TEACHING MULTIPLICATION OF NUMBERS FROM 1 TO 10 TO STKIP SURYA STUDENTS USING MATEMATIKA GASING

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Abstract

Multiplication of numbers from 1 to 10 is very important as it provides the basis for learning multiplication of other larger numbers as well as other related mathematical operations. How do students learn multiplication? Usually students just memorize the results of multiplication. This is often performed without a complete comprehension of the concept of multiplication. This study aimed to discuss how to teach multiplication of numbers from 1 to 10 to STKIP Surya students using Matematika GASING. GASING stands for Gampang, ASyIk dan menyenaNGkan which is translated as easy, fun and enjoyable. The materials are taught according to its unique way of teaching mathematics which follows three stages: concrete, abstract and mental calculation. The first stage (concrete) encourages students to explore using concrete objects. This is done prior to the second stage called the abstract stage. Students are then able to move on to the third stage where they can do mathematical calculation mentally and instantly. By following these stages in this order, students can understand mathematics more easily and clearly. The research method used in this study was design research. It consists of three phases; they are preliminary design, teaching experiment and retrospective analysis. The sample was fourteen first-year undergraduate students (matriculation level) at STKIP Surya (Surya College of Education), Tangerang, Banten. The instruments used were both oral and written tests. They were used to measure the ability of performing mental computation as well as the ability to teach this material. This study showed that Matematika GASING helped these students to understand and be able to teach multiplication of numbers from 1 to 10 better.

Keywords: multiplication of numbers from 1 to 10, Matematika GASING, design research

Abstrak

Perkalian bilangan 1 sampai 10 sangat penting sebagai dasar untuk pembelajaran perkalian bilangan-bilangan yang lebih besar atau operasi-operasi yang berkaitan. Bagaimanakah siswa belajar tentang perkalian? Biasanya siswa hanya mengingat hasil dari perkalian. Hal ini sering terjadi tanpa adanya pemahaman yang lengkap tentang konsep perkalin. Penelitian ini bertujuan untuk mengetahui bagaimanakah mengajarkan perkalian bilangan 1 sampai 10 kepada mahasiswa STKIP Surya menggunakan Matematika GASING. GASING merupakan singkatan dari Gampang, ASyIk dan MenyenaNGkan. Pembelajaran GASING dilakukan dengan tahapan-tahapan yang unik yaitu konkret, abstrak, dan mencongak. Tahap konkret adalah mendorong siswa untuk mengeksplorasi dengan benda-benda konkret. Setelah tahap konkret selesai, selanjutnya adalah tahap abstrak. Selanjutnya, siswa menuju tahap ketiga yaitu mereka dapat menghitung mencongak dan cepat. Dengan tahapan-tahapan tersebut, siswa dapat memahami matematika dengan jelas dan mudah. Metode yang digunakan dalam penelitian ini adalah *design research*. Metode ini terdiri tiga tahapan, yaitu *preliminary design*, *teaching experiment* dan *retrospective analysis*. Sampel dari penelitian ini adalah 14 orang mahasiswa tahun pertama (kelas matrikulasi) di STKIP Surya, Tangerang, Banten. Instrumen yang digunakan dalam penelitian ini berupa instrument tes tertulis dan tes lisan. Instrumen ini digunakan untuk mengukur kemampuan mencongak dan kemampuan menajarkan materi. Hasil dari penelitian ini menunjukkan bahwa Matematika GASING dapat membantu mahasiswa dalam memahami perkalian bilangan 1 sampai 10 dan mampu mengajarkan perkalian bilangan 1 sampai 10 dengan lebih baik.

Kata Kunci: perkalian bilangan 1 sampai 10, Matematika GASING, *design research*

Multiplication is one of many important mathematical topics taught at elementary schools. It has many applications in our daily life. Learning multiplication begins with learning multiplication of numbers from 1 to 10 which provides the basis for learning multiplication of other larger numbers as well as other related mathematical operations. Students are still having difficulties in learning multiplication and division (Raharjo et al., 2009). They do not remember basic multiplication (multiplication of two numbers where each number is of one digit) which means multiplication of numbers from 1 to 10.

Currently a problem that continues to arise in the field is how to teach students in such a way that they could do basic multiplication aptly. Matematika GASING provides one of possible solutions to this problem; it emphasizes in using logics when learning mathematics. This way students tend to have a fuller grasp of mathematics and do not have to merely memorize or depend on mathematical formulas. A critical point in Matematika GASING is defined as basic materials to be learnt and mastered in each mathematical topics taught to students. Once students reach the critical point, they will be able to do mathematical questions for that particular topic with no difficulties. Being able to do multiplication of numbers from 1 to 10 in Matematika GASING is a critical point that students have to reach in order to be able to do multiplication in general. There are five steps to be ascended in order to reach this critical point.

This study aimed to show these five steps as a learning process of multiplication of numbers from 1 to 10 using Matematika GASING. An instructional sequence was designed according to these five steps. It was then implemented in the teaching of this material to a matriculation class of fourteen first-year undergraduate students at STKIP Surya. The first research question was: what was the learning trajectory of multiplication of numbers from 1 to 10 like using Matematika GASING? This study also intended to show the computational competence of these students as well as their capability to teach the material after the learning process. Therefore the second and third research questions were: how competent were STKIP Surya students in multiplication of numbers from 1 to 10? How capable were they of teaching multiplication of numbers 1 to 10 using Matematika GASING? Hence the research method used in this study was design research. The main purpose of design research is to investigate possible educational improvements by generating and studying new forms of learning (Gravemeijer & Van Eerde, 2009). It also allows researchers to analyze the actual process of students' learning and mental activities performed when participating in the instructional activities in a classroom (Bustang, et al., 2013).

Matematika GASING

GASING stands for *Gampang, ASyIk dan menyenaNGkan*, which is translated as easy, fun and enjoyable (Surya, 2012). There are three stages in learning mathematics using GASING. During the first stage (the concrete stage) students are introduced to mathematical concept using concrete objects. Concrete objects help students to understand the mathematical concepts and thus students could do the mathematical calculations easily. This is done prior to the second stage (the abstract stage), where students are then introduced to write mathematical symbols. This is consistent with the theories of Dienes, Piaget & Bruner (Ibrahim & Suparmi, 2012).

During the final stage students are encouraged and expected to be able to do mathematical calculations mentally without using any aid such as calculating or recording device and produce the answers instantly (the mental calculation stage). This is done using mental computation or mental arithmetic. Mental computation is defined as a computation done to produce exact numerical answers without using any aid such as calculating or recording device (Reys, 1985). Mental computation is usually linked to mental arithmetic. Whilst mental arithmetic mainly focuses on mental recall of basic numerical facts and procedures and mainly counts on the memory of the students, mental computation emphasizes on the mental processes used to obtain the answers. Moreover, mental computation includes mental arithmetic. Mental arithmetic is an important element that may be used in performing mental computation (Beishuizen & Anghileri, 1998).

Multiplication of Numbers from 1 to 10 in Matematika GASING

Multiplication is a mathematical operation which involves adding a number to itself a certain number of times (repetitive addition). Let a and b be numbers. The result of adding a to itself b number of times is called the product of a by b. It is written as $a \times b$ or $a.b$ or ab. It is often called as "a times b".

As explained above, there is a critical point which needs to be reached to be able to master multiplication in Matematika GASING. There are five steps to be ascended prior to reaching this critical point.

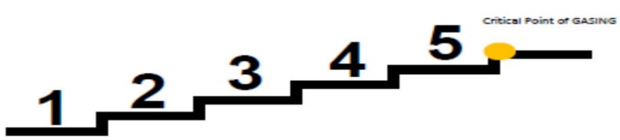

Figure 1. The GASING Critical Point for Multiplication (Surya, 2013)

They are: (1) multiplication Concept, (2) multiplication of numbers 1, 10, 9, 2 and 5, (3) multiplication of two same numbers, such as $1 \times 1, 2 \times 2, \ldots, 10 \times 10$, (4) multiplication of numbers 3 and 4, (5) multiplication of numbers 8, 7 and 6.

METHOD

The participants were 14 first year undergraduate students at the matriculation mathematics class of the academic year 2013 - 2014 at STKIP Surya (Surya College of Education). They belonged to the study program of Informatics and Computer Science Education (*Pendidikan Teknik Informatika dan Komputer* – shortened as *TIK*).

The research method used in this study was design research. Apart from its main purpose, which is to investigate possible educational improvements by generating and studying new forms of learning, it also has other purposes such as the teacher's role, role of symbols, role of language. The main purpose of this research however emphasized on the learning trajectory formulated for multiplication of numbers from 1 to 10 using Matematika GASING and its implementation in a classroom.

As a research method design research has two notable characteristics, they are the specific role of design and the specific role of the experiment. There are three phases in conducting design research. They are: (1) preliminary design, (2) teaching experiment, (3) retrospective analysis (Van den Akker, et al., 2006).

The main objective of the teaching experiment phase is to explore, probe and investigate in order to adapt and refine the conjectured local instruction theory. There is a reflexive connection between the theory and the experiment, whereby an iterative or a cyclic process of testing and improving the conjectured local instruction theory may take place. The conjectured local instruction theory is used by the teachers and the researchers as a guide when conducting the teaching experiment, on the other hand it is adapted to the activities during the actual learning process. In this study, the conjectured local instruction theory formulated at the preliminary design phase was implemented in the classroom once. The next cycle where the revised version of the local instruction theory could be implemented should take place in the matriculation mathematics class at STKIP Surya in the next academic year.

During this teaching experiment, data collections were also conducted to answer the research questions. Data collections were obtained using video observations, tests and field notes. Video observations were used to capture the activities done during the learning process. The videos were also used to support the findings and allow the researchers to investigate what students did and how the activities went during the experiment as well as to ensure the validity of the data; researchers conveyed what actually happened during the experiment. Microteaching test, written and oral tests were also given. Written and oral tests were given to measure the computational competence of students in multiplication of numbers from 1 to 10. Microteaching and teaching written tests were given to measure the capability of students teaching the material. The tests questions were prepared and consulted with an expert in Matematika GASING at STKIP Surya.

From these data, we were able to conclude how students absorbed the materials and how they could present the materials both orally and in writings. We could then see which parts of the materials were or were not understood fully by students. The analysis of these data was thoroughly done during the last phase of design research namely analysis retrospective. The conjectured local instruction theory was compared to the actual learning activities taking place in the class and hence revised accordingly. However, we may add that the analysis began during the teaching experiment, (in fact the boundaries between the three phases in design research are not fixed). The most important rule is that the closer the instructional activities designed to what actually happens in the classroom the better.

The conjectured local instruction theory in this study contained the learning goals, an instructional sequence of five steps to be ascended in order to reach the critical point in Matematika

GASING needed for mastering multiplication. It also contained the conjectures of students' reasoning when participating in the designed activities. In this study, the conjectured local instruction theory was designed according to the Matematika GASING training module for elementary school level (Surya, 2013). This was an initial answer to the first research question, which would be revised to create a better fit to the actual learning process.

An overview of the conjectured local instruction theory is given in Table 1 below.

Table 1. An Overview of the Conjectured Local Instruction Theory

Activity	Main Goals	Description of Activity	Questions to be Asked	Conjectures of Students' Answers
Multipli ca-tion Concept.	Students understa nd the concept of multipli ca-tion.	• The teacher introduces the meaning of multiplication using real objects such as containers and markers (concrete). • The teacher explains the mathematical writings of multiplication (abstract). • The teacher points out the commutative law for multiplication.	• How to explain multiplicatio n; what is 1 x 5; how about 2 x 5? • Explain 6 x 3 and 3 x 6 using real objects! What can be deduced from multiplication 6 x 3 and 3 x 6? Do they have the same meaning?	• Multiplication is when you add repeatedly; 1 x 5 means there are 5 number 1's or 1 number 5; 2 x 5 means there are 2 number 5's or 5 number 2's. • Same results so same meaning; same results but not the same concretely; same results but different calculations.
Multiplicati on of Number 1.	Students understa nd, able to compute and	• The teacher explains multiplication of number 1 concretely. • The teacher	• Anyone knows how to explain multiplication of number 1 concretely?	• Yes – one or two students are encouraged to come forward and explain using

	teach multipli ca-tion of number 1.	explains the mathematical writings of multiplication of number 1 (abstract). • The teacher encourages students to pick up on the pattern of the results of multiplication of number 1; this helps students to memorize the multiplication easily.	• What is the next stage after explaining concretely? How to teach multiplication of number 1 at this stage? • What can be deduced from the results of this multiplication so that they can be memorized easily?	concrete objects; no. • Abstract stage – just write the numbers and the multiplication results; no answers. • The results are the numbers themselves; there is a difference by 1; just add 1 to each result in ascending order; no answers.
Multiplication of two same numbers.	Students understand, able to compute and teach multipli ca-tion of two same numbers.	• The teacher explains multiplication of two same numbers concretely. • The teacher explains the mathematical writings of multiplication of two same numbers (abstract). • The teacher encourages students to find	• Based on what you have learnt about multiplication of numbers 1, 10, 9, 2 and 5 so far; anyone knows how to explain multiplication of two same numbers? • How about the abstract stage? • How would	• Yes – one or two students are encouraged to come forward and explain using concrete objects; no. • Just write down the results. • Just memorize the results; by recognizing the patterns.

		a way to memorize the results easily.	you memorize the results?	
Multiplication of number 3.	Students understand, able to compute and teach multiplica-tion of number 3.	• The teacher explains multiplication of number 3 concretely. • The teacher explains the mathematical writings of multiplication of number 3 (abstract). • The teacher encourages students to find a way to memorize the results easily.	• How do you explain multiplication of number 3 concretely? • How about the mathematical writings? • How would you memorize the results?	• For examples: 1×3 means there is 1 box containing 3 markers or 3 stones, 2×3 means there are 2 boxes containing 3 markers or 3 stones each, etc.; no answers. • Just write down the results. • Just memorize the results; by recognizing the patterns; by using fingers.
Multiplication of number 8.	Students understand, able to compute and teach multiplica-tion of number 8.	• The teacher explains multiplication of number 8 concretely. • The teacher explains the mathematical writings of multiplication of number 8 (abstract). • The teacher	• How do you explain multiplication of number 8 concretely? • How about the mathematical writings? • How would you memorize the results?	• For examples: 1×8 means there is 1 box containing 8 apples or 8 bananas, 2×8 means there are 2 boxes containing 8 apples or 8 bananas each, etc.; no answers.

	encourages students to find a way to memorize the results easily.	• Just write down the results. • Just memorize the results; by recognizing the patterns; by using fingers; by using card games; by singing songs.

RESULTS AND ANALYSIS

Prior to teaching multiplication, students had been taught addition using GASING. Teaching multiplication of numbers from 1 to 10 using GASING was conducted using the steps explained above, which started with teaching the concept of multiplication.

The Teaching Experiment

A selection of important events or conversations were noted and hence compared and adapted to the conjectured local instructional theory.

Introduction of the GASING Critical Point for Multiplication

The teacher explained that there was a critical point students have to reach in order to master multiplication in Matematika GASING. The critical point in this case was multiplication of numbers from 1 to 10, which are divided into five steps.

The First Step: The Multiplication Concept

In this learning session, students already had an understanding about stages of learning a mathematical topic in Matematika GASING from the previous topic, namely addition. They knew that in Matematika GASING it always started with the concrete stage.

The teacher started the learning process by asking what the meaning of multiplication was.

Teacher : What is multiplication?

Raja : Repetitive addition.

Every students agreed that multiplication is repetitive addition. The teacher then asked the next question.

Teacher : What does it mean by 3×5?

Raja & Nyong : Keep adding number 3 for 5 times.

Ferry : Add number 5 for 3 times.

There were obviously two different understandings of 3×5, which would be addressed to after the teacher explained what multiplication meant concretely.

The teacher showed two cards with pictures of 3 bananas on each card to illustrate multiplication 2×3. Afterwards, the teacher showed what it meant by 3×5 using picture cards.

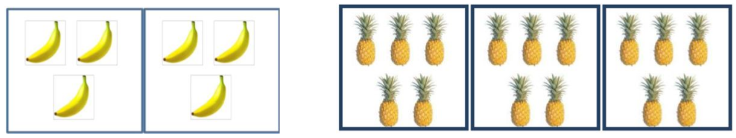

Figure 2. 2×3 and 3×5 Using Picture Cards

After a few more examples, it was concluded that 2×3 means 2 boxes where each box contains 3 objects. The word "box(es)" was (were) replaced by the symbol ☐. Hence 2×3 could be written as $2\square_3$, which meant $3 + 3 = 6$. 3×5 meant 3 boxes, where each box contained 5 objects. It could be written as $3\square_5$. Then the teacher asked one of the students to explain the meaning of 5×2 using cards at the board. Afterwards, the teacher asked another students to explain the meanings of multiplication 2×5 and 5×2. Here, students were lead to conclude that these multiplication produced the same results but they had different meanings.

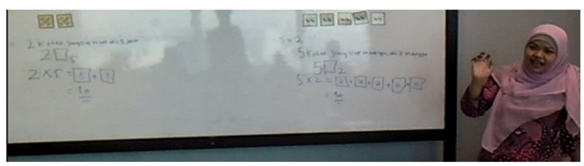

Figure 3. Multiplication of 2×5 and 5×2 Using Picture Cards

The Second Step: The Multiplication of Numbers 1, 10, 9, 2 and 5

The Multiplication of Number 1

The students already understood that a concrete explanation had to be given at the beginning of each topic of multiplication. The teacher then gave a demonstration of how to explain multiplication of number 1 using cards with pictures of apples. Afterwards, a student was asked to express the concrete understanding of the concept by writing it down as in Figure 5. Then the teacher wrote down the abstract form (mathematical writings) of multiplication of number 1 as in Figure 5.

The Concrete Concept in Writings

$1 \times 1 = 1\square_1 = 1$
$2 \times 1 = 2\square_1 = 1 + 1 = 2$
$3 \times 1 = 3\square_1 = 1 + 1 + 1 = 3$
$4 \times 1 = 4\square_1 = 1 + 1 + 1 + 1 = 4$
$5 \times 1 = 5\square_1 = 1 + 1 + 1 + 1 + 1 = 5$
$6 \times 1 = 6\square_1 = 1 + 1 + 1 + 1 + 1 + 1 = 6$
$7 \times 1 = 7\square_1 = 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7$
$8 \times 1 = 8\square_1 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8$
$9 \times 1 = 9\square_1 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9$
$10 \times 1 = 10\square_1 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10$

The Abstract Form

$1 \times 1 = 1$
$2 \times 1 = 2$
$3 \times 1 = 3$
$4 \times 1 = 4$
$5 \times 1 = 5$
$6 \times 1 = 6$
$7 \times 1 = 7$
$8 \times 1 = 8$
$9 \times 1 = 9$
$10 \times 1 = 10$

Figure 5. The Concrete Concept in Writings and the Abstract Form of Multiplication of Number 1

After students understood the concrete concept and the abstract writings, the teacher moved on to the mental calculation stage.

Teacher : How do you think the results of multiplication of number 1 can be memorized easily?

Raja : By using a song?

Teacher : Any other ways?

Ferry : The answers are the same as the number of boxes.

Adit : Each answer is the same as the number itself.

Teacher : That's right, each result is the same as itself.

The teacher then encouraged the students to observe the abstract forms and recognize the pattern formed by each result. Hence it could be concluded that a number multiplied by 1 produced the number itself. Students could then provide the answers of multiplication of number 1 orally and instantly by memorizing them (mental arithmetic). It could be seen that students had no difficulties in memorizing this particular multiplication. It was noted that at the end of the session, students were drilled with questions of multiplication of number 1 orally by the teacher (the drilling activity was not included in the conjectured local instructional theory).

The Multiplication of Numbers 10, 9, 2, and 5

The multiplication to be taught next was of number 10, followed by numbers 9, 2 and 5 in that order. The learning of multiplication of each number began with a concrete explanation (concrete stage), followed by the abstract stage as in the learning of multiplication of number 1 and ended with the mental calculation stage. The first two stages were similar to the stages in multiplication of number 1, students were expected to understand the concrete concept as well as writing down the multiplication of each number (similar to Figure 5). The last stage (mental calculation stage) was where students were taught the easy ways to memorize the results of each multiplication. There were different ways to do this depending on the multiplication. A few notes on how the teaching experiment went are given below.

It was noted that students began to understand how to write down the abstract forms at the learning of multiplication of number 10. The results of multiplication of number 10 could be memorized by recognizing the pattern. Students were asked if they knew how to memorize this multiplication easily. Almost everyone suggested to notice the pattern and to just add 0 at then end of each number. The teacher confirmed that this was the right observation. Students could then answer the questions of multiplication of number 10 easily and instantly by memorizing them (mental arithmetic).

When learning multiplication of number 9, a student who was asked to demonstrate the multiplication concretely using picture cards was confused about the concept of "boxes" and "objects". The teacher then straightened the misconception by explaining again which number represented the number of boxes and which number represented the number of objects contained in each box. Students were also reminded how to calculate the results of multiplication of number 9 using what we called “the streak system”, which was already taught to students previously in addition, i.e. to calculate $9 + 9 = 18$ we streaked the second 9: $9 + 9\,^{8}$. We then remembered the last digit of the result which was 8. The number of streaks represented the number of tens; in this case there was 1 streak so there was one ten, combining it with the last digit gave the result as 18.

At the mental calculation stage of multiplication of number 9, the teacher asked the students if they knew how to do the mathematical calculations mentally. A student guessed that using fingers could help but he did not know how to do this exactly. The teacher explained that there were 2 ways to do this. One way was by using fingers, another way was to see the pattern.

Using fingers follows a rule. First, number the fingers from left to right (see Figure 6). For example, we would like to calculate 3×9, we folded finger number 3, hence we saw that there were 2 fingers to the left of that folded finger and there were 7 fingers to the right of that folded finger. This meant that the result was 27. Students were then very keen on using their fingers when trying to calculate the results of multiplication of number 9.

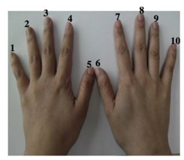

Figure 6. Using Fingers for Multiplication of Number 9

Another way was by looking at the pattern. First of all, consider two numbers a and b such that $a + b = 9$. Then a was called the 9’s pair of b and vice versa. For example, 4 was the 9’s pair of 5. Now, if we wanted to calculate 7×9, first we looked for a number which was 1 less than 7, namely 6. Afterwards we looked for the 9’s pair of 6, namely 3, so the result was 63. By doing this, students would be able to mentally calculate the results of multiplication of number 9 easily. The teacher reminded the students that by then the results of 1×9 and 10×9 had already been memorized (multiplication of numbers 1 and 10), so students only needed to focus on the remaining multiplication: $2 \times 9, \dots, 9 \times 9$.

At the learning of multiplication of number 2, the student asked to demonstrate the multiplication concretely was already able to fully grasp the meaning of concrete objects; objects used could be anything from bananas, apples, durians to pens and markers. The student was also able to differentiate between “boxes” and “objects contained in the boxes”. It was noted that the students also became more accustomed to the stages concrete, abstract, and mental calculation in Matematika GASING here.

At the mental calculation stage of this multiplication, the teacher reminded the students that the results of 1×2, 9×2 and 10×2 had already been memorized previously. The teacher then asked the students if they could think of a way to memorize the remaining multiplication of number 2 easily.

A few students suggested to add 2 each time. The teacher then explained using an example of 3×2 which meant $2 + 2 + 2 = 6$ and that 2×3 means $3 + 3 = 6$. Both multiplication gave the same results although they had different meanings. This way could be used to enable the students to calculate the results of this multiplication easily and hence to memorize them easily.

At the stage of mental calculation of multiplication of number 5, the teacher reminded the students that the results of 1×5, 2×5, 9×5 and 10×5 had already been memorized previously. The teacher hence explained how the results of the remaining of this multiplication could be memorized using fingers. For example for 4×5, the teacher showed 4 fingers. She then grouped every 2 fingers together. Each group of 2 fingers was worth 10. Since there were 2 groups of 2 fingers here, the result was 20. Another example given was 7×5. There were 3 groups of 2 fingers which were worth 30, another remaining finger was worth 5, so in total the result is 35. This way, the students could memorize the results easily.

The Third Step: The Multiplication of Same Numbers

This was the third step to be ascended. Same three stages of learning process were applied here. The concrete and abstract stages were similar to previous multiplications and students had no difficulties at these stages.

At mental calculation stage the teacher asked if the students knew about the way to calculate the results mentally. The students had no idea how to do this. The teacher proceeded by firstly reminding the students that 1×1, 2×2, 5×5, 9×9 and 10×5 had been memorized previously, so they only needed to focus on 3×3, 4×4, 6×6, 7×7 and 8×8. 3×3 was calculated as follows: $3 + 3 = 6$, whereas 4×4 was done by first calculating $4 + 4 = 8$ and then $8 + 8 = 16$. These calculations were done repetitively by students until they could memorize the results well. 6×6, 7×7 and 8×8 were calculated using fingers. First, consider two numbers a and b such that $a + b = 10$. Then a was called the 10's pair of b and vice versa. For example, 4 was the 10's pair of 6. So for 6×6, the teacher showed 4 fingers (the 10's pair of 6) of each hand. There were 2 folded fingers which represented 20. The unfolded fingers which were 4 in each hand were then multiplied by each other, $4 \times 4 = 16$. Hence, the result was $20 + 16 = 36$. Similarly for 7×7 and 8×8 (see Figure 7).

6 × 6 7 × 7 8 × 8

Figure 7. Using Fingers for 6×6, 7×7 and 8×8 (Source: Surya, 2013)

Students were encouraged to do these calculations using fingers repetitively until they could memorize the results well. They were very eager to do this. This would enable them to do mental calculation of this multiplication and provide the results instantly.

The Fourth Step: The Multiplication of Numbers 3 and 4

Same three stages of learning process were applied here. The concrete and abstract stages were similar to previous multiplication.

At mental calculation stage of multiplication of number 3, the teacher explained the way to calculate the results mentally to students. The teacher reminded the students that only 4×3, 6×3, 7×3 and 8×3 that were needed to be memorized here. 4×3 was calculated by first calculating $3 + 3 = 6$ and then $6 + 6 = 12$.

6×3, 7×3 and 8×3 are difficult to memorize. To memorize 6×3, 7×3 and 8×3 the teacher introduced a song called *"Lagu Perkalian 3"*, which can be translated as "The Song of the Multiplication of Number 3". The music used was of a children's folk song called *"Bintang Kecil"*, which can be translated as "The Little Star". The lyrics contained the results of multiplication of number 3. For example, when the word "three" was sung, the teacher showed her little finger on her left hand which meant to show that 3 is the result of 1×3. The song was intended to help student to memorize 6×3, 7×3 and 8×3 in particular, so the part of the song that mentioned the results of these multiplication had to be repeated. This was done until the students could memorize the multiplication well.

Another way was to use a card game. The teacher took out a card of number 6 and said "times 3", hence the students had to show a card of number 18. Another card taken out was 7 and the teacher said "times 3", hence the students had to show a card of number 21, similarly for 8 times 3.

At the end of this session students were expected to be able to perform mental calculation and provide the results instantly.

At the mental calculation stage of multiplication of number 4, students mentioned that they had already memorized 1×4, 2×4, 3×4, 4×4, 5×4, 9×4 and 10×4. So they only needed to memorize 6×4, 7×4 and 8×4. The teacher then asked the students if they could guess how to do this. Some said by singing a song, which was a correct guess. Another song and lyrics similar to the song of multiplicaation of number 3 were introduced. The teacher then trained the students on how to memorize 6×4, 7×4 and 8×4 in particular by singing the song and using fingers (similar to the way they did in the learning of multiplication of number 3).

The Fifth Step: The Multiplication of Numbers 8, 7 and 6

This was the last step to be ascended in order to reach the critical point for multiplication in Matematika GASING. Same three stages of learning process were applied. The concrete and abstract stages were similar to previous multiplication.

At the mental calculation stage of the multiplication of number 8, there were only 6×8 and 7×8 left to memorize. Students had noted this. They then suggested that these could be memorized by using fingers, singing a song, recognizing patterns. The teacher then explained that these could be memorized by using fingers (similar to multiplication of two same numbers). Another way to memorize 7×8 is to observe 78 56 (the order was reversed from 56 78). Whereas for 6×8, we could recall that $5 \times 8 = 40$, add 8 to it to get 48.

At the mental stage of the multiplication of number 7, only 6×7 was left to memorize. The students then suggested all sorts of ways to memorize this. However, the teacher told them that since there was only one multiplication left, they just had to memorize this directly. This ended the learning process as there were no more multiplication left to go through and students had reached the critical point for multiplication in Matematika GASING.

It was noted that at the end of every learning of each multiplication, students were orally drilled with questions by the teacher (the drilling activity was not included in the conjectured local instructional theory). Moreover, students were very enthusiastic and very active at trying to answer the questions instantly. It was also noted that a fun and exciting environment was created when learning multiplication of numbers from 1 to 10 using Matematika GASING.

Retrospective Analysis

After conducting the teaching experiment, the results of the tests given during the teaching experiment were analyzed. A few things were also concluded by comparing the conjectured local instructional theory with the actual learning process.

One of the research questions was to find out the computational competence of the students in multiplication of numbers from 1 to 10 using Matematika GASING. To measure this, there were written and oral tests given during the teaching experiment. 100 questions was given in the written test. Score of 1 was given to a correct answer and 0 was given to a wrong answer. See Table 2 for the results of this written test.

There were two things that could be analyzed from the results, they were the scores and the time taken to do the test. The average score was 94. The test was done under 5 minutes 6 seconds on average. There were 6 students who were able to score perfectly in 3 minutes and 30 seconds on average. This showed 94% of competency level in doing multiplication of numbers from 1 to 10 using Matematika GASING in writing.

50 questions out of 100 questions of the written test were chosen and used to test the students orally. Each student was individually tested, did the calculations without using any aid and provided the answer orally and instantly. The score of 1 was given for a correct and instant answer and 0 was given otherwise. See Table 2 for the results. The average score was 42.50 out of 50 and the average time was 3 minutes and 8 seconds. This showed 84% of competency level in performing calculation of multiplication of numbers from 1 to 10 orally and instantly.

Table 2. The Results of the Tests

Sample	Computational Capability				Teaching Capability	
	Written Test		Oral Test		Microteaching test	Written test
	Score	Time	Score	Time	Score	Score
S1	100	2’03”	49	1’55”	97.5	55
S2	97	2’26”	46	1’58”	70	25
S3	100	4’32”	44	2’46”	95	65
S4	100	4’00”	49	1’55”	82.5	35
S5	100	2’43”	46	2’24”	95	75
S6	100	3’00”	46	2’29”	87.5	65
S7	99	3’52”	39	2’33”	90	45
S8	70	8’00”	38	4’44”	85	20
S9	91	7’34”	36	4’24”	85	50
S10	94	7’03”	43	3’16”	82.5	50
S11	100	4’45”	44	2’39”	92.5	40
S12	92	8’00”	34	4’37”	82.5	20
S13	95	6’19”	40	3’53”	90	50
S14	78	7’02”	37	4’25”	27.5	15
Average	**94**	**5’06”**	**42.50**	**3’08”**	**83.04**	**43.57**

It could be concluded from both results that the level of competency of students in doing multiplication of numbers from 1 to 10 using Matematika GASING in this case is 89%.

Another research question was to find out the students’ capability to teach multiplication of numbers from 1 to 10. There were two tests given to measure this during the teaching experiment; they were a written test and a microteaching test. From these tests, the students’ level of absorbance of the materials could also be seen.

There were 5 topics of multiplication of numbers from 1 to 10 chosen for the microteaching test; they were multiplication of numbers 10, 9, 4, 8 and multiplication of two same numbers. Each student was assigned to explain and to redeliver the materials of one of these five subtopics. Each student was given 20 minutes to do the microteaching.

An assessment rubric was developed by the researchers by collaborating with the teachers. The assessment scores assigned to each sample by two different researchers were consistent. There were 4 criterias to be assessed and the scores were 1 to 5 for each criteria (see table 3). The average score of microteaching test was 83.04. However there was one student, S14, whose score was very low, 27.5. S14 had not absorbed nor understood the topic well and hence could not deliver the materials. Her score in both written and oral tests which measured computational competence were below average. It could be concluded that S14 was weak even in addition; S14 had a difficulty in calculating the results of multiplication of number 4 by repetitive addition. The stages of learning in GASING is very important; in order to master a particular topic, a student must first master previous topics. In this case, S14 would have to go back to addition and restart from there.

Table 3. Assessment Rubric for Microteaching Test

	5	4	3	2	1
Delivering the materials clearly according to the correct stages in Matematika GASING	Able to deliver all materials perfectly without any mistakes according to stages in GASING	Able to deliver all materials correctly without any mistakes but not perfectly	Only able to deliver correctly 2/3 of all materials	Only able to deliver correctly 1/3 of all materials	Unclear delivery of materials and stages of GASING are not obvious
Giving suitable examples that can help students understanding the material	Giving relevant and complete examples (both at concrete and mental calculation stages)	Giving only some relevant examples (both at concrete and mental calculation stages)	Giving only some relevant examples (at concrete stage only or at mental calculatio n stage only)	Giving irrelevant examples	Giving no examples
Giving questions about the mathematical content (1×1, 7×8, etc.) to students	Giving questions at every topics perfectly according to the stages in GASING	Giving questions at every topics according to the stages in GASING but not perfectly	Giving questions at 2/3 of the materials	Giving questions at 1/3 of the materials	Not giving any questions
Giving emphasis in certain things during learning	Giving an emphasis perfectly in	Giving an emphasis in every part of	Giving an emphasis in 2/3 of	Giving an emphasis in 1/3 of	Giving no emphasis

process that enable students to understand the material better	every part of material according to the stages in GASING	the material according to the stages in GASING but not perfectly	the material	the material	

In general however, the scores were relatively high. There was no perfect score, but the highest one was 97.5. The common mistake made when explaining how to perform mental calculation or mental arithmetic as the last stage of each topic in Matematika GASING. For example, S3 only explained how to memorize multiplication of number 9 using fingers. S3 forgot to explain further that it could also be memorized by recognizing patterns. This suggested the needs to give emphasizes about ways to memorize multiplication of numbers from 1 to 10 at the mental calculation stage during the learning process.

Students were also given a written test which measured their capability to teach. The test was composed of 5 questions and were consulted with an expert of Matematika GASING at STKIP Surya. The aspects assessed in this written test were: (1) understanding the order of the steps to be ascended to reach the critical point for multiplication in Matematika GASING, (2) understanding the multiplication concept, (3) understanding how to be able to memorize and mentally calculate multiplication of number 9, (4) understanding the stages of learning multiplication of two same numbers in Matematika GASING, (5) understanding multiplication of numbers 3 and 4 in Matematika GASING. Students were given an hour to do the test. The assessment was done by developing another assessment rubric, the scores are 0 to 4 (see table 4). The scores given to each answer to each question by each student were consistent.

The average was 43.57, which was a weak result. This contrasted the results of the microteaching test. Further observations of the students' written work and the videos of students' microteaching test sessions were then conducted. According to these, students tended to find it more difficult to write the learning process of the materials as opposed to delivering the materials orally. It is possible that students may have had a problem in writing skills especially in mathematical writing.

Table 4. Assessment Rubric for Written Test to Measure Teaching Capability

Score	Criteria
4	The answer is substantially correct and complete
3	The answer contains one mistake or a significant omission
2	Some parts of the answer are correct, but there are one or more mistakes or significant omissions
1	Most parts of the answer are not complete, but there is at least one correct part
0	The answer is incorrect or no answer

CONCLUSION

Matematika GASING was able to create a fun and exciting environment when learning multiplication of numbers from 1 to 10. Students and the teacher were able to enjoy the learning activities together. The students participated in the learning activities enthusiastically and actively tried to answer the questions given by the teacher. They often competed to provide the right answers orally as instantly as possible during mental calculation stage. Matematika GASING helped students to understand the concept of multiplication better. In general, students were able to calculate and perform mental calculation of multiplication of numbers from 1 to 10 as well as teaching the materials relatively well. In the case where a student performed poorly in the tests suggesting that previous materials were not fully understood, the student should then go back to the previous material and restart from there.

The fact that the students had a problem with their mathematical writing skills suggests further research that could be done to investigate this problem further. Possible factors that may have influenced this could include language, writing skills in general and so on.

It was noted that the revised local instructional theory should contain added instructional activities such as drilling at the end of every session (at the end of the learning process of each topic in multiplication of numbers from 1 to 10). Drilling with questions of the materials should allow students to produce the results of these multiplication orally and instantly by performing mental computation or mental arithmetic. It could also be improved by adding learning activities that focus on the emphasis of the ways to memorize multiplication of numbers from 1 to 10 using Matematika GASING. Moreover, concrete objects such as boxes and items may be used at the concrete stage of the learning of each multiplication prior to using picture cards so that students could experience the actual materials and grasp the idea better.

Further research may be done in the next academic year where the revised local instructional theory could be used as a guidance. A possible refinement of the revised local instructional theory may also take place then.

Acknowledgement. We would like to express our gratitude to the tutors, Novi Murniati, S.Si. and Indra Bayu Muktyas, S.Si. at TIK matriculation class for their help in conducting the teaching experiement for this research. We would also like to thank the students. We are also thankful to Ali Godjali, Ph.D. for his consultation about Matematika GASING.

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