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UNDERSTANDING OF PROSPECTIVE MATHEMATICS TEACHERS OF THE CONCEPT OF DIAGONAL

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Abstract

This study aims to investigate the concept images of prospective mathematics teachers about the concept of diagonal. With this aim, case study method was used in the study. The participants of the study were consisted of 7 prospective teachers educating at the Department of Mathematics Education. Criterion sampling method was used to select the participants and the criterion was determined as taking the course of geometry in the graduate program. Data was collected in two steps: a diagnostic test form about the definition and features of diagonal was applied to participants firstly and according to the answers of the participants to the diagnostic test form, semi-structured interviews were carried out. Data collected form the diagnostic test form and the semi-structured interviews were analyzed with descriptive analysis. According to the results of the study, it is understood that the prospective teachers had difficulties with the diagonals of parallelogram, rhombus and deltoid. Moreover, it is also seen that the prospective teachers were inadequate to support their ideas with further explanations although they could answer correctly. İt is thought that the inadequacy of the prospective teachers stems from the inadequacy related to proof.

Keywords: Diagonal, Quadrilateral, Prospective Teachers, Mathematics Education

Abstrak

Penelitian ini bertujuan untuk mengetahui gambaran pengetahuan calon guru matematika tentang konsep diagonal. Oleh karena ini, penelitian ini menggunakan metode studi kasus. Subjek penelitian terdiri dari 7 mahasiswa calon guru matematika. Metode sampling kriteria digunakan untuk memilih subjek penelitian dengan kriteria mahasiswa yang mengambil mata kuliah geometri. Data dikumpulkan dalam dua tahap yaitu uji diagnostik tentang definisi dan bagian-bagian diagonal dan wawancara semi terstruktur berdasarkan hasil uji diagnostik yang telah dilakukan. Data yang terkumpul dianalisis secara deskriptif. Hasil penelitian menunjukkan bahwa mahasiswa calon guru mengalami kesulitan dengan diagonal jajar genjang, belah ketupat dan *deltoid*. Selanjutnya, hasil ini juga menunjukkan bahwa mahasiswa calon guru tidak memiliki alasan yang baik untuk mendukung gagasan mereka, walaupun mereka dapat menjawab dengan benar. Hal ini disebabkan atas kurangnya kemampuan mereka terkait materi pembuktian.

Kata kunci: Diagonal, Quadrilateral, Calon Guru, Pendidikan Matematika

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Geometry is defined as “the branch of mathematics on the interrelations, measurements, properties of line, angle, surface and field.” (Türk Dil Kurumu, 2016). According to Baki (2008), geometry, in general, seeks to identify the properties of geometric objects both in the three-dimensional space and the plane and the relations between these objects, to determine the geometric locus of these shapes, to represent transformations, and to prove geometrical propositions. In line with such general objective, geometry teaching is expected to enable students to understand geometrical shapes and their properties, to be able to solve the questions related to the subject and to apply geometrical properties into real-life situations (Aktaş and Cansız Aktaş, 2012). Therefore, geometry is considered as an important branch of mathematics (Altun, 2004). However, international assessments such as TIMSS indicated that Turkish students are not adequate in terms of the knowledge, skill and level of thinking required by the branch of geometry (Abazaoğlu, 2012; Uzun, Bütüner & Yiğit, 2010). The research, which have been performed in the national context, emphasized that students are not competent in geometry in a similar way (Cansız Aktaş & Aktaş, 2011; Toluk, Olkun & Durmuş 2002; Türnüklü, Gündoğdu-Alaylı & Akkaş, 2013).

It is notable that the model of concept image and concept definition, which was used in the study by Tall and Vinner (1981), were frequently utilized in these research on geometry (Cansız Aktaş & Aktaş, 2011; Aktaş & Cansız Aktaş, 2012; Pickreign, 2007; et al.,2012; Türnüklü, Akkaş & Gündoğdu Alaylı, 2013; Usiskin, Griffin, Witonsky & Willmore, 2008). Tall and Vinner (1981) describe the concept image as a cognitive structure involving mental pictures, features and processes of the concept and the concept definition as a formal definition approved by a scientific community. The concept definition includes the consistent information with the concept; on the other hand, Rusken and Rolka (2007) stated that such information is not necessary to create a concept image and that the concept image may also include the contradictory information that students unconsciously develop in relation to the concept. Studies show that during the development of a concept, students primarily utilize the concept images and then develop the concept definition (Tall&Vinner, 1981; Vinner, 1992). An analysis on the relevant research notably demonstrates that there is a plenty number of research on quadrilateral (Erşen& Karakuş, 2013). For instance, in a study, performed with prospective classroom teachers, they were asked to draw the given shapes and to define these shapes in order to analyze the concept images in relation to square, rectangle, trapezoid, and parallelogram (Erşen& Karakuş, 2013). The results revealed that prospective teachers were relatively successful at defining the shapes, but they had difficulty in drawing them, which may follow that prospective teachers have incomplete knowledge on the given quadrilaterals. In a similar way, in the study by Pickreign (2007) with prospective classroom teachers, it was found that the prospective teachers came up with incomplete or completely wrong definitions in regard to various geometrical objects such as "the object with 2 parallel edges of the same length (rectangle)", "a kind of cube (rhombus)" and thus they had incomplete concept knowledge. While these research were conducted with prospective classroom teachers, Türnüklü et al. (2013) examined the perceptions and images of prospective primary school mathematics teachers in relation to quadrangles. The study by Türnüklü et al. (2013) reported that the majority of the teachers were successful at drawing whereas only 19% of them were able to define square correctly, 12% of them were able to define rectangle and parallelogram, 11% were able to define rhombus and 7% were able to define trapezoid correctly. Other students defined these rectangles incompletely, by stating additional properties or incorrectly, or failed to do so. For that reason, it is apparent that prospective mathematics teachers, like prospective classroom teachers, do not have adequate information on geometrical objects.

Considering the results in these research that the knowledge of prospective teachers on geometrical object is not adequate, it would be interesting to reveal what prospective teachers know about different properties of these shapes. An analysis on the literature indicated that some studies examined the concept of diagonal, which is an important concept in geometry and defined as "the line segment joining two non-adjacent corners in a polygon" (TDK, 2016). For instance, Duatepe-Paksu, İymen and Pakmak (2013) analyzed the concept images of a total of 47 prospective classroom teachers, studying at 4th grade, in relation to the diagonals of quadrilaterals. In the study, two questions on the perpendicular intersection of the diagonals of trapezoid, rhombus, rectangle and parallelogram and their diagonal lengths were asked to the prospective teachers. The interviews with the prospective teachers found out that 31.91% of them do not know the concept of diagonal and mistake it for edge and corner. Whereas 46.80% of the prospective teachers answered the first question, which was about the perpendicular intersection of the diagonals perpendicular intersection of the quadrilaterals correctly, the answer of 46.80% of them was inaccurate; the rest of them, 6.39%, did not answer the question. It was reported that more than half of the prospective teachers (63.5%) correctly answered the second question on the information regarding the diagonal lengths of the given shapes and 32% of them did not have the correct information on the diagonal lengths. Another study which was performed with prospective classroom teachers examined their answers to the questions on the concept of height in triangles and the concept of diagonal in polygons (Cunningham& Roberts, 2010). In the study with a pre-test and post-test design, although the definitions of height and diagonal were provided in the pre-test for the prospective teachers, the analyses indicated that they did not have adequate knowledge on these concepts. Though the prospective teachers showed better performance in the post-test, the number of correct answers was low for the questions except for the prototypes.

Roberts (1995) utilized "Van Hiele Geometry Test" in his study with classroom teachers. The test was developed by Usiskin (1982) and frequently used to determine the level of geometrical thinking. The 6th, 7th and 8th items in the test are about the diagonals of square, rectangle and rhombus whereas the 14th item is about the diagonals of parallelogram. The analysis of the data showed that the answer of 26,3% of the prospective teachers to the 6th question, which is about a property applying to every square. For that reason, it is obvious that the prospective teachers do not know that the diagonals in square perpendicularly intersect. The prospective teachers were asked to determine a property not applying to rectangles in 7th question and a result similar to that in 6th question was obtained. In 8th question, the prospective teachers were provided with the definition on rhombus and asked about the property which does not apply to every rhombus. 55.3% of the prospective teachers answered correctly to the question. In 14th question, the prospective teachers were asked to determine the property which applies to every rectangle but is not valid for certain parallelograms. Only 20.4% of the prospective teachers correctly answered to this question on the third level. Considering the answers of the prospective teachers to the questions, it is apparent that the prospective teachers have incomplete or inaccurate knowledge on the diagonals of rhombus.

In the studies which were reviewed as a part of this study, both students (Cansız Aktaş & Aktaş, 2011; Aktaş & Cansız Aktaş, 2012; Ergün, 2010; Fujita, 2012; Pickreign, 2007; Toluk et al., 2002) and teachers or prospective teachers (Duatepe, 2000; Pickreign, 2007; Duatepe-Paksu, İymen & Pakmak, 2012; Türnüklü et al., 2013; Prahmana & Suwasti, 2014) have been selected as the participant group in the studies exploring geometrical objects and the properties of these objects. Yet, it is notable that the studies performed with prospective teachers are less in number. However, given the gaps of students and the difficulties that they experience in geometry, it is believed that teachers and, thus, prospective teachers have the greatest responsibility in overcoming these problems (Erşen & Karakuş, 2013). For that reason, it is essential to determine the level of knowledge of prospective teachers in regard to geometrical concepts and to make the necessary adjustments accordingly. In this sense, this study aims to reveal the concept knowledge and concept images of prospective teachers in regard to the diagonals of quadrilaterals.

METHOD

This is a case study performed in order to identify and observe the details of an existing phenomenon, to develop possible explanations for the phenomenon and evaluate them (Gall, Borg and Gall, 1996). In line with this purpose, this study seeks to analyse the concept knowledge of prospective teachers in regard to the concept of diagonal in some geometrical shapes and the properties of diagonals in depth. Therefore, a case study design was used in the study.

Participants

The study was conducted with 7 prospective teachers, who are currently in the second year of elementary mathematics teaching program at a public university. The method of criterion sampling, which was one of the purposeful sampling methods, was used in the selection of the participants. In line with the purpose of the study, having taken the course of geometry was determined as a criterion and the sample group was selected accordingly.

Data Collection Tools

The data collection process of this study was divided into two phases. In this first phase, a diagnostic test form consisting of the questions on the concept of diagonal and its properties was administered to the participants. The questions on the form were prepared in a way that makes them easy to understand, prevents them from suggesting a specific answer, and includes different types of questions. The opinions of 3 field experts were obtained in regard to the questions. Thus, the questions to be included in the diagnostic test form were determined.

The diagnostic test form consists of two parts. In the first part, the students were asked to define the concept of diagonal whereas in the second part, there are different properties of diagonals and the students were asked whether different geometrical shapes have these properties or not. The geometrical shapes included rectangular, square, rhombus, deltoid and parallelogram. The questions in the diagnostic test form are presented in Table 1.

Table 1. The questions in the opinion form

Part I:
1) What is the concept of diagonal in your opinion? Please explain.
Part II
2) Are the lengths of diagonals in the given geometrical shape equal?
3) Do the diagonals of the given geometrical shape intersect perpendicularly?
4) Do the diagonals of the given geometrical shape bisect each other?
5) Are the diagonals of the given geometrical shape angle bisectors?
6) Are the diagonals of the given geometrical shape medians?

As shown in the Table 1, the questions in the second part were asked for each geometrical shape to the students. The answers of the questions were expected to be yes, no or I do not know. In the second phase of the data collection process, a clinical interview was performed in order to reveal the opinions of the students in depth. The answers of the students to the questions on the diagnostic test form were examined prior to the interview and a total of four students were selected. The interview questions consisted of the questions prepared according to the answers of the students on the diagnostic test form and the questions created during the interview. Some general questions such as “Could you explain your answer further?”, “What did you mean here?” and “How did you decide on this answer?” and some other specific questions in line with the answers of the students were asked to them. The interviews were carried out by a researcher over a period of 20-25 minutes. Further, the interviews were recorded by the researcher via voice recorder and decoded for the analysis.

Data Analysis

The data regarding the sub-problems in the study were qualitative data based on the data obtained from the diagnostic test form and interviews. The data analysis was performed after the voice records from the interviews were transcribed. Firstly, the descriptive analysis of the data obtained from the diagnostic test form was carried out. In the analysis, certain categories were formed based on the answers of the participants for each question. The concepts, mainly associated with each other in terms of their meaning, were grouped under the same category by considering the relevant concepts in the literature in the framework of the research questions. The data obtained from the interviews were analysed under these categories.

The data analysis was conducted simultaneously by the two researchers. Following the determination of the categories, the researchers encoded 10% of the data, which were randomly selected, in order to test the inter-encoder reliability. The percentage of agreement between the researchers was calculated to be 89%. Following that, the differences between the researchers were eliminated and the analyses were performed based on the specified categories. The results were stated in the findings.

RESULTS AND DISCUSSION

This study aims to reveal the concept knowledge of the prospective teachers studying in the elementary mathematics teaching program in regard to the concept of diagonal. In line with this purpose, the findings obtained from the study are respectively provided for each question under separate headings.

The Findings on the Question "What is a Diagonal?"

The participants were asked to define the concept of diagonal in this question. The prospective teachers generally described the concept of diagonal as follows: "it combines two corners of an object." Some of the participants used the term of "line" whereas some of them utilized the term of "line segment" in their definition. For example, the answers of some participants are as follows:

> *A quadrilateral is a line, which is somehow drawn from one diagonal to another from opposite corners (P1)*
> *It is the line segments which combine the opposite corners of a geometrical shape (P2)*
> *It is a line segment extending from one corner to the other. It helps us to determine the distance between two corners (P3)*
> *Let us have a polygon in the plane. A diagonal is the line segments, which are formed by combining the non-consecutive corners of the polygon (P4).*

As seen in the definitions above, the prospective teachers utilized similar expressions in defining the concept of diagonal. One of the participants described the concept of diagonal differently: "*A diagonal is the line combining the two angles of a polygon*. (P4)"

The definition of the concept of diagonal in the literature is "*the line segment combining non-adjacent corners of a polygon*" (Çoker & Karaçay, 1983). The definitions of the participants in regard to diagonal are generally close to the true definition, but it is notable that they could not specify the corners combining with the other corners to create a diagonal. In this sense, only P2 and P4 specified the corners that combine with the others. Moreover, the participants P1 and P4 utilized the term of line, rather than line segment, in their definition of diagonal. In that regard, it can be stated that the participants have failed in understanding the concept of diagonal.

The Findings on the Answers of the Participants in regard to the Properties of Diagonal

The second question on the diagnostic test form was about the diagonal properties of the given geometrical shapes. The frequency of the answers of the participants in regard to the properties is presented in a table; following that, the categories obtained from the interviews are provided.

The Findings on the Question "Are the lengths of diagonals in the given geometrical shape equal?"

Table 2 demonstrates the frequencies of the answers of the prospective teachers in regard to the lengths of diagonals.

Table 2. The answers of the prospective teachers in regard to the lengths of diagonals

Property	**Geometrical Shape**	**Yes**	**No**	**I do not know**
The Lengths of Diagonals are equal.	*Rectangle*	P1, P2, P3, P4, P5, P6, P7		
	Square	P1, P2, P3, P4, P5, P6, P7		
	Rhombus	P4, P6	P1, P2, P3, P5, P7	
	Deltoid		P1, P2, P3, P4, P5, P6, P7	
	Parallelogram	P2, P3, P5, P7	P1, P4, P6	

Based on the answers of the prospective teachers on the diagnostic test form, all of the prospective teachers stated that the lengths of diagonals are equal for rectangles and squares. The reasons of some of the prospective teachers for this answer are as follows:

> *The lengths of the diagonals are equal in rectangle because of angle similarity (edge-to-edge-to-edge) (P2).*
> *The lengths of the diagonals are equal in rectangle since the opposite edges are equal to each other (P4).*

As seen in the statements of the participants above, they concluded the result that the lengths of the diagonals are equal due to similarity based on angles. In a similar way, they stated that the lengths of the diagonals in square are equal as well due to angle similarity. Further, one of the participants, in regard to the reason of equality in the diagonals in square, reflected that "*The lengths of the diagonals in square are equal since the diagonals intersect perpendicularly in square*."

Whereas two of the participants reported that the lengths of the diagonals are equal in rhombus, five of them reflected that they are not equal. Some of the answers of the participants are as follows:

> *The lengths of the diagonals are equal in rhombus since edges are equal (P4).*
> *As the diagonals have the same edge and angle, their lengths are equal (P3).*
> *Since the angles that the diagonals make with each other are different from each other, the lengths of diagonals are not of equal length (P2).*

All of the participants stated that the lengths of diagonals are different in the concept of deltoid. Similarly, the participants reported that the edges of deltoid are not equal or that the angles that the diagonals make with each other are different. Lastly for parallelogram, 4 participants reflected that the lengths of the diagonals are equal whereas 3 of them stated that they are not equal. Some of the answers of the participants are as follows:

Since the lengths of the diagonals in parallelogram bisect each other, the lengths of the diagonals are equal (P5).
Diagonals are of equal length as they have the same edge and angle (P3).
As the opposite edges in parallelogram are same, the length of its diagonals are equal (P2).
As the diagonals are not bisector, the lengths of the diagonals are not equal (P5).
Since the diagonals in a parallelogram are not perpendicular to each other, they are not of equal length (P4).

Whereas some participants reflected that the lengths of diagonals are not equal based on edges and angles, others stated that the diagonals are not of equal length on the grounds that the diagonals are not bisector or perpendicular.

The Findings on the Question "Do the diagonals of the given geometrical shape intersect perpendicularly?"

Table 3 shows the frequencies of the answers of the prospective teachers in regard to the perpendicular intersection of diagonals.

Table 3. The answers of the participants in regard to the perpendicular intersection of diagonals

Property	**Geometrical Shape**	**Yes**	**No**	**I do not know**
The diagonals perpendiculary intersect	*Rectangle*	P3, P4, P5	P1, P2, P6, P7	
	Square	P1, P2, P3, P4, P5, P6, P7		
	Rhombus	P2, P5	P1, P3, P4	P6, P7
	Deltoid	P1, P3, P4, P5, P6, P7	P2	
	Parallelogram		P1, P2, P3, P4, P5, P6, P7	

Based on the answers of the prospective teachers on the diagnostic test form in regard to the perpendicular intersection of diagonals, 3 participants stated that the diagonals perpendicularly intersect whereas 4 participants reflected that they do not perpendicularly intersect. P4, who thinks that the diagonals perpendicularly intersect, explained his/her reflection as follows: "*As the angles of the rectangle are equal to each other, the diagonals perpendicularly intersect?"* On the other hand, two of the participants, who believe that the diagonals do not perpendicularly intersect, explained their reflection as follows:

The diagonals of quadrangles do not perpendicularly intersect because of the angles (P3).
The diagonals of a rectangle do not perpendicularly intersect since they do not exactly bisect the angle at the starting point (P2, P5).

When the participants were asked whether the diagonals of a square perpendicularly intersect or not, all of the participants reflected that they perpendicularly intersect. P4 explained the perpendicular intersection of the diagonals as follows: "*Since a square consists of isosceles triangles (45-45), the diagonals of a square perpendicularly intersect.*"

In regard to the answers of the participants on deltoid, 6 participants stated that the diagonals of a deltoid perpendicularly intersect while 1 participant reflected that they do not perpendicularly intersect. P2, who believes that the diagonals perpendicularly intersect, explained it as follows: "*The diagonals in a deltoid perpendicularly intersect as they bisect the angle at the starting point*."

When the participants were asked whether the diagonals of a parallelogram perpendicularly intersect or not, all of the students reported that they do not perpendicularly intersect. P2, who stated that the diagonals do not perpendicularly intersect in rectangle as well and also specified its reason, attempted to explain his/her claim as follows: "*The diagonals of a parallelogram do not perpendicularly intersect since it resembles to a rectangle.*"

The Findings on the Question "Do the diagonals of the given geometrical shape bisect each other?"

Table 4 indicates the frequencies of the answers of the prospective teachers in regard to the question of whether the diagonals bisect each other or not.

Table 4. The answers of the participants in regard to the question of whether the diagonals bisect each other

Property	Geometrical Shape	Yes	No	I do not know
The diagonals bisect each other	*Rectangle*	P1, P2, P3, P4, P5, P6, P7		
	Square	P1, P2, P3, P4, P5, P6, P7		
	Rhombus	P1, P2, P3, P5, P6	P4, P7	
	Deltoid	P1, P2, P3, P4, P5, P6, P7		
	Parallelogram	P2, P3, P5, P6, P7	P1, P4	

Based on the answers of the participants in regard to the question of whether the diagonals bisect each other, all of the participants reflected that the diagonals of a rectangle, square and deltoid bisect each other. The participants generally explained the reason that the diagonals of a rectangle bisect each other by stating that the lengths of the diagonals are equal. In this sense, P3 reflected his or her opinion as follows:

Figure 1. Rectangular drawing of P3

I: Do the diagonals of a rectangular bisect each other? What do you think about that?
P: Yes, they do.
I: Could you explain your answer further?
P: They do since their lengths (the lengths of the diagonals) are equal, so these two diagonals are of equal length.
I: Well, does the diagonals of equal length indicate that they bisect each other?
P: In this case, the angles are same as well, 90-90 (showing the angle that two diagonals intersect). In other words, the angles as well as the lengths are equal.
I: Could you show us by drawing it?
P: (Drawing the figure…) (Showing the two diagonals) Again, this one is equal to that. That one is equal to the other, but…
I: What is the basis for your claim that the diagonals are equal?
P: Since the angles are equal.
I: What do you mean by the angles are equal? Please clarify.
P: Here is 30 60 and there is 30, that one is obviously 60. (Writing the angles on the figure)
I: So, could you expand on it?
P: It is likened to a butterfly. The opposing triangles are isosceles and their angles are equal to each other. Therefore, the diagonals bisect each other.
I: You are reflecting that the diagonals bisect each other based on its resemblance to a butterfly, right?
P: Yes.

As seen in Figure 1, P3 tried to show that the diagonals bisect each other on the figure by means of its resemblance to a butterfly. On the other hand, the participants reported that the diagonals in square bisect each other based on similar grounds. P3 explained that as follows: "*As square is similar to rectangle, the diagonals in square bisect each other as well*."

Based on the answers of the participants in regard to the question of whether the diagonals in rhombus bisect each other, 5 participants reflected that the diagonals of a rhombus bisect each other whereas 2 participants said that they do not bisect each other. The explanations of the participants, who stated that the diagonals of a rhombus bisect each other, are as follows:

The diagonals bisect each other due to the resemblance of the shape to a butterfly. (P3).
The diagonals bisect each other because the lengths of edges are equal (P4).
As the diagonals of a rhombus perpendicularly intersect, they bisect each other (P2).

When the participants were asked whether the diagonals of a deltoid bisect each other, all of them stated that they bisect each other. Some of these participants reflected that only a diagonal bisects another diagonal. The relevant explanations of the participants are as follows:

The diagonals of a deltoid bisect each other given the angle-angle similarity (P2).
As there is an isosceles triangle in deltoid, the diagonals bisect each other (P5).
Whereas one of the diagonals of a deltoid bisects the other, the other one is not bisected by the first one (only one of the diagonals is bisected) (P3, P5).

When the participants were asked about the diagonals of a parallelogram, 5 of them stated that the diagonals of a parallelogram bisect each other while 2 of them expressed that they do not bisect each other. The participants demonstrated their answers on the figures that they drew.

The Findings on the Question "Are the diagonals of the given geometrical shape angle bisectors?"

Table 5 indicates the frequencies of the answers of the prospective teachers in regard to the question of whether the diagonals are angle bisectors or not.

Table 5. The answers of the participants in regard to the question of whether the diagonals are angle bisectors or not

Property	Geometrical Shape	Yes	No	I do not know
The diagonals are angle bisectors.	*Rectangle*	P2, P4	P1, P3, P5, P6, P7	
	Square	P1, P2, P3, P4, P5, P6, P7		
	Rhombus	P1, P2, P3, P5, P6, P7	P4	
	Deltoid	P1, P2, P3, P4, P5, P6, P7		
	Parallelogram	P2, P3	P1, P4, P5, P6, P7	

The participants were further asked whether the diagonals are angle bisectors or not. Based on their answers, all of the participants reported that the diagonals of a square and a deltoid are angle bisectors while one of them stated that the diagonals of a rhombus are not angle bisectors. The participants explained their claims in the interviews, and the opinions of P3 and P2 in regard to the expression that the diagonals of a square are angle bisectors are as follows:

The diagonals of a square are also angle bisectors. Since their angles are 45 degrees (P3).
The diagonals in a square are angle bisectors since they bisect the angle at the starting point (P2).

Based on the answers of the participants regarding other geometrical shapes, 2 of them stated that the diagonals are angle bisectors whereas 5 of them reported that they are not angle bisectors. The opinions of the participants who stated that the diagonals are angle bisectors are as follows:

The diagonals are angle bisectors since all of the angles are equal to each other (P4).
They are angle bisectors as the diagonals would bisect each other (P5).

On the other hand, the participants, who reported that the diagonals of a rectangle are not angle bisectors, were asked to explain the reasons for their answers, which are provided below:

As the corners of a rectangle are perpendicular to each other, they are not angle bisectors (P3).
The diagonals of a rectangle bisect the angle at the starting point, and thus, they are not angle bisectors (P2).

In regard to the question of whether the diagonals of a square are angle bisectors, all of the participants expressed that they are angle bisectors. The participants explained their answers as follows:

The diagonals of a square are angle bisectors as well. As their angles are 45 degrees (P3).
The diagonals in a square are angle bisectors since they bisect the angle at the starting point (P2).

Moreover, during the interview with P2, P2 explained his or her opinions about the question of whether the diagonals are angle bisectors or not as follows:

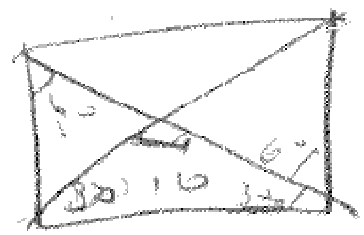

Figure 2. Square drawing of P2

I: What do you think about the question of whether the diagonals of a square are angle bisectors or not?
P2: They are angle bisectors.
I: Could you explain your answer further?
P2: As the opposite edges are perpendicular in this way. That edge is equal to that one, and that angle is equal to this one.
I: Yes.
P2: As the same thing applies to that one as well, this one would be equal to that one. For that reason, all of them are equal to each other, and they are 45 degrees.

When the participants were asked about the diagonals of a rhombus, all of the participants, except 1 of them, stated that the diagonals of a rhombus are angle bisectors. In the interviews, the participants, who reported that the diagonals are angle bisectors, elaborated their opinions as follows:

As rhombus is a parallelogram, the diagonals are angle bisectors (P3).
One can find that the diagonals of a rhombus are angle bisectors by means of angle-edge-angle property (P2).
As the lengths of edges are equal to each other, the diagonals are angle bisectors (P4).

All of the participants reported that the diagonals of a deltoid are angle bisectors, like that of a square. Although all of the participants stated that the diagonals are angle bisectors, P3 expressed that only one of the diagonals is angle bisector and explained his or her opinions as follows: *"A deltoid consists of two isosceles triangles diametrically opposite. Therefore, its diagonals are angle bisectors."*

When the question of whether the diagonals of a parallelogram are angle bisectors or not was asked, 2 of the participants reported that they are angle bisectors whereas 5 of them stated that they are not.

The Findings on the Question "Are the diagonals of the given geometrical shape median?"

Table 6 indicates the frequencies of the answers of the prospective teachers in regard to the question of whether the diagonals are medians or not.

Table 6. The answers of the participants in regard to the question of whether the diagonals are medians or not

Property	Geometrical Shape	Yes	No	I do not know
The diagonals are medians.	*Rectangle*	P4, P5	P1, P2, P3, P6, P7	
	Square	P1, P4, P5	P2, P3, P6, P7	
	Rhombus	P5	P1, P2, P3, P4, P6, P7	
	Deltoid	P5	P1, P2, P3, P4, P6, P7	
	Parallelogram	P5	P1, P2, P3, P4, P6, P7	

The last question asked to the participants was whether the diagonals of the given geometrical shapes are medians or not. Based on the answers of the participants, only P5 stated that the diagonals of all of the geometrical shapes are medians and explained his or her opinion in regard to rectangle as follows:

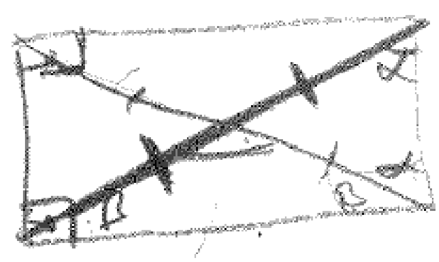

Figure 3. Rectangle drawing of P5

I: Are the diagonals of a rectangle are medians?
P: Yes, they are in a rectangle.
I: Could you explain your answer further?
P: As they bisect the edge, the diagonals are medians.
I: Which edge do they bisect?
P: (Drawing the figure and showing it on the figure…) This edge on this triangle (the participant drew a diagonal in rectangle and creating two triangles, and then, bisected an edge of the triangle, which is the corner of the rectangle, as he or she stated.)

On the other hand, whereas P4 reflected that the diagonals of rectangle and square are medians, P1 stated that only the diagonals of square are medians. Other participants reported that the diagonals of other geometrical objects are not medians. The participants explained the reason of their claim generally as follows: "As diagonals connects the corners, none of them are medians."

This study aims to examine the concept knowledge of prospective teachers on the properties of diagonals and their concept images in regard to the concept of diagonal. Accordingly, firstly, the question of what a diagonal is asked to the participants, and their level of knowledge regarding some of the properties of the diagonals of rectangles, squares, rhombus, deltoids and parallelograms such as perpendicular intersection, bisecting, was evaluated in the study. In line with their answers, it is obvious that the concept knowledge of the prospective teachers in regard to the diagonals of the given geometrical objects was inadequate.

The analysis on the definitions of the prospective teachers in relation to diagonal demonstrated that they have good knowledge of what a diagonal is in general or have a concept image regarding diagonal; however, it is notable that they had difficulties in defining it. Such difficulties may result from the inadequate knowledge of the teachers in regard to the concept of diagonal. Since, some of the participants did not take into consideration the property of "*combining two non-adjacent corners*" in their definition of the concept. Based on the answers of these participants, a diagonal combines any two corners. Furthermore, the study revealed that the participants confused the concepts of line and line segment in defining the concept. Such findings indicated that the prospective teachers did not have a completely accurate concept knowledge regarding the concept of diagonal. In a similar way, Duatepe-Paksu et al. (2013) concluded that the prospective teachers have inadequate knowledge of the concept of diagonal and further some prospective teachers did not know the concept at all.

When the question of whether the diagonals of rectangle are equal to each other was asked to the prospective teachers, the study found out that all of the prospective teachers correctly answer to the question in regard to rectangle, square and deltoid; yet, the majority of the prospective teachers had difficulties in terms of parallelogram in particular. Although the diagonals of parallelogram are not equal, some of the teachers reported that they are equal and, it is notable that they memorized the information they utilized in explaining their answers and failed to justify their explanations accurately. Similarly, Roberts (1995) reported that prospective teachers have difficulty when it comes to the diagonals of parallelogram. Moreover, when the prospective teachers were asked to define or to draw parallelogram in other studies, only a small number of the teachers was able to define the concept or draw it accurately (Çetin & Dane, 2004; Erşen& Karakuş, 2013; Türnüklü et al. 2013). As they would not be able to correctly identify the properties of a concept which they could not define or draw, it is expected that they experience difficulties in regard to this geometrical shape.

Another question on the properties of the diagonals of the given geometrical shapes was about the perpendicular intersection of the diagonals. In this regard, the study showed that the prospective teachers did not have adequate information on the perpendicular intersection of the diagonals of rectangle and rhombus, whereas almost all of the participants correctly answered to the questions on the other geometrical shapes. Although the diagonals of rhombus perpendicularly intersect, only two of the seven participants correctly answered to the relevant question. Less emphasis is given to rhombus, in comparison to square or rectangle, during the teaching of geometrical objects; for that reason, it can be stated that students have less chance of exploring this object and its properties, which may result from that the prospective teachers have less conceptual knowledge. However, it is highly surprising that the prospective teachers inaccurately answered to the question on rectangle, which is one of the most emphasized geometrical objects in geometry. Since the experience of students with rectangle is relatively more than their experience with other geometrical objects, it is expected that their knowledge on the concept of rectangle would be better. Indeed, Roberts (1995) demonstrated that they performed better in the question on the properties of rectangle. That being said, it is obvious that such finding is not consistent with the results of this study. On the other hand, given that square is a special form of rectangle, it is surprising that the prospective teachers correctly identified the properties of a special form of rectangle whereas they failed to do so when it comes to rectangle. This may result from the finding of Türnüklü et al. (2013) that mathematics prospective teachers think rectangle as separate from square. The finding of this study might follow that the prospective teachers were not able to establish a relationship between these two shapes. For that reason, it can be said that the prospective teachers had difficulty in establishing a relationship between the groups of geometrical objects as well as the given properties of the diagonals of geometrical objects, which might be an indicator of that the prospective teachers learnt geometrical knowledge not in a relational manner, but individually.

The prospective teachers were asked whether the diagonals of geometrical objects are angle bisectors or not. The study found that the majority of the prospective teachers correctly answered the question; however, though the diagonals of a rectangle are not angle bisectors, two of them stated that they are angle bisectors. Further, in regard to rhombus, all of them except one, provided the accurate answer that the diagonals are angle bisectors. The prospective teachers, surprisingly, were mistaken about rectangle in regard to the property of diagonals of being medians like in the property of diagonals of intersecting perpendicularly, which supports a previous finding of this study that they have inadequate conceptual knowledge on the properties of rectangle. In addition to that, all of the prospective teachers reported that the diagonals of deltoid are angle bisectors; however, it is notably known that only one of the diagonals of deltoid is angle bisector whereas the other diagonal is not an angle bisector. The lack of knowledge of the participants on this property of the diagonals of deltoid may indicate that their conceptual images in relation to deltoid or diagonal are not sufficient. Since, one of the diagonals of deltoid, defined as “a quadrilateral formed by two isosceles triangles joined at their bases”, is an angle bisector as it combines the peak of the isosceles triangles, but, the other diagonal is not an angle bisector. It can be stated that the prospective teachers, who stated that both of the diagonals are angle bisectors, have inaccurate conceptual images as it is possible that they might think that both of them combine the peak of the isosceles triangles in deltoid. Furthermore, it is remarkable that the prospective teachers often used analogy while expressing the reasons for their claims. Whereas some of them justified their answers by means of the classifications of geometrical shapes, others explained their answers in a simpler manner, such as by stating that “as they divide the angle, and thus they are angle bisectors”, which shows that the prospective teachers memorize what they learn and they are not competent in their conceptual knowledge.

In the question on whether the diagonals bisect each other or not, the prospective teachers in general answered correctly; on the other hand, they failed to answer the question on the diagonals of deltoid bisecting each other correctly. Although only one of the diagonals of a deltoid bisects, merely two of the participants reflected that only one of the diagonals of a deltoid bisects the other. Such finding may follow that the participants have incomplete or wrong information on deltoid, just like in the property of diagonals being angle bisectors. Still, it was observed that the prospective teachers were more comfortable in answering whether the diagonals bisect each other, in comparison to other questions. The participants generally used analogy in explaining their arguments and attempted to answer the question of whether the diagonals of the objects bisect each other or not, based on the analogies they made. For that reason, it can be stated that the prospective teachers are relatively more competent at analogies.

The question of whether the diagonals of the geometrical objects are medians or not was the last question asked to the participants in the study. It is blatantly obvious that diagonals cannot be medians due to their nature and definition. Therefore, the participants are expected to be aware of such fact and the number of accurate answers to be high. Indeed, the majority of the participants answered that any diagonal in any geometrical object is not a median. These participants justified their answers by stating that as a diagonal is from one corner to the other, it cannot be a median. However, remarkably, the number of the participants who reflected that the diagonals are medians was three in rectangle, two in square, one in deltoid, parallelogram and rhombus. It is believed that the participants who reported that they are medians confused the concept of edge and diagonal and made a mistake, which is a result consistent with the finding of the study by Duatepe-Paksu et al. (2013).

An analysis on the results of the study indicates that the prospective teachers have various problems in identifying the properties of the diagonals of the given geometrical objects. It can be said that the problems that they experience with deltoid, rhombus and parallelogram are more notable. Such finding is consistent with the findings of the other studies with prospective teachers, which claim that they do not have the required level of knowledge on geometrical concepts (Çetin and Dane, 2004; Dane, 2008; Pickreign, 2007; Sandt and Nieuwoudt, 2003). The reason may be that prospective teachers have not had enough experience with the given objects. Since, while supporting their claims, they generally utilized a kind of information without conceptual foundation, which they seem to memorize. Duatepe-Paksu et al. (2013) also concluded a similar finding and emphasized that the prospective teachers used the information that they memorize. This might indicate that the prospective teachers learn about diagonals in an environment in which the properties of diagonals are directly shown or told by instructor, rather than an environment where they can explore the properties of diagonals. Since it is noticed that the prospective teachers were inadequate in supporting their claims on geometrical properties.

The inadequacy of the prospective teachers may result from the insufficient knowledge on the concept of diagonal as well as their incompetency in proving their claims. In support of such hypothesis, different studies revealed that prospective teachers have difficulty in proving (Alkan & Bukova Güzel, 2005; Almedia, 2000; Harel ve Sowder, 1998; Jones, 2000; Knapp, 2005; Moore, 1990). Thus, teaching process should feature not only the rectangles with diagonals perpendicularly intersecting or being equal but also the rectangles with other properties and a comparison between these rectangles should be provided along with their reasons, in order to enhance the understanding of prospective teachers in regard to the concept of diagonal. It is believed that the use of various technological tools in such comparisons, due to the dynamic nature of these tools, would enable the prospective teachers to explore the relevant properties in different perspectives and to develop more accurate conceptual images in line with their increased conceptual knowledge. In fact, the findings of the studies conducted by Köse, Tanışlı, Erdoğan and Ada (2012) are consistent with such argument.

On the other hand, it is observed that another aspect that the prospective teachers fall behind was reasoning skill. Since the prospective teachers could not demonstrate the required level of reasoning skill in supporting or proving their claims.

CONCLUSION

The study by Kuzinak and Rauscher (2007) supports such argument; the researchers, in their study on diagonals, concluded that students have low reasoning skill levels. For that reason, given the positive contributions of proving to reasoning skills, it is believed that the inclusion of proving in teacher training would play an important role in improving reasoning skills to the required level. In this sense, further studies may re-evaluate the level of the prospective teachers, following a teaching process including proving, reveal whether they progress on the subject or not, and find out whether such teaching process has a positive impact on the reasoning skills of the prospective teachers. In addition teachers should choose appropriate strategies to improve students' mathematical literacy about geometry (Putra & Notiva, 2015).

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