IndoMS. J.M.E
Vol.1 No. 1 Juli 2010, pp. 17-40

LEVELING STUDENTS' CREATIVE THINKING IN SOLVING AND POSING MATHEMATICAL PROBLEM

Tatag Yuli Eko Siswono

Abstract

Many researchers assume that people are creative, but their degree of creativity is different. The notion of creative thinking level has been discussed .by experts. The perspective of mathematics creative thinking refers to a combination of logical and divergent thinking which is based on intuition but has a conscious aim. The divergent thinking is focused on flexibility, fluency, and novelty in mathematical problem solving and problem posing. As students have various backgrounds and different abilities, they possess different potential in thinking patterns, imagination, fantasy and performance; therefore, students have different levels of creative thinking. A research study was conducted in order to develop a framework for students' levels of creative thinking in mathematics. This research used a qualitative approach to describe the characteristics of the levels of creative thinking. Task-based interviews were conducted to collect data with ten 8th grade junior secondary school students. The results distinguished five levels of creative thinking, namely level 0 to level 4 with different characteristics in each level. These differences are based on fluency, flexibility, and novelty in mathematical problem solving and problem posing.

Keywords: student's creative thinking, problem posing, flexibility, fluency, novelty

INTRODUCTION

Many researchers assume that people are creative, but their degree of creativity is different (Beghetto & Kauffman, 2009; Craft, 2003; Isaksen, 1987; Lumdaine & Lumsdaine, 1995; Pehkonen, 1997; Solso, 1995). That is shown by someone being able to create ideas, technology or knowledge whilst others merely use these or just accepted. Mathematics creative thinking is a combination of logical and divergent thinking which is based on intuition but has a conscious aim (Pehkonen, 1997). When one is applying creative thinking in a practical problem

A Mathematics Lecturer in Surabaya State University

solving situation or problem posing context, divergent thinking produces many ideas. Some of these seem to be useful for finding solutions. A process of logical thinking is used to examined an appropriate solutions. A logical thinking involved systematic and rational process to verify and make a valid conclusion. Divergent thinking is focused on flexibility, fluency, and novelty in mathematical problem solving and problem posing (Haylock, 1997; Krutetskii, 1976; Haylock, 1997; Silver, 1997). This observation points to the existence of different levels or degrees of creativity or creative thinking for different students. The purpose of this paper is to provide a hierarchical framework for distinguishing students' levels of creative thinking during mathematical activities such as problem solving and problem posing.

The idea of level of students' creative thinking has been examined by experts. De Bono (in Barak & Doppelt, 2000) defined four achievement levels of creative thinking skills development. These are awareness of thinking, observation of thinking, thinking strategy, and reflection on thinking. Awareness of thinking is a general awareness of thinking as a skill. Someone has a willingness to think about something, to investigate a particular subject, and to listen to others. Level 2 indicated as an observe the implications of action and choice, consider peers' points of view, comparison of alternatives. Level 3 is classified as thinking strategy which is intentional use of a number of thinking tools, organization of thinking as a sequence of steps; and reinforcing the sense of purpose in thinking. Level 4 is reflection on thinking which involves structured use of tools, clear awareness of reflective thinking, assessment of thinking by the thinker himself, and planning thinking tasks and methods to perform them. These levels are based on the viewpoint that creative thinking is a synthesis of lateral thinking and vertical thinking, each complementing the other. Lateral thinking refers to discovering new directions of thinking in the quest for a wealth of ideas, whilst vertical thinking deals with the development of ideas and checking them against objective criteria (De Bono, 1995). Barak & Doppelt (2000) used these levels for assessing students' portfolio. A portfolio is a record of a pupil's learning process: what a student has learned and how he or she has gone about learning; how he or she thinks, questions, analyzes, synthesizes, produces, and creates; also, how one interacts intellectually, emotionally, and socially. So, these levels try to capture the breadth students' characters in many activities but are too general and not easily recognized in mathematical problem solving and problem posing. However, Barak & Doppelt (2000) still consider the product design such as originality, authenticity, usefulness, unique design and functionality, reliability, accuracy, geometric structure, scientific principle.

Gotoh (2004) described three stages of development of mathematical thinking in problem solving. There is the empirical or informal activity (stage 1), the algorithmic or formal activity (stage 2), and the constructive or creative activity (stage 3). In the first stage, some kind of technical or practical application of mathematical rules and procedures is used to solve problems without a certain kind of awareness. The second stage, mathematical techniques are used explicitly for carrying out mathematical operations, calculating, manipulating and solving. The third stage, a non-algorithmic decision making, is performed to solve non-routine problem such as a problem of finding and constructing some rule.

In similar terms, Ervynck (cited in Sriraman, 2004) presented three stages of mathematical creativity: preliminary technical stages (stage 0), algorithmic activity (stage 1), and creative (conceptual, constructive) activity (stages 2). The preliminary technical stage consists of some kind of technical or practical application of mathematical rules and procedures, without any awareness of the theoretical foundation from the user. Algorithmic activity consists primarily of performing mathematical techniques, such as explicitly applying an algorithm repeatedly. Creative activity consists of non-algorithmic decision making.

Neither Gotohnor Ervynck discussed the levels of students' creative thinking. These described the characteristics students in which work out the mathematics problems. However, these stages could be used to classify and categorize students' creativity when we believe that students have potential creativity. When they are in stage 1 or 2, for example, we might consider them as not creative or quite creative respectively.

Krulik & Rudnick (1999) described levels of thinking as recall, basic, critical, and creative thinking. Recall includes those skills that are almost automatic or reflexive. Basic includes the understanding and recognition of mathematical concepts like addition and subtraction as well as the application of these in problems. Critical thinking is thinking that examines, relates, and evaluates all aspects of a situation or problem. Creative thinking is thinking that is original and reflective and that produces a complex product. Krulik & Rudnick (1999) classified students' reasoning in trying to understand mathematical problems and overcome all aspects of situations and conditions hierarchically. These levels assign to students in different positions when they were not creative using levels, they might be categorized as critical. It will be difficult to distinguish them because they could be viewed as a complementary aspect or a different domain. Critical and creative thinking is discussed as if students should succeed solving a complex mathematical problem. However, these levels pointed out that a hierarchy of thinking might exist in mathematics classrooms and creative thinking is indicated by originality and producing products of thinking.

Silver (1997) pointed out that a suitable approach to identifying students' creative thinking was to use problem solving and problem posing. The three components (fluency, flexibility, and novelty) of creative thinking respectively assess different parts of thinking and are independent of each other. Students have various backgrounds and different abilities. They possess different potentials in thinking patterns, imagination, fantasy and performance. Consequently, it is reasonable to posit that students have different levels of creative thinking. A student may demonstrate all three components, two components, or only one component during problem solving and problem posing.

Problem posing activities have positive influences on students' ability to solve or resolve mathematical problems and provide a chance to gain insight into students' understanding of mathematical processes and concepts (Bonotto, 2009; Christou, et.al. 2005; English, 2003; Leung, 1997). In these studies, mathematics problem posing is a task which asks students to pose or construct a mathematical problem based on given information, and then solve the problem.

Problem solving is often seen as one of a number of skills to be taught in mathematics classrooms. Problem solving consists of activities such as understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1957). A problem for a task situation is where students are required to connect the known information in a way that is new (for them) to do the task (Pehkonen, 1997).

In previous work the author has developed a set of levels for students' creative thinking in terms of problem posing in mathematics (Siswono, 2004). These levels emphasized the product and the creative process. The product of creative thinking that is, the problem solution or the problem posed, allows the researcher to determine the presence of the three aspects that are fluency, flexibility, and novelty (Haylock, 1997; Krutetskii, 1976; Pehkonen, 1997; Silver, 1997). The creative process involves mechanisms such as the synthesis of ideas, generating new ideas, and applying ideas (Airasian et al, 2001; Isaksen, 2003; Lumsdaine & Lumsdaine, 1995; Krulik & Rudnick, 1999). The description of these levels is as the following:

Level 5: Result of students' task satisfies *all* criteria of creativity product. Student can synthesize ideas, generate new ideas from mathematical concepts and real life experience, and apply the ideas to construct some problems. Student is also able to revise it when a hindrance is met.

Level 4: Result of students' task satisfies *all* criteria of creativity product. Student can synthesize idea; generate new ideas from mathematical concepts but few from *real life experience*. Student is able to apply the ideas to construct some problems and is able to revise these when a hindrance is met.

Level 3: Result of students' task satisfies *all* criteria of creativity product. Student can synthesize ideas, generate new ideas *only* from mathematical concepts, and apply the ideas to construct some problems. Student is also able to revise these when a hindrance is met.

Level 2: Result of students' task satisfies *just one or two* criteria of creativity product. Student can synthesize ideas from mathematical concepts or real life experience, and generate new ideas from either mathematical concepts or real life experience, but not both. Student hasn't applied all ideas to construct some problems, but is able to revise a problem when a hindrance is met.

Level 1: Result of students' task satisfies *just one or two* criteria of creativity product. Student can not synthesize ideas from mathematical concepts or real life experience, and generate new ideas *only* from mathematical concepts or real life experience. Student hasn't applied all ideas to construct some problems, or revised a problem when a hindrance is met.

Level 0: Result of students' task did *not* satisfy *any* criterion of creativity product. Student can not synthesize ideas from mathematical concepts or real life experience, and can't generate new ideas. *Student merely recalls* own ideas.

These levels were verified with a total of 128 students at two junior high schools at grade 7. They were 41 students in Class I-A, 40 students in Class I-I of Junior Secondary schools of SMP Negeri 4, 43 students in Class I-D and 44 students in Class I-E of SMP Negeri 26 Surabaya. All students were given the problem posing tasks for which the information is based on a picture/diagram or word problem (verbal) situation using a semi-structured situation (Stoyanova, 1998) similar to other problem posing studies (e.g. Christou et.al., 2005). If students showed evidence of satisfying all three aspects (i.e., fluency, flexibility, novelty), they are said to be creative; if they satisfied one or two aspects, they are said to be quite creative; if they do not satisfy any aspects, they are said to be not creative. Of 167 participants, 24 were said to be creative, 127 quite creative and 16 not creative. Subsequently, 23 students of the creative group, 22 students of the quite creative group, and 6 students of the not creative group were chosen to be interviewed in-depth. Students in the creative group were interviewed more than others, because they gave much more information and further questioning was needed to establish levels. Interviews gave insight into the creative process of students, so the students' levels of creative thinking could be determined exactly.

Interviewed students were able to be placed at all levels, there were 6 students at level 5; 4 students at level 4; 13 students at level 3; 13 students at level 2; 9 students at level 1; and 6 students at level 0. This was not impacted by students' achievement level or gender. These results indicated that problem posing tasks were appropriate for classifying the level of students' creative thinking as indicated by Leung (1997) and Silver (1997. Other research by Siswono & Novitasari (2006) has shown that problem posing activities using the "what's another way?" strategy could improve students' abilities in creative thinking. However, using problem posing has some shortcomings. As teaching using problem posing tasks is not common in Indonesian classrooms, when students face these tasks in a research situation, they could not understand directly what they were doing. Students always needed encouraging to construct a complex problem. Sometimes, they constructed many problems but by using the same pattern and easy (common) problems such as what is the area of a rectangle or what is the perimeter of a rectangle? (Siswono, 2004). Therefore, problem posing as a stand alone activity makes less sense in mathematical activity than a situation when it is combined with problem solving. Reflecting on these conditions, I considered problem solving might be a better strategy for classifying and categorizing of students' creative thinking. Problem solving already had been considered as promoting creativity by others (Davis, 1984; Haylock, 1997; Hwang, et.al., 2007; Pehkonen, 1997).

As indicated by Silver (1997), creative thinking was not only in mathematical problem posing, but also emphasized in mathematical problem solving, it was prudent to think that these levels may be in need of revision when used in problem solving situations. I then hypothesized that levels of creativity should include the three components of flexibility, fluency, and novelty, but in both mathematical problem solving and problem posing. It was proposed that the new draft levels of creative thinking (LCT) would consist of 5 levels, namely level 4, level 3, level 2, level 1, and level 0. Students in level 4 satisfied the three aspects (i.e. fluency, flexibility, and novelty); level 3 if students were able to show novelty and flexibility, or novelty and fluency; level 2 if students were able to show novelty only, or fluency and flexibility; level 1 if students were able to show flexibility or fluency not both; level 0 if students did not show any of the three aspects. These levels emphasized divergent thinking where the highest position was novelty, then flexibility and the lowest aspect was fluency. Novelty was placed at the highest position because it was believed to be the main characteristic to assess the product of creative thinking (Isaksen, 2003; Isaksen & Puccio, 1988; Lumsdaine & Lumsdaine, 1995; Solso, 1995). Flexibility was placed at the next important position because it referred to the manner of production of some ideas which were used to overcome a task. Students express and justify in one way, then other ways. Students also pose problems that are solved in different ways. Fluency was indicated when the student produced and generated different ideas which were appropriate to the question task. Students explored problems with many solutions and answers, then generated many problems to be solved.

The draft of the LCT was verified by using student data (Siswono & Budayasa, 2006) as the initial research. It was found that students displayed characteristics of LCT in levels 4, 1 and 0, but not in levels 2 and 3. Even though not all levels were evident among the students, there was enough to suggest further investigation of the theory was warranted. In this present research the draft framework is revised so that novelty and flexibility are important components, but not higher than others as in the previous theory. Flexibility and novelty can be rewarded as features of creative thinking in mathematics classroom including for solving or posing problems. This set of levels is called the revised draft of creative thinking levels. They consist of 5 levels: level 4 if students have satisfied the three aspects (i.e., fluency, flexibility, and novelty) or novelty and flexibility only; level 3 if students are able to show flexibility and fluency, or novelty and fluency; level 2 if students are able to show novelty or flexibility but not both; level 1 if students are able to show fluency; level 0 if students are not able to show any of the three aspects. The existence of there 5 levels is a hypothesis which will be verified in the mathematics classroom.

The focus of this research is to describe the characteristics of students' creative thinking levels. In this research, creative thinking is taken to be the mental process which someone uses to come up with "new" ideas as fluently and flexibly as possible. By "Idea" is meant a thought for solving and posing a problem. The quality of problem solving and problem posing instructional tasks with regards to creativity used the three components of creative thinking (Silver, 1997). However, these were modified in order to overcome difficulties of overlapping with other definitions. Students demonstrate *fluency in problem solving* when they are able to obtain many solutions. As an example: "Given a rectangle 12 centimetres x 8 centimetres draw some plane figure with an area the same as the rectangle!" Students' responses might be o draw a triangle with base 8 centimetres and length 24 centimetres; triangle with base 12 centimetres and length 16 centimetres; parallelogram with base 4 centimetres and length 24 centimetres; or trapezoid with two parallel sides 4 centimetres and 8 centimetres, and the length is 16 centimetres. Students explore open ended problem with many interpretations or answers.

Students were said to be *flexible in problem solving* when they were able to solve a problem using many different methods or express a solution in one way then in other ways. An example from the previous example, the next task is "How do you find them? Use other ways presenting the solution!" Students might think and give a reason firstly "I use and modify the formula of area of a rectangle to be the formula of other plane figures. I use these sizes drawing the plane figure". Then use other ways such as "I use a picture of the rectangle. Just transform it to become an other plane figure" or "I am cutting a rectangle to be other shapes". Students explain and present different strategies drawing triangles, trapezoids, parallelograms, or combining forms.

Students were said to demonstrate *novelty in problem solving* when they were able to examine a problem and answer with many solutions or answers, then generate another that is different. Other solutions are considered to be "different from others", when they have a different pattern or are not usual for students of this grade level, such as students can construct a combination of other shapes, or they could be finding an original solution which is not common for that student's grade or their knowledge level. From the previous example, when students produce rectangles with different sizes such as 6 centimetres x 16 centimetres, 4 centimetres x 24 centimetres, or 2 centimetres x 48 centimetres, or create triangle with base and length 12 centimetres, 16 centimetres; 8 centimetres, 24 centimetres; 4 centimetres, 48 centimetres; they can not be said to be showing novelty because their solutions follow certain patterns. Students show a novel solution if they have drawn a triangle with base 12 centimetres and length 16 centimetres, then a trapezoid with the lengths of parallel sides as 4 centimetres and 8 centimetres and the height 32 centimetres, and combine shapes of triangle and rectangle with the total area of 96 square centimetres.

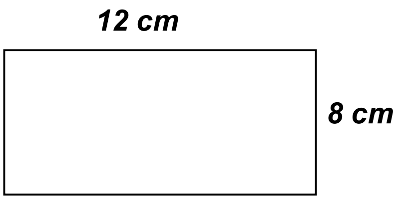
Silver (1997) explained that *fluency in problem posing* is indicated by an ability of students to generate many problems with correct solutions. An example from the previous task, occurs when students are asked to "Construct two problems related to the rectangle above and solve them!" Students might construct problems such as "Find the area of the rectangle!", "What is the perimeter of triangle?", "What is the length of the diagonal?", or "A photo frame has rectangle form with size 12 cm and 8 cm. What is the area of it?" Here, students make the solution relevant with the problems. Students were said to be fluent, if they ask or pose the problems and solve themselves. The problems might have similar concepts, but using different attributes of the problem is a common indicator of fluency which is recognized by many students.

Flexibility in problem posing also refers to a student's ability to pose or construct problems with divergent solutions. A follow up task to the previous example is: "Examine your initial problems. Was there a problem with many different solutions or ways? If there was, you have to express other solutions and if there was not, you have to create the problem with different solutions or ways". Students might have created the problem "The picture was 12 cm x 8 cm, what size of frame could be needed by this picture?" The students might then explain there were many solutions such as the size of frame could be 10 cm x 12 cm, or 16 cm x 12 cm by adding 1 cm or 2 cm to the length and width of the picture. To answer in a different way is to use twice the area of the picture which is 192 square centimetres, so the size might be about 14 cm x 14 cm.

Novelty in problem posing refers to a student's ability to pose or construct a problem different from others. Students examine several posed problems then pose a problem that is different in contexts and concepts or is unfamiliar for them. When students pose problems such as finding area, perimeter, or diagonals, they have not shown that they have produced novel problem, yet. However, students could be considered to be able to design a novel problem, when they create a problem such as: "Many decorated stickers of size 12 x 8 cm^2 were used to cover a floor with area 96 m^2. How many stickers were used to cover without overlapping among them?" Then they solve correctly using one way or many different ways. This problem is different in mathematical context and content from previous problems. Several students might be unfamiliar with this problem.

METHOD

The research approach is qualitative aiming to identify the characteristics of the levels of students' creative thinking in mathematics to verify the draft of the LCT. Data were collected through two task-based interviews with thirteen 8^{th} grades of Junior Secondary School students. Snowball sampling (Licthman, 2009; Merriam, 1998) was used to select ten students from Junior Secondary School at Sidoarjo and three students from one private school in Surabaya. Students were aged 12-13 years old and consist of five boys and eighth girls. The students were from the higher ability group in mathematics at their school as determined by teacher nomination and they had good communication skills a requirement by the researcher for their selection. The task for interview is an open-ended task which is divergent in its solution and methods. The task is as follows:
Given the rectangle below



a. Draw some plane figures with area equal to the rectangle!

b. Construct a minimum of two other shapes with area equal to the rectangle above.
c. Examine one shape and explain your way to find the solution? Use an other way to present the solution!
d. Create at least two problems related to a rectangle and find the solutions!
e. Examine your initial problems. Was there a problem with many difference solutions or ways of solving? If there was, you have to express other solutions and if there was not, you have to create the problem with different solutions or ways of solving.

(Second task is modified from these which discuss about perimeter of rectangle such as the question: Draw some plane figures with perimeter equal to the rectangle. Construct a minimum of two other shapes with perimeter equal to the rectangle.)

Triangulation is conducted by giving students a second equivalent task and interviewing them again deeply. The students' work is analyzed by identifying the correctness of the answers then checking for aspects of creative thinking (fluency, flexibility, and novelty) in problem posing and solving. Data were analyzed by the method of constant comparison (Licthman, 2009; Merriam, 1998). For the following, students' level was estimated by applying a qualitative analysis method to determine the level of a student's creative thinking.

RESULTS

Although there were 13 students from sampling process; it turned out that 10 students provided the best data to inform characteristics of students' creative thinking. All samples are shown in Table 1.

Table 1: Characteristics of Students' sample

No.	Pseudonyms	Math Score	IQ	Gender	LCT
1	Adi	93	102	Male	0
2	Bejo	89	108	Male	0
3	Jono*	87	108	Male	0
4	Nini	93	109	Female	1

5	Titi	90	108	Female	1
6	Tono*	90	110	Male	1
7	Rini	91	102	Female	2
8	Wati	90	--	Female	2
9	Susi	93	108	Female	3
10	Rudi	82	107	Male	3
11	Warni*	98	110	Female	4
12	Siti	90	105	Female	4
13	Tira	89	114	Female	4

Note: *They are not including in this description

In the following the products of several of these students' problem solving and problem posing is described some students identifying and how the characteristics of them were used to place them at those levels.

Adi is a boy from the highest group with mean mathematics score of 93 (score interval 0-100) and with an IQ score of 102 (Using the Stanford-Binet scale and conducted by Psychometric Institution). His work is categorized at level 0 of creative thinking. I give, shows his response to task 1.

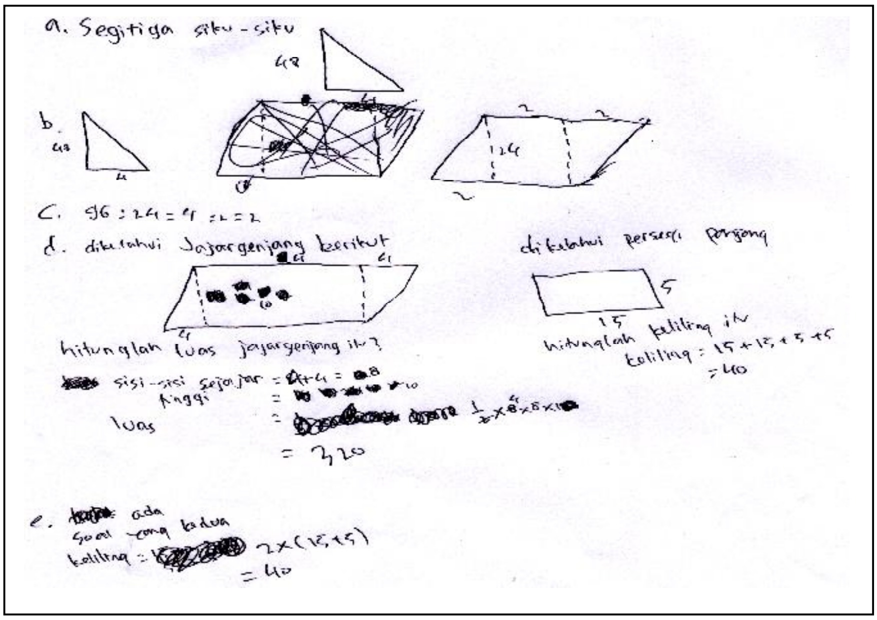


Figure 1: Adi's Solutions of First Task

Adi made a triangle with base 4 and the height 48. He then drew a triangle again, and parallelogram with base 4 and the height 24. It is unclear whatever the student is displaying fluency or not. He was not considered to be showing a flexible strategy, because he expressed one strategy that is using the formula of area to find the size of the shape. So, it needs clarification. He doesn't make a novel solution, because his solution is common for his classroom level. In problem posing, Adi is fluent in constructing a problem but the method for solving his own problem is wrong. He used as the formula for the area of a parallelogram ½ of b x h. He didn't make a problem with many different ways and a novel problem, because Adi used a common problem in mathematics context and didn't connect with the real life or camouflage the situation. Using this data, Adi is placed at level 0 or level 1. It is possible his work is at level 1 because he might be fluent in solving and posing a problem. Adi was interviewed further without showing his initial work with the same task.

Based on data from interview and the paper test, Adi was not fluent making many solutions, did not develop different methods or ways to solve a problem, and did not express other solutions. However, he could create many problems although the problems were common and did not have different solutions. These data are still unclear, so a second task was used confirming this situation. Subsequent to activities using the second task and interview data, finally Adi was placed in level 0, because he did not perform fluently, flexibly, or construct a novel solution or problem.

Using same process, Bejo is also in level 0. In the classroom he is in the medium group of students in his classroom with a mean mathematics score of 89 and IQ score of 108. Adi and Bejo have similar and different characteristics. The characteristics of the two students were compared using the method of constant comparison. Neither was able to generate alternative solutions and create problems fluently and flexibly, and then they did not develop novel solutions and novel problems. Moreover, they could not construct solutions because of a lack of remembering formulae area of shapes. They thought that constructing problems was easier than solving problems and different representations of formulae such as $P = 2(p+l)$ and $P = 2p + 2l$ were different methods or solutions. Actually, these are not different. Adi was, however different in character in performing problem solving and posing problems from Bejo. Adi could solve some problem but he was not fluent posing a problem. His problems always involved mathematical content without it being related to daily life. This condition is opposite to what Bejo did. He was able to make an easy problem connected with real situations. Adi did not remember exactly the formula of parallelogram but he could find this using a combination formula for area of rectangles and triangles.

Following their completion of two tasks and being interviewed twice, Nini and Titi were considered as being at level 1. In made a solving the second task, Nini made a triangle but the size is wrong. It is impossible to use 15 cm, 20 cm, and 5 cm as side of the lengths a triangle. However, from interviewed and confirmed, she realizes then can answer questions. She could be hypothesized to be in level 1 or level 0 because she didn't show flexibility and novelty in problem solving and posing. The researcher gave her first task and in-depth interview. Finally she could be considered in level 1 because she is fluent to solve and pose problems common in her mathematics classroom. Titi is a girl in the highest group of students in her classroom with mean mathematics score f 90 and IQ score of 108. Nini and Titi have the same characteristics in regard to creative thinking. They were able to generate the alternative solution and create the problem fluently. Creating many problems is easier than finding many solutions but it does not mean that both are simple. When constructing a problem, they realized and anticipated the solutions that were different in problem solving. Nini need more time to think up the solutions than Titi. Titi just thought to find solutions was appropriate for the problems not to expand her mind.

Wati and Rini were placed at level 2. Wati from the first task created several plane figures such as a triangle with base 16 cm and the height 12 cm, trapezoid with the length of parallel sides 18 cm and 30 cm and the height 4 cm, and parallelogram with base 32 cm and the height 3 cm. She was alike to a novel solutions but she was still asking common problems like "what is the area of …" or "what is the perimeter of…". This was confirmed when her responses were triangulated with the other task and in-depth interviewed. She was struggling to solve and construct problems; however, she could create uncommon problems and find a solution. Rini was categorized as being at level 2 similar to Wati. Although this level is hypothesized as being indicated by students displaying novelty or flexibility but not both, no students yet have displayed just flexibility in solving and posing a problem. They think that constructing problems is more difficult than solving problems because they have to think for solutions, making good sentences, and mathematical content in relationship with the given information. The difference between them is that, Rini constructed a problem related to daily life (word problem) but Wati made sense in the mathematics concerned.

Budi and Susi were considered to be at level 3. In made a solving the first task Budi made a triangle with base 8 cm and height 24 cm, a rectangle with length 48 cm and width 2 cm, and another rectangle with length 32 cm and width 3 cm. He modified the plane figure using the formula of area for each the plane figures. He is fluent in constructing problems and can make a problem with many solutions. So, he might be displaying fluency, flexibility or novelty. After triangulation through interviewed and the second task, he was considered as being at level 3. He has abilities solving and posing problems flexibly and fluently. Susi displayed different characteristics of this level. She can produce a novel solution and create an original problem and she is fluent making other plane figures and producing problems. She makes a triangle, trapezoid, square, and combines a rectangle and a triangle. She can construct problems connected to daily life with themes such as the classroom, cloth, or a house. She also constructs common problems like the area of a rectangle or its perimeter. Both Budi and Susi are in level 3 because they show flexibility and fluency (i.e., Budi) or novelty and fluency (i.e., Susi). Budi always focuses on mathematical content without relating to daily life. Susi can make an easy problem connected with real situations. From second task, Budi construct the plane figure using two strategies. First, he draws a polygon and gives the size with the perimeters 40 cm equal of the rectangle perimeter. Second, he divides 40 cm to be units and combines them as the plane figures. Susi could construct a problem with many solutions with regard to using the formula or divided them in a other known figures and find the area for each.

At level 4, we have Tira and Siti. Tira from the first task creates plane figures such as a triangle with base 16 cm and height 12 cm, a trapezoid with the length of parallel sides 10 cm and 14 cm and height 8 cm, a combined triangle and rectangle, and combined trapezoid and rectangle (See in Figure 2).

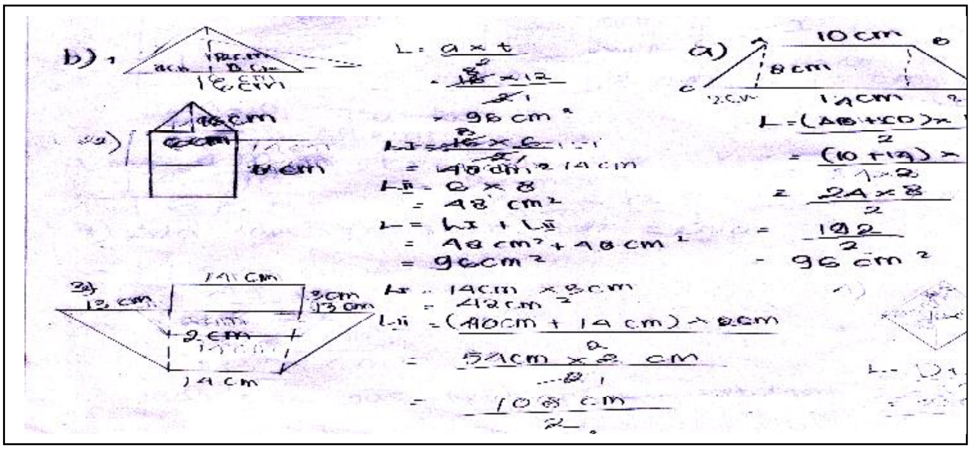


Figure 2: The First Task of Tira for parts a and b.

She has solved problems flexibly because she first used a plane figure and fixes appropriate sizes with the area equal to the area of triangle; then she uses using a paper as model of a triangle, and cuts to form other shapes (see Figure 5). This is seen in the interview below.

Interviewer: Okay. You can make the trapezoid that equals to a rectangle 12 x 8 cm like this. Explain… What was your strategy to make it?

Tira: I draw its shape....then trying checked and revised the sizes. Its height is 8 then...gives name *A*, *B*, *C*, *D*...*AB* is equal with 10 cm and *CD* is 14 cm...I check it using trapezoid formula. either this… 24 times 4 equal with 96 square centimetres.

Interviewer: Hm…yes good. Now, explain the other way to make this trapezoid! (She thinks for awhile, then say)

Tira: By cutting....

Interviewer: Well, could you demonstrate it?

Tira: (She takes a paper and writes down the sizes of the given rectangle and cuts it with scissors to make a trapezoid. Tira takes scissors herself without prompting by the researcher).

Figure 3: Tira makes cutting strategy

Tira can make other plane figures alike below (Figure 4).

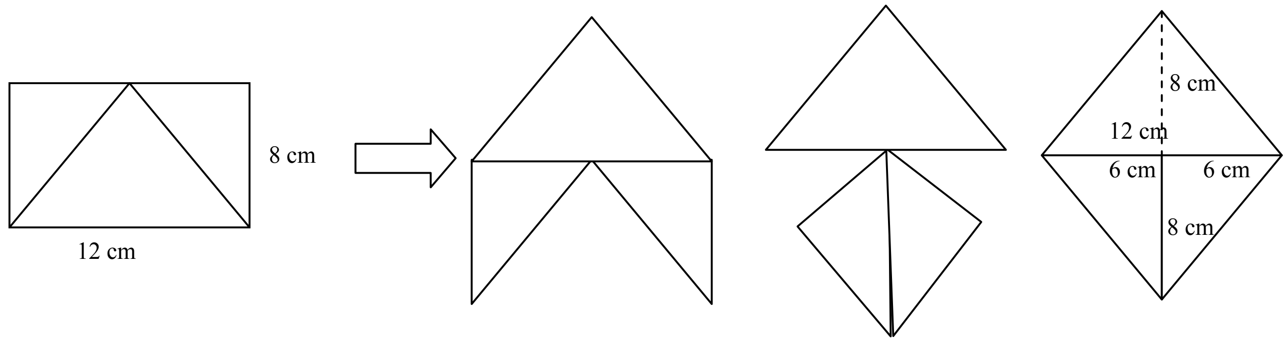


Figure 4: Different shapes of plane figures

Tira makes problems related to real life. She makes the form of polygon as a traffic sign, or a window. According to Tira, it is not hard to relate mathematics concepts and questions to daily life. Her responses are triangulated with those for the second task and she interview in-depth. She is still consistently to be fluent, flexible, and makes a novel solution and new problems.

Siti takes as long solving and posing problems as Tira. Siti can make novel plane figures like in Figure 5.

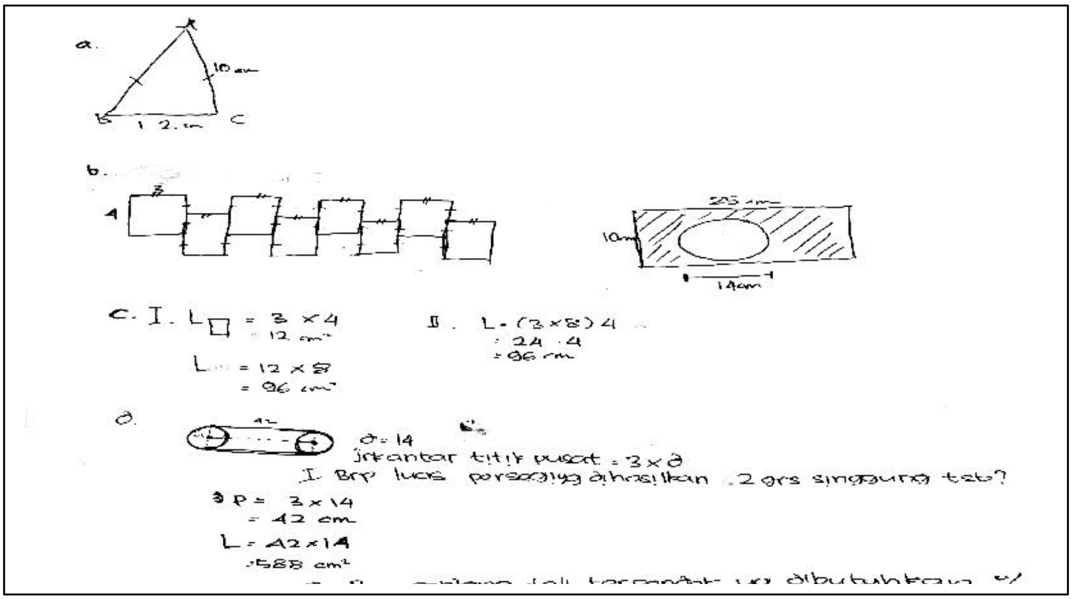


Figure 5: Siti's result of the First Task

She is flexible solving problems. Firstly, she determined the plane figure and gave sizes equal to the area of the rectangle. Secondly, she imagined the model of rectangle and made a signing area and sides then developed the plane figure without actual cutting it. She is fluent in creating problems although not those related to real life, because she thinks a sentence is too long and not easy to solve. To confirm these data and interpretations, her responses are triangulated with those to the second task and in-depth interviewed. She is considered to be at level 4. Tira and Siti think that constructing problems is more difficult than solving is problems because they must think of solutions, make good sentences, and mathematical content in relationship with the given information.

Based on these data, I considered the five hierarchical levels of students' creative thinking in mathematics classroom to have the characteristics shown in Table 2.

Level	Characteristic of Creative Thinking Level
Level 4 (Very Creative)	Students satisfied *all* components of creative thinking or only flexibility and novelty in solving and posing problems. They tend to say that constructing a problem is more difficult than solving a problem, because they must have a certain way to make solutions. They tend to say that finding the solution methods is more difficult than searching for other answers or solutions.
Level 3 (Creative)	Students were fluent and then they were flexible or demonstrate novelty, but not both in solving and posing problems. They tend to say that constructing a problem is more difficult than solving a problem, because they must have a certain way to make solutions. They tend to say that finding the solution method is more difficult than searching for other answers or solutions.
Level 2 (Quite Creative)	Students were able to show flexibility and novelty in solving and posing problems without fluency. They tend to say that constructing a problem is more difficult than solving a problem, because they are unfamiliar with the task and find it difficult to estimate numbers, formulae or solutions. They have understood that the different methods or strategies in solving problems can be represented by another formula with different representation but in reality these attempts are not different.
Level 1 (Almost Not Creative)	Students were able to show fluency without novelty and flexibility in solving and posing problems. They tend to say that constructing a problem is more difficult than solving a problem, because it depends on the complexity of the problem. They make problems mathematically without connecting to real life.
Level 0 (Not Creative)	Students were not able to show any components of creativity. They tend to say that constructing problems is easier than solving a problem, because they already know its solutions.

Table 2: Characteristics of a student's creative thinking level

DISCUSSION AND CONCLUSION

The characteristics of the levels of creative thinking as shown on Table 2 contained different aspects for each level. Although this level uses the indicator of divergent thinking as a major perspective, it was not determined to be general (Craft, 2003; Plucker and Zabelina, 2009). Leveling is assumed to be in the specific domain of the mathematical classroom when students partake in the activities of solving and posing problems. The author believes that creative thinking of students cannot be described only by giving levels to responses to assigned problems without concern for other perspectives. In addition students' creative thinking should not be decided without determining interaction between individual, domain, and field the so called the "systems approach" (Sriraman, 2004). Using divergent thinking as an indicator of creative thinking will be useful as a stepping stone to changing the paradigm to one that encourages mathematical thinking. These levels are more a pragmatic approach than the cognitive or the sociality-personality approach (Sriraman, 2004).

Students in classroom are seldom faced with solving a difficult mathematics problem and rarely work on something that require them to be creative; Instead, they are able to "problem solve" with tasks that common to their level of schooling. When they learn the mathematics topics in general classroom such as fractions, plane figures, statistics, or system of linear equation, teachers have the opportunity to promote mathematics creative thinking using indicators such as novelty, fluency, and flexibility. They then can assess students and put them in levels as shown on Table 2. When students have progressed such as moving from level 1 to level 3, teachers could conclude students are progressing or their creative thinking abilities have increased. Tasks for this research were not used to classify students' creative thinking in general but these can be modified to assess students' creative thinking. Students in level 0 to 1 have a tendency to say that constructing problems is easier than solving a problem, because they already know its solutions. This indicates students' were not encouraged to be more creative and so construct complex problems.

This research was in line with earlier studies (Haylock, 1997; Krutetskii, 1976; Plucker and Zalbelina, 2009) who believe that flexible thinking is a valued mathematics outcome in order to promote creative thinking. The revised levels in Table 2 by considering flexibility in solving and posing problems has resulted in identification of students struggling apply to information from a different area to other area. Students had difficulty showing flexibility. Flexibility thus stands out as a key component of creative thinking parallel with novelty as a general indicator. The levels are in fact hierarchical components'.

Similar to Beghetto & Kaufman (2009), these levels are meant to categorize and elaborate "little-c" creativity that pertains to the novel and personally meaningful insight in learning the regular academic curriculum. "Little-c" levels are useful in that nearly all students can find ways to express this in mathematics projects, tasks, and activities. The highest levels of "little-c" in term of this research when students are expressed solve and pose problems fluently, flexibly, and with novelty; the lowest level occurs when they do not express these three aspects. This occurs because students are assumed as having multicreative potential (Beghetto & Kaufman, 2009). The levels could be compared to those of De Bono (citied in Barak & Doppelt, 2000), Gotoh (2004), and Krulik & Rudnick (1999) as in Table 3.

De Bono (Barak and Doppelt, 2000)	Gotoh (2004)	Krulik & Rudnick (1999)	LCT
Awareness of Thinking,	Empirical	Recall	Level 0
Observation of Thinking	Formal	Basic	Level 1
Thinking Strategy		Critical Thinking	Level 2
	Constructive (Creative)		Level 3
Reflection on Thinking		Creative Thinking	Level 4

Table 3: Comparing some levels

When students are in level 0, they recall the knowledge and use applying of rules and procedures empirically; they might not have any awareness of theoretical foundation generally but cover awareness of thinking. Students in level 1 actually have understanding and recognition of mathematics concepts and formal mathematics. They could observe implications of action and choice, consideration of peers' points of view, comparison of alternatives. Level 2 and 3, students could examine, evaluate, and relate aspects of problems, and might construct novel products. They apply thinking strategies and organize them to reinforce the sense of purpose in their thinking. Level 4, students fulfill the highest category in each level.

This research has described the characteristics of junior high school student's creative thinking levels in classroom mathematics tasks. The distinguishing of levels is based on fluency, flexibility, and novelty in mathematical problem solving and problem posing. Students at level 4 fulfilled three components of creative thinking indicators; at level 3, they fulfilled two components, flexibility and fluency, or novelty and fluency. Students at level 2 only satisfied one aspect that is flexibility or novelty not both, and at level 1, they satisfied only the fluency aspect. Students at level 0 did not fulfill any aspects. These levels are easy to apply in the mathematics classroom because teachers can examine the product of classroom task if their objective is to develop students' creative thinking in mathematics.

This research is but one of the approaches to assess, identify or classify students' creative thinking in mathematics. The study of creative thinking or creativity has many limitations because creative thinking or creativity is a multi-faceted phenomenon. It arises from many definitions, criteria, or concepts. However, it is quite possible to focus on certain aspects, as pointed out by Isaksen (2003) that "*It is quite possible that various researchers and writers emphasize certain facets of creativity in their definitions because of the focus of their work*". Finally, hopefully this research will stimulate others to continue the research, to verify, modify, or apply it.

ACKNOWLEDGEMENTS

I would like to thank Dr. Gloria Stillmann, University of Melbourne for her guidance in listening to me and revising this manuscript. I would also like to thank Directorate General of Higher Education, Department of National Education, Indonesia for their support and aid under project of Program of Academic Recharging (PAR) for visiting the University of Melbourne.

REFERENCES

Airasan, P.W., Anderson, L.W., Krathwol, D.R., Cruikshank, K.A., Mayer, R.E., Pintrich, P.R., Raths, J., & Wittrock, M.C. (2001). *A taxonomy for learning, teaching and assessing: A revision of Bloom's taxonomy of educational objectives.* New York: Addison Wesley Longman.

Barak, M., & Doppelt, Y. (2000). Using portfolio to enhance creative thinking. *The Journal of Technology Studies Summer-Fall 2000*, Volume XXVI, Number 2. Retrieved December 27, 2004, from http://scholar.lib.vt.edu/ejournals.

Begheto, R. A., & Kaufman, J. C. (2009). Do we all have muticreative potential? *ZDM the International Journal on Mathematics Education*, 41(1), 39-44.

Bonotto, C. (2009). Realistic mathematical modeling and problem posing. Paper presented at *ICTMA 13 in Bloomington, USA*. Retrieved November 24 , 2008, from http://site.edu.indiana.edu/portals/161/Public/Bonotto.pdf.

Christou, C., Mousoulides, N., Pittalis. M., Pitta-Pantazi, D., Sriraman, B. (2005). An empirical taxonomy of problem posing processes. *Zentralblatt für Didaktik der Mathematik*, 37 (3), 149-158.

Craft, A. (2003). The Limits of Creativity in Eduacation: Dillemas for Educator. *British Journal of Educational Studies*. Volume 51. No. 2 (Jun. 2003). pp. 113-127.

Davis, R. E. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. Sidney: Croom helm Australia.

De Bono, E. (1995). *Serius creativity: Using the power of lateral thinking to create new ideas*. Hammersmith, London: HarperCollinsPublisher.

English, L. D. (1997). Promoting a problem posing classroom. *Teaching Children Mathematics*, November 1997. p.172-179

Gotoh, G. (2004). The quality of the reasoning in problem solving processes. *The 10th International Congress on Mathematical Education*, July 4-11, 2004. Copenhagen, Denmark. Retrieved November 12, 2004, from http://www.icme-10.com/conference/2_paperreports/3_section.

Haylock, D. (1997). Recognizing mathematical creativity in schoolchildren. *ZDM Volum 29 (June 1997) Number 3. Electronic Edition ISSN 1615-679X.*

Retrieved August 6, 2002, from http://www.fiz.karlsruhe.de/fiz/publications/zdm

Isaksen, S. G. (2003). *CPS: Linking creativity and problem solving.* Retrieved August 22, 2004, from www.cpsb.com.

Isaksen, S.G. (1987). A new dimension for creativity research: Examining style and level of creativity. A paper presented at *The KAI Conference, Hertfordshire, UK June 30-July 2, 1987*. Retrieved August 22, 2004, from www.cpsb.com.

Isaksen, S.G, and Puccio, G.J. (1988). *Adaption-innovation and the Torrance test of creative thinking: The level style issue revisited.* Psychological Reports, 1988, 63, p.659-670

Krulik, S., & Rudnick, J.A., (1999). Innovative tasks to improve critical and creative thinking skills. In Lee V. Stiff & Frances R Curcio (Eds). from *Developing Mathematical reasoning in Grades K-12* (pp.138-145). Reston, Virginia: The National Council of Teachers of Mathematics.

Krutetskii, V.A. (1976). *The psychology of mathematical abilities in schoolchildren.* Chicago: The University of Chicago Press.

Leung, S. S. (1997). On the role of creative thinking in problem posing. *ZDM* Volum 29 (June 1997) Number 3. Electronic Edition ISSN 1615-679X. Retrieved August 6, 2002, from http://www.fiz.karlsruhe.de/fiz/publications/zdm.

Litchman, M. (2009). *Qualitative research in education: A user guide. second edition*. Thousand Oaks, CA: Sage.

Lumsdaine, E., & Lumsdine, M.(1995). *Creative problem solving: Thinking skills for a changing world.* Singapore: McGraw-Hill.

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education*. San Fransisco: Jossey-Bass

Pehkonen, E. (1997). The state-of-art in mathematical creativity. *ZDM Volume 29 (June 1997*) Number 3. Electronic Edition ISSN 1615-679X Retrieved August 6, 2002, from http://www.fiz.karlsruhe.de/fiz/publications/zdm.

Polya, G. (1957). *How to solve it*. second edition. Princeton: Princeton University Press.

Silver, E. A. (1997). Fostering Creativity through Instruction Rich in Mathematical Problem Solving and Thinking in Problem Posing. *ZDM Volume 29 (June*

1997) Number 3. Electronic Edition ISSN 1615-679X. Retrieved August 6, 2002, from http://www.fiz.karlsruhe.de/fiz/ publications/zdm

Siswono, T. Y. E. (2004). Identifying creative thinking process of students through mathematics problem posing. *"MATEMATIKA" Jurnal Teori dan Terapan Matematika* Volume 4 Nomor 1. November 2004. ISSN: 1412-5056, pp. 201-206.

Siswono, T. Y.E., & Budayasa, K. (2006). Theoretical implementation of students' creative thinking levels in mathematics (in Indonesian). *A paper presented at the 12th annual conference of the Indonesian Mathematics Society.* Semarang State University, 24-27 July 2006.

Siswono, T.Y.E., & Novitasari, W. (2007). Improving students creative thinking abilities through problem solving of"What's Another Way" type. (in Indonesian*). Journal of Pendidikan Matematika "Transformasi"*. ISSN: 1978-7847. Volume 1 Number 1, October 2007.

Sriraman, B. (2004). The Characteristics of Mathematical Creativity. *The Mathematics Educator.* Volume 14. Number I, 2004 pp.19-34. Retrieved September 10, 2005, from http://jwilson.coe.uga.edu/ DEPT/TME/Issues/v14n1.sriraman.pdf.

Solso, R. L. (1995). *Cognitive Psychology*. Needham Heights, MA: Allyn & Bacon.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. – In: A. McIntosh & N. Ellerton (Eds.), *Research in Mathematics Education: a contemporary perspective.* Edith Cowan University: MASTEC, p. 164-185.