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<https://doi.org/10.1016/j.conbuildmat.2019.117591>

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# Tensile Stress-Strain Characteristics of Rubberised Concrete from Flexural Tests

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**Abstract:** The tensile characteristics of rubberised concrete are practically impossible to obtain from direct tensile tests, due to the non-uniform distribution of aggregates and stiffness. In this paper, notched three-point bending tests are used to characterise Mode I fracture behaviour of concrete incorporating high volume of rubber particles obtained from post-consumer tyres. The test results show that rubber particles enhance energy absorption capacity and ductility of concrete. Inverse finite element analysis is performed to indirectly determine tensile stress-strain curves of rubberised concrete. The key material parameters introduced in the constitutive model are tensile strength, fracture energy and crack band width. The spurious mesh dependency is resolved by adopting a simple modification to the softening modulus as a function of element size. The performance of the proposed tensile stress-strain relation is compared with that of Model Code 2010 using ABAQUS concrete damaged plasticity model and shows considerably better accuracy. The proposed model can be used to improve the reliability of numerical analyses of rubberised concrete elements and structures.

**Author keywords:** Rubberised concrete; Three-point bending test; Fracture energy; Tension softening; Inverse finite element analysis; Crack band.

## 1. Introduction

Vulcanised rubber material extracted from end-of-life tyres is highly durable, has good strength and deformability, and can maintain its volume under stress, thus making it an ideal candidate to replace mineral aggregate for highly

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20 deformable concrete. The feasibility of utilising recycled rubber as aggregate in concrete has been assessed by many  
21 studies in the past [1-4]. The majority of those investigations focused on compressive performance and found that  
22 the replacement of mineral aggregates with rubber can reduce concrete density, workability, compressive strength  
23 and elastic modulus. This means that rubberised concrete (RuC) can only be used for non-structural applications,  
24 such as heat, sound and vibration isolation. More recently, it was also found that RuC has enhanced lateral strain [5,  
25 6] under axial loading. This has led researchers during the EU-funded project “Anagennisi” [7] to confine rubberised  
26 concrete (RuC) in an attempt to enhance axial strength and strain capacity [6]. The hypothesis was that confined  
27 RuC could be used in regions with high deformation demands, such as base isolation columns and bridge bearings.  
28 To model such elements, the full mechanical characteristics of RuC are required [8].

29 The compressive characteristics of RuC can be determined from uniaxial and triaxial compressive tests. However,  
30 its tensile characteristics are difficult to obtain from direct tensile tests as the uneven distribution of rubber particles  
31 results in a non-uniform distribution of tensile strength, even for small linear elements [1]. Hence, the tensile  
32 performance of RuC is normally derived from splitting or flexural tensile tests. Experimental evidence [1, 5, 9] has  
33 shown that the tensile strength (splitting or flexural) of RuC decreases with increasing rubber content. Furthermore,  
34 the tensile strength values obtained from the two types of tests are not equivalent. The maximum strength derived  
35 from flexural tests is governed by the concrete strength on the tension surface of the beam and can be affected by its  
36 boundary and moisture conditions [10]. In the case of splitting tests, the majority of the concrete perpendicular to the  
37 loading direction is under constant tensile stress, thus the splitting tensile strength is closer to the strength  
38 determined from direct tensile tests [11]. On the other hand, flexural tests produce a more stable load-deflection  
39 curve than splitting tests and can be used to obtain a more reliable estimate of the energy dissipation ability of RuC  
40 after cracking [1]. Such test results prove that the flexural post-cracking performance of RuC is enhanced by the  
41 replacement of aggregates with rubber particles [12]. This can be explained by the ability of rubber particles to store  
42 elastic energy and dissipate it through pull-out [13]. Since both indirect tensile tests do not provide a direct measure  
43 of the tensile stress-strain or stress-crack opening relations, there is a lack of a tensile constitutive model for this  
44 novel material. This limits the development of numerical models that could demonstrate the applicability of RuC in  
45 structural applications.

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4 46 The Concrete Damage Plasticity (CDP) model is widely used to simulate the failure behaviour of concrete in finite  
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6 47 element analysis [14-16]. The CDP treats cracked concrete as a continuum and cracks are represented by cracking  
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8 48 strains smeared over a certain width of fracture zone, or the width of a finite element. Thus the constitutive  
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10 49 behaviour of cracked concrete is modelled using the stress-strain relations combined with a damage parameter [17].  
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12 50 The successful finite element modelling of RuC using CDP also requires the complete uniaxial tensile stress-strain  
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14 51 relationship. Although this cannot be obtained from direct tensile tests, it is possible to obtain the tensile stress-strain  
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16 52 relationship by inverse finite element analysis (FEA) on the flexural test result. However, if a softening material  
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18 53 model is only described on the basis of stress-strain relationships, strain-softening can lead to spurious sensitivity  
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20 54 with respect to the size of elements due to the localisation of deformation [17-19]. Mesh refinement leads to a  
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22 55 smaller strain localisation band width and reduces global energy dissipation. One remedy for this spurious mesh  
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24 56 sensitivity issue is to use the crack band technique [17, 20]. The basic idea of this technique is to modify the strain-  
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26 57 softening modulus to the chosen size of the finite element. This adjustment ensures that different size elements  
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28 58 dissipate identical fracture energies [21].

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32 59 This study proposes a new tensile constitutive model for RuC obtained through inverse FEA and implemented in the  
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34 60 CDP model to predict the tensile behaviour of RuC. The paper initially describes an experimental program on which  
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36 61 three-point bending (TPB) and splitting tensile tests are utilised to determine the tensile behaviour of RuC. The next  
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38 62 sections describe the methodology applied for the inverse FEA and mesh sensitivity studies. Finally, to demonstrate  
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40 63 the efficiency of the proposed model, the predicted mechanical response using the proposed tensile  $\sigma$ - $\varepsilon$  relationships  
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42 64 and relationships obtained from Model Code 2010 are compared. This work will lead to a better understanding of  
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44 65 the behaviour of RuC and will assist in developing applications for highly deformable elements.

## 48 66 **2. Experimental program**

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51 67 Forty-five samples consisting of thirty-six cylindrical specimens ( $\varnothing 100 \times 200$  mm) and nine prisms  
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53 68 (L500 $\times$ W150 $\times$ H150 mm) were manufactured to examine the compressive and tensile characteristics of RuC. The  
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55 69 samples were divided into three sets by rubber contents ( $\rho_{vr} = 0\%$ , 30% and 60%) and labelled as R0, R30 and R60,  
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57 70 respectively. The rubber content  $\rho_{vr}$  is defined as the volume of mineral aggregates in the reference regular concrete  
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59 71 mix replaced by rubber [5, 22].

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## 2.1 Rubber Particles

The recycled rubber particles were extracted from car (sizes up to 10 mm) and truck (sizes in the range of 10–20 mm) end-of-life tyres [23]. Fig. 1 shows the particles size range and used ratio in the concrete mix. These proportions were adopted from an experimental study on RuC mixes conducted by Raffoul et al.[22], in which acceptable strength loss and desirable workability were achieved.

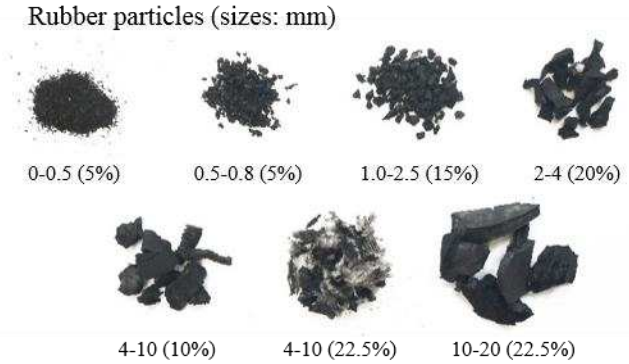


Fig. 1. Rubber particles used in the concrete mix to replace sand (size:0-4 mm) and mineral aggregate (sizes: 4-10 mm and 10-20 mm).

## 2.2 Mix designs

A reference regular concrete mix ( $\rho_{vr} = 0$ ) for bridge pier design with target 28-day compressive strength of 60 MPa was initially selected and modified for rubberised concrete mixing. In the RuC mixes, 20% of the cement (CEM II - 52.5N) was replaced by 50% fly ash [24] and 50% silica fume [25]. Silica fume and fly ash were added to increase flowability and strength and to improve particles packing. Two RuC mixes were produced with 30% and 60% of rubber contents, respectively. The specific gravity of mineral aggregates was considered to be 2.65, and that of rubber was 0.8 [22]. The quantities of rubber particles used in the mixes were 165 kg/m<sup>3</sup> and 330 kg/m<sup>3</sup>, respectively. Table 1 summarises the mix proportions used in this study.

Table 1. Mix proportions for optimised RuC mix.

$\rho_{vr}$	CEM	Silica	Pulverised	Aggregates	Rubber Particles	Plasticiser	Superplasticiser	Water	Designed
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	II 52.5	Fume	Fuel Ash	kg/m <sup>3</sup>			l/m <sup>3</sup>			density	kg/m <sup>3</sup>		
				0/5 mm	5/10 mm	10/20 mm	0/4 mm	4/10 mm	10/20 mm				
0	340	42.5	42.5	820	364	637	-	-	-	2.5	5.1	150	2396
30%	340	42.5	42.5	574	255	456	74	17	75	2.5	5.1	150	2026
60%	340	42.5	42.5	328	146	255	149	33	148	2.5	5.1	150	1634

## 2.3 Test procedure and methodology

### 2.3.1 Compressive and Splitting tests

The compressive (see Fig. 2a) and splitting tests (see Fig. 2b) on cylindrical specimens were conducted under load control at loading rates of 0.4 MPa/s [26] and 0.01 MPa/s [27], respectively. For the compressive tests, three laser sensors (accuracy of 0.005 mm) were used to measure global axial displacement. The sensors were mounted radially at 120° on two aluminium rings with 100 mm gauge length. Local axial strains were measured by using strain gauges placed at mid-height of the specimen. The possible initial rotations were accommodated by a top loading plate fitted with a hinge. The dimensions of each specimen were measured before testing. The specimen ends were strengthened with post-tensioned metal straps with a thickness of 0.8 mm and a width of 25 mm to avoid local failure.

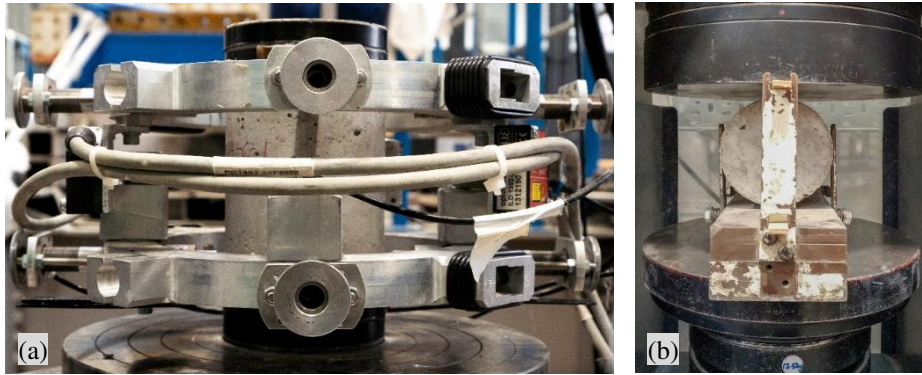


Fig. 2. View of testing for a) Compression tests and b) Splitting tests.

The splitting tensile strength ( $f_{ct,sp}$ ) can be calculated by Eq. (1) considering the maximum load ( $P_{max}$ ) [27]:

$$f_{ct,sp} = \frac{2P_{max}}{\pi L d_{cy}} \quad (1)$$

where  $L$  is the length of the specimen,  $d_{cy}$  is the diameter of the cylinder.

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107 The load in Eq. (1) is assumed to be concentrated along a line. However, the load is practically applied on a sheet of  
108 plywood or compact cardboard. Rocco et al. [28] showed that the loading area width ( $t$ ) can affect the tensile  
109 strength of the cylinders up to 25%. Tang et al. [29] proposed the following equation to consider the effect of the  
110 loading area width on the splitting tensile strength.

$$f_{ct,sp} = \frac{2P_{max}}{\pi L d_{cy}} \left( 1 - \left( \frac{2t}{d_{cy}} \right)^2 \right)^{1.5} \quad (2)$$

112 Bompa et al. [5] proposed Eqs. (3) to predict the splitting tensile strength ( $f_{ct,sp}$ ), tensile strength ( $f_{ct}$ ) and elastic  
113 modulus ( $E_c$ ) as a function of the compressive strength ( $f_{co}$ ) of RuC. The performance of these equations will be  
114 evaluated against the test data obtained in this study.

$$\begin{aligned} f_{ct,sp} &= 0.26 f_{co}^{2/3} \\ f_{ct} &= 0.24 f_{co}^{2/3} \\ E_c &= 12 \left( \frac{f_{co}}{10} \right)^{2/3} \end{aligned} \quad (3)$$

### 116 2.3.2 Three-point bending tests

117 Before testing, the geometrical dimensions of each specimen were measured and a 5mm wide and 25 mm deep  
118 notch was sawn at mid-span of the prisms and perpendicular to the cast surface using a diamond blade [30]. All  
119 specimens were tested under a TPB loading arrangement using a 300 kN electromagnetic testing machine (see Fig.  
120 3). Two Linear Variable Differential Transducers (LVDTs) were mounted on an aluminium yoke to measure central  
121 deflections [31] and a clip gauge was mounted across the notch to obtain the crack mouth opening displacement  
122 (CMOD) [32]. In order to achieve a stable result, the tests were CMOD-controlled at a constant rate of 0.02 mm/min  
123 [33]. The load, central deflection and CMOD were recorded throughout loading history.

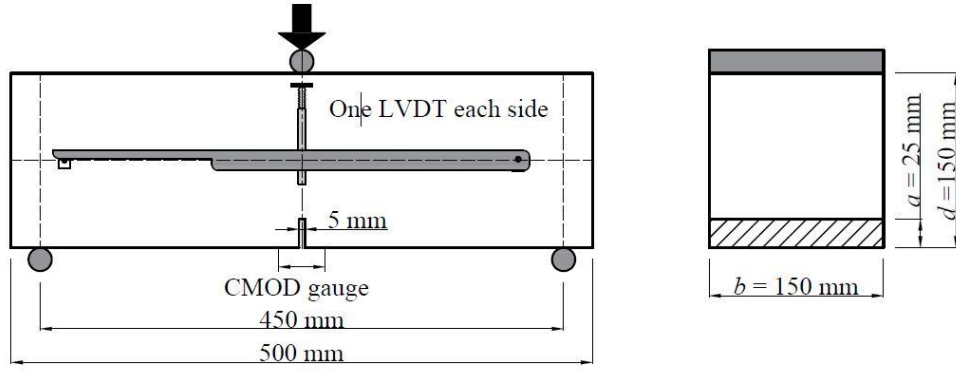


Fig. 3. Set-up of the three-point bending tests.

The flexural tensile strength ( $f_{ctm,fl}$ ) can be calculated as the equivalent elastic tensile flexural stress corresponding to the maximum load recorded, as:

$$f_{ctm,fl} = \frac{3P_{max}l}{2b(d-a)^2} \quad (4)$$

where  $l$  is the span of the prism,  $b$  is the width of the prism,  $d$  is the depth of the prism, and  $a$  is the effective depth of the notch.

The flexural modulus of elasticity ( $E_{fm}$ ) is determined by using the elastic flexural deformations, as given below:

$$E_{fm} = \frac{l^3P}{48I\delta} \quad (5)$$

where  $P/\delta$  (N/mm) is the slope of the load-deflection curve at 40% of the peak load and  $I$  ( $\text{mm}^4$ ) is the second moment of area of the mid-span cross-section.

### 2.3.3 Total fracture energy using work-of-fracture method

Hillerborg's work-of-fracture method assumes that the energy required for crack formation is equivalent to the work of the external load. Thus the complete load-deflection curve of the specimen is necessary to estimate the work of the external load, which can be represented as the area under the load-deflection curve. The fracture energy ( $G_F$ ) can be then calculated using the following equation [34]:



$$G_F = \frac{W_0 + \left(m_1 \frac{l}{L} + 2m_2\right) g \delta_0}{A_f} \quad (6)$$

where  $W_0$  is energy absorption capacity, which represents the area under the load-deflection curve during the bending test;  $m_1$  is the mass of the beam between supports;  $m_2$  is the mass of the unattached loading apparatus;  $L$  and  $l$  are the length and span of the specimen, respectively;  $g$  is the gravity acceleration;  $\delta_0$  represents the maximum recorded deflection; and  $A_f = (d - a)b$  denotes the area of the fracture surface.

## 2.4. Experimental results

### 2.4.1 Changes in unit weight

As a result of the low unit weight of rubber particles, the unit weight of rubberised concrete reduces by increasing the rubber content. Fig. 4 shows the expected linear relationship between unit weight ( $d_{RuC}$ ) and rubber contents ( $\rho_{vr}$ ). Compared with the unit weight of concrete mixes without rubber, the average unit weight of the concrete mix with a rubber content of 30% and 60% decreased by 15.4% and 31.8%, respectively.

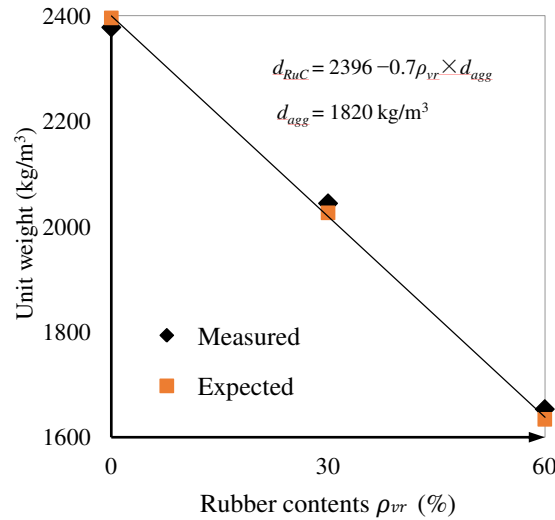


Fig. 4. Unit weight of RuC with different rubber contents  $\rho_{vr}$ .

### 2.4.2 Compressive and splitting tensile strength

The compressive test results are summarised in Table 2. The mean cylindrical compressive strengths ( $f_{co}$ ) of R30 and R60 were about 29% and 10% of that of regular concrete R0 (75 MPa), respectively. The compressive stress-strain

curves are shown in Fig. 5. The elastic moduli ( $E_c$ ) of R30 and R60 were 41% and 21% of that of R0 (41 GPa), respectively. The post-peak behaviour was also significantly influenced by the percentage of rubber content. The typical compressive failure patterns for the tested cylinders are shown in Fig. 6a. As expected, all plain ( $\rho_{vr} = 0\%$ ) specimens failed suddenly in a brittle manner. The failure of RuC cylinders with  $\rho_{vr} = 30\%$  and  $60\%$  was more gradual as the cylinders developed a large number of macro-cracks prior to failure. Rubber particles dilate more at lower displacements than mineral aggregates due to their high Poisson's ratio and low modulus of elasticity. This dilation generates internal splitting tensile stresses perpendicular to the loading direction and causes the concrete to fail at an earlier stage.

**Table 2.** Experimental results of examined concrete mixes

$\rho_{vr}$	$f_{co}$ MPa	$E_c$ GPa	$f_{ct,sp}$ MPa	$f_{ctm,fl}$ MPa	$E_{fm}$ GPa	$G_F$ N/mm	$W_0$ N·m
0	75 (5.6)	41 (2.3)	4.1 (0.32)	5.9 (0.5)	44 (9)	0.12 (0.02)	2.2 (0.02)
30	21.9 (3.6)	17 (3.5)	1.8 (1.29)	3.9 (1.86)	20 (8)	0.46 (0.14)	8.1 (0.14)
60	7.7 (0.3)	8.5 (1.2)	0.9 (0.52)	2.1 (0.22)	10 (0.3)	0.93 (0.06)	16.8 (0.06)

Note: Standard Deviation is presented in brackets.

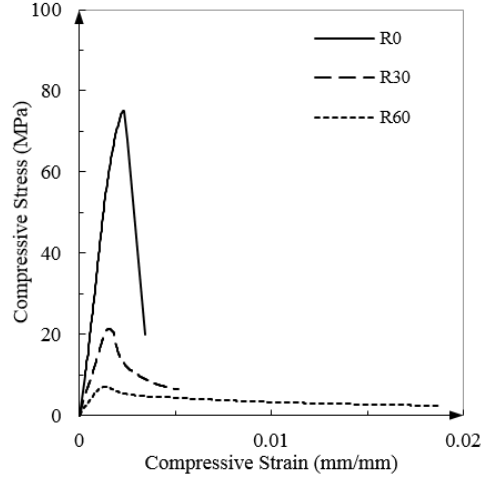
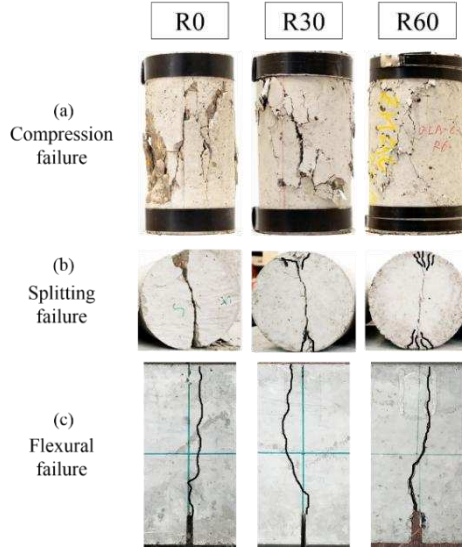


Fig. 5. Compressive stress-strain curves of R0, R30 and R60.



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169 Fig. 6. Typical failure modes for a) Compression; b) Splitting tension and c) Flexural.

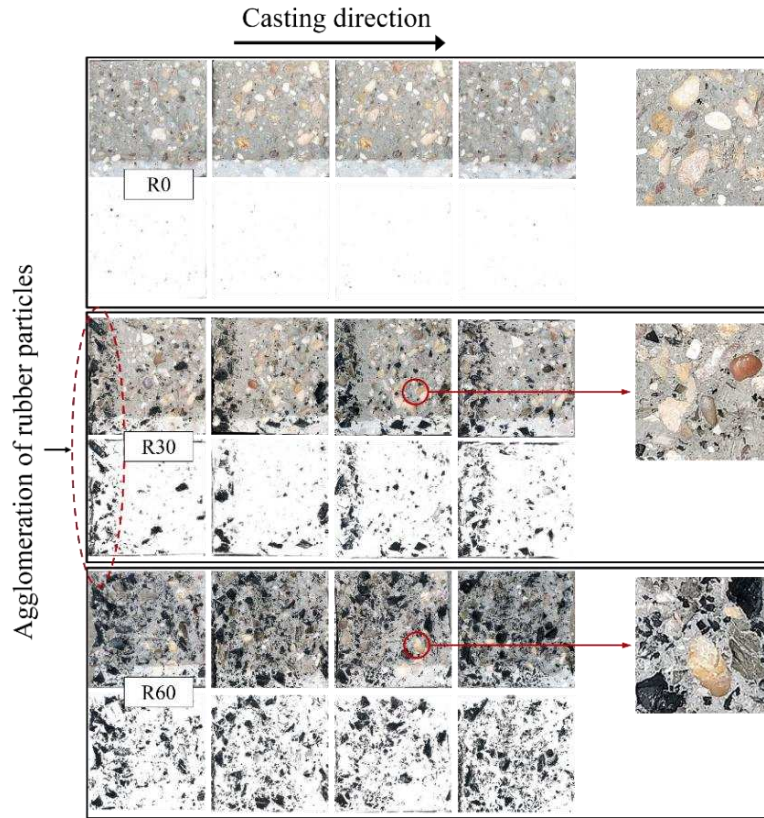
170 During the splitting tests, as expected, all samples failed due to wedge formation and splitting as shown in Fig. 6b.

171 Compared to the brittle failure mode of plain concrete, the RuC samples exhibited a slower axial splitting process  
 172 and higher capacity in absorbing plastic energy. The regular concrete specimens split into two halves immediately  
 173 upon reaching their ultimate load capacity, while the RuC specimens were capable of withstanding at least 80% of  
 174 the ultimate load for about 30 seconds (R30) and 55 seconds (R60) after splitting. The RuC specimens underwent  
 175 large displacements before the loading stopped, and the deformation was partially recovered upon unloading. All the  
 176 RuC specimens maintained their integrity even for highly cracked specimens. The splitting tensile strength ( $f_{ct,sp}$ )  
 177 results are reported in Table 2, which show an apparent degradation in tensile strength with increasing rubber  
 178 content. However, the trend in tensile strength loss is less severe than in compressive strength. The splitting tensile  
 179 strengths of R30 and R60 were 44% and 22% of that of R0 (4.1 MPa), respectively.

#### 180 2.4.3 Three-point bending tests

181 The flexural failure patterns of all specimens are shown in Fig. 6c. The average flexural strength ( $f_{cm,fl}$ ) and modulus  
 182 ( $E_{fm}$ ) of three replicates are listed in Table 2. Fig. 7 shows images of typical fracture surfaces obtained after the  
 183 bending tests (upper row for each mix) as well as the results from the digital image processing of these sections  
 184 (lower row) to examine rubber particle distribution along the casting direction. The images show that the rubber  
 185 tended to float towards the cast face (left in the pictures) during vibration of the fresh concrete leading to an

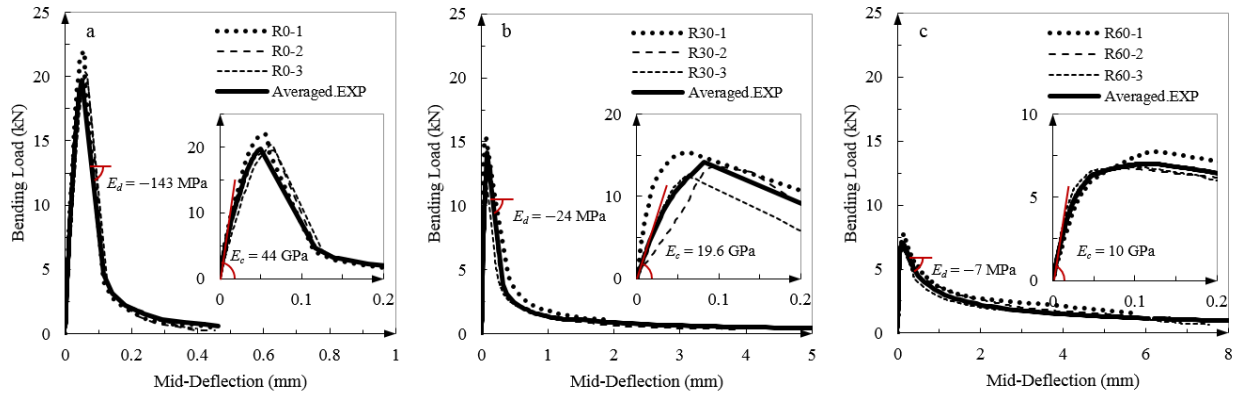
186 agglomeration of rubber particles. However, this tendency is less obvious in the specimens with a rubber content of  
 187  $\rho_{vr} = 60\%$ .



189 Fig. 7. The fracture surface of three-point bending tests.

190 Fig. 8 shows the individual and average  $P-\delta$  curves from three TPB specimens for each of the  $\rho_{vr}$  tested. The mid-  
 191 span deflection of each prism was determined by averaging the measurements from LVDTs. The response of the  
 192 TPB test can be described by three typical phases: 1) an elastic phase up to cracking; 2) a short flexural hardening  
 193 response up to the maximum flexural capacity; and 3) a reduction of the load with increasing CMOD/deflection. The  
 194 comparative assessment of load-deflection curves from Fig. 8 shows a reduction in flexural strength and stiffness  
 195 with increasing rubber content. The flexural strengths ( $f_{cm,\eta}$ ) of concrete with  $\rho_{vr} = 30\%$  and  $60\%$  are 66% and  
 196 35.6% of the reference concrete, respectively. The flexural modulus ( $E_{fm}$ ) of R30 and R60 were reduced to 45% and  
 197 20% of that of R0, respectively. However, the ratio of cracking stress to flexural strength ( $f_{cr}/f_{cm,\eta}$ ) are 54%, 68%  
 198 and 72% for R0, R30 and R60, respectively. At load levels close to the peak, RuC specimens exhibited a stable  
 199 transition from the pre-cracking to post-cracking regimes, while the reference (plain concrete) beams showed a

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4 200 brittle behaviour after the maximum capacity was reached. Beyond the peak, the descending branch showed  
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6 201 reducing gradients ( $E_{d,R0} = -143$ ,  $E_{d,R30} = -24$  and  $E_{d,R60} = -7$  MPa) and increasing energy absorption capacity ( $W_0 =$   
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8 202 2.2;  $W_{0,R30} = 8.1$  and  $W_{0,R60} = 16.8$  N·m) and fracture energy ( $G_{F,R0} = 0.15$ ,  $G_{F,R30} = 0.51$  and  $G_{F,R60} = 0.93$  N/mm<sup>2</sup>) as  
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10 203 the rubber content increased (see Fig .9). This enhancement indicates that the capability of rubberised concrete in  
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12 204 energy absorption is significantly better than regular concrete and RuC specimens developed up to 20 times larger  
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14 205 deflections. The higher deformability of rubberised concrete elements can potentially result in more resilient  
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16 206 structural systems and improve structural integrity under extreme load conditions.



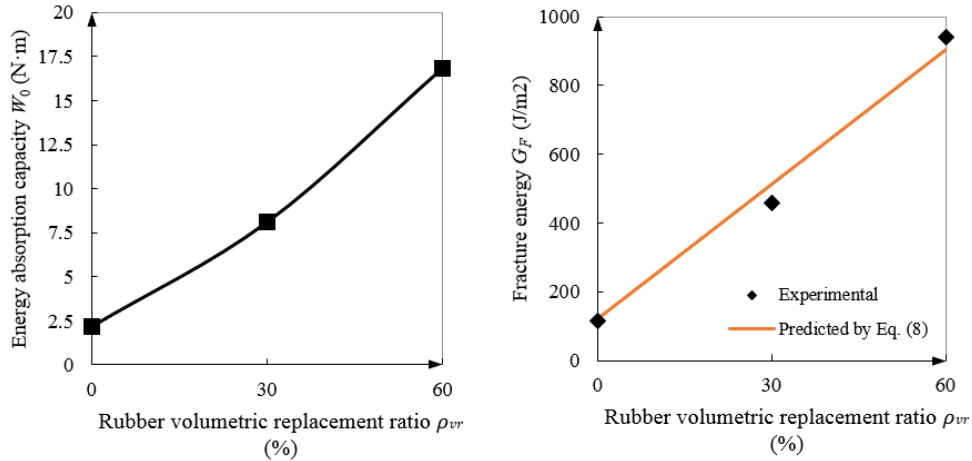
207  
208 Fig. 8. Load-deflections of TPB specimens: (a) R0, (b) R30 and (c) R60.

209 The fracture energy ( $G_F$ ) in N/mm<sup>2</sup> for regular concrete can be determined using Model Code 2010 [35]:

$$G_F = 0.073 \times f_{co}^{0.18} \quad (7)$$

210  
211 Using the results of this study, a parametric equation was developed to calculate the fracture energy ( $G_{F,RuC}$ ) of RuC  
212 based on Eq. (7) and as a function of rubber content ( $\rho_{vr}$ ) (Eq. (8) and Fig. 9):

$$G_{F,RuC} = (1 + 10.8\rho_{vr}) \cdot G_F \quad (8)$$



215 Fig. 9. Energy absorption capacity and fracture energy with different rubber contents.

216 Further inspection of the fractured surfaces (see Fig. 6) revealed a different degree of roughness depending on the  
 217 percentage of rubber volume replacement. At 0% rubber replacement, almost all the aggregates were found broken.  
 218 The flexural failure mode of RuC can be explained as follows. At 30% rubber replacement, the majority of mineral  
 219 aggregates were found broken on the fracture surface. At 60% rubber replacement, no broken mineral aggregates  
 220 were found on the fracture surface. When a tension crack propagates through RuC and reaches a rubber particle, the  
 221 crack keeps developing in the same direction as the rubber has the ability to withstand large elastic deformation  
 222 under small tensile stress. As rubber can withstand much higher tensile stress and strain until failure than cement and  
 223 mineral aggregates, the primary failure mechanism of rubber particles is pull-out. This failure mechanism results in  
 224 the energy dissipation enhancement discussed above.

### 225 3. Inverse FEA

226 Inverse fracture energy based FEA is developed and utilised in this section to obtain the tensile stress-strain curves  
 227 of RuC by using the experimental results presented in Section 2.

#### 228 3.1 Assumptions:

229 The following assumptions are used, the first three proposed by Uchida et al. [36]

- 230 i. A single main crack exists through a fracture area (see Fig. 10). A rotational axis exists on the top of the  
 231 fracture area. The distribution of the crack opening is linear.

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- 232 ii. The mean crack opening ( $w_c$ ) at the fracture area is represented by half of the crack mouth opening displacement (CMOD) at the notch. This opening can be smeared over a crack band width to obtain strain.
- 234 iii. The externally applied energy is equivalent to the energy needed for crack development.
- 235 iv. The shape of the uniaxial tension softening curve of concrete is exponential, and the function proposed by Krätzig and Pölling [37] can be adopted (Eq. (9)):

$$y = e^{-\frac{\epsilon_{cr} x}{\gamma_t}} \quad (x > 1) \quad (9)$$

238 where  $x = \epsilon_{ck}/\epsilon_{cr}$ ,  $y = \sigma_t/f_{ct}$ ;  $\sigma_t$  and  $\epsilon_{ck}$  are tensile stress and strain, respectively;  $\epsilon_{cr}$  is cracking strain; the tensile strength ( $f_{ct}$ ), and elastic modulus ( $E_c$ ) can be calculated by Eqs. (3);  $\gamma_t$  adjusts the area under the stress-strain curve and can be determined by:

$$\gamma_t = \frac{g_t}{f_{ct}} - \frac{1}{2} \frac{f_{ct}}{E_c} \quad (10)$$

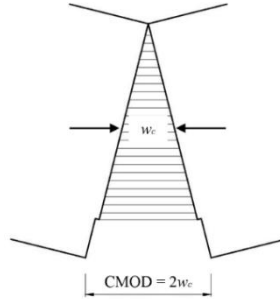


Fig. 10. Model of fracture area reproduced from Uchida et al. [33]

### 3.2 Definitions:

- $G_{ch} = w_{cb} \times g_c$  is energy dissipated during crushing in a compressive test;
- $G_F = w_{cb} \times g_t$  is energy required for crack propagation in a flexural test;
- Coefficients  $g_c$  and  $g_t$  are equal to the areas under  $\sigma-\epsilon^p$  curves in uniaxial tension and compression, respectively, as shown in Fig. 11;

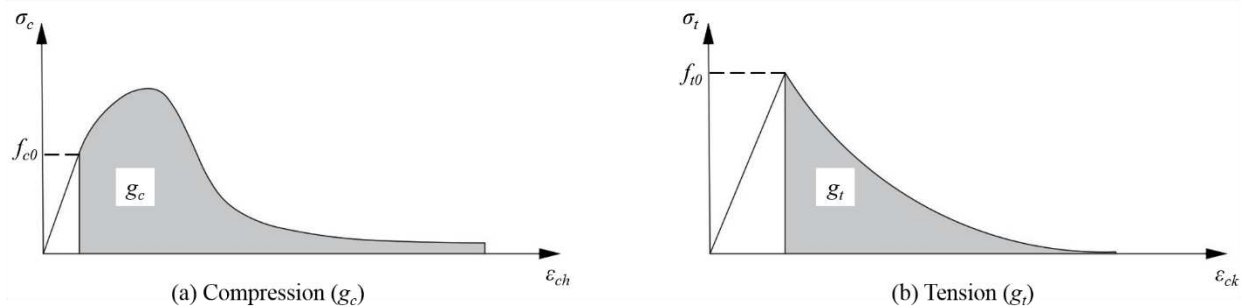


Fig. 11. Definition of coefficient  $g_c$  and  $g_t$ , after [8].

- $w_{cb}$  is the crack band width of a heterogeneous material and represents the effective width of the strain-softening damage zone [38, 39]. This length can be expressed as the ratio of the energy dissipated per unit area ( $\text{N/mm}^2$ ) to the energy dissipated per unit volume ( $\text{N/mm}^3$ ).

Bazant and Oh [18] claimed that  $w_{cb}$  is approximately 2.7-3 times the maximum aggregate size  $d_a$ . The value of  $w_{cb}$  is also dependent on concrete strength. By reducing the difference between the elastic modulus of the coarse aggregates and the mortar (e.g. in high strength concrete), the value of  $w_{cb}/d_a$  decreases and a more homogeneous material is obtained. The opposite is expected for low strength concrete or when rubber aggregates are used as the material becomes more heterogeneous.

Since direct tensile tests on rubberised concrete are neither easy to perform nor available, the crack band width cannot be calculated via  $G_F / g_t$ . However, it can be estimated through the determination of the  $\sigma_c$ - $\epsilon_{ch}$  curve and crushing energy ( $G_{ch}$ ) of RuC with various rubber contents from carefully instrumented compressive tests according to [8]. For the RuC mix examined, compressive test results are available from this study and Raffoul et al. [6], and an analytical  $\sigma$ - $\epsilon$  model is given by Bompa et al. [5]. These results are used to determine  $w_{cb}$  as shown in Table 3. It is interesting to note that the value of  $w_{cb}$  is higher for RuC and increases with increasing rubber content, which confirms the hypothesis that  $w_{cb}$  increases with increasing heterogeneity.

Table 3. Crushing energy ( $G_{ch}$ ) and coefficient ( $g_t$ ) of concrete with different rubber contents ( $\rho_{vr}$ )

$\rho_{vr}$	$G_{ch}$ $\text{N/mm}^2$	$g_c$ $\text{N/mm}^3$	$\epsilon_{ult}$ $\text{mm/mm}$	$g_t$ $\text{N/mm}^3$	$w_{cb} / d_a$
0	9.1	0.16	0.006	0.0027	2.8
30%	8.7	0.11	0.055	0.0065	3.9
60%	10.5	0.10	0.075	0.0088	5.0

Note:  $d_a$  is maximum aggregate size, which is 20 mm in this study.



### 3.3 Steps for the Inverse FEA

1. Determination of total external work ( $W_0$ ) through the integration of the load-deflection curve.
2. Determination of fracture energy ( $G_F$ ) by Eq. (6). Table 2 shows the results.
3. Determination of the ultimate cracking strain ( $\varepsilon_{ult}$ ) of concrete by dividing the ultimate mean crack mouth opening ( $w_{c,ult}$ ) by  $w_{cb}$ . Results are shown in Table 3.

$$\varepsilon_{ult} = \frac{w_{c,ult}}{w_{cb}} \quad (11)$$

4. Determination of  $g_t$  by using the fracture energy ( $G_F$ ) obtained from bending tests. Results are shown in Table 3.

$$g_t = \frac{G_F}{w_{cb}} \quad (12)$$

5. Determination of the shape factor  $\gamma_t$  in Eq. (10) by using  $g_t$ . Results are shown in Fig. 12.

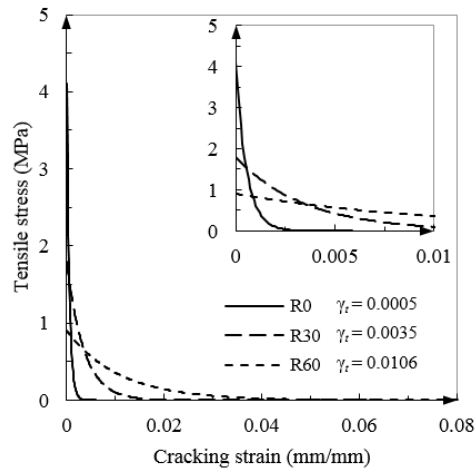


Fig. 12. Tensile stress-strain curves for concrete with various rubber contents.

## 4. Numerical modelling

A finite element model has been developed using Abaqus/Standard finite element software package [40]. The “Concrete Damaged Plasticity (CDP)” [41] developed by Lubliner et al. [8] and elaborated by Lee and Fenves [42]

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283 is adopted as it allows the use of a custom tensile stress-strain model for concrete in tension. CDP uses the modified  
284 Drucker-Prager criterion and the yield surface  $F$  in the deviatoric plane and assumes isotropic damage  $d$ . The details  
285 can be found in [43].

286 The CDP model requires the following material functions and parameters:

- 287 • Stress-strain relations for uniaxial behaviour in compression and tension. The compressive stress-strain  
288 relations proposed by Bompa et al. [5] are adopted in this study, and the tensile stress-strain relations are  
289 obtained through inverse FEA (see Fig. 12).
- 290 • The damage variables  $d_c$  and  $d_t$  represent the portion of normalised energy dissipation for damage in  
291 compression and tension, respectively [8]. The values of  $d_c$  and  $d_t$  are determined using Eqs. (13), and the  
292 curves of  $d_t$  vs  $\varepsilon_{ck}$  are shown in Fig. 13 (zero means no damage and one means fully damaged).

$$d_c = \int_0^{\varepsilon_{ch}} \sigma_c d\varepsilon_{ch} / g_c; d_t = \int_0^{\varepsilon_{ck}} \sigma_t d\varepsilon_{ck} / g_t \tag{13}$$

- 294 • The default values of 2/3, 0.1 and 1.16 are assigned to shape parameter  $K_c$ , eccentricity  $\epsilon$ , and the biaxial to  
295 uniaxial compressive strength ratio  $f_{bo}/f_{co}$ , respectively [40]. A low viscosity parameter of  $2 \times 10^{-6}$  was used  
296 to avoid convergence problems. It should be noted that the non-associative plastic volume deformation of  
297 the concrete is controlled by the dilation angle  $\psi$ , while the lateral dilation of concrete increases with  
298 increasing  $\psi$ . As replacing the mineral aggregates with rubber particles can dramatically increase the lateral  
299 dilation of RuC by up to 300% over the plain concrete [6], a relatively high dilation angle  $\psi = 40^\circ$  was  
300 chosen for RuC [44, 45].

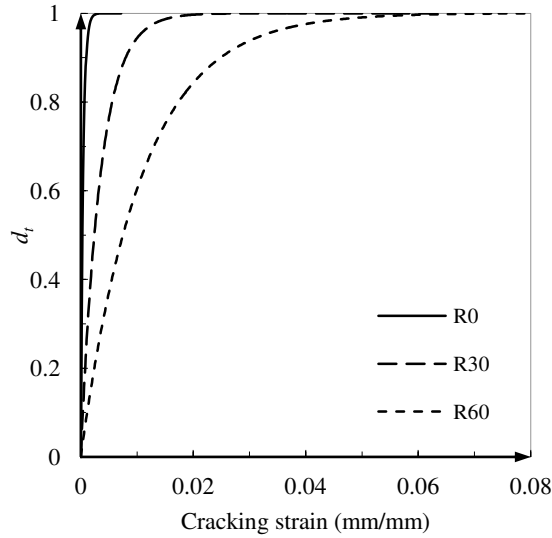


Fig. 13. Tensile damage variables ( $d_t$ ) vs cracking strain ( $\varepsilon_{ck}$ ).

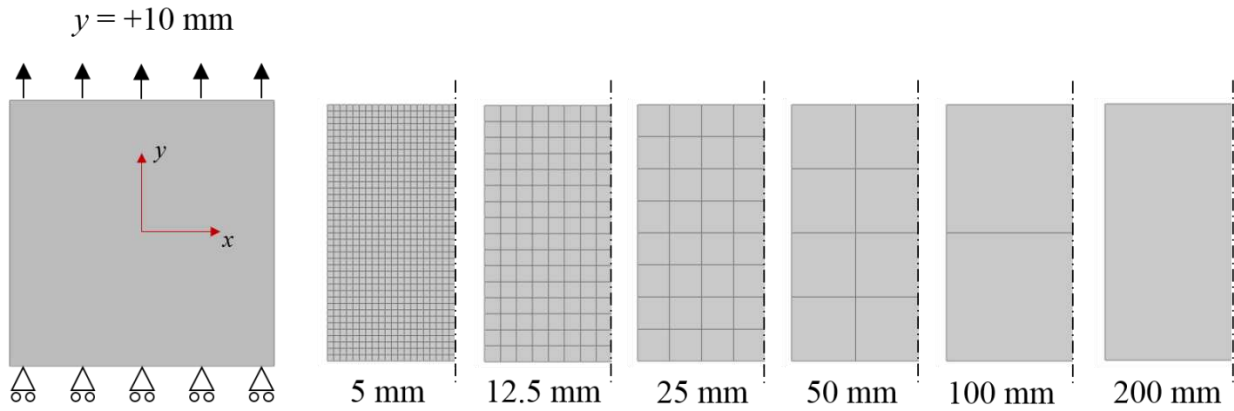
#### 4.1 Mesh-sensitivity study

The lower bound value of finite element size that can be attributed to  $w_{cb}$  is shown in Table 3, while an upper bound value of  $G_F E_c J_f^2$  is recommended by Bažant and Oh [38]. It can be seen that the lower bound values are significantly larger when compared with the dimension of the specimens examined in this study, thus to obtain a detailed resolution within the strain-softening region, the element mesh size ( $h_e$ ) needs to be smaller than the experimentally derived  $w_{cb}$ . However, as discussed before, this would lead to mesh sensitivity issues as the dissipation of energy depends on the width of the element size used in the area where the damage is localised. To investigate the mesh-sensitivity problem, the numerical response of a square plate (200×200 mm) subjected to plane stress conditions (Fig. 14) was examined. A monotonically increasing displacement in the positive  $y$ -direction was applied to the upper boundary ( $y = +10$  mm), while a traction-free boundary condition was applied to the bottom edge. The numerical calculation was performed using four-node elements with four integration points (CPS4) and six different mesh sizes (see Fig. 14). The reduced integration option was not considered in this study as it is not reliable for problems involving crack formation [46]. The tensile stress-strain relation of R60 with a value of  $w_{cb} = 100$  mm was used.

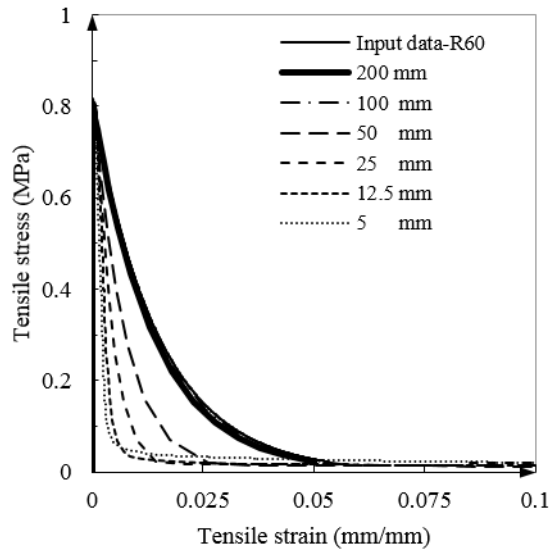
Fig. 15 illustrates the average resulting stress on the upper boundary vs average tensile strain, which clearly shows the sensitivity of the results to the selected mesh size. It can be noted that only the stress-strain curves obtained from

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319 the plate with the element sizes above 100 mm are similar to the curves derived from the input data (i.e.  
320 experimental test results). Fig. 16 shows that the total energy dissipation is mesh-dependent as a result of the  
321 localisation of deformations (see Fig. 16).



322  
323 Fig. 14. Plate subjected to uniaxial tension: boundary conditions and examined mesh geometries.



324  
325 Fig. 15. Tensile response of a plate subjected to uniaxial tension and modelled using different mesh sizes.

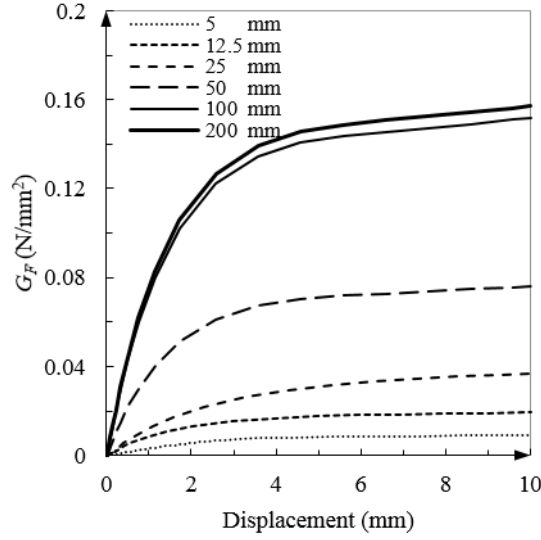


Fig. 16. Total energy dissipation of a plate subjected to uniaxial tension and modelled using different mesh sizes.

## 4.2 Modified procedure

Based on the numerical observation above, the element mesh size ( $h_e$ ) used in the fracture zone should be carefully selected when defining the tensile behaviour of the material. The crack band model can provide substantially the same results regardless of the width of the strain localisation zone when the energy dissipation per unit area of the crack band plane is correct [17]. The softening modulus of the post-peak softening curve can be adjusted by incorporating the ratio of element size ( $h_e$ ) to the crack band width ( $w_{cb}$ ) so that the correct energy dissipation per unit crack surface can be obtained for all cases. In a uniaxial tensile experiment on concrete, the dissipated energy ( $G_F$ ) per unit crack surface can be written as

$$G_F = w_{cb} \left[ \int_0^{\varepsilon_{ult}} e^{-\frac{\varepsilon_{cr}}{\gamma_t}} d\varepsilon_{cr} \right] = h_e \left[ \int_0^{\varepsilon_{ult, h_e}} e^{-\frac{\varepsilon_{cr}}{\gamma_{t, h_e}}} d\varepsilon_{cr} \right] \quad (14)$$

The shape factor  $\gamma_{t, h_e}$  for different mesh sizes can be obtained as:

$$\gamma_{t, h_e} = \lambda \frac{g_t}{f_{ct}} - \frac{1}{2} \frac{f_{ct}}{E_c} \quad (15)$$

Hence, the corresponding ultimate cracking strain is:

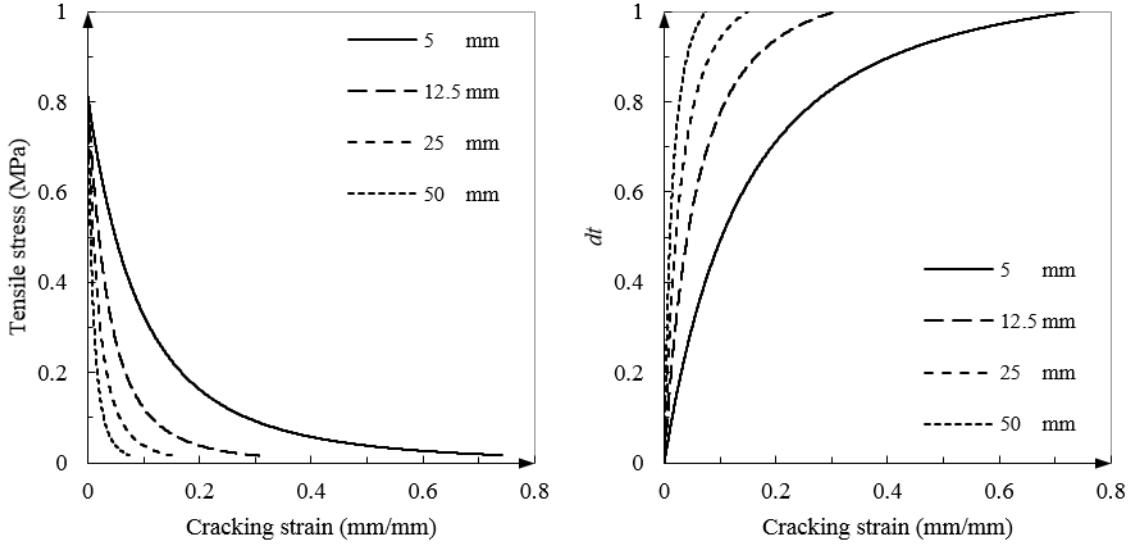
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$$\varepsilon_{ult,h_e} = \lambda \varepsilon_{ult} \tag{16}$$

341 where  $\lambda = w_{cb}/h_e$ .  $h_e$  is the element size. For a square element with a surface area  $A_e$ , the value of  $h_e$  is equal to  $\sqrt{A_e}$ .

342 A detailed discussion on element size can be found in [20, 47].

343 The modified tensile stress/damage variable vs cracking strain curves of R60 for different mesh sizes are shown in  
344 Fig. 17. The effectiveness of the modification expressed by Eqs. 16 and 17 are illustrated in Fig. 18, which confirms  
345 that similar total energy dissipation can be obtained with different mesh sizes.



346  
347 Fig. 17. Modified tensile constitutive laws of R60 for different mesh sizes: (a) tensile stress-cracking strain curves;  
348 (b) damage variables-cracking strain curves.

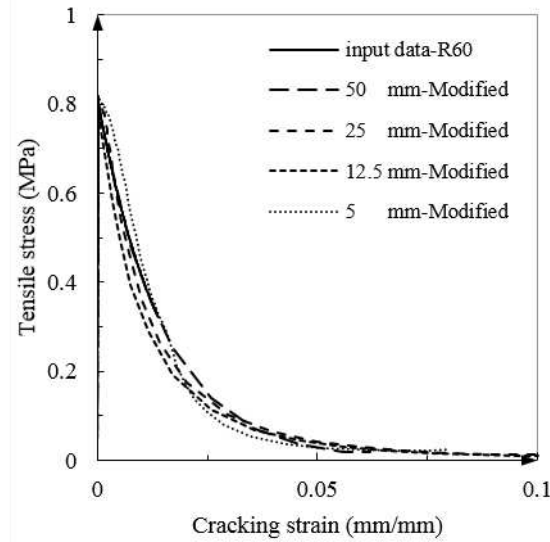


Fig. 18. Results of the modified tensile constitutive laws of R60 with respect to different mesh sizes under uniaxial tensile loading.

The proposed methodology enables the derivation of a more reliable tensile constitutive law for both conventional and rubberised concrete solely based on values obtained from simple compression tests. Eqs. (3) and (7) or (8) can be used to calculate tensile strength ( $f_{ct}$ ) and fracture energy ( $G_f$ ), respectively; while the crack band width ( $w_{cb}$ ) of rubberised concrete tested in this study was found to be directly proportional to the rubber content ( $\rho_{vr}$ ). A linear relationship between  $w_{cb}/d_a$  and  $\rho_{vr}$  ( $w_{cb}/d_a = 3.67\rho_{vr} + 2.8$ ) was found to capture reasonably well the crack band width variation for the concrete tested in this study, but further work should be carried out to validate these findings for different values of  $\rho_{vr}$  or different particles size distributions. For any given mesh size ( $h_e$ ), the complete tensile constitutive law can be described by Eqs. (14 - 16).

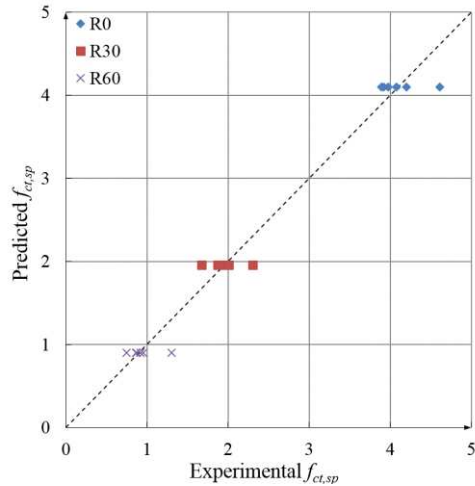
### 4.3 Verification of the material model

#### 4.3.1 Splitting tests

Finite element modelling of cylinders subjected to splitting tests was performed to examine whether Eqs. (3) can provide a reasonable prediction of the splitting tensile strength. As the diametric compression load was applied experimentally through a plywood strip (width: 10 mm and thickness: 3 mm), the same area was used to apply the load on the surface of the cylinder in the numerical analysis. Since maximum capacity is mesh-independent, an

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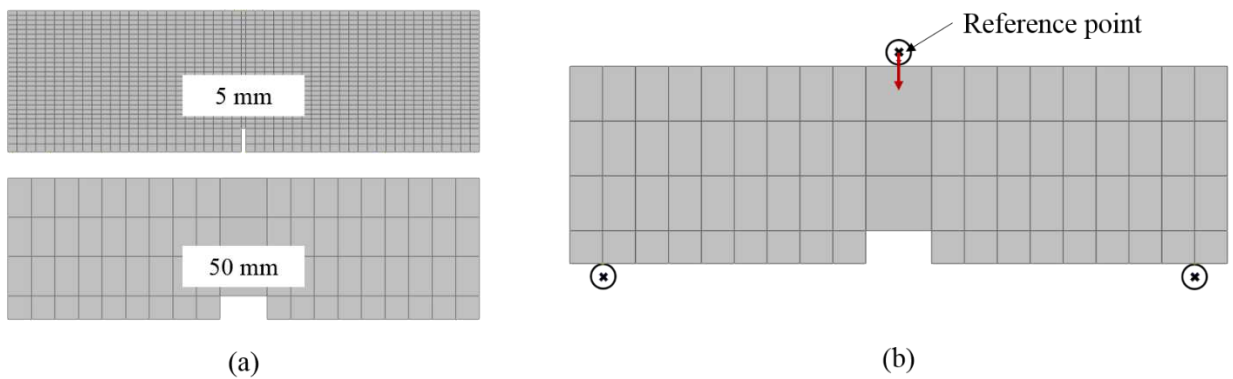
366 arbitrary mesh size of 15 mm was chosen [20]. Fig. 19 shows the comparison between numerical and experimental  
367 results, which confirms the effectiveness of Eqs. (3).



369 Fig. 19. Comparison of the predicted splitting tensile strength with experimental results.

#### 370 4.3.2 TPB tests

371 Two finite element meshes (5 mm and 50 mm in fracture area) were used to model the beams subjected to TPB, as  
372 illustrated in Fig.20a. In the TPB tests, the load was applied via a steel roller. In the numerical study, the steel roller  
373 was modelled as a rigid body and a load was applied as a displacement at the relative reference point (see Fig. 20b).  
374 A frictionless surface-to-surface interaction was assigned to the steel roller-concrete interface.



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376 Fig. 20. Discretisation of a beam subjected to a three-point bending test.



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377 For each rubber content, the load-deflection curves were calculated by using the general stress-strain relations  
 378 suggested by Model Code 2010 [35] and by the proposed model with the adaptable softening modulus (obtained  
 379 from inverse FEA). The Model Code stress-strain relation was derived from the bilinear stress-crack opening  
 380 relation accounting for the selected mesh size ( $h_e$ ) as follows:

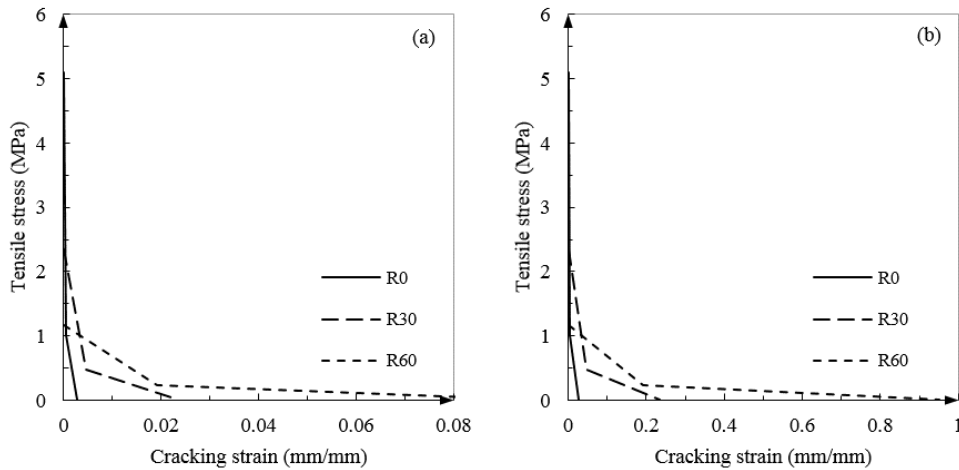
$$\begin{aligned} \sigma_t &= f_{ct} \cdot (1.0 - 0.8 \cdot \frac{w}{w_1}) & \text{for } w \leq w_1 \\ \sigma_t &= f_{ct} \cdot (0.25 - 0.05 \cdot \frac{w}{w_1}) & \text{for } w_1 < w \leq w_c \end{aligned} \quad (17)$$

382 where  $w_1 = G_F/f_{ct}$  is the crack opening at  $\sigma_t = 0.2f_{ct}$ ,  $w_2 = 5G_F/f_{ct}$  is the crack opening at  $\sigma_t = 0$  and  $f_{ct} = 0.3(f_{co})^{2/3}$ .

383 Fig. 21 shows the converted tensile stress-strain curves for mesh sizes 50 mm and 5 mm, respectively. The accuracy  
 384 of the two models was evaluated by the errors in the FE predictions for peak load and energy absorption capacity as  
 385 well as the Mean Square Error (MSE) between the predicted load-deflection behaviour over the entire loading range  
 386 and the corresponding experimental results (average of three tests).

$$MSE = \frac{\sum_{i=1}^N [(\chi)_i^{anal} - (\chi)_i^{exp}]^2}{N} \quad (18)$$

388 where  $(\chi)_i^{anal}$  represents the predicted values of flexural load ( $f_{cm,\beta}$ ),  $(\chi)_i^{exp}$  shows the corresponding experimental  
 389 values and  $N$  is the total number of collection data.



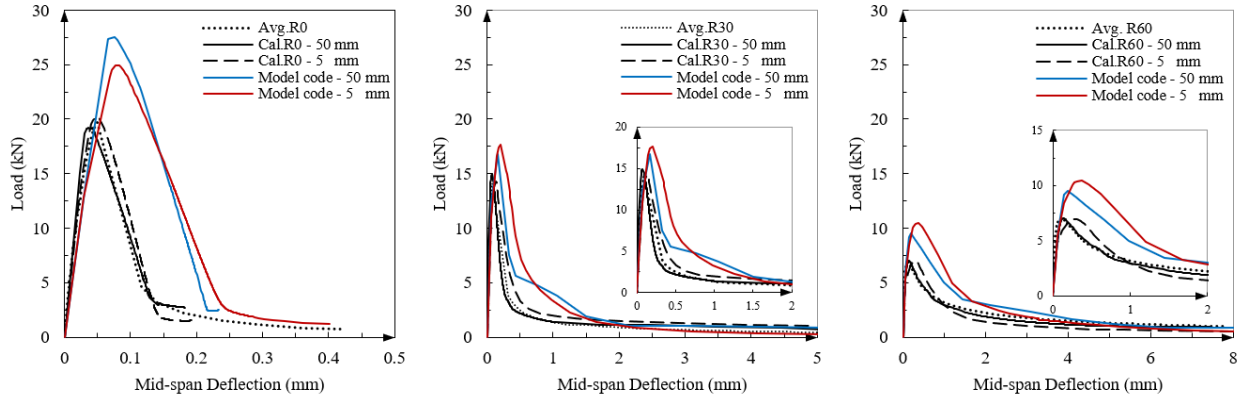
391 Fig. 21. Stress-strain curves according to Model Code 2010: (a) mesh size 50 mm; (b) mesh size 5 mm

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392 In Fig. 22, the average experimental load-deflection curves for all tested TPB specimens are compared with the  
393 numerical predictions. It is shown that the numerical analyses performed using the inverse FEA stress-strain  
394 relations show a considerably better agreement with the experimental data for both rubberised and conventional  
395 concrete specimens. It can be seen that when the Model Code relations are adopted, higher flexural strength is  
396 obtained, as well as larger fracture energy. This can be attributed to the fact that the bilinear stress-strain curve of  
397 Model Code yields a higher value for the coefficient  $g_t$  (see Fig. 23). Moreover, the load-deflection curve obtained  
398 from implementing the proposed relations with different mesh sizes can achieve nearly identical results, thus  
399 confirming that proposed relations describe more accurately the evolution of  $g_t$  at increasing levels of strain.

400 For better comparison, the calculated errors are also summarised in Table 4. The results indicate that the stress-strain  
401 relation recommended in Model Code led to, on average, 32% and 63% overestimation of the peak flexural load and  
402 the energy absorption capacity, respectively. It should be mentioned that similar observations were reported in [48,  
403 49]. However, by using the modified stress-strain relation with adaptable softening modulus, the average errors  
404 reduced to 11% and 3% for energy absorption capacity and peak flexural load, respectively. The MSE results also  
405 confirm that the proposed adaptive stress-strain relation can provide an accurate simulation of the flexural behaviour  
406 of concrete over the entire loading range.

407 Fig. 24 shows the variation of the damage parameter ( $d_t$ ) vs CMOD for R0, R30 and R60. The value of  $d_t$  for the  
408 reference concrete R0 increases rapidly for relatively small CMOD values due to the relative low toughness and  
409 higher brittleness of regular concrete. The  $d_t$  of RuC (R30 and R60), however, increases at a slower rate once the  
410 crack develops, and the severity of damage evolution decreases with increasing rubber content. This can be  
411 attributed to the fact that the rubber particles can control crack development and lead to an increase in ultimate  
412 CMOD and overall ductility.



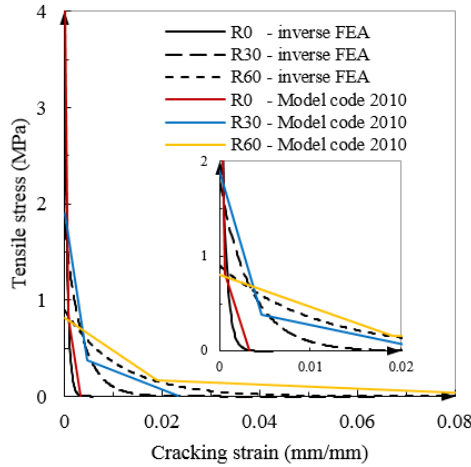
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414 Fig. 22. Comparison of numerically derived load-deflection curves with experimental data.

415 Table 4. Errors in predicting load-deflection response, peak load and energy absorption.

$h_e$ (mm)	$\rho_{vr}$ (%)	Error in peak load		Error in energy absorption		MSE of load-deflection response	
		MC 2010	Inverse FEA	MC 2010	Inverse FEA	MC 2010	Inverse FEA
5	R0	26%	2.6%	70%	10%	78%	1.7%
	R30	25%	5.6%	58%	17%	21%	2.4%
	R60	49%	0.2%	50%	16%	9%	0.8%
50	R0	39%	2.6%	67%	13%	95%	1.3%
	R30	19%	5.8%	83%	3%	18%	1.1%
	R60	35%	0.3%	49%	8%	4%	0.3%

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418 Fig. 23. Comparison of stress-strain curves for Model Code 2010 and modified model obtained based on inverse  
419 FEA.

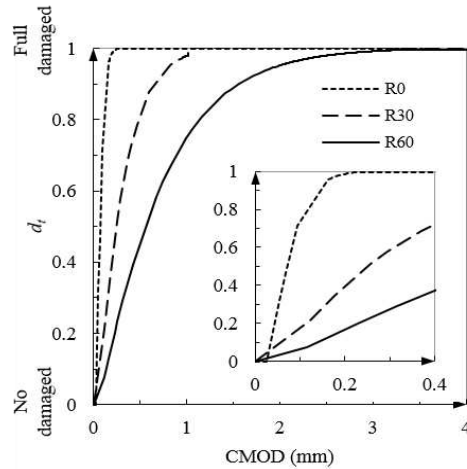


Fig. 24. Evolution of the damage variable,  $d_t$ .

## 5. Conclusions

In this paper, the fracture energy of rubberised concrete has been investigated by conducting three-point bending tests. The primary parameter investigated was the total aggregate replacement volume by rubber particles. The experimental results show that replacing mineral aggregates with rubber particles leads to an apparent reduction in strength (compressive, tensile and flexural) and elastic modulus. However, a less severe reduction is observed in the tensile and flexural properties. Replacing aggregates with rubber also enhanced the fracture energy and energy absorption capacity and led to a more ductile post-cracking behaviour. The test results enabled the implementation of an energy-based inverse finite element analysis method to obtain the tensile stress-strain relation of rubberised concrete indirectly. The proposed constitutive laws were implemented in the concrete damage plasticity model in ABAQUS to analyse the tensile strain-softening behaviour numerically. The influence of mesh size on load-displacement and energy dissipation was examined and an adaptive method was proposed to eliminate mesh dependency by introducing element size and fracture energy as parameters in the tensile constitutive model. It is shown that the proposed methodology can provide significantly more accurate predictions compared to the implementation of the strain-softening relationship proposed in Model Code 2010 for regular concrete in terms of peak load, energy absorption capacity and load-deflection response over the entire loading range.

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## **Acknowledgements**

The study contributes to the FP7 EU-funded project “Anagennisi” [FP7/2007- 2013] under grant agreement n° 603722. The first author also appreciates the support from China Scholarship Council (CSC) under Grant CSC n° 201608060059.

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