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PRIMARY SCHOOL STUDENTS' ABSTRACTION LEVELS OF WHOLE-HALF-QUARTER CONCEPTS ACCORDING TO RBC THEORY

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Abstract

Whole-half-quarter are important mathematical concepts that form the basis of fractions and should be well understood for advancing mathematical topics. The aim of this study is to determine the primary school students' abstraction levels of whole-half-quarter concepts according to RBC theory. The participants of the study are six students (8 age group) from the second grade of primary school. The data of the research which is a case study were collected through worksheets and semi-structured interviews. The data obtained from interviews were analyzed by qualitative data analysis steps. The abstraction levels of students were evaluated according to RBC theory. As a result of the study, it was seen that many of the students could not abstract the whole, half and quarter concepts. It was determined that difficulties of students to abstract the whole-half-quarter concepts resulted from reasons such as not understanding the half and quarter concepts, not being able to divide the whole into two equal parts, not being able to divide one dimensional shapes into half and quarter, generalizing dividing into quarter as putting a "+", not being able to divide into four equal parts for quarter.

Keywords: Abstraction, Fractions, Mathematics Education, RBC Theory, Whole-Half-Quarter Concepts

Abstrak

Konsep *whole-half-quarter* adalah konsep penting matematika yang membentuk dasar pecahan dan harus dipahami dengan baik untuk menguasai materi matematika. Tujuan dari penelitian ini adalah untuk menentukan tingkat abstraksi siswa sekolah dasar dari konsep *whole-half-quarter* menurut teori RBC. Subjek penelitian pada penelitian ini adalah enam siswa (usia 8 tahun) kelas dua sekolah dasar. Data untuk penelitian studi kasus ini dikumpulkan melalui lembar kerja siswa dan wawancara semi-terstruktur. Data yang diperoleh dari wawancara dianalisis menggunakan analisis data kualitatif. Tingkat abstraksi siswa dievaluasi sesuai dengan teori RBC. Hasil penelitian ini menunjukkan bahwa banyak dari siswa tidak dapat memahami bentuk abstrak dari konsep keseluruhan, setengah, dan seperempat. Hasil ini menunjukkan bahwa kesulitan siswa untuk mengabstraksi konsep *whole-half-quarter* yang dihasilkan dari sejumlah sebab, diantaranya tidak memahami konsep setengah dan seperempat, tidak mampu membagi keseluruhan menjadi dua bagian yang sama, tidak mampu membagi satu bentuk bangun datar menjadi setengah dan seperempat, menggeneralisasi pembagian menjadi seperempat sebagai menempatkan "+", tidak dapat membagi menjadi empat bagian yang sama untuk seperempat.

Kata kunci: Abstraksi, Pecahan, Pendidikan Matematika, Teori RBC, Konsep *Whole-Half-Quarter*

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Mathematics is a system consisting of ideas (structures) and relations developed as successive abstraction and generalization processes (Baykul, 2011). Abstraction is defined as a cultural activity that leads to the creation of new meanings in the process of reorganizing and restructuring known mathematical knowledge as a new structure (Bikner-Ahsbahs, 2004). Thus, understanding mathematical concepts for primary school students can often be difficult.

Abstraction is the process of vertically reorganizing previous constructs that lead to the emergence of a new mathematical construct (Dreyfus, 2007). Vertical mathematization is the process of building a new mathematical structure that incorporates mathematics itself and its mathematical

meanings and usually involves reorganizing previous structures, establishing relationships and connections between them, using a single mathematical thinking process to achieve a new mathematical structure (Dreyfus, 2007). The abstraction process consists of three observable epistemic actions: Recognizing – Building with – Constructing (RBC). According to the RBC model, the most important actions in the abstraction processes are mental actions and they cannot be directly observed. The idea of epistemic actions helps us to overcome this obstacle. Epistemic actions are the visualization of mental actions through verbal expressions or physical actions of students (Dreyfus, 2007). In this model of abstraction, which is discussed in terms of sociocultural viewpoint, it is the prevailing thought that the abstraction process has come to life with abstract thought, that the experiential thinking for scientific concepts does not lead to the formation of abstract knowledge, and therefore the dialectic rationale is necessary for the process of abstracting scientific concepts (Dreyfus, 2007; Dreyfus & Tsamir, 2004; Hershkowitz, Schwarz & Dreyfus, 2001).

The three epistemic actions in the RBC model are subjective, that is, a student can recognize a structure he has constructed in the previous activity. For this reason, the student's subjective knowledge determines which structures can be recognized and whether a particular task will lead to building-with or constructing. A structure recognized by a student may cause another student to construct a new structure (Dreyfus & Tsamir, 2004). These actions are not linear but intrinsic, in other words, the creative process does not follow the recognizing and building-with, but it simultaneously requires these epistemic actions (Dooley, 2007). The student behaviors required by the three actions of this theory in the abstraction process can be summarized as follows:

“Recognizing” action is recognizing the properties of a previously solved problem (Hershkowitz et al. 2001), explaining the results of past activities related to the subject (Schwarz, Dreyfus, Hadas & Hershkowitz, 2004), recognizing the existence of a familiar mathematical structure (Bikner-Ahsbahs, 2004). It contains recognizing a familiar mathematical notion, process or idea occurs when a student realizes that the given mathematical situation is inherent in (Dreyfus & Tsamir, 2004; Monaghan & Özmantar, 2006).

“Building with” action is using of the mathematical structure that was created before, to achieve the desired goal (Schwarz et al. 2004) and the process of combining familiar pieces of knowledge into a new context (Bikner-Ahsbahs, 2004). This action consists of combining existing artifacts in order to meet a goal such as solving a problem or justifying a statement (Dreyfus & Tsamir, 2004). It emerges not when the situation is enriched with more complex structural information, but when the students use their existing structural knowledge to deal with the problem at hand (Dooley, 2007).

“Constructing” can be expressed as the process of gathering information structures that allow information to be reorganized vertically (Schwarz et al. 2004). It is the process of reorganizing and restructuring what is recognized and known to construct new meanings (Bikner-Ahsbahs, 2004). In other words, it means bringing together elements of information to produce a new structure (Dreyfus, 2007). In the abstraction process, constructing is more important and rarer than other actions, and it

involves gathering the information to create a new information structure that is familiar to the learner (Monaghan & Özmantar, 2006). Students construct more complex structures from simpler ones, involving the reorganization of mathematical elements, resulting in a more refined structure (Dooley, 2007).

“Constructing incorporates the other two epistemic actions in such a way that building-with actions are nested in constructing actions and recognizing actions are nested in building-with actions and in constructing actions” (Dreyfus & Tsamir, 2004, p. 273). Because of nested epistemic actions of abstracting these three actions, the model is called "the dynamically nested RBC model of abstraction". According to the model, abstraction emerges through the following three steps; the need for a new structure, the construction of a new abstract structure, and the consolidation of the abstract structure through the repeated recognition and use of the new structure (Dreyfus & Tsamir, 2004). In this respect Dreyfus, Hadas, Hershkowitz and Schwarz (2006) stated that the stages of construction and consolidation are mostly intertwined and in order to support this, they studied the students' abstraction process of the subject of probability. They analyzed the mechanisms that students used to consolidate new knowledge structures and found three mechanisms that showed epistemic movements; consolidation during building-with process with constructing, consolidation during projection of constructing, consolidation and in order to recognize the structure with regard to constructing more structures during a new abstraction process. Dreyfus (2007) has added consolidation into the RBC model because of the centralization of consolidation in learning processes in such research results, and RBC + C (consolidation) model has emerged. According to this model, the constructing phase of abstraction does not mean consolidation. An unconsolidated abstract mental structure can be fragile. Consolidated knowledge has become an integral part of the student's current knowledge. The constructed knowledge is that the student uses a specific problem only under specific circumstances in a specific context. Consolidation allows the student to use abstract thinking in a fluid and safe way in a variety of situations. In other words, consolidation gives students a sense of flexibility, trust and clarity (Dreyfus & Tsamir, 2004).

The whole-half-quarter concept is a matter of critical subject that must be comprehended at a young age because it forms the basis of fractions. Fraction is also required in many advanced mathematical studies and students should experience many concepts including parts, ratio and division to fully understand fractions (Van de Walle, Karp & Bay Williams, 2012). The students come across expressions with part-whole relations such as "whole, half, quarter" in the preschool period and they interpret these phrases as different sizes (Olkun & Toluk, 2003). In addition, the researches show that the students have difficulty in learning fractions, ratio and proportion and that their comprehension level is insufficient (Alacacı, 2010; Brown & Quinn, 2006; Kaplan, İşleyen & Öztürk, 2011; Sowder & Wearne, 2006; Stafylidou & Vosniadou, 2004). The concept of fractions can be constructed on the bases of abstracting different meanings of fractions in the student (Temur, 2015). The use of multiple physical representations of translations between pictorial, manipulative, verbal,

real-world and symbolic representations in the initial comprehension of fractions significantly enhances the achievement of students in terms of fractions (Cramer, Post & del Mas, 2002). Different methods can be applied in the comprehension of the whole-half-quarter subject to attract younger students. Also, many types of research about different subjects on RBC abstraction theory have been made (Bikner Ahsbahs, 2019; Dooley, 2007; Dreyfus, 2007; Dreyfus, Hershkowitz & Schwarz, 2001; Gilboa, Kidron & Dreyfus, 2019; Guler & Gurbuz, 2018; Halverscheid, 2008; Kidron & Dreyfus, 2008; Monaghan & Özmantar, 2006; Özmantar & Monaghan, 2007). However, there is no study which analyzes the abstracting process of the whole-half-quarter concepts of students according to the RBC theory. Therefore, in this study, the primary school students' abstraction levels of whole-half-quarter subjects were examined.

METHOD

Research design

The case study of qualitative research methods was used in the study. The case study is an in-depth description and analysis of a limited system. The most important characteristic feature of the case study is the limitation of the object, that is the case of the study (Merriam, 2013). In some studies, researchers choose more than one case to analyze and compare, while in other studies a single case is analyzed (Creswell, 2013). In this study, students' abstraction levels of whole-half-quarter concepts were analyzed.

Participants

Participants of the study consisted of six students (8 age group) training in the second grade of a state primary school in Turkey. According to the primary education program in Turkey, the whole-half relation is the subject of the first grade of primary school. The aim of the program in this regard is *"The student shows the whole and half with appropriate models, and explains the relationship between the whole and half"* (National Ministry of Education [NME], 2018, p. 28). In the second grade, quarters are taught. The aim in the second-grade education program is *"The student shows the whole, half and quarter with appropriate models, and explains the relationship between the whole, half and quarter"* (NME, 2018, p. 34). Thus, the second grade was selected. Participants were selected with purposive sampling method from among the most successful and unsuccessful students in the class in accordance with the views of the classroom teacher. Three successful students (Ceyda, Yaren, Berkay) and three low achievers (Kadirhan, Yunus Emre, Kenan) in mathematics course participated in the study. Two of the participants were female students (Ceyda, Yaren) while four were males (Kadirhan, Yunus Emre, Kenan, Berkay). For ethical purposes, the students' pseudonyms names were used.

Data collection tools

To determine students' abstraction levels of the whole-half-quarter concepts, two worksheets were developed consisting of open-ended questions. The first worksheet contained questions about the

half concept and the second one was about quarter. The questions on the worksheet were prepared in accordance with the RBC theory on the basis of the opinions of the classroom teacher. Especially the examples about two-dimensional forms (such as square, triangle, rectangle) and one-dimensional shapes (straight line) were included in the worksheets. Since the teacher told that she taught the subject with examples of two-dimensional forms (such as square, triangle, rectangle), such shapes were included in the worksheet. The teacher stated that he did not teach how to divide a straight line. For this reason, the straight line (one-dimensional shape) is also included in the worksheet as a shape that students have never split half and quarter before. An interview form was also developed to determine the students' abstraction levels of these concepts based on the worksheets. The worksheets and interview form were reorganized through the expert opinions working in the field of mathematics. The students solved the questions on the worksheets and they were interviewed about their solutions.

Data analysis

The data obtained from interviews were analyzed using qualitative data analysis steps. Merriam (2013) states that data analysis has two steps in case study; case analysis and cross-case analysis. In the case analysis, each situation is first seen as a comprehensive situation within itself, the data are collected, and the researcher can learn as much as possible about the contextual variables. Cross-case analysis begins when case analysis is complete for each case. Cross-case analysis can result in a sample description, cause the theme or categories that conceptualize the data in all cases or result in a fixed theory that provides an integrated framework covering many cases. The RBC abstraction theory was used in determining the level of abstraction of the students. Interviews were analyzed by considering the epistemic mental actions required by the recognizing, building-with, and constructing stages of the RBC theory. The data were analyzed by two different researchers, the results of both analyses were compared, and the consensus of them was determined.

Reliability and validity arise from the presence of the investigator, the mutual communication of the investigator and the participants, the interpretation of data perceptions and the rich explanatory triangles, as opposed to the experimental designs described before the research (Merriam, 2013). In this study, the data collected from the students were analyzed according to the steps of the qualitative data analysis and explained in detail in the findings section. In addition, findings were supported by direct quotations from the student interviews.

RESULTS AND DISCUSSION

When the interviews were analyzed, it was observed that students abstracted whole-half-quarter concepts at different levels. The first student interviewed was Ceyda. Ceyda correctly answered the questions about the half and quarter on worksheets and showed in the interviews that she fully understood these concepts. Ceyda recognized the information required by the questions and she could divide the shapes into half and quarter by using her existing knowledge. The fact that she could even divide the one-dimensional straight line, which she did not know before, accurately shows that she reached the right

conclusion, that is, she constructed half and quarter concepts by interpreting and reorganizing her existing knowledge on the subject with new examples. The answers of Ceyda to the questions were as follows:

Researcher: What is half, Ceyda?

Ceyda: Half of a whole.

Researcher: So, how much of the whole?

Ceyda: We divide the whole into two pieces. And, it becomes half. The equal parts of the whole are called half.

Researcher: What do you call a quarter, Ceyda?

Ceyda: We divide the half into equal parts and it becomes quarter.

Researcher: How many parts do we divide the whole into?

Ceyda: Four.

Researcher: Can we divide the whole randomly?

Ceyda: No. They must all be equal.

Kadirhan divided into half all the shapes in the first worksheet. He divided most of the shapes in the second worksheet into quarters. But he could not divide the straight line into quarters. Figure 1 shows the second worksheet of Kadirhan. After analyzing Kadirhan's answers about quarters in Figure 1, it was seen that he divided all shapes into quarters by adding a "+" (plus) sign. However, the way he tried to divide a straight line by adding a plus sign showed that he could not comprehend the quarter concept. In fact, Kadirhan did not correctly answer any of the questions about what the whole, half, and quarters are, although he could divide all shapes into half and quarters (except for the straight line). His answers to the questions about the half and quarter showed that Kadirhan could not grasp the half and quarter and that he could divide them correctly by adding + rotely. He has not figured out how the half and quarters are formed and the whole-half-quarter relationship and memorized the subject of dividing shapes into half and quarters in a formal way. Kadirhan is able to think based on the visual characteristics of shapes and cannot make any conclusion about dividing in half and quarter independently. It is possible to understand this from his expression "it becomes a quarter when we put a +". In this case, it is understood that half and quarter knowledge was not obtained by Kadirhan and he could not abstract these concepts at all.

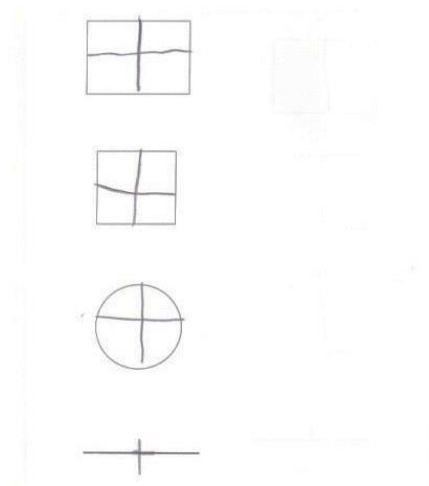


Figure 1. The answers of Kadirhan to the quarter worksheet

The interview between researcher and Kadirhan was as follows:

Researcher: What do you call a whole, Kadirhan?

Kadirhan: Quarter

Researcher: Can you divide this into half?

[After the student divides correctly]

Researcher: What is this called?

Kadirhan: A circle.

Researcher: What do you call a quarter?

Kadirhan: A line.

Researcher: What do you mean? Can you divide this into quarters?

[After the student correctly divides the two-dimensional shapes]

Researcher: What are these parts of the whole?

Kadirhan: Rectangular.

Researcher: Can you divide this straight line into quarters?

[After student puts a "+" on the straight line]

Researcher: Is this a quarter?

Kadirhan: Yes.

Researcher: How? What do you call a quarter, Kadirhan?

Kadirhan: Rectangular.

Researcher: How did you make the quarter?

Kadirhan: By putting a +. When we put a +, it becomes a quarter.

Figure 2 shows Kenan's answers to the questions in the quarter worksheet. Kenan could divide two-dimensional shapes into halves and quarters. But it was seen that he could not divide the straight line into halves and divided into quarters by putting three "+". The interviews showed that he had no understanding of what half and quarter were. He explained how he could divide the shape into quarters by saying "We put + to divide into quarters". This shows that Kenan only memorized how to divide the shape into halves and quarters instead of comprehending the whole-half-quarter relation. The student adheres visually to formal characteristics of half and quarter in the questions and as a result, cannot make any conclusions about these concepts. It was understood that Kenan could not construct whole-half-quarter concepts.

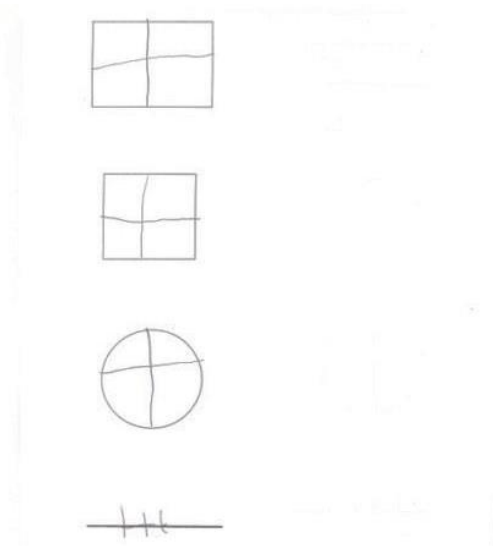


Figure 2. Kenan's worksheet about quarters

Berkay's second worksheet about quarter and his answers in the interviews can be seen in Figure 3. Berkay divided all the shapes into halves, but could not divide anything into quarters. His expressions in the interviews showed that he fully comprehended and constructed the half concept.

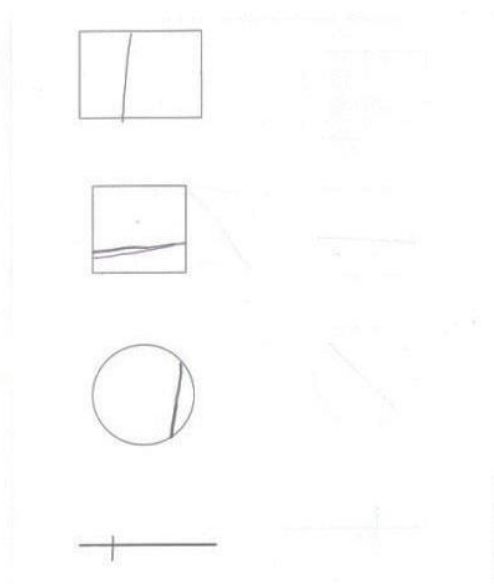


Figure 3. Berkay's quarter worksheet

The student correctly divided all shapes into half using his knowledge of the subject, moreover, he showed that he abstracted this subject by even dividing the straight line into half which was a shape he had not learned before. However, Berkay described the quarter as "quarter is a part of the whole", so he divided the shapes false on the worksheet (Figure 3). It was understood that Berkay could not construct the quarter concept. The interview between researcher and Berkay was as follows:

Researcher: Berkay, what do we call a whole?

Berkay: The entire part of a shape.

Researcher: What do you call a half, then?

Berkay: If you cut a plank of wood in the middle, you get two halves.

Researcher: Can you divide these shapes into half?

[After the student divides]

Researcher: So, you divided them into what?

Berkay: Into half.

Researcher: How many pieces did you get? And, what do we call each one of them?

Berkay: Two halves.

Researcher: What is this?

Bannon: A line.

Researcher: Can you divide this into half?

Berkay: Yes.

[After the student divides]

Berkay: I couldn't divide exactly.

Researcher: How should it be?

Berkay: Right in the middle.

Researcher: What do you call a quarter?

Berkay: This, a little shorter than half.

Researcher: How do we divide into the quarter? If I get this piece, can I call it a quarter?

Berkay: Yes.

Researcher: Can you divide this into quarters?

[After he divides]

Researcher: How did you divide it?

Berkay: I divided it a little towards the front of the middle.

Researcher: Berkay, how do you divide into the quarter?

Berkay: Small, a little smaller than a half.

Yunus Emre divided the two-dimensional shapes correctly into half in the questions posed to him, but could not divide the one-dimensional shape into the half, and did not answer any questions about quarters. When asked what the half was, he answered correctly. His answer "*If we divide it into two right in the middle, it will be half*" showed that he comprehended the whole-half relation. The answers given to the first worksheet showed that Yunus Emre recognized and used the half concept in the questions. However, the fact that he could not divide the straight line, which was a new kind of question that he had not encountered before, showed that he could not construct the half concept and therefore could not completely abstract this concept. Also, the student did not answer any questions about the quarters. Failure to questions about quarters in the interview also indicates that the student could not obtain the quarter knowledge. Yaren's answers to the questions about the half and quarter were as follows:

Researcher: Yaren, what is a whole?

Yaren: The whole is like a cube. If we cut the cube, it becomes a half, if we do not cut it, it becomes whole.

Researcher: How should we cut it so that it becomes a half?

Yaren: We should cut it right in the middle.

Researcher: Can you divide those shapes into half?

[After the student divides]

Researcher: What is this now?

Yaren: A half.

Researcher: How many halves do a whole consist of?

Yaren: Two.

Researcher: Can you divide this? [Showing the straight line] Can you divide this shape into half?

Yaren: It cannot be divided.

Researcher: Why not?

Yaren: Because it is a line.

Researcher: So, how should it be so that it can be divided into half?

Yaren: It needs to be a shape like a cube or a triangle.

Researcher: What do you call a quarter?

Yaren:

Yaren's worksheet about half was presented in Figure 4. Yaren says that two-dimensional shapes such as square and cube can be divided into half, but the straight line cannot be divided. Yaren responded correctly to the question of what the half was, she actually knows that the half should be equal two halves of the whole, but she cannot grasp how a straight line is divided. When she was asked to divide a straight line, she replied: "*this shape cannot be divided*". Yaren recognized the half concept and used her knowledge about the halves, used it to divide two-dimensional shapes. However,

she stated that one-dimensional straight line, which was a new shape for her, could not be divided into half. Not being able to use her existing knowledge on a newly encountered question shows that she could not construct and abstract the knowledge. Yaren did not answer any questions about quarters expressing that she did not know how to divide into quarters. Not having any familiar knowledge indicates that the concept of quarters was not constructed, too.

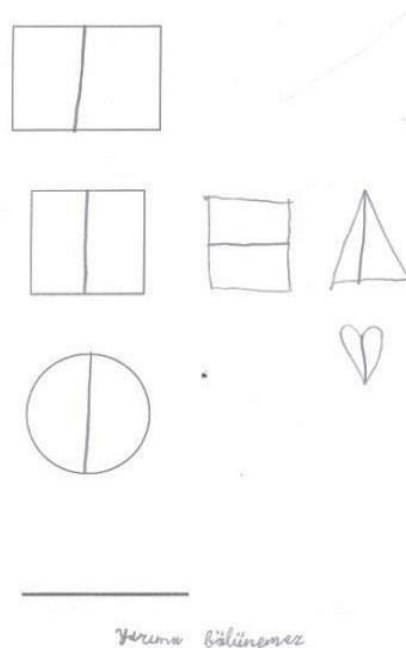


Figure 4. Yaren's first worksheet about half

The abstraction levels of the students are revealed as in Table 1. As it can be seen in Table 1, it is understood from Ceyda's responses that she constructed and completely abstracted the half and quarter concepts. While only Ceyda and Berkay abstracted the concepts of the half, nobody abstracted quarter concept except Ceyda. However, it was determined that students had difficulty in understanding these subjects, they could not comprehend and abstract the whole-half-quarter concepts in the interviews even though they could divide them correctly on the worksheets.

Table 1. Students' levels of abstraction for the whole-half-quarter concepts

	Abstraction of Concepts	
	Half	Quarter
Ceyda	+	+
Yaren	-	-
Berkay	+	-
Kadirhan	-	-
Yunus Emre	-	-
Kenan	-	-

This study was conducted to examine the abstraction levels of whole-half and quarter concepts of primary school students. According to RBC theory, the abstraction steps are Recognizing – Building with – Constructing and there are properties of these steps like “recognizing the existence of a familiar mathematical structure” for recognizing; “combining familiar pieces of knowledge into a new context” for building with and “reorganizing what is recognized and known to construct new meanings” (Bikner-Ahsbals, 2004) as mentioned above. In the interviews, the students’ abstraction levels were examined according to abstraction steps and as a result of the study, it has been seen that most of the students could not abstract whole-half-quarter concepts.

When students’ abstraction levels were analyzed, it can be seen that most of them failed. Only two of them were able to answer the questions and abstract the “half” concept accurately. However, the interviews showed that only Ceyda could construct and comprehend the whole-half-quarter concepts correctly. It can be concluded that while a student abstracted all the concepts, many of them memorized the definitions of these concepts, but they did not understand the subject at all, they adhere to the definitions and could not abstract the concepts.

They have difficulties in learning and understanding these concepts. It was understood that difficulties of students to abstract the whole-half-quarter concepts resulted from reasons such as not understanding the half and quarter concepts, not being able to divide the whole into two equal parts, not being able to divide one dimensional shapes into half and quarter, generalizing dividing into quarter as putting a "+", not being able to divide into four equal parts for quarter. It was concluded that the reasons for these learning difficulties were mostly caused by rote learning.

Some students memorized half-and-quarter concepts as shapes (putting + to divide into quartiles), while others have not been able to make any definitions and answering correctly the questions, some of them could not generate any information even at the level of recognition. It can be assumed that these students could not abstract these concepts because of memorizing the definitions or mentally adhering to the shapes that the classroom teacher had shown in the courses before. Because students could not make any conclusions when they were asked to divide half and quarter of a different shape (one-dimensional straight line) that has not been taught by their teacher before, and they could not divide it into half and quarter. Moreover, not only unsuccessful but also successful students experienced these mistakes.

The results of the study are consistent with the Van Hiele Geometric thinking levels. This is because students can make inferences appropriate for the zero and one level they belong to according to this model. It has been understood that they could not comprehend the division into half and quarter independently of the definition of the shape being taught. As a matter of fact, in the literature, it has been found that the misconceptions about fractions are caused due to not being divided equally (Alacacı, 2010; Stafylidou & Vosniadou, 2004) and that these misconceptions can be avoided by making meaningful activities in the first, second and third classes in which the concept of fraction is introduced (Erbilgin, Şahin & Arıkan, 2017).

CONCLUSION

For the correct learning of the concept of fractions, points to note were stated in the literature. In order for the fractions to be grasped correctly, it is especially important for students to understand that the whole is divided, the pieces must be of equal size, and that a region can be divided equally into different groups (Charalambos & Pitta-Pantazi, 2007; Temur, 2015). It is aimed to accomplish effective fraction teaching by using rules of real-life situations and concrete tools instead of teaching the rules of fractions (Temur, 2015). It can be claimed that different teaching methods or computer programs can be beneficial in this regard and will enable students to correctly structure their future learning. However, there are points that need attention. The method used by the teacher should not prevent students from thinking and structuring their forms of understanding, so the teacher needs to be careful (Van de Walle et al. 2012). For this reason, the abstraction levels of students and the success of mathematics education depends on the characteristics of teacher. The teachers need to be careful to achieve the goals of the mathematics topics.

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REFERENCES

- Alacacı, C. (2010). *Öğrencilerin kesirler konusundaki kavram yanılgıları*. In E. Bingölbali ve M. F. Özmentar (Eds). *İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri*, (p. 63-94), Ankara: Pegem Akademi.
- Baykul, Y. (2011). *İlköğretimde matematik öğretimi (10. Baskı)*. Ankara: Pegem Akademi.
- Bikner-Ahsbahs, A. (2004). *Towards the emergence of constructing mathematical meanings*. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2, 119-126.
- Bikner-Ahsbahs, A. (2019). *The research pentagon: A diagram with which to think about research*. In Compendium for Early Career Researchers in Mathematics Education (pp. 153-180). Springer, Cham.
- Brown, G., & Quinn, R.J. (2006). Algebra students' difficulty with fractions: An error analysis. *Australian Mathematics Teacher*, 62(4), 28-40.
- Charalambous, C.Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational studies in mathematics*, 64(3), 293-316. <https://doi.org/10.1007/s10649-006-9036-2>.
- Cramer, K.A., Post, T.R., & Delmas, R.C. (2002). Initial fraction learning by fourth-and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education*, 33(2), 111-144. <https://doi.org/10.2307/749646>.
- Creswell, J.W. (2013). *Nitel araştırma yöntemleri, beş yaklaşıma göre nitel araştırma ve araştırma*

- deseni (3. baskıdan çeviri)*. Çev. Ed: M. Bütün, S. B. Demir. Ankara: Siyasal Kitabevi.
- Dooley, T. (2007). *Construction of knowledge by primary pupils: The role of whole-class interaction*. In Proceedings of CERME, 5, 1658-1668.
- Dreyfus, T. (2007). Processes of abstraction in context the nested epistemic actions model. Retrieved from <https://pdfs.semanticscholar.org/d190/0be9d6a043ac815c81344caa8c2713dcc329.pdf>.
- Dreyfus, T., Hadas, N., Hershkowitz, R. & Schwarz, B. (2006). *Mechanisms for consolidating knowledge constructs*. Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education. Prague, Czech Republic.
- Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2001). Abstraction in context II: The case of peer interaction. *Cognitive Science Quarterly*, 1(3/4), 307-368.
- Dreyfus, T., & Tsamir, P. (2004). Ben's consolidation of knowledge structures about infinite sets. *The Journal of Mathematical Behavior*, 23(3), 271-300. <https://doi.org/10.1016/j.jmathb.2004.06.002>.
- Erbilgin, E., Şahin, B., & Arıkan, S. (2017). Pasta dilimlerinin fiyatı ne kadar?. *Journal of Inquiry Based Activities*, 5(1), 34-47.
- Gilboa, N., Kidron, I., & Dreyfus, T. (2019). Constructing a mathematical definition: the case of the tangent. *International Journal of Mathematical Education in Science and Technology*, 50(3), 421-446. <https://doi.org/10.1080/0020739X.2018.1516824>.
- Guler, H.K., & Gurbuz, M.C. (2018). Construction process of the length of [cube root of 2] by paper folding. *International Journal of Research in Education and Science*, 4(1), 121-135.
- Halverscheid, S. (2008). Building a local conceptual framework for epistemic actions in a modelling environment with experiments. *ZDM Mathematics Education*, 40(2), 225-234. <https://doi.org/10.1007/s11858-008-0088-x>.
- Hershkowitz, R., Schwarz, B., & Dreyfus, T., (2001). Abstraction in context: epistemic actions. *Journal for Research in Mathematics Education*, 32(2), 195-222. <https://doi.org/10.2307/749673>.
- Kaplan, A., İşleyen, T., & Öztürk, M. (2011). The misconceptions in ratio and proportion concept among 6th grade students. *Kastamonu Education Journal*, 19(3), 953-968.
- Kidron, I., & Dreyfus, T. (2009). *Justification, enlightenment and the explanatory nature of proof*. In Proceedings of the ICMI Study 19 Conference: Proof and proving in mathematics education, 1, 244-249.
- Merriam, S.B. (2013). *Nitel araştırma desen ve uygulama için bir rehber (3. baskıdan çeviri)*. Çev. Ed: S. Turan. Ankara: Nobel Yayıncılık.
- Monaghan, J., & Ozmantar, M.F. (2006). Abstraction and consolidation. *Educational Studies in Mathematics*, 62(3), 233-258. <https://doi.org/10.1007/s10649-006-8753-x>.
- National Ministry of Education. (2018). Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar). Retrieved from <http://mufredat.meb.gov.tr/ProgramDetay.aspx?PID=329>.
- Olkun, S. & Toluk, Z. (2003). *Matematik öğretimi*. Ankara: Anı Yayıncılık.
- Ozmantar, M.F., & Monaghan, J. (2007). A dialectical approach to the formation of mathematical abstractions. *Mathematics Education Research Journal*, 19(2), 89-112. <https://doi.org/10.1007/BF03217457>.

- Schwarz, B., Dreyfus, T., Hadas, N. & Hershkowitz, R. (2004). *Teacher guidance of knowledge construction*. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education. Bergen, Norway.
- Sowder, J., & Wearne, D. (2006). What do we know about eighth – grade achievement? *Mathematics Teaching in the Middle School, 11*(6), 285-293.
- Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*(5), 503-518. <https://doi.org/10.1016/j.learninstruc.2004.06.015>.
- Temur, Ö.D. (2015). Opinions of teachers of fourth and fifth grade about teaching fractions: A phenomenographic research. *Dumlupınar Üniversitesi Sosyal Bilimler Dergisi, 29*.
- Van de Walle, J.A., Karp, K.S. & Bay Williams, J.M. (2012). *İlkokul ve ortaokul matematiği gelişimsel yaklaşımla öğretim* (Yedinci baskıdan çeviri, S. Durmuş Çev. Ed.). Ankara: Nobel Yayıncılık.