's claim, it is clear why these participating mathematics teachers do not give the best response related to how their students should show their mathematical thinking through their written works. In the curriculum documents, the government does not encourage teachers to request their students to show their thinking process. The national exam form, which uses multiple choice for most of the years, also do not encourage mathematics teachers to encourage their students to show their mathematical thinking.

Ketelaar (2012) identified three crucial factors that influence teachers' readiness in implementing innovation. The factors are sense-making, ownership, and agency. In the sense-making stage, the teachers relate the innovation to their existing knowledge, experiences, and expectations. If the teachers feel that innovation makes sense to them, there is a big chance for the teachers to develop ownership. Unfortunately, encouraging mathematical thinking, visible innovation seems not to make sense for these participating teachers. Their experiences during the pre-service mathematics teachers program which focused more on content acquisition did not give a chance for them to make-sense this innovation. The multiple-choice types of national examination that should be taken by students could also be the reasons why teachers feel that encouraging students to show their thinking processes through their written work is not a make sense activity.

Call for encouraging students' mathematical thinking through their students' written work could be new for the participating teachers. This call is not coming from themselves. There is also no formal training given to these participating teachers. Ownerships from these participating teachers could be very low. Therefore, it makes sense if their responses are not encouraging mathematical thinking.

In term of agency, although government, through Ministry of National Education and Culture (MOEC) has trained almost every mathematics teacher in Indonesia (Puri & Ilma, 2014), there are no specific agencies that were designed specially to ensure the encouragement of mathematical thinking processes through students' written work. There were no agencies who train teachers on how to encourage students' mathematical thinking through students' written work, monitor and supervise its implementation, assess and evaluate its achievement. Leadership who can direct and lead the implementation of teaching mathematical thinking is not available. Therefore, what is Cook (2014) said as the most important is absent for this encouraging mathematical thinking into written works.

Based on the above discussion, the authors recommend others to follow up this research by comparing teachers' expectation about thinking processes to be incorporated into students works based on teachers' familiarity with mathematical thinking, and their involvement in educational innovation. Finally, the national government could also conduct researches to evaluate the effectiveness of having a national examination system which students thinking can be made visible, toward students' thinking development, and teachers' tendencies in implementing teaching and learning activities in the classroom.

CONCLUSION

Students' thinking processes are not expected to be included in students' written work yet. Teachers' focus on students' written work is still on its mathematical content only. Teachers do not see the importance of making their students' thinking visible. Lack understanding of mathematical thinking, teachers' perception on the innovation especially things that are related to change their practices are among factors that could make the teachers do not include students' mathematical thinking in written works. Sense-making about encouraging students to show their mathematical thinking processes in their written works should be improved. Teachers' ownership of these innovative practices and the existence of agents who can monitor and supervise the practices are also needed to make this innovation happens.

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