An efficient Bouc & Wen approach for seismic analysis of masonry tower

Luca Facchini, Michele Betti

Department of Civil and Environmental Engineering (DICEA), University of Florence
luca.facchini@unifi.it, mbetti@dicea.unifi.it

ABSTRACT. The assessment of existing masonry towers under exceptional loads, such as earthquake loads, requires reliable, expedite and efficient methods of analysis. These approaches should take into account both the randomness that affects the masonry properties (in some cases also the distribution of the elastic parameters) and, of course, the nonlinear behavior of masonry. Considering the need of simplified but effective methods to assess the seismic response of such structures, the paper proposes an efficient approach for seismic assessment of masonry towers assuming the material properties as a stochastic field. As a prototype of masonry towers a cantilever beam is analyzed assuming that the first modal shape governs the structural motion. With this hypothesis a nonlinear hysteretic Bouc & Wen model is employed to reproduce the system response which is subsequently employed to evaluate the response bounds. The results of the simplified approach are compared with the results of a finite element model to show the effectiveness of the method.

KEYWORDS. Bouc & Wen model; Masonry towers; Nonlinear dynamics; Perturbation methods.

INTRODUCTION

The analysis of the structural response of slender masonry towers, especially in case of seismic loading, should take into account both the randomness that affects the masonry material (even the elastic parameters distribution along the height of the beam) and the specific nonlinear masonry behavior (the small tensile strength). Several methods have been proposed in recent years to analyze the structural response of such structural systems. Betti *et al.* [1] analyzed the response of a slender masonry wall under turbulent wind proposing an approach based on the modal reduction (MORE: Modal Reduction). The material was assumed as no tensile resistant (NTR), but the mechanical properties were assumed as deterministic. An analysis method to solve this typology of mechanical problems, assuming the properties of the material as a stochastic field, was further proposed by Facchini *et al.* [2] based on a Galerkin approach. Additional possible approaches are the perturbation methods, originally proposed by Liu *et al.* [3].

To offers an insight on this background, the paper proposes an expeditious and effective method to assess the seismic response of slender masonry towers where the masonry is assumed as a random material by means of an approach based on a nonlinear hysteretic Bouc [4] & Wen [5, 6] model. The Bouc & Wen hysteretic model has been found great interest in nonlinear stochastic dynamics being a smooth and multipurpose hysteresis model that allows for explicit form calculation of the linearization terms. The versatility of the Bouc & Wen model is also demonstrated by the large number of applications reported in the scientific literature concerning, f.i., steel and concrete [7] and wood structures [8]. Despite the spreading of the model only few case studies, to the authors' knowledge, are reported concerning applications with masonry structures.

In the following, a masonry cantilever beam, as a prototype of masonry towers, is analyzed assuming that the first modal shape governs the structural motion. With this hypothesis the nonlinear hysteretic Bouc & Wen model is employed to reproduce the system response, which is subsequently employed to evaluate the bounds of the structural response through a perturbation approach. The results of the proposed approach are compared with the results of a finite element (FE) model to show the effectiveness of the method.

IDENTIFICATION OF BOUC & WEN MODEL FOR THE TOWER

The Bouc [4] & Wen [5, 6] model (in the following BW for short) has been extensively employed to model the dynamic behavior of hysteretic systems. As masonry is a no-tensile resistant material, it may on occasion exhibit such kind of behavior. To assess the employability of the BW model as an efficient approach for seismic analysis of masonry towers the paper analyzes the clarifying example of a masonry cantilever beam.
To identify the BW model parameters several methods have been proposed. Herein the tuning of the BW parameters is performed by comparing the behavior of the BW model with the response of the masonry cantilever beam acted upon by a static transversal force as evaluated through a FE analysis.
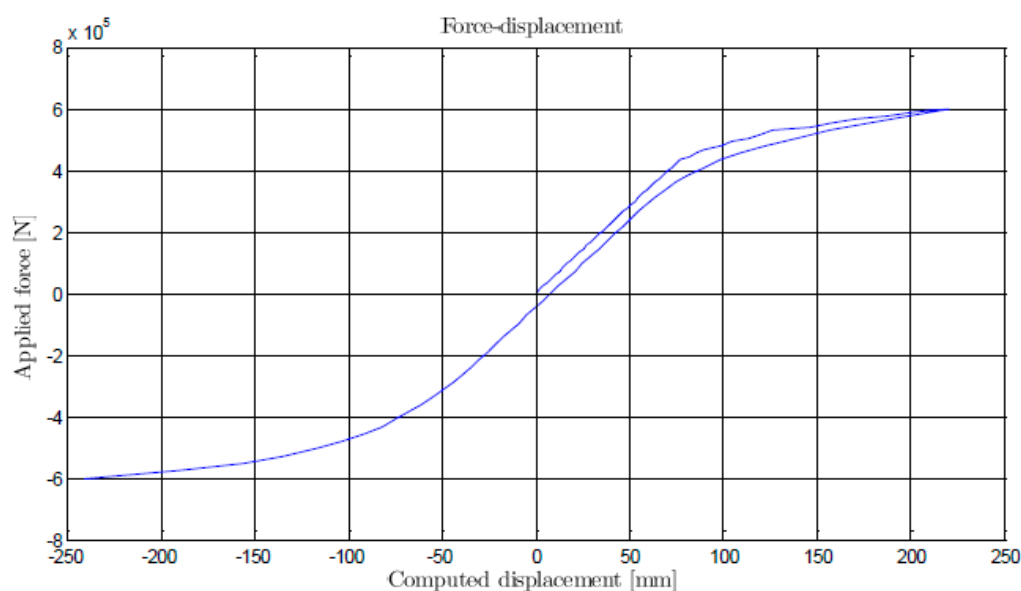

Figure 1: The curve "applied force – computed displacement" (ANSYS analysis under cyclic loading).

Numerical FE modeling

As a reference case the structural behavior of a cantilever masonry beam with dimensions 10 (length) × 40 (height) × 1 (thickness) m was investigated by the ANSYS FE code [9]. The behavior of the masonry tower under seismic loads was analyzed by means of both static nonlinear pushover analysis and dynamic nonlinear analysis.
To model with the ANSYS code the nonlinear behavior of masonry, a proper combination of two failure domains was employed. On the one hand the Drucker-Prager (DP) plasticity criterion [10], originally proposed for geo-materials, was employed. The material parameters required to define the plasticity model, the cohesion c and the internal angle of friction φ, were introduced in such a way that the circular cone yield surface of the DP model corresponds to the outer vertex of the hexagonal Mohr-Coulomb yield surface. The DP criterion was combined with the Willam and Warnke (WW) failure criterion, originally proposed for concrete [11], which accounts for both cracking and crushing failure modes through a smeared model. With these assumptions the masonry is modeled as an isotropic continuous medium capable to exhibit plastic deformation, to crack by traction and to crush by compression. The assignment of the mechanical parameters required by the DP and WW criterion requires a careful calibration [12, 13], especially if a macro-element approach is used. In the present study, being the analyses aimed to assess the effectiveness of a disordered BW system, these parameters were assumed on the basis of available literature results for stone masonry walls [14].
The final three-dimensional model consisted of 902 joints and 400 solid elements corresponding to 2,640 degrees of freedom (dofs). The nonlinear system of equations was solved by an incremental Newton-Raphson method. Details of the

analyses are reported in [15]. If for instance the force is harmonic, then the top displacement of the cantilever beam exhibits a slight hysteretic behavior, as shown in Fig. 1.

Adopted simplifications
From the results of the structural analyses performed by means of ANSYS (Fig.1), it can be inferred that the dynamic behavior of the cantilever beam can be modeled accurately enough by means of its first modal shape. Therefore, a nonlinear single degree of freedom (SDOF) system can be defined whose stiffness is determined by means of the performed nonlinear numerical analyses. The analysis performed with ANSYS applying a horizontal displacement on the top of the masonry cantilever beam, in such a way to obtain the first-loading curve and the successive unloading curve, was hence employed to approximate the system behavior with the BW model, as reported in the following.

Approximation of the tower response with Bouc & Wen oscillator
Within the first part of the research, the simplified SDOF BW model for the restoring force of the cantilever beam was identified. The behavior of the oscillator is described by an incremental equation of the form:

$$
\begin{aligned}
&m\ddot{x}(t) + c\dot{x}(t) + kg(t) = f(t) \\
&g(t) = \alpha x(t) + (1-\alpha)z(t) \\
&\dot{z}(t) = \dot{x}(t)\left\{A - |z|^n \cdot \left[\beta + \gamma \, sign(\dot{x})\, sign(z)\right]\right\}
\end{aligned}
\tag{1}
$$

This model represents the restoring $g(t)$ function with a linear combination of a linearly elastic force and a history-dependent term: $z(t)$. The variable $x(t)$ represents the displacement of the top of the cantilever beam.

The first set of parameters that must be identified is composed by k, α, A, n, β and γ. It can be shown [15] that parameter A in eq. (1) is redundant, and hence in the following it will be considered as unitary: $A=1$. The tangent stiffness of the oscillator can be obtained by deriving the nonlinear restoring function $g(t)$ with respect to the displacement x, obtaining:

$$
k_t = k\frac{\partial g}{\partial x} = k\left[\alpha + (1-\alpha)\frac{\partial z}{\partial x}\right] = k\left[\alpha + (1-\alpha)\frac{\dot{z}}{\dot{x}}\right]
\tag{2}
$$

Being the parameter A assumed unitary without loss of generality, parameter k reduces to the initial stiffness of the system and the initial stiffness can be computed substituting initial conditions $z(0)=0$ in Eq. (2), obtaining:

$$
z(0) = 0 \quad \Rightarrow \quad k_i = k_t \mid_{z=0} = k\left[\alpha + (1-\alpha)\right] = k
\tag{3}
$$

And asymptotically (according Marano and Greco [16] the post-elastic stiffness is given as $k_f = \alpha k$):

$$
\dot{z} = 0 \quad \Rightarrow \quad k_t = k_f = \alpha k \quad \Rightarrow \quad \alpha = \frac{k_f}{k_i}
\tag{4}
$$

When the maximum displacement is reached, and the unloading process begins, following expression holds:

$$
\left.\begin{aligned}
&z = \hat{z} = 1/(\gamma+\beta)^{1/n} \\
&sgn(z) = 1; \quad sgn(\dot{x}) = -1
\end{aligned}\right\} \quad k_u = k\left\{\alpha + (1-\alpha)\left[1 - \frac{\beta-\gamma}{\beta+\gamma}\right]\right\}
\tag{5}
$$

and:

$$
\frac{\beta-\gamma}{\beta+\gamma} = \frac{1-(k_u/k)}{1-\alpha} = \frac{k_i - k_u}{k_i - k_f}
\tag{6}
$$

Finally, the elastic limit displacement can be expressed, according to Cunha [17], as: $\beta + \gamma = x_Y^{-n}$.

The ratio β/γ affects the transitions from the loading curve to the unloading one. Therefore, only one parameter, n, or alternatively β or γ, remain undetermined. It is possible to observe that the parameter n influences the transition from elastic to post-elastic behavior and the distance of the unloading path from the first loading.

As a first attempt, imposing the exponent *n*=5 and the limit elastic displacement x_Y = 90 mm, the behavior shown with a red line in Fig. 2 is obtained. From the numerical analyses carried out by means of ANSYS, an initial stiffness of k_i = 5654 N/mm and a ratio α = k_f / k_i = 0.1395 can be estimated (Tab. 1). It is possible to observe that *n*=5 reproduce correctly the transition from the elastic to the post-elastic branch, but the overall behavior of the oscillator is very poor.
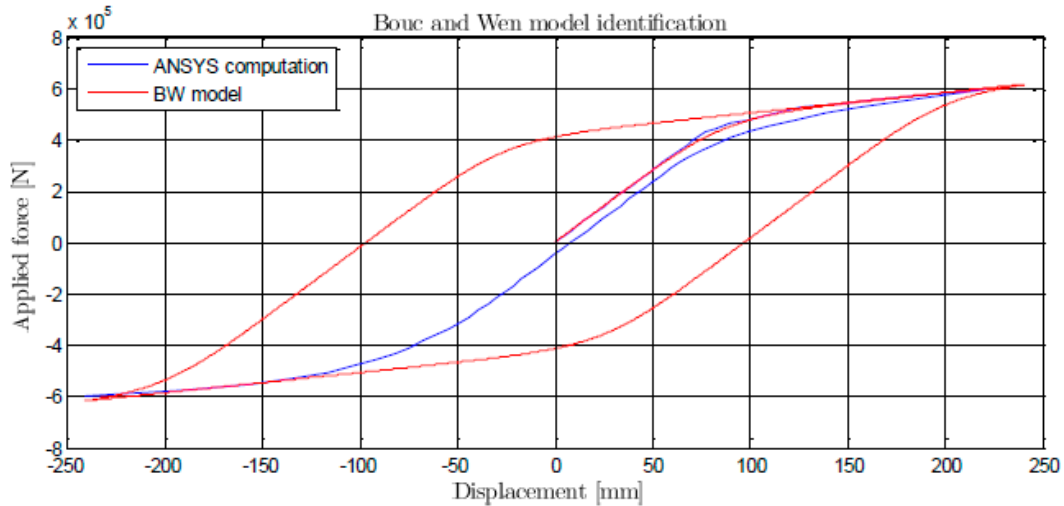

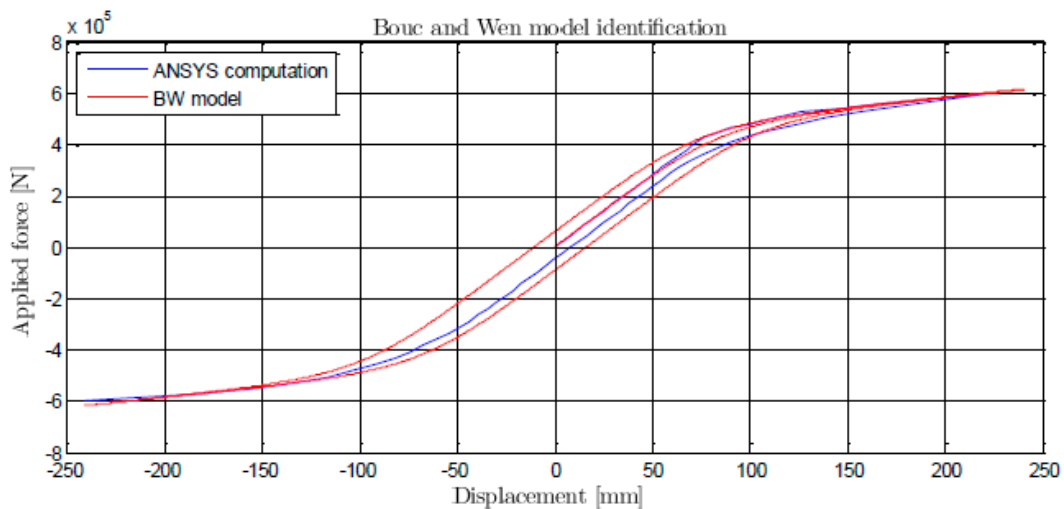Figure 2: Comparison of the identified BW model with the ANSYS results (1st attempt).

Figure 3: Comparison of the identified BW model with the ANSYS results (2nd attempt).

As a second attempt, *n* and the unloading initial stiffness k_u were both lowered (Tab. 1), obtaining the behavior reported in Fig. 3. These choice gave better results, with an energy dissipation cycle close to the one obtained with the FE code.

	k_i (N/mm)	k_f (N/mm)	k_u (N/mm)	x_Y (mm)	*n*	α	β	γ	k (N/mm)
1st	5654	789	1171	90	5.0	0.1395	$1.627 \cdot 10^{-10}$	$6.656 \cdot 10^{-12}$	5654
2nd	5654	789	793	90	4.0	0.1395	$1.523 \cdot 10^{-8}$	$6.646 \cdot 10^{-12}$	5654

Table 1: BW model parameters identification.

Once the restoring function was modeled, the mass *m* and the damping *c* in eq. (1) are two further parameters that need to be evaluated. A "hint" can be drawn from the FE model which has a participating mass of 165 tons. Assuming these

values, the dynamic response of the identified BW oscillator was compared with the dynamic response of the cantilever beam evaluated with ANSYS, and the comparison is showed in Fig. 4 (to perform the dynamic nonlinear analyses the El Centro 1940 earthquake was employed).

It can be seen that the SDOF BW oscillator compares well enough with the more complex and computational demanding models elaborated with ANSYS (Fig. 4); however the worst error committed on the response peaks is about 20%. To encompass this error, a possibility is to identify mass and damping of the BW system by means of the analysis of the seismic response of the cantilever beam. According to this approach, the system mass m and damping c are optimized in order to minimize the difference between the cantilever beam response computed with ANSYS and the response of the BW oscillator. Results of this optimization are reported in Fig. 5.
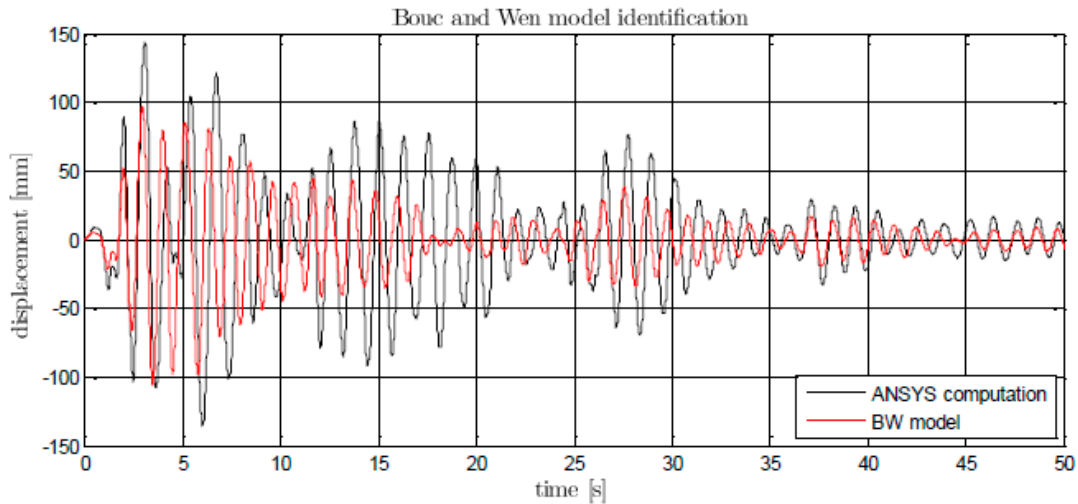

Figure 4: Displacement time history: comparison between BW identified model and ANSYS results (2nd attempt).
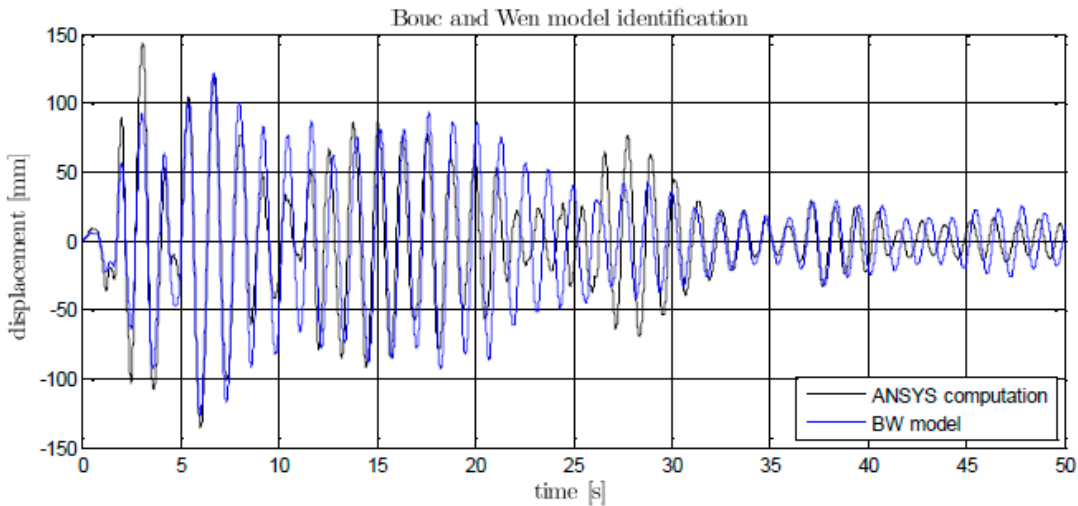

Figure 5: Displacement time history: comparison between BW identified model and ANSYS results (optimization of mass and damping of the BW SDOF system).

INFLUENCE OF RANDOM STIFFNESS ON BW RESPONSE

After calibration of the BW oscillator parameters (k, α, A, n, β, γ, m and c), the BW model was employed to analyze the behavior of the disordered oscillator. Masonry mechanical parameters, such as Young's modulus, are often difficult to estimate, especially for monuments and historical buildings. Possibilities can be in-situ tests by means of flat jacks and even laboratory tests on small samples of the examined masonry, but the results can only

give an estimate of the parameters, which are necessarily affected by a more or less pronounced randomness. Hence the problem has significant impact on seismic assessment.

Being one of the aims of the research to determine the influence of the material parameters (e.g. resistance, stiffness etc.) on a tower response, the Liu *et al.* [3] approach (successively developed by Chiostrini and Facchini [18]) was employed and a variation of material stiffness was considered. Based on this perturbative approach both the system response *x(t)* and the history-dependent term *z(t)* were approximate by means of a series expansion on the random parameter *k*:

$$x(t,k) \simeq \overline{x} + (k - \overline{k})\overline{x}_k + 1/2(k - \overline{k})^2 \overline{x}_{kk}$$
$$z(t,k) \simeq \overline{z} + (k - \overline{k})\overline{z}_k + 1/2(k - \overline{k})^2 \overline{z}_{kk}$$

(7)

The $\overline{x}_k$ and $\overline{z}_k$ represent the sensitivity vectors, and must be computed with their derivatives. The solving equations are the derivatives of the equation of motion with respect to the random parameter *k:*

$$m\ddot{\overline{x}}(t) + c\dot{\overline{x}}(t) + kg_0(t) = f(t)$$
$$g_0(t) = \alpha\overline{x}(t) + (1-\alpha)\overline{z}(t)$$
$$\dot{\overline{z}}(t) = \dot{\overline{x}}(t)\left\{1 - |\overline{z}|^n \cdot \left[\beta + \gamma sign(\dot{\overline{x}})sign(\overline{z})\right]\right\}$$

(8)

$$m\ddot{\overline{x}}_k(t) + c\dot{\overline{x}}_k(t) + kg_1(t) = -g_0(t)$$
$$g_1(t) = \alpha\overline{x}_k(t) + (1-\alpha)\overline{z}_k(t)$$
$$\dot{\overline{z}}_k(t) = \dot{\overline{x}}_k(t)\left\{1 - |\overline{z}|^n \cdot \left[\beta + \gamma sign(\dot{\overline{x}})sign(\overline{z})\right]\right\} +$$
$$\dot{\overline{x}}(t)\left\{1 - n|\overline{z}|^{n-1} sign(\overline{z})\overline{z}_k \cdot \left[\beta + \gamma sign(\dot{\overline{x}})sign(\overline{z})\right] -$$
$$|\overline{z}|^n \cdot \left[\beta + \gamma D_k\left(sign(\dot{\overline{x}})sign(\overline{z})\right)\right]\right\}$$

(9)

The computation of the sensitivity vectors requires the solving of Eq.. (8)-(9), where a derivative of the signum function appears: it implies differentiation with respect to *k* of a product of two signum functions, which involves considerable numerical difficulties. To overcome this problem, the signum function was approximated with $s(x) = \frac{2}{\pi}\tan^{-1}(cx)$ with $c=5.5 \cdot 10^{-4}$.
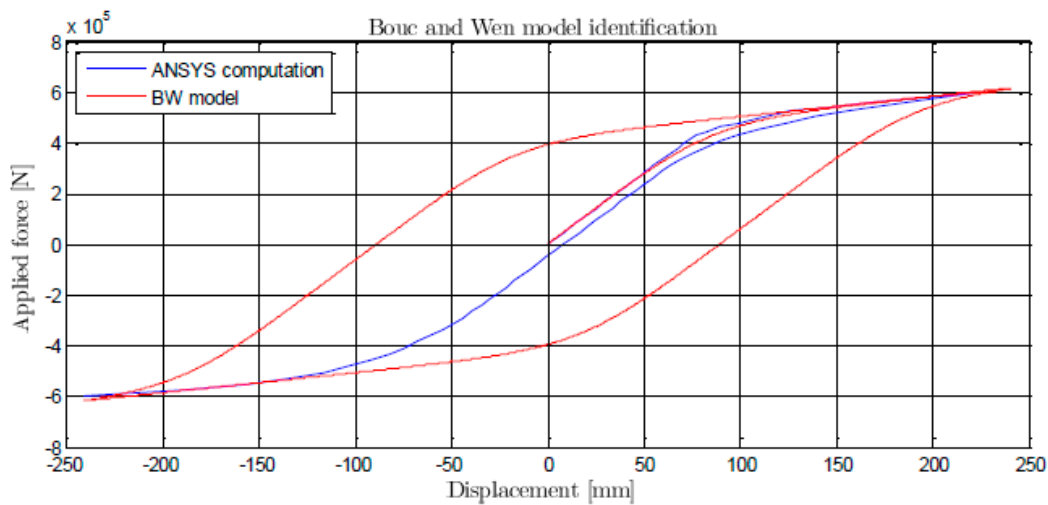

Figure 6: Comparison of the identified BW model with the ANSYS results (signum function approximated with s)

It can be seen that the replacement of the signum function with $s(x)$ causes the cycle of the BW oscillator to enlarge (Fig. 6), thus missing the match with the reference response (Fig. 1); on the other hand, it was seen that the re-introduction of the condition upon the unloading stiffness brought back the cycle of the BW system to match the reference response again ((Fig. 7). A final check was made taking into account the dynamic response of this modified version of BW system to the El Centro 1940 earthquake, and the comparison with the FE results is shown in Fig. 8.
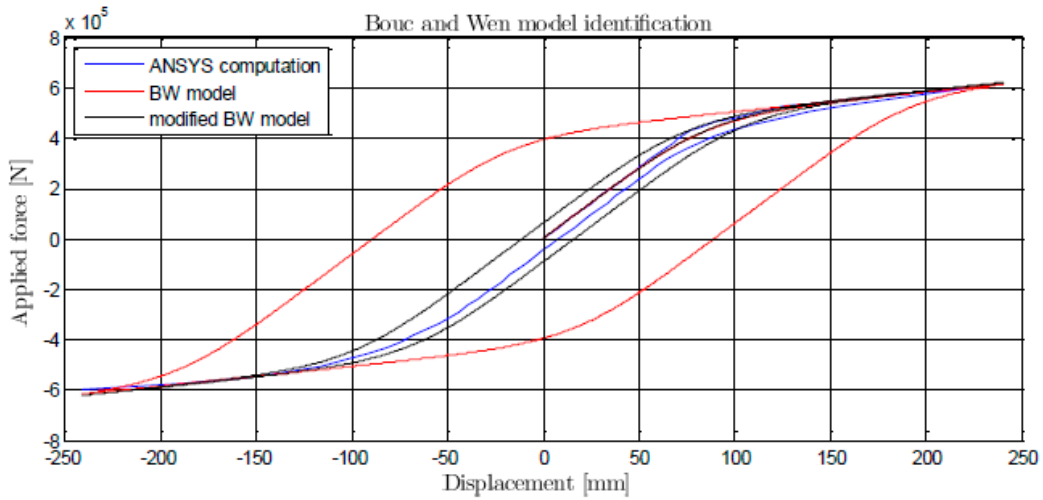

Figure 7: Comparison of the identified BW model with the ANSYS results.
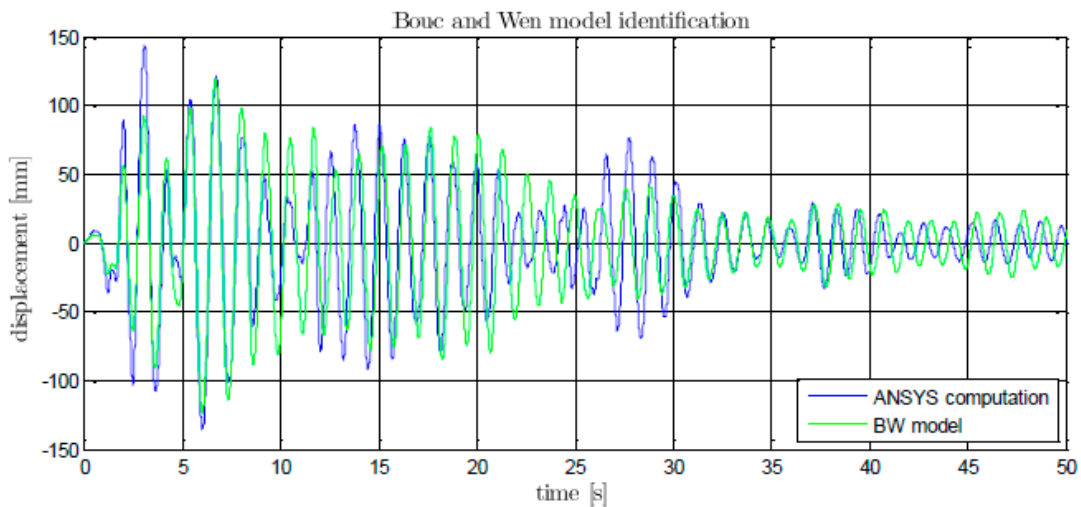

Figure 8: Displacement time history: comparison between BW identified model and ANSYS results (El Centro 1940 earthquake).

Taking into account the proposed approximation for the signum function, Eq. (8)-(9) can be written as follows:

$$
\begin{aligned}
& m\ddot{\overline{x}}(t) + c\dot{\overline{x}}(t) + kg(\overline{x}, \overline{z}) = f(t) \\
& g(x_1, x_2) = \alpha x_1 + (1-\alpha)x_2 \\
& \dot{\overline{z}}(t) = \dot{\overline{x}}(t)\left[1 - h(\dot{\overline{x}}, \overline{z})\right] \\
& \text{where:} \quad h(x_1, x_2) = |x_2|^n \cdot \left[\beta + \gamma s(x_1) s(x_2)\right]
\end{aligned}
\tag{10}
$$

$$m\ddot{\overline{x}}_k(t) + c\dot{\overline{x}}_k(t) + kg(\overline{x}_k, \overline{z}_k) = -g(\overline{x}, \overline{z})$$

$$\dot{\overline{z}}_k(t) = \dot{\overline{x}}_k(t)\left[1 - h(\dot{\overline{x}}, \overline{z})\right] -$$

$$\dot{\overline{x}}(t)\left[h_{,1}(\dot{\overline{x}}, \overline{z})\dot{\overline{x}}_k + h_{,2}(\dot{\overline{x}}, \overline{z})\overline{z}_k\right] \tag{11}$$
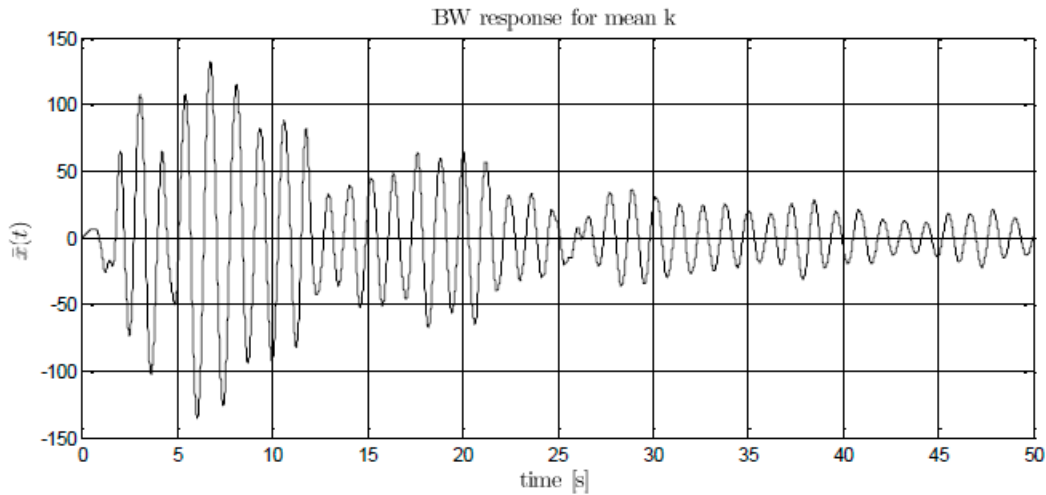

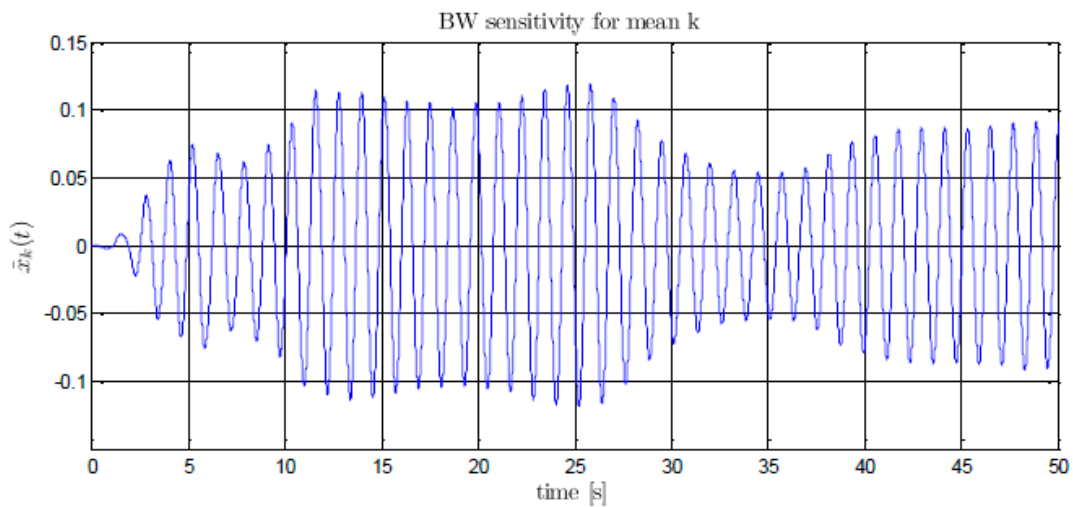Figure 9: Response $\overline{x}(t)$ evaluated for $\overline{k} = E[k]$.

Figure 10: Sensitivity of the response $\overline{x}_k(t)$.

Moreover, a further differentiation with respect to k gives the equation of motion of the second derivative of BW response to the earthquake, as follows:

$$m\ddot{\overline{x}}_{kk}(t) + c\dot{\overline{x}}_{kk}(t) + kg(\overline{x}_{kk}, \overline{z}_{kk}) = -2g(\overline{x}_k, \overline{z}_k)$$

$$\dot{\overline{z}}_{kk}(t) = \dot{\overline{x}}_{kk}(t)\left[1 - h(\dot{\overline{x}}, \overline{z})\right] -$$

$$2\dot{\overline{x}}_k(t)\left[h_{,1}(\dot{\overline{x}}, \overline{z})\dot{\overline{x}}_k + h_{,2}(\dot{\overline{x}}, \overline{z})\overline{z}_k\right] - \tag{12}$$

$$\dot{\overline{x}}(t)\left[h_{,11}(\dot{\overline{x}}, \overline{z})\dot{\overline{x}}_k^2 + 2h_{,12}(\dot{\overline{x}}, \overline{z})\dot{\overline{x}}_k\overline{z}_k + h_{,22}(\dot{\overline{x}}, \overline{z})\overline{z}_k^2 +\right.$$

$$\left. h_{,1}(\dot{\overline{x}}, \overline{z})\dot{\overline{x}}_{kk} + h_{,2}(\dot{\overline{x}}, \overline{z})\overline{z}_{kk}\right]$$

By means of the integration of Eq. (10)-(11)-(12), that constitute a set of coupled deterministic nonlinear equations, it is possible to obtains the sensitivity vectors and to estimate: i) the response $\overline{x}(t)$ evaluated for $\overline{k} = E[k]$ (Fig. 9); ii) the sensitivity of the response $\overline{x}_k(t)$ (Fig. 10) and iii) the derivative of the sensitivity $\overline{x}_{kk}(t)$ (Fig. 11).
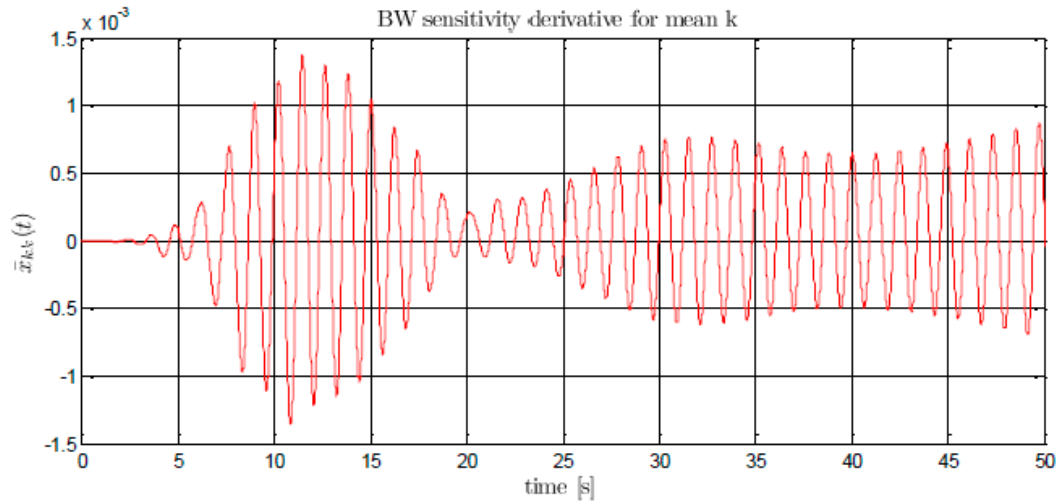

Figure 11: Derivative of the sensitivity $\overline{x}_{kk}(t)$.

And eventually, the approximations for the expected value of the response and its standard deviation are obtained considering a second-order expansion for the mean displacement and a first-order expansion for the variance, by:

$$E[x(t)] \simeq \overline{x}(t) + 1/2\,\overline{x}_{kk}(t)\sigma_k^2$$
$$\sigma_x(t) \simeq \overline{x}_k(t)\sigma_k \tag{13}$$

The expected response, together with the 3σ bounds, is shown in Fig. 12.
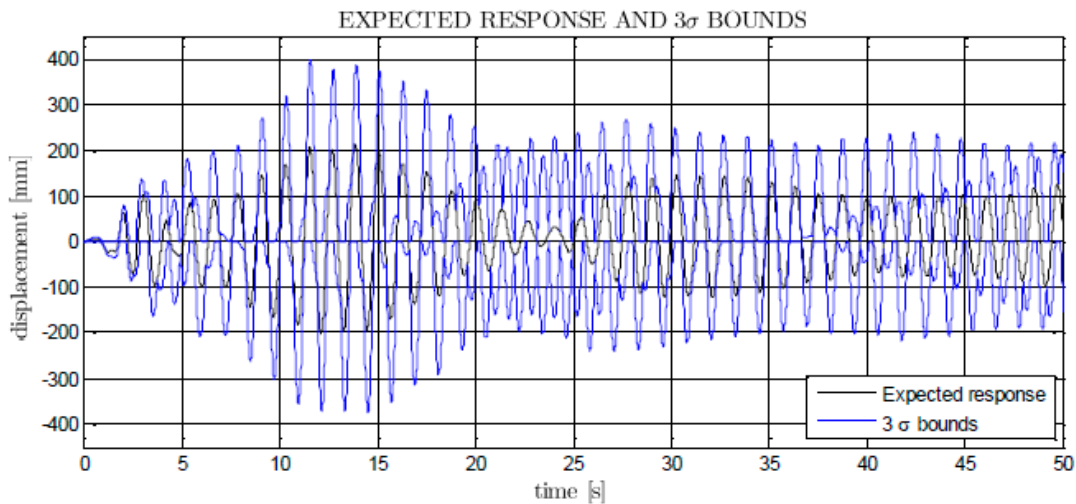

Figure 12: Expected response and bounds of the disordered BW system.

CONCLUSIONS

The paper showed the application of a simplified methodology for the seismic analysis of masonry towers. The method is based on the assumption that such response is mainly due to the first modal shape of the tower, and takes into account the randomness of the tower parameters. With this assumption, the behavior of the first modal

shape is modeled by means of a nonlinear hysteretic Bouc & Wen model, where the parameters of the model are identified on the results of a numerical computation. The paper demonstrates that the tower seismic response can be efficiently approximated by a SDOF Bouc & Wen system, provided that a proper identification of the parameters is performed.

The influence of a random initial stiffness of the Bouc & Wen model on its seismic response is subsequently analyzed by means of a perturbation approach, and the bounds of the response of the system are computed. It is possible to comment that such approach works well with a single accelerogram, but it cannot be applied with a seismic stochastic process yet. Further improvements of the proposed approach can take into account the response of the disordered system to a stochastic process, instead of only one accelerogram, and the dependence of the response on the randomness of other parameters, such as the yielding point.

ACKNOWLEDGEMENTS

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