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USE OF COMPUTER SOFTWARE TO DO MATHEMATICS AND THE MATHEMATICS ACHIEVEMENT OF STUDENTS IN PUERTO RICO USING RESTRICTED 2015 NAEP

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Abstract: Use of Computer Software to Do Mathematics and the Mathematics Achievement of Students in Puerto Rico Using Restricted 2015 NAEP Data

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This quantitative study explored the relationship between the mathematics achievement patterns of eighth grade students in Puerto Rico and their use of computer software application programs for doing mathematics. The theoretical framework used is the educational production function, which allowed the use of a function to analyze this relationship. The researcher analyzed 2015 restricted National Assessment of Educational Progress (NAEP) mathematics data. Data analysis consisted of descriptive statistical analysis and multilevel modeling analysis. Control variables to measure socioeconomic status and absenteeism were included in the multilevel model. Results of this study showed that average scores on NAEP 2015 were higher for students who use computer programs to do mathematics with less frequency than students who use it with more frequency. Understanding the relationship between the use of computer programs to do mathematics and the mathematics achievement of these students help the mathematics education community to cautiously create policies that do not focused on frequency of using technology. The researcher provided a discussion of the results and implications for researchers, administrators and teachers that would help them to target on the improvement of mathematics achievement of students in Puerto Rico.

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Resumen (Abstract in Spanish): Uso de Software de Computadoras para Hacer Matemática y el Aprovechamiento Académico de los Estudiantes en Puerto Rico Usando Data Restringida de NAEP en 2015

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Este trabajo cuantitativo exploró la relación entre patrones de aprovechamiento matemático de estudiantes de octavo grado en Puerto Rico y el uso de programas de computadora para hacer matemáticas. El marco teórico es la función de producción educativa, el cual permitió el uso de una función para explicar esta relación. La investigadora analizó datos restringidos del 2015 de la Evaluación Nacional del Progreso Educativo de Matemáticas (NAEP, por sus siglas en inglés). El análisis de datos consistió en estadística descriptiva y análisis multinivel. En este último, la investigadora utilizó variables control para medir el nivel socioeconómico y el ausentismo de los estudiantes. Los resultados de este estudio mostraron que los estudiantes que usaron programas matemáticos con mayor frecuencia obtuvieron puntajes promedio más altos en NAEP 2015 que los estudiantes que los usaron con menor frecuencia. Entender la relación entre el uso de programas de computadora y el aprovechamiento académico de estos estudiantes ayuda a la comunidad de educadores en matemática a crear, con cautela, políticas educativas que no se enfoquen en la frecuencia del uso de tecnología. La investigadora incluyó una discusión de los resultados así como implicaciones para investigadorxs, administradorxs y maestrxs que pueden ayudarlos a identificar prácticas que mejorarán el aprovechamiento matemático de estudiantes en Puerto Rico.

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Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

The purpose of this study is to investigate the mathematics achievement patterns related to the use of computer software application programs to *do mathematics* measured by the National Assessment of Educational Progress (NAEP) (National Center for Education Statistics [NCES], 2012a) of students in Puerto Rico. This is a quantitative study using multilevel modeling on restricted 2015 P.R. NAEP Mathematics data. The main goal is to help the mathematics education community understand the relationship between the students' practice of using computer software application programs to do mathematics and the mathematics performance of students in Puerto Rico. Uncovering the relationship and the achievement patterns will provide suggestions and guidance on policies for schools in Puerto Rico.

This chapter includes demographic and background information about Puerto Rico. To provide a foundation for understanding the educational system in Puerto Rico the researcher presents information about the Puerto Rico Native Americans, and the colonial status implications for education. Other information about schools and assessments is included. At the end of this chapter, the researcher presents the research question that guided the investigation.

Puerto Rican Demographic Information

According to the U.S. Census Bureau (2010a), the population of Puerto Rico is 3,725,789. Of this population, 99% are Latinx[1], and 95.4% are Puerto Rican (U.S. Census Bureau, 2010b). Dominicans are the second largest population of Latinxs in Puerto Rico, 1.8% of

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[1] Latinx is gender inclusive and refers to the Spanish speaking communities in Latin America, also known as Hispanic. Latinx is an ethnicity, but not a race. This means that each Latinx is identify with a race or a combination of races such as Black, White, and Native American.

the population is Dominican, followed by Cuban (0.5%), Mexican (0.3%), Colombian (0.1%), Venezuelan (0.1%), and 0.8% from other Latinx communities (U.S. Census Bureau, 2010b). Fifty two percent of the population are females and forty eight percent are males. The average household size is 2.68 people (U.S. Census Bureau, 2010a). In terms of religion, 85% of the population are Roman Catholic, and the rest of the population are identified as Christian-non-Catholic, or other religions (Metcalfe, Bergo, & Holde, 2019).

The median household income in Puerto Rico is $19,350 (U.S. Census Bureau, 2015). The cost of living in Puerto Rico is lower than the cost of living in the United States. For the last eighteen months, it is estimated that the rent prices in Puerto Rico are less than half the prices in the United States, the childcare prices are about 40% the prices in the United States, and the house prices are about 55% the prices in the United States (Adamovic, 2019).

The percent of the population that graduated from high school is estimated to be 73.0% as reported by the 2011-2015 American Community Survey (U.S. Census Bureau, 2015). This number is higher than the rest of the Latinx population in the United States that completed high school, 64.9%, but lower than the United States general population, 86.7% (U.S. Census Bureau, 2015).

Students in Puerto Rican schools. The Department of Education of Puerto Rico (DEPR) provides demographic information on the students in Puerto Rico. The reports include the number of students by nationality, and economic status, as well as the number of participants in the language support program for students with Spanish language limitations. The researcher includes the most recent available demographic information starting in the academic year of 2011-2012 up to the academic year 2015-2016.

Almost every student in Puerto Rico is Latinx. The DEPR (2015, 2016) reported that between 2012 and 2016 more than 99% of their students were Latinx. About 97.3-98.0% of all students in Puerto Rico are Puerto Ricans, while 1.7-1.8% are Latinx but not Puerto Rican. Table 1 shows that the number of students decreased each academic year between 2011-2016 from 452,740 to 379,818. This trend reflects an increase in emigration from the island in the 21st century (León López, 2013) due to economic problems (Center for Puerto Rican Studies, 2017).

Table 1

Enrollment of Latinx students in Puerto Rico

	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016
Total of students	452,740	434,609	423,934	410,950	379,818
Puerto Rican	98.0%	97.9%	97.7%	97.3%	97.8%
Latinx (not Puerto Rican)	1.6%	1.7%	1.8%	1.8%	1.8%
Non Latinx	0.4%	0.4%	0.5%	0.9%	0.4%

The DEPR (2015, 2016) also reported on the economic situation of students. According to their report, about three quarters of the population have an economically disadvantaged status. Table 2 presents the specific percentages of economically disadvantaged students per year. In the academic year of 2014-2015 there was a non-typical percentage of students from economically disadvantaged groups, which decreased about eleven percent from the previous year and increased again about next year. This might reflect the economic status of the migrating students.

The proportion of students with disabilities has been presenting an increasing pattern since 2012 from 19.3% to 27.5%. The proportion of students in the program for Spanish Language Learners has also increased from 0.1% to 0.4%. The percentage of students that are Spanish Language Learners in Table 2 is usually lower than the non-Latinx students in Table 1.

However, in the academic year of 2015-2016, the percentage of Spanish Language Learners matches the percentage of non-Latinx on the island.

Table 2

Proportion of students in Puerto Rico with economic disadvantages, disabilities, and Spanish language limitations

	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016
Total of students	452740	434609	423934	410950	379818
Economically disadvantaged students	71.3%	75.4%	76.5%	65.6%	76.4%
Students with disabilities	19.3%	23.6%	24.5%	26.6%	27.5%
Spanish Language Learner students	0.1%	0.2%	0.3%	0.4%	0.4%

Historical Background of Puerto Rico

Borikén: The island of Taínos. To understand Puerto Ricans, it is important to understand their ancestors. According to the oldest archeological findings, it is estimated that the island was first populated 200 years BCE (García Leduc, 2002). It is not clear how the island was populated, but the most accepted theory states that multiple groups from Venezuela and Colombia migrated to the Antilles in canoes until some of them arrived in the island of Borikén, currently known as Puerto Rico (Rodríguez Ramos, 2010). Groups of migrants included the Huecoides from the north coast of Venezuela and Colombia, who arrived around 200 years BCE; and the Salaloides from the Orinoco River in Venezuela, who arrived between 1 CE and 500 CE (García Leduc, 2002). The modification of their original life styles in South America, and the integration of their cultures built the pre-Taíno community (700 CE-1200 CE), which eventually formed the Taíno culture on the island (García Leduc, 2002).

The Taíno ceremonies and social activities happened in a *batey.* The *batey* was an open space in the tribe to celebrate *areytos* and play *batú.* These *areyto* ceremonies shown in Figure 1, were opportunities for the Taínos to socially interact and celebrate as a community. *Behíques* used *areytos* to tell stories that preserved the Taíno oral traditional knowledge. Other celebrations during the *areytos* included weddings and religious ceremonies. Taínos also danced and sang in the *areytos*. Another activity celebrated in the *batey* was playing *batú,* a Taíno game that consisted of hitting a ball without touching it with the hands.

Figure 1. Taíno Council Guatu-Ma-Cu A Borikén celebrating an areyto ceremony. (El Concilio Taíno Guatu-Ma-cu A Borikén, 2019)

Taínos were well known around the Caribbean because of their kindness and peaceful personalities. Cristobal Colón, in his journal (Colón, 2006), described Taínos as lovely and peaceful, saying that he believed there were not better people in the world. He added that Taínos were the sweetest persons in the world, and were always smiling. In his description, he also described some of their practices, such as being naked, painting their bodies and using accessories such as necklaces, earrings, and bracelets made of bones or stones. Taínos were polytheistic and practiced polygamy.

The colony of Puerto Rico. In 1493, Puerto Rico suffered the colonization of Spain. Thus, after 1493, Taínos were forced to work as slaves, and to change their language and religious beliefs to Catholicism. Spanish people also brought African slaves to the island. Borikén became a colony of Spain and was named Puerto Rico. Though Taínos suffered the colonization, they coexisted with the Spanish people (Martínez-Cruzado et al., 2005). As with the rest of Latin America, the population in Puerto Rico started to mix their races[2] and their cultures[3]. Thus, during the Spanish colonization period, Puerto Ricans developed a strong cultural identity merging the cultures of Spain, Africa, and Taínos. In 1868, Puerto Ricans fought for their independence in the Lares rebellion, but they were not successful.

In 1898, United States and Spain fought the Spanish-American War in the Caribbean. As a result, Puerto Rico became a colony of the United States in the same year. When the United States took the island, this provoked a cultural shock and transformation.

Puerto Rico is still a colony of the United States, but Puerto Ricans have been able to elect their governor since 1948. The chief of state is the President of the United States and the head of the government is an elected governor. Puerto Ricans are United States citizens since 1917; however residents of Puerto Rico still do not have federal voting rights. To avoid

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[2] Omi and Winant (1994) propose that " . . . race is a concept which signifies and symbolizes social conflicts and interests by referring to different types of human bodies. Although the concept of race invokes biologically based human characteristics (so-called "phenotypes"), selection of these particular human features for purposes of racial signification is always and necessarily a social and historical process" (p. 55).

[3] Culture is considered as a cultural practice, in other words, the incidence or prevalence of behavior or the actions of groups and organizations (Biglan & Embry, 2013).

ambiguity, the term United States in this dissertation is referring to the fifty states and the District of Columbia not including Puerto Rico.

History of education in Puerto Rico. Before 1493, Taínos, the Indigenous people that lived on the island, led education in Puerto Rico and structured the teaching and learning around traditional knowledge and the ways of living. Taínos learned to work in agriculture, fishing, and small hunting from their elders. The behíque, who was the medicine man of the tribe, also had the role of preserving the knowledge of the tribe in the *areytos* ceremonial celebrations. In Figure 2, a behíque is preparing for a ceremony. He transmitted the tribal knowledge to the tribe, and taught the children of the *cacique*, the chief of the tribe (Medicina Taína, 2010).

Figure 2. The *behíque* was the medicine man of the Taíno culture who also served as a teacher (Medicina Taína, 2010).

In 1493, as a consequence of the Spain invasion, European traditional school models started to emerge in the island with the purpose of evangelization (Rosario, McGee, López, Quintero, & Hernández, 2015). The teaching of European mathematics started in 1512 with the foundation of Spanish grammar schools. However, this education was limited. Only males of high socioeconomic status were able to participate in these schools and the teachers were all from Spain. By the end of the 18th century there were some efforts at non-Spanish and female integration in schools. At the beginning of the 19th century, Rafael Cordero, a Puerto Rican who is also known as the father of public education in Puerto Rico, started to teach economically disadvantaged communities in the South West of Puerto Rico. Since then, education has become more inclusive of females, non-Caucasian, and low socioeconomic status communities (Quintero, n.d.).

Education in Puerto Rico started to change dramatically in 1898 as a consequence of the United States invasion. Some teachers migrated to Spain, and the United States started a process of Americanization by bringing United States teachers to the island. In 1937, the United States mandated every class to be taught in English. In 1948, Puerto Ricans were allowed to elect their own governor; the new governor named a secretary of education, who changed school language back to Spanish (Resnick, 1993). The new administration not only changed the language back to Spanish, but also brought mandatory and free K-12 education to the island (Quintero, n.d.).

In 1952, Puerto Rico became the *Estado Libre Asociado de Puerto Rico*, which translates to Associated Free State of Puerto Rico, but is commonly known as the Commonwealth of Puerto Rico. Thus, after 1952, education in Puerto Rico has been parallel with the United States education curriculum and law changes. Important United States laws applied to Puerto Rico as a United States commonwealth including the American with Disabilities Act (1990) and the No Child Left Behind Act (NCLB, 2002).

Puerto Rican identity. Puerto Rican history and culture are unique. Puerto Ricans' culture is merged in the cultures of the Taínos, Africans, Spanish, and Americans.

The first three shape the multi-racial and cultural aspects of Puerto Ricans, and give them the tri-racial Latinx identity that bonds with the rest of Latin America. Puerto Ricans preserve the

Spanish language on the island, as well as their traditions. For example, Puerto Ricans still use musical instruments such as the guitar from Spain, the maracas from the Taínos, and the drums from Africa. Puerto Rican food also mixes these three cultures, such as the inclusion of rice from the Spanish, coconut dishes from the Africans and the yucca root dishes from the Taínos.

On the other hand, the United States influences Puerto Rico by laws, citizenship, and education. Puerto Rico follows the American education model, including the academic calendar starting in August and ending in May. Puerto Rico also uses United States standardized assessments and reports the status of their schools to the United States federal government. These peculiarities make Puerto Rico a unique United States territory with a Latinx population and an American educational system in Spanish.

Educational System in Puerto Rico

The colonial condition of the island has been a factor that not only impacts the economy and politics of the island, but also its education. Education in Puerto Rico has been influenced by the United States over the last century. Though mathematics education in Puerto Rico has been managed similarly as with United States education, there is a need to understand the factors that differentiate education in Puerto Rico from education in the United States.

The educational system in Puerto Rico follows the American educational model but it is in the Spanish language. Though the official language for public schools is Spanish, a course in English is required in each grade level. Students in Puerto Rico not only learn in Spanish, but also learn the Puerto Rican culture at school through history classes and extracurricular activities. For example, on November 19, the day of the Spanish colonization of the island, each school commemorates Puerto Rican Day where they celebrate the mix of the three cultures that built today's Puerto Rican culture.

The Department of Education of Puerto Rico is divided into seven regions: Arecibo, Bayamón, Caguas, Humacao, Mayagüez, Ponce, and San Juan. These regions had a total of 2,652 schools during the 2009-2010 academic year (Disdier-Flores & Marazzi-Santiago, 2011). Fifty-seven percent of these schools were public and hosted 68% of students in Puerto Rico (Disdier-Flores & Marazzi-Santiago, 2011). During this academic year 39,102 teachers worked in public schools and 11,829 teachers in private schools. The student-teacher ratio in public schools was 12.6 students per teacher, while in private schools it was 19.8 students per teacher. A typical classroom in Puerto Rico has between 20-25 students; however there is a lot of variation depending on the school location (J. Figueroa, phone interview, December 27, 2017).

Mathematics curriculum in Puerto Rico. Since 2014, Puerto Rico has adopted the Mathematics Puerto Rico Core Standards (DEPR, 2014). These standards are a Spanish version of the Mathematics Common Core State Standard (Math-CCSS) (National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010). Through conversation, one of the collaborators of the Mathematics Puerto Rico Core Standards agreed that these standards are a translation from the Math-CCSS (J. Figueroa, phone interview, December 27, 2017). However, Puerto Rico is not on the list of states and territories that have adopted the CCSS (Association for Supervision and Curriculum Development, n.d.). The researcher suspects that the reason might be that though Puerto Rico has the Spanish version of the Math-CCSS, the English CCSS were not adopted because the official language in schools is Spanish.

Before the 2014 Mathematics Puerto Rico Core Standards, schools in Puerto Rico used the 2007 Mathematics Content Standards and Grade Level Expectations, which replaced the Standards of Excellence of 1996 (DEPR, 2007). These standards reflected the National Council of Teaching of Mathematics (NCTM) Standards (NCTM, 1989) and the progressive movement of education. For example, one of the eighth grade 2007 standards in Puerto Rico was:

"A.MO.8.5.1: Model a real world situation with an equation or inequality using multiple methods and representations" (DEPR, 2007, p. 57).

Standardized assessments in Puerto Rico. The rise of the use of standardized tests in Puerto Rico and the United States is associated with the requirements of the No Child Left Behind Act (NCLB, 2002). The NCLB is the act to close the achievement gap so that no child is left behind (NCLB, 2002). This act was an amendment of the Elementary and Secondary Act of 1965 (NCLB, 2002). In 2015, President Obama signed the Every Student Succeeds Act (ESSA, 2015), which was also an amendment of the Elementary and Secondary Act of 1965 and replaced the NCLB. The ESSA (2015) still requires Puerto Rico to report student performance through standardized tests.

There are two standardized tests that are currently taken by students in Puerto Rican schools. One of them, NAEP (NCES, 2012a), is at the United States national level and is taken across the United States by a representative sample of students in each state (or territory). The second standardized test is specialized for students in Puerto Rico called the *Medición y Evaluación para la Transformación Académica de Puerto Rico* (META-PR), which translates to Evaluation and Measurement for the Academic Transformation of Puerto Rico. This test was created in 2016 with the purpose of replacing the *Pruebas Puertorriqueñas de Aprovechamiento Académico* (PPAA), that is, the Puerto Rican Test of Academic Achievement. This replacement was due to validation issues, in particular, the alignment between the test scores and students' achievement (Quiles, 2015).

To fulfill the standardized test requirements to report student performance, Puerto Rico currently uses the META-PR. This test is used to report adequate yearly progress (AYP) for each school, which is an indicator to measure the annual progress of schools. Prior to META-PR, Puerto Rico used the PPAA to calculate AYP.

Statement of the Problem

Standardized tests in Puerto Rico have shown low mathematics achievement of students. In the academic year of 2011-2012, 91% of the public schools were under an improvement plan (DEPR, 2012). This status is measured by not meeting the required level of AYP for two consecutive years. This result caused much tension for mathematics teachers, who had to start teaching to improve the standardized test results (Vázquez Pérez & Bonilla Rodríguez, 2007). Teachers also indicated that the PPAA threatened students' motivation for learning mathematics and was not a valid standardized test to measure student mathematics achievement (Ortiz Franco, 2013).

NAEP has also shown results with problematic mathematics achievement levels for students in Puerto Rico. This standardized test has been taken in Puerto Rico since 2003. The U.S. NAEP report cards (e.g., NCES, 2016b, 2016c) showed that Puerto Rico has more mathematics educational needs than other states in the United States. NAEP report cards heavily focused on the gap between Puerto Rico and the United States, since the nature of the test allows comparisons of students from Puerto Rico and the states. It also allows comparisons of groups of students by levels of achievement. These achievement levels in NAEP are (1) *Basic*: "denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade" (2) *Proficient*: "solid academic performance for each grade assessed", and (3) *Advanced*: "superior performance." NAEP also registers students *below* the *Basic* level when they do not meet the requirements of at least the Basic level.

At a glance, looking at the 2011-2017 NAEP reports, in Puerto Rico the percentage of fourth and eighth grade students who performed at *Proficient* or *Advanced* levels is significantly lower than in the United States. In fact, the NCES reported that less than 1% of students in Puerto Rico performed at these two levels (e.g., NCES, 2016b, 2016c, 2018c, 2018d). As a consequence, Puerto Rico has the lowest percentage of students at these levels among all the states and jurisdictions. Alabama follows Puerto Rico with 24% and 2% of students in *Proficient* and *Advanced* levels, respectively. This shows an achievement gap between Puerto Rico and the United States.

In addition, Puerto Rico is the jurisdiction with the largest percentage of students performing *below* the *Basic* level. Based on the released public reports from NAEP 2011, 2013, and 2015, approximately 94% of the eighth grade students' mathematics scores are below the *Basic* level (NCES, 2016c). In 2017, P.R. NAEP Mathematics reflected approximately 91% below the *Basic* level in eighth grade. In contrast, the national percentage of eighth grade students scoring below *Basic* is, on average, less than 30%. This information indicates a problem in the mathematics achievement of eighth grade students in Puerto Rico.

Significance of the Study

Studies in different countries have shown that the use of computer software application programs for learning mathematics is associated with deeper mathematical understanding (e.g., Bakker, 2004; Ruthven, Deaney, & Hennessy, 2009; Saha, Ayub, & Tarmizi, 2010; Sutherland & Rojano, 1993; Yerushalmy, 2006). The researcher of this study believes that the use of computer software application programs to do mathematics can be associated with the learning of mathematics. The learning of mathematics is expected to impact student mathematics achievement. Thus, the researcher expects that the frequent use of computer software application programs to do mathematics will be associated with the improvement of the mathematics achievement of students in Puerto Rico.

The purpose of this study is to investigate the relationship between the use of computer software application programs to do mathematics and mathematics achievement. To the mathematics education community of researchers, this work will be a cornerstone for exploring technology and the mathematics achievement patterns of students in Puerto Rico.

Research Question

The focus of this investigation is to understand the relationship of the frequency of using computer software application programs on the mathematics achievement of students in Puerto Rico. The research question to be explored is:

RQ. *How does the use of computer software application programs to do mathematics by students relate to the 2015 NAEP Mathematics scores of eighth grade students in Puerto Rico?*

Summary

Puerto Ricans have a Latinx identity based on history and culture, but have been influenced by the government and laws of the United States. For this reason, understanding Puerto Rican education requires knowledge of the relationship of the island with the United States and its history with Spain. Since 1898, Puerto Rico has been part of the United States. Fifty-four years later, Puerto Rico became a Commonwealth with K-12 mandatory education for all. Thus, the influence of the United States has impacted significantly the education in Puerto Rico. For example, the educational system of Puerto Ricans follows the American model. However, the official language of education in Puerto Rico is Spanish, and so is its education (e.g., curriculum, assessment).

As a consequence of the No Child Left Behind Act (NCLB, 2002) and the Every Student Succeeds Act (ESSA, 2015), students in Puerto Rico are required to participate in standardized tests. The META-PR started in 2016 in replacement of the PPAA. These two tests have been used for standardized assessment requirements. Results of these standardized tests have shown low mathematics achievement of students in Puerto Rico including mathematics (DEPR, 2016). NAEP, a national United States standardized test, results have also shown alarming results of mathematics achievement of students in Puerto Rico.

Research shows that mathematics achievement of students is positively related to the use of technology such as computer software application programs to do mathematics. In this study the researcher wants to explore the relationship between the use of this technology and the mathematics achievement of students in Puerto Rico.

Chapter 2

Literature Review

This chapter includes the theoretical framework, a literature review on mathematics achievement of students in Puerto Rico, and a literature review on the use of computer software application programs to do mathematics.

The theoretical framework for the study is the educational production function (EPF), which serves as a lens for examining and explaining the mathematics achievement of students in Puerto Rico. In addition, the researcher avoids the use of the deficit comparison of the achievement gap between Puerto Rico and the United States.

Previous research studies on the mathematics achievement of students in Puerto Rico and the United States provide a strong base to build this dissertation. Because the available research studies in Puerto Rico are limited, the researcher also considers the mathematics achievement patterns of ethnic minorities[4] in the United States. Though students in Puerto Rico are not the same as students in the United States, these studies can provide guidance on the selection of variables to help explain the mathematics achievement of students in Puerto Rico.

Lastly, this chapter includes research about the use of computer software application programs, which have been shown to impact positively on student learning of mathematics.

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[4] Ethnicity is used in terms of groups that are characterized in terms of a common nationality, culture or language (Betancourt & López, 1993). Ethnic minorities are seen as the underrepresented ethnic minorities in sciences, technology, engineering, and mathematics fields in the United States (National Action Council for minorities in Engineering, 2019). These minorities are Blacks, Native Americans and Hispanics.

Based on the literature, the investigator developed the hypothesis that the use of computer software by students in Puerto Rico is positively associated with their mathematics achievement.

Theoretical Framework

This study is shaped under the umbrella of the EPF theory, which allows analysis of the relationship of variables in education by examining changes in the values of individual predictor variables to study their relationship with the response variable (such as mathematics achievement).

Educational production function. The origins of EPF theory started with the Coleman Report (Coleman, 1968), which was a study undertaken by the federal government of the United States in the 1960s. The purpose of this study was to fulfill a mandate from the Title IV Section 402 *Survey and Report of Educational Opportunities* in the Civil Rights Act of 1964 to write a report of the "availability of equal educational opportunities for individuals by reason of race, color, religion, or national origin in public educational institutions at all levels in the United States" (Civil Rights Act, 1964, p. 4). The Coleman Report consisted of collecting data on United States students' achievement. Results from the study were used for changing policies, such as the reallocation of resources.

The Coleman Report implemented an input-output analysis to model the relationship of students' achievement and education quality. Eventually this analysis adopted the name of EPF, which is an analysis that attempts to improve the achievement output by changing the inputs (Bowles, 1970). This analysis is based on the input and output point of view of the economists (Krueger, 1999). In education, the EPF analysis is commonly used for analyzing big data sets to identify general patterns of students' achievement.

The EPF is a mathematical model that measures school output that represents "the relationship between school and student inputs" (Bowles, 1970). Bowles (1970) defines the EPF as $A = f(X_1, \ldots, X_m, X_{m+1}, \ldots X_{m+n}, X_{m+n+1}, \ldots, X_{m+n+p})$, where m, n, p are positive integers and:

A measures a school output, such as student achievement (Hanushek, 2008);

$X_1, \ldots, X_m$ are m explanatory variables measuring school environment (e.g., teaching practices, school resources, teachers qualifications);

$X_{m+1}, \ldots, X_{m+n}$ are n explanatory variables measuring environmental influences on learning outside of school (e.g., parental education, parental support, family income);

$X_{m+n+1}, \ldots, X_{m+n+p}$ are p explanatory variables measuring students' ability and initial level of learning (e.g., students' IQ, verbal ability).

Hanushek (2008) used the same model, but considered different variables. The model used by Hanushek had three categories for the explanatory variables: school resources, teacher quality, and family attributes. The school resources and teacher quality were also measured in Bowles' model in the school environment category, while the family attributes were merged in the environmental influences on learning outside of school.

The EPF allows the researcher to obtain results that can determine the relationship of different factors on student achievement. Student achievement provides a basis for describing an "efficient production" (Hanushek, 1979, p. 353). For example, the analysis of school resource effect on student achievement could be used to explore appropriate changes in education costs. The family attributes and the environmental influences on learning outside of school can also help in understanding the socio-demographic characteristics of students. Analyzing the relationship of the explanatory variables and student achievement allows the EPF to improve education by exploring possible changes to educational policies based on the relationship (Hanushek, 2008).

Educational production function in Puerto Rico. The EPF is applied in this study to the population of Puerto Rico. This means that a function is used to understand the relationship of variables of interest with the mathematics achievement of students in Puerto Rico.

Given the current colonial status of Puerto Rico, students on the island are considered students of the United States. For this reason, they are exposed to performance comparisons with the rest of the nation such as the NAEP snapshot reports (e.g., NCES, 2016b, 2016c, 2018c, 2018d). In these reports, a gap between Puerto Rico and the United States is shown. The researcher uses the information on the existent gap to recognize a problem that needs attention in Puerto Rico. However, the researcher avoids the examination of this gap and analyzes the achievement patterns within the students in Puerto Rico through an EPF model.

Mathematics Achievement of United States Ethnic Minorities and Puerto Rico

Researchers have studied factors that affect the mathematics achievement of diverse groups of ethnic minority students in the United States. This research provides guidance on the factors that need to be considered when investigating patterns of mathematics performance of students in Puerto Rico.

Factors associated with the mathematics achievement of ethnic minorities in the United States. Some of the factors that are affecting the mathematics achievement of Black, Native Americans and Latinxs are: the culture (e.g., Mejía-Colindrés, 2015; Nasir, 2000; Pacheco Sosa, 1993), socioeconomic status (Byrnes, 2003; McGraw, Lubienski, & Strutchens, 2006), stereotypes (Gutstein, 2003; McGee, 2015), parental education and support (e.g., Barton & Coley, 2007; Harrison, 2015), and teaching practices (e.g., Young, 2017, pp. 69-89).

Culture is a key consideration in the mathematics achievement of students in the United States, especially when considering ethnic minorities. For example, Nasir (2000) explored the cultural shift of African American basketball players when they moved from middle school to high school and their understanding of mathematical concepts such as average and percentage through their participation in sports. Nasir (2000) found that the practice of basketball differs at these two levels of play corresponding to differences in mathematics linked to play. Demmert, Grissmer, and Towner (2006) argued that most family and community characteristics that are linked to lower achievement for *all* racial/ethnic groups are also linked to lower achievement for Native Americans.

Another cultural aspect affecting education is language. When students are learning English as a second language, they are considered English Language Learners (ELL). Various researchers have found that ELL status affects the mathematics achievement of Latinxs (Mejía-Colindrés, 2015; Pacheco Sosa, 1993). Specifically, students learning mathematics in bilingual schools can experience different frequencies of using English or Spanish by their teachers. Teachers who used more Spanish than English facilitated stronger mathematical concept connections for their ELL Latinx students than those teachers who used more English than Spanish (Mejia-Colindrés,2015).

Another factor that has been attached to the mathematics achievement of multiple minority and non-minority groups is the socioeconomic status of students. Studies have found that many of the differences in the mathematics achievement of ethnic minorities on standardized exams are explained by socioeconomic status (e.g., Byrnes, 2003; McGraw, Lubienski, & Strutchens, 2006). Byrnes (2003) used NAEP to study White, Black, and Hispanic 12th grade students and found that socioeconomic status was one of the main aspects explaining the variability in mathematics achievement. In addition, McGraw, Lubienski, and Strutchens (2006) found that the gap between females and males was mostly explained by socioeconomic status.

Factors related with family structure and support also affect the mathematics achievement of ethnic minorities. Barton and Coley (2007) conducted a study by race/ethnic groups of the United States, including Latinxs, highlighting the importance of family in the education of this group of students. They showed that the factors affecting Latinx students are parent-pupil ratio, family finances, literacy development, child-care disparities, resources available at home, and parental support. Parental support also affects the mathematics achievement of Black students (Harrison, 2015).

Students from ethnic minorities in the United States also suffer from stereotypes and racism. Particularly, aspects of racism have affected the mathematics achievement of Black (e.g., McGee, 2015; McGee & Martin, 2011) and Latinx students (Osborne, 2001). Osborne (2001) studied a group of 12th grade students and found differences between Whites and two minority groups –African Americans and Latinxs. The study showed that negative stereotypes in testing situations of these minorities increase the anxiety of students in comparison to White students, which can explain deficiencies in their mathematics achievement. However, McGee and Martin (2011) showed that even when stereotypes exist and affect students, some students can overcome these racial stereotypes with appropriate management. These researchers studied a group of Black mathematics and engineering college students and found successful Black students demonstrated patterns of appropriate management such as focusing on defining their own reasons to achieve, instead of proving stereotypes wrong.

Teaching practices have been shown to affect mathematics achievement of ethnic minorities. For example, Paris (2012) states that teaching practices should be culturally sustaining pedagogies, which means that they need to be more relevant to the cultural experiences of students. Thus, teaching practices should honor the diversity of experiences of Latinx (Moschkovich, 1999), Black (Young, 2017), and Native American (Kellermeier, 2012; Lipka, Wong, Andrew-Ihrke, & Yanez, 2012) students. Young (2017), for example, created a dancing learning activity for understanding graphing points. This activity helped Black girls to develop a deep understanding of graphing points in the coordinate plane. Moschkovich (1999) highlighted that there is a need to value the resources that these students bring to the classroom and to provide mathematical discussion opportunities for them.

In summary, factors that could be related to the mathematics performance for United States ethnic minorities are culture, socioeconomic status, stereotypes, parental education and support, and culturally sustaining teaching practices.

Factors associated with the mathematics achievement of Puerto Ricans in the United States. Puerto Ricans living in the states are part of the Hispanic ethnic minority in the United States. In this study, the researcher uses the term “U.S. Puerto Ricans” to refer to this group. The factors affecting the mathematics achievement of U.S. Puerto Rican students are mainly parental support (Lestch, 1984), absenteeism (Alsace & Samora, 2008), and culture (Alsace & Samora, 2008).

Studies on the mathematics achievement of this population are limited. For this reason, the researcher includes investigation studies on the general academic achievement of U.S. Puerto Rican students, including but not limited to the mathematics achievement patterns of this group of students. Research on the general achievement patterns have also shown that, in addition to the factors affecting mathematics achievement listed above, socioeconomic status is an important consideration for the achievement of U.S. Puerto Rican students (e.g., Díaz, 1998; Nieto, 2000).

Specifically, Díaz (1998) found that financial limitation in students' households is related with their underachievement.

As with many other groups of students, U.S. Puerto Rican students highly value their families. Thus, parental support and family structure affect the mathematics achievement of this group. Lestch (1984) investigated the parental influence and cognitive style in the mathematics achievement of U.S. Puerto Rican students by assessing children's perception of parental child-rearing behaviors. Lestch (1984) found that maternal support impacted the mathematics achievement of Puerto Rican boys in the study. Other studies, for the general achievement of U.S. Puerto Rican students have also found that maternal support impacts students' motivation to succeed in school (Antrop-González, Vélez, & Garrett, 2005, 2008; Garrett, Antrop-González, & Vélez, 2010). In addition, researchers have found that family structure also influences students' achievement (Díaz, 1998; Hidalgo, 2000). For example, Díaz (1998) found that an unhappy home climate due to parents' absence or poor parents' relationship could be a factor leading to underachievement. On the other hand, Hidalgo (2000) conducted a qualitative study and found that students' acknowledgement of the effort of their grandmothers and single mothers to raise them would motivate them to graduate from school. Díaz-Soto (1988) highlighted the importance of family support and parental reinforcement of aspirations in Puerto Rican children for their academic achievement.

The attendance and retention of students can also affect the mathematics achievement of U.S. Puerto Rican students. Alsace and Samora (2008) investigated the factors that influence the academic achievement of mathematics (and English) of U.S. Puerto Rican ELL students. They found that students' attendance and consistency in school programs of study are positively associated with the mathematics (and English) achievement of U.S. Puerto Rican students.

Another factor to be considered is culture. Having an education in a country with a different culture can influence the mathematics achievement of students. The special case of Puerto Rican students, who come from a territory of the United States with a different culture and language, has captured the attention of researchers and educators. One of the factors shown to affect the learning of mathematics is language, specifically the ELL status of students. For example, Alsace and Samora (2008) studied a group of bilingual students and found that some students whose English reading level was higher than their Spanish reading level performed better in mathematics when it was assessed in Spanish instead of English. Another cultural factor considered in mathematics achievement is the students' identity as Puerto Ricans. Having a strong identity as Puerto Ricans has been observed to positively affect the mathematics achievement of students (Antrop-González, Vélez, & Garrett, 2005). However, Flores-González (1999) interviewed a group of eleven high achieving senior Puerto Rican students in a Chicago High School and found that students did not view their academic success as associated to a particular ethnic group.

Factors associated with the mathematics achievement of students in Puerto Rico. Even when Puerto Ricans in the states are from Puerto Rico and share a similar identity with those on the island, their experiences and education can be different, especially because classrooms in Puerto Rico are not racially or culturally diverse. In other words, more than 97% of students in Puerto Rico are Puerto Ricans, and the language in schools is Spanish.

The factors associated with the mathematics achievement of students in Puerto Rico began to be a topic of research in the 1980s. Rivera (1987) studied two public schools and two private schools in Puerto Rico to determine the patterns of effective teaching of eighth grade mathematics. He identified factors affecting the mathematics achievement of students in Puerto

Rico and classified them in four categories: planning, modes of presenting the mathematics content, classroom management, and student behavior. Planning was affected by factors such as school policies and standards, school leadership, and curricular material. Factors affecting the modes of presenting mathematics content are: teaching style and interaction of students in the classroom. Factors affecting the classroom management are general school policies and student-teacher relationships; while the factor affecting students' behavior was the students' level of engagement in class (Rivera, 1987).

Regarding mathematics achievement patterns between male and female students in Puerto Rico, the NCES Reports (NCES, 2007a, 2016c, 2018c) indicated no significant sex differences in the overall mathematics results of NAEP. However, on average, fourth grade females scored higher than males in geometry and spatial sense content in 2003 NAEP standardized test (NCES, 2007a). In 2005, eighth grade female students scored higher than male in the area of probability and data analysis.

The NCES Reports (NCES, 2016c, 2018c) indicated an impact of absenteeism on mathematics achievement reflected in the NAEP scores. In 2015, eighth grade students who were absent more than ten days in the last month by the time they took the test, had an average score of 214, while the students that were absent less than two days had an average score of 227 (NCES, 2016c). Similar patterns are shown for the 2017 P.R. NAEP Mathematics Report.

Recent research on the mathematics achievement of students in Puerto Rico includes the *perspectives of teachers*. Álvarez Suárez (2014) investigated the perspective of 100 teachers in Puerto Rico by conducting a survey about the mathematics achievement of students in Puerto Rico. Twenty of those teachers also participated in a focus group. According to this study, teachers in Puerto Rico think that the standardized test PPAA was not totally aligned with the mathematics curriculum in each grade. Teachers also identified other factors that are affecting the mathematics achievement of students on PPAA such as parental support, missed school days (caused by faculty meetings, professional development, natural disasters, etc.), absenteeism, school disciplinary climate, failing grades in the previous class, and students' apathy towards mathematics.

Because of the limited research on the mathematics achievement of students in Puerto Rico, the researcher expanded the literature review to include newspaper articles. The approach of using teachers' perspectives and opinions for explaining the mathematical achievement of students in Puerto Rico has been a common approach in recent investigations and also in newspaper articles. Two newspaper articles discussed factors that *might* be affecting the education of students in Puerto Rico, including mathematics. For example, in one of the newspaper articles, Ayala-Reyes (2012) suggested that education is affected by the lack of teaching tools (e.g., textbooks), school desertion, school building deficient structures, and the suspension of classes because of contamination problems (e.g., gas leaks). On the other hand, another newspaper article suggests that education is affected by the obsolete curriculum and teachers' lack of motivation to provide high-quality teaching (Velázquez, 2012). These statements are reflecting the perspectives of the newspaper authors and need further research considerations, however, these opinions give the researcher an idea of the popular opinion and the educational environment in Puerto Rico.

In summary, little research has been done to explore the mathematics performance patterns of K-12 grade students in Puerto Rico. The focus of recent research for students in Puerto Rico has been on the perspectives of teachers, which are not necessarily factual, and the research in the 1980s needs to be updated. There is also no recent research on the relationship between the use of computer software application programs to do mathematics and the mathematics achievement of students in Puerto Rico. This study will fill in this gap in the literature.

Technology in Mathematics Classrooms

Increasing numbers of jobs require the use of technology, specifically the ability to use computer software application programs and solve problems with appropriate technological tools. The development of technology has changed people's ways of living, including teaching and learning. As a result, the use of technology is now an important topic in the United States education community. Federal legislation, such as the Enhancing Education Through Technology Act of 2001, recommends that by eighth grade all students should be technologically literate regardless of the student's race, ethnicity, sex, family income, geographic location or disability. This recommendation is addressed by the Mathematics Common Core State Standards (Math-CCSS) (National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010), which suggests that teachers provide experiences for their students to use appropriate technology for solving mathematical problems.

The use of technology can reinforce active learning. When a classroom environment embraces active learning, "students are able to engage actively in rich, worthwhile mathematical activity" (Henningsen & Stein, 1997, p. 524). Technology can have multiple levels of engagement for students and teachers. It is expected that the use of computer software application programs to do mathematics, by definition, will reflect an active learning environment.

Technology in mathematics standards. The Math-CCSS (NGACBP & CCSSO, 2010) and the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report

(Franklin et al., 2007) provide learning goals and suggest practices to teach the mathematics standards which include the use of technology.

The Math-CCSS content standards (NGACBP & CCSSO, 2010) are divided in domains, which are large groups of related standards such as operations and algebraic thinking; geometry; measurement and data; the number system; and statistics and probability. Table 3 shows that the use of technology is *explicitly* mentioned in some of the Math-CCSS content standards. For example, when working with functions the Math-CCSS recommends the use of technology for graphing. The standards also recommend the use of technology for making models because it can provide support for building varying assumptions, exploring consequences, and comparing data predictions (NGABP & CCSSO, 2010). There are not separate standards for modeling: the adequate *modeling* standards are identified with a (*) in the Math-CCSS standards, and are included in other domain standards. For statistics, the Math-CCSS recommends the use of technology to generate regression functions and correlation coefficients.

Table 3

The use of technology in mathematics content standards (NGACBP & CCSSO, 2010)

Grade & Domain	Content Standard
Grade 7: Geometry	7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
Grade 8: Expressions and Equations	8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. 8.G.A Understand congruence and similarity using physical models, transparencies, or geometric software.

	1. Verify experimentally the properties of rotations, reflections, and translations 2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures describe a sequence that exhibits the congruence between them. 3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. 4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. 5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so*.
High School: Algebra	HSA.REI.C.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater). HSA.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
High School: Functions	HSF.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* A. Graph linear and quadratic functions and show intercepts, maxima, and minima B. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

	C. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. D. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. E. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. HSF.LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. HSF.TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*
High School: Modeling	HSA.REI.D.11 (See High School: Algebra) HSF.IF.C.7 (See High School: Functions) HSF.TF.B.7 (See High School: Functions)
High School: Statistics and Probability	HSS.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

In 2007, the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report highlighted the need for statistical literacy (Franklin et al., 2007). Among the goals presented in the GAISE Report, the authors state that technology is a tool that can help introduce statistical concepts. The report recommends appropriate use of computer software for analyzing and representing data. For example, students should be given the opportunity to identify the misuse of graphs, and then use a statistics software program to draw a corrected graph representation (Franklin et al., 2007). Also, the use of statistics software programs can provide tools for analyzing data such as creating a scatter plot, fitting a line of regression, and computing the standard deviation of the residuals (Franklin et al., 2007). When explaining the role of probability in statistics, Franklin et al. (2007) indicates that students should be familiar with how to use appropriate technology to find areas under the normal curve.

The CCSS standards of mathematical practices are a set of habits that mathematics educators at all levels should seek to develop in their students so that they become mathematically proficient (NGACBP & CCSSO, 2010). There are eight standards of mathematical practices as shown in Table 4. These practices provide tools for teachers to help students develop mathematical maturity when learning mathematics content standards. The standard MP5 *Use appropriate tools strategically* explicitly recommends the use of technology, which can enable students to visualize results and compare predictions with data.

Table 4

Common Core State Standards of Mathematical Practices (NGACBP & CCSSO, 2010)

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning

Using appropriate technology (MP5) not only helps students to explore and deepen the understanding of the content standards, it can also provide connections to other mathematical practices while learning mathematics. For example, the use of appropriate technological tools (MP5), such as spreadsheets, could help students make sense of a problem (MP1), or to build arguments based on spreadsheet results to critique the reasoning of others (MP3).

Defining computer software application programs to do mathematics. The use of technology in mathematics classrooms has an enormous existing literature. In this study, the researcher delimits the term technology to guide the reader to the specific aspect of using computer software application programs to do mathematics.

First, technology in education includes a diverse range of physical tools such as computers, calculators, phones, cameras, projectors, and abacuses. One way to classify these tools is by considering the general purpose of the technology. For example, information technologies (IT) include any equipment that is used for managing or delivering data or information. These IT include computer software programs because they are a set of instructions to tell the computer how to perform a task. Computer software programs can have two classifications depending on their purposes. The first one is software for the use of the operating system and the second one is for the use of an application. The researcher is specifically interested in the use of computer software application programs (CSAPs).

In this study, the use of CSAPs is studied in the context of mathematics. The use of CSAPs for education includes the use of these programs in other subjects such as history, science and English. For example, CSAPs can be used in any class for formative assessments such as online homework or quizzes. It can also be used as a tool for delivering the content of the course in an online platform like Moodle or for creating educational videos. All of these tools are innovative and present different ways of engagement by the students and the educator. However, the use of CSAPs for this study is bounded by the exclusivity of using the programs for a mathematics class. This means that the tools examined in this study are specifically used for working with mathematics, and excluding non-mathematical CSAPs, such as word processing and presentation programs.

The last distinction for delimiting the technology of interest is to define the meaning of doing mathematics. This is a philosophical matter and a subjective task. To define the doing of mathematics, first the researcher will use a definition for mathematics from Schoenfeld (1994):

> Mathematics is an inherently social activity, in which a community of trained practitioners engage in the science of patterns—systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure math") or models of system abstracted from real-world objects ("applied math"). These tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. (p. 60)

The learning of mathematics involves the understanding of these tools. Specifically, Schoenfeld (1992) connects the learning of mathematics, as a social activity, with learning to think mathematically. This mathematical thinking includes development of a mathematical point of view, competency with the mathematical tools, and effective use of these tools for making sense of mathematics (Schoenfeld, 1994). The doing of mathematics is increasingly coming to be seen as a social and collaborative act (Schoenfeld, 1992). Thus, the doing of mathematics is used for making sense of mathematics and developing a mathematical point of view, which are key points to developing mathematical thinking, and therefore learning mathematics.

The researcher will also use the didactical functionality point of view of the use of technology in mathematics education by Drijvers (2013). Drijvers uses the term *do mathematics* as a classification that describes the functionalities of technology; a technology is used to *do mathematics* when it is outsourcing work that could also be done by hand. This perspective complements the definition given by Schoenfeld (1994) of doing mathematics by providing a functionality perspective, which can be adapted to the specific use of CSAPs to do mathematics.

These perspectives of *doing mathematics* match the Common Core State Standards of Mathematical Practices (NGACBP & CCSSO, 2010). Specifically MP5, *Use appropriate tools strategically*, states:

> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, *a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software*. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations... (NGACBP & CCSSO, 2010, p. 7)

In this mathematical practice standard, the appropriate CSAP tools include dynamic geometric software programs, spreadsheets, computer algebra systems, or statistical packages. These are examples of tools for outsourcing work that could also be done by hand to solve a mathematical problem. Roschelle, Noss, Blikstein, and Jackiw (2017) state that these types of technology, identified here as CSAPs to do mathematics, can enhance productivity and effectiveness, as well as provide opportunities for extending learning experiences.

The use of CSAPs facilitate students in justifying and generalizing solutions, which help them to spend more time on solving the mathematical problems instead of just focusing on procedures (Roschelle, Noss, Blikstein, & Jackiw, 2017). Technology to do mathematics also provides suitable tools for learning of mathematics and for everyday life (Roschelle, Noss, Blikstein, & Jackiw, 2017).

Following the recommendations of the MP5 in the CCSS (NGACBP & CCSSO, 2010), the researcher is interested in the geometric, spreadsheet, graphing, and statistics CSAPs to do mathematics. These tools improve the student understanding of mathematics, as well as student motivation.

Geometric CSAPs. Geometric CSAPs have been used in mathematics classrooms to improve the learning of geometry. Examples of geometric software are: *Geometer's Sketchpad*, *Cabri Geometry*, *GeoGebra*, and *Autograph*.

Features of these geometric CSAPs include the manipulation of geometrical elements such as points and segments. They also provide an environment to assign specific properties to geometrical objects for keeping during manipulation. This is an important base to support students in discovering and making generalizations of geometrical facts. The use of these CSAPs also facilitates compass and straightedge constructions such as the bisection of an angle. These features make the geometric software programs effective supporting tools for students, enhancing their mathematics learning and enthusiasm.

The use of geometric CSAPs has facilitated important tools for teaching mathematics, particularly for the teaching of geometry. In this case, the use of geometric CSAPs has facilitated exploration (e.g., Oner, 2008; Shadaan & Leong, 2013), visualization (e.g., Bulut, Akçakın, Kaya, & Akçakın, 2016; Shadaan & Leong, 2013), generalizations (e.g., Oner, 2008), and proofs (e.g., Jackiw, 2003; Oner, 2008).

The use of geometric CSAPs enhances student geometric learning. Multiple experimental studies have compared a group using a geometric CSAP for teaching geometry and a control group not using a geometric CSAP. They have found that students working with geometric CSAPs had better mathematics achievement and learning than students working on other non-CSAP classroom environments (e.g., Saha, Ayub, & Tarmizi, 2010; Shadaan & Leong, 2013; Zengin, Furkan, & Kutluca, 2012). For example, a group of researchers in Turkey studied a sample of 51 students in a trigonometry course (Zengin, Furkan, & Kutluca, 2012). They assigned 25 students to an experimental group using *GeoGebra* and 26 students to a control group using a constructivist approach without a CSAP. They found that the experimental group outperformed the control group in learning trigonometric concepts. Studies also found that the use of geometric CSAPs enhances the learning of specific concepts in geometry such as coordinate geometry (Saha, Ayub, & Tarmizi, 2010) and circles (Shaadan & Leong, 2013).

The use of geometric CSAPs is also relevant in other fields of mathematics, such as statistics, number theory, and complex analysis. In the field of statistics, for example, students used a geometric CSAP to understand the construction of a regression line by visualizing the distance of each point to the line (Lesh, Caylor, & Gupta, 2007). Thambi and Eu (2013) used an experimental design on third grade students in Turkey to investigate the use of *GeoGebra* to visualize fractions. They found that students using the geometric CSAP performed better in the posttest fraction assessment compared to students taught in a traditional way. In the field of complex analysis, students used a geometric CSAP to visualize a two dimensional structure of complex numbers by providing didactic trajectories through the geometric interpretation of complex numbers, and dynamically generalized visualizations (Jackiw, 2003).

The use of geometric CSAPs not only enhances the learning of mathematics, but also the enthusiasm of students. Students have shown more positive perceptions toward the learning of geometry (Arbain & Shukor, 2015), and statistics (Emaikwu, Iji, & Abari, 2015) when they use geometric CSAPs. They have also shown enthusiasm for using geometric CSAPs in their geometry courses (Isiksal & Askar, 2005; Shadaan & Leong, 2013).

Spreadsheet CSAPs. Spreadsheet CSAPs have been used in mathematics classrooms as tools for algebra and statistics. Examples of spreadsheet CSAPs are *Microsoft Excel*, *Google Sheets*, *LibreOffice*, and *Numbers*.

A spreadsheet is a grid with an indefinite number of rows and columns. Other than the use of rows and columns to organize a data set, spreadsheet CSAPs facilitate students compute an operation per row or per column. Common basic functions in spreadsheets include the sum, average, round, and count. Spreadsheet CSAPs also allow the user to use functions involving exponents, absolute values, and modules. More advanced features are the use of conditional environments to limit the use of a function to data with specific characteristics. Students can also use spreadsheet CSAPs to work with probability and data analysis by using functions such as the random number generator, or to manipulate probability density functions. Spreadsheet CSAPs also provide tools for students to perform descriptive statistical analysis and to obtain charts to visualize the data set of interest.

Spreadsheet CSAPs also enhance students' learning of algebra. Specifically, spreadsheets help students to transition from specific to general thinking (e.g., Friedlander, 1998; Sutherland & Rojano, 1993), explore and solve problems without being concerned about calculations and algebraic manipulations (Friedlander, 1998), and develop conceptual understanding of functional relationships (e.g., Sutherland & Rojano, 1993) and variables (e.g., Friedlander, 1998; Rojano, 1996). For example, Friedlander (1998) studied the teaching of the *variable* concept to seventh grade students in Israel and found that the use of spreadsheets "build an ideal bridge between arithmetic and algebra" (Friedlander, 1998, p. 383). Spreadsheet CSAPs are also effective for students to solve mathematical problems (Rojano, 1996) and solve equations in informal settings (Dettori, Garuti, & Lemut, 2001). Ainley (1996) conducted a qualitative study about the use of spreadsheets in an introductory algebra course in a primary school in the United Kingdom, and reported that students who used a spreadsheet CSAP were motivated and persistent.

The use of spreadsheet CSAPs in statistics also enhances student understanding of statistical concepts. Researchers have highlighted the uses of spreadsheet CSAPs for learning statistical concepts such as multiple regression, the F-test, t-test, and multicollinearity (Martin, 2008), and for graphing (Wu & Wong, 2007). The use of these CSAP tools supports students' statistical conceptual understanding (e.g., Pace & Barchard, 2006; Warner & Meehan, 2001; Wu & Wong, 2007), reduces students' anxiety in statistics learning (Pace & Barchard, 2006), and improves their computer skills (Warner & Meehan, 2001).

Graphing CSAPs. Graphing technology tools have been used in mathematics classrooms for improving the learning of mathematics. The research on the use of graphing CSAPs usually focuses on either secondary school or college level. Examples of graphing CSAPs are: *Desmos*, *GeoGebra Graphic View*, *Visual Math* and *Grapher*. The *Desmos* and *GeoGebra* software are not exclusively graphing CSAPs. However, for this graphing CSAPs section, the researcher is only considering the graphing features of these programs.

Features of these graphing CSAPs include graphing multiple functions at the same time and establishing parameters in functions. Students can manipulate the parameters in a function to understand their effect on the graph. For example, by manipulating a parameter a in the function $f(x) = a\sin(x)$, the student can visualize the effect of changing the amplitude of a sinusoidal function. In Calculus courses, graphing CSAPs can be used to understand the concepts of limits (Liang, 2016), derivatives (Hohenwarter, Preiner, & Yi, 2017), and Riemann sums (Hohenwarter, Preiner, & Yi, 2017). These features make the graphing CSAPs effective tools for students, enhancing their mathematics learning experiences and enthusiasm.

The use of graphing CSAPs enhances student mathematics conceptual learning. Researchers have found that students working with graphing CSAPs had better understanding of concepts than students working in other non-graphing CSAP classroom environments (e.g., Carreira, Amado, & Canário, 2013; Heid, 1988; Thompson, Byerley, & Hatfield, 2013; Zulnaidi & Zakaria, 2012). Specifically, studies have found that the use of graphing CSAPs enhances the learning of functions (Koştur & Yılmaz, 2017; Zulnaidi & Zakaria, 2012), problem solving (Carreira, Amado, & Canário, 2013; Yerushalmy, 2006), and asymptotes (Öçal, 2017). For example, Zulnaidi and Zakaria (2012) conducted an experimental study on 124 students in Indonesia, and found that conceptual understanding of functions was better in the posttest for students who used the *GeoGebra* graphing software in comparison to those that did not use this graphing CSAP.

Also, the use of graphing CSAPs supports exploring and verifying solutions (e.g., Koştur & Yılmaz, 2017; Yerushalmy, 2006), modeling mathematical problems (e.g., Carreira, Amado, & Canário, 2013), and overcoming difficult algebraic manipulations (e.g., Ruthven, Deaney, & Hennessy, 2009; Yerushalmy, 2006). Koştur and Yılmaz (2017) found that the use of the *Desmos* graphing CSAP was beneficial for students' understanding of exponential functions, because it compensated the lack of procedural knowledge and provided opportunities for exploration.

The use of graphing CSAPs also enhances the motivation of students. Tedious written work is reduced when students use a graphing CSAP (Ruthven, Deaney, & Hennessy, 2009). Students can also benefit from the conceptual understanding without getting distracted by their procedural knowledge (Koştur & Yılmaz, 2017). These outcomes indicate that a graphing CSAP enhances the engagement of students with the task.

Statistics CSAPs. The use of statistics CSAPs helps students learn statistics. Some of the statistics CSAPs used in mathematics classrooms are *Fathom*, *Tinkerplot*, *Minitab*, and *Statistical Package for Social Science* (SPSS) software.

Features that are usually included in statistics CSAPs are the easy manipulation and analysis of data. Students can type data by hand, upload data, or copy and paste data into a spreadsheet-like table. Some programs also read data directly from the web in html format. Features of statistics CSAPs are very convenient to immediately obtain results of descriptive statistics, graphs, regression analysis and hypothesis testing. Students can also use statistics CSAPs to generate data and manipulate probability density functions. These features help students focus on the statistical reasoning, instead of being overwhelmed by long and tedious computations.

The use of statistics CSAPs has facilitated students learning of statistics. Specifically in statistical reasoning (e.g., Abrahamson & Wilensky, 2007; Lehrer, Kim, & Schauble, 2007; Meletiou-Mavrotheris, 2003), probability conceptual understanding (e.g., Kazak, 2015; Prodromou, 2014), visualization (e.g., Abrahamson & Wilensky, 2007; Prodromou, 2014), and exploration (e.g., Abrahamson & Wilensky, 2007; Prodromou, 2014). In the case of probability, students can also use games in statistics CSAP environments to understand uncertainty and fairness (Kazak, 2015). Statistics CSAPs are effective for introductory statistics courses at the university level (Meletiou-Mavrotheris, 2003; Rosen, Feeney, & Petty, 1994; Wassertheil, 1969). In addition, the use of statistics CSAPs provides appropriate tools for students to develop models and simulations to explain sample variability (Lehrer, Kim, & Schauble, 2007) and to study probability distributions (Prodromou, 2014).

Other than learning statistics, the use of statistics CSAPs can help create active learning environments. Researchers have shown that the use of statistics CSAPs can improve the engagement of students for learning statistics (e.g., Dimitrova, Persell, & Maisel, 1993; Prodromou, 2014). These CSAPs can also help facilitate discussions with peers about statistical results obtained in a statistics CSAP (e.g., Prodromou, 2014).

Summary

The theoretical framework that shapes this investigation is the EPF. This is appropriate because the data analysis in this dissertation will use a mathematical function for explaining the mathematics achievement of a group of students, in this case, students in Puerto Rico. Given the colonial situation of students in Puerto Rico, the mathematics achievement of this group of students is usually presented as a comparison with the United States. However, the researcher is not focusing on the gap between Puerto Rico and the United States students.

Previous research about mathematics achievement of students in Puerto Rico is limited. Research on the teachers' perspectives about the mathematics achievement of students in Puerto Rico suggests that factors affecting the mathematics achievement of students in Puerto Rico include sex, parental support, absenteeism, school disciplinary climate, students' apathy toward math, lack of teaching tools, school desertion, and curriculum. A study in the 1980s identified that planning, modes of presenting the mathematics content, classroom management, and student behavior are also affecting mathematics achievement of students in Puerto Rico.

The Math-CCSS recommends the use of appropriate tools to learn mathematics, and research has shown that the use of CSAPs (such as geometric, spreadsheet, graphing and statistics programs) to do mathematics positively affects the mathematics achievement of students. However, there is no research study in the existing literature that investigates the relationship of mathematics achievement of students in Puerto Rico and the use of these CSAPs. This research sheds light on exploring this relationship.

Chapter 3

Methodology

This chapter describes the methodology for the investigation including the research questions, research design, data set, variables of interest, and data analysis procedures. The research question that guided this study is:

RQ *- How does the use of computer software application programs to do mathematics by students relate to the 2015 NAEP Mathematics scores of eighth grade students in Puerto Rico?*

The problem of mathematics achievement of students in Puerto Rico needs further and appropriate exploration. Quantitative studies allow the use of big samples to discern a statistical generalization (Middleton, Cai, & Hwang, 2015). A quantitative study allows the researcher to identify patterns representing students in Puerto Rico thus suggesting effective policies and practices for mathematics education in Puerto Rico.

There is no research in the existing literature to explore and understand the mathematics achievement patterns of students in Puerto Rico. However, the data set, 2015 P.R. NAEP Mathematics, provides a valid standardized test for analyzing this mathematics achievement. NAEP also has variables that enable the researcher to answer the research question, as it provides information about the use of computer software application programs (CSAPs) to do mathematics.

Research Design and Methods

This is a large-scale (Middleton, Cai, & Hwang, 2015), non-experimental (Johnson, 2001) quantitative (Creswell, 2011; Johnson & Christensen, 2014) study to explore the mathematics performance patterns related to the frequency of using computer programs by students in Puerto Rico. Because the use of geometric, spreadsheet, graphing, and statistics

CSAPs to do mathematics has been shown to improve mathematics learning, this study will explore the relationship with mathematics achievement of eighth grade students in Puerto Rico.

This study is non-experimental because the group of subjects could not be manipulated by the researcher to consider control and treatment groups (Johnson, 2001). Also the researcher has no control over the predictor variables, such as students' socio-economic factors or the frequency of using CSAPs.

In this case, a quantitative study is appropriate to answer the RQ because this is a closed ended question, and the answer to this question is quantifiable (Creswell, 2011). Quantitative research uses measurable variables to uncover patterns. It allows the investigator of this study to quantify the relationship among variables (Creswell, 2011) measuring frequency of using CSAPs and mathematics achievement of students in Puerto Rico. Through this quantification, the researcher utilizes a statistical analysis (Creswell, 2011) to test the specific hypothesis (Johnson & Christensen, 2014) that the frequency of using CSAPs for the population of interest is related to the mathematics achievement of students in Puerto Rico.

One of the characteristics of quantitative research is to collect numeric data from a large number of members of the population of interest (Creswell, 2011). Middleton, Cai, and Hwang (2015) indicated that there is a need for large-scale studies in mathematics education. Large-scale studies help identify patterns of equity (or inequity) in the educational system or curriculum (Middleton, Cai, & Hwang, 2015). This type of study can also help researchers see new patterns that are impossible to discern using small-scale studies, and to check findings drawn from small-scale exploratory studies (Middleton, Cai, & Hwang, 2015). This dissertation uses a large-scale representative sample of the population of students in Puerto Rico.

In summary, this research is a quantitative, large-scale, and non-experimental study to unpack statistically significant effects of the predictor variables on the variable of interest. This captures the relationship between the use of computer programs and the mathematics achievement of students in Puerto Rico.

Data Set - National Assessment of Educational Progress (NAEP)

NAEP is the largest nationally representative and continuing assessment in mathematics to measure achievement of United States students (NCES, 2017b). The researcher used the 2015 Mathematics NAEP data of eighth grade students in Puerto Rico. The use of NAEP has implications in the methodology. Thus, NAEP background information such as the assessment main purpose, item selection, survey structure, data collection processes, and data analysis considerations helps to set the grounds for understanding this assessment. Also, the researcher includes specific NAEP implications in Puerto Rico.

For allowing generalization, a quantitative study needs to have a rigorous and complex data collection process that ensures a well-represented sample of all subpopulations such as students in urban vs. rural schools. NAEP implements a careful selection of a large representative sample of students per states, or in this case for the territory of Puerto Rico. This data set also includes variables that reflect the frequency of the use of CSAPs to do mathematics reported by eighth grade students in Puerto Rico. Thus, NAEP provides a valid standardized test for analyzing mathematics achievement patterns of students in Puerto Rico and allows the researcher to conduct quantitative analysis to answer the research question.

Overview of NAEP. The Exploratory Committee for the Assessment Progress in Education (ECAPE) was established in 1964 and has held national assessments since 1969. These assessments, now known as NAEP, assessed student achievement at the national level.

Some state level student achievement reports started in 1990 including states that agreed to participate in NAEP (NCES, 2012b). In 2001, the reports began to include fourth and eighth grade mathematics and reading assessment at the state and national level, including *all* states in the United States (NCES, 2012b).

At present, NAEP mathematics assessment is taken every other year by fourth and eighth grade students. The NCES carefully selects a probabilistic large sample of students that allows representation of the student population at the school district, state, and national levels. The mathematics education community uses NAEP results to monitor progress and help develop ways to improve education policies in the United States.

The National Assessment Governing Board (NAGB) works with the National Center for Education Statistics (NCES) to prepare this assessment and ensure that it is valuable for the United States. The NAGB has authority over NAEP policies and oversights including the development of the framework of what skills and knowledge should be assessed in each subject area, the review of test items, and the set of the levels of achievement based on student performance on the test. On the other hand, the NCES manages the administration of NAEP and its operations such as designing, analyzing, and reporting the results of the assessment. The NCES is also in charge of developing items, sampling students, and collecting data. There is also a group of contractors that are in charge of implementing NAEP in the selected schools (NCES, 2018b).

NAEP was a paper-based assessment in 2015. In 2016, NAEP mathematics and reading assessments were piloted on tablets with an attached keyboard. To protect trend reporting, NAEP is using a multistep process to transition from paper to digital technology. At this moment, the *mathematics* NAEP is in digital form, but the *general* NAEP transition to digital form is still ongoing.

Survey instruments of NAEP. The survey instruments consist of data collected from students, teachers, and school administrators. Students report non-cognitive and cognitive data. The non-cognitive data include demographic information and classroom experiences. The cognitive data, in the case of mathematics, includes measures from five sub-content areas: algebra; geometry; measurement; number properties and operations; data analysis, statistics and probability. NAEP also surveys teachers and school administrators. Teachers report background questions such as classroom practices and teacher's academic preparation. School administrators report information about the school, teachers, and students. For example, the school information includes the percentage of students in special education and school location. The teachers' information includes the percentage of teachers absent, and the number of part-time teachers in school. The student's background information includes identifying the student's disability status, and English Language Learner (ELL) status.

For the students' cognitive questions, items are divided by sub-content areas, complexity levels, and format such as multiple choice or short constructed response. In the case of eighth grade mathematics, in the 2015 NAEP, there were a total of 150 items that were either modified from previous years, or developed and reviewed by the NCES (Beaton et al., 2011). The purpose of the review process is to check the questions' alignment with the framework, the mathematical accuracy, the appropriateness for grade level, the clarity of language, and the avoidance of political sensitivity bias (NCES, 2007b). This process also has the purpose of checking the answers and creating appropriate scoring guides (NCES, 2007b).

The cognitive questions are then grouped into ten blocks and randomly assigned into fifty booklets. The purpose of this process is to minimize the order, context and fatigue effects, which are environmental factors that can affect the item performance of a student. For example, if a specific item is always the last item in every exam for every student, this item might have a fatigue effect that reflects that the student is tired.

Participants are only assessed using one booklet of approximately thirty to forty items. For ensuring that the process is fair, each of the ten blocks appears in booklets for an equal number of students. Also, each of the fifty booklets is taken by an equal number of students. The purpose of taking this small portion of questions is to minimize the time it takes students to answer the test, but in a way that it ensures the validity of the scale (Rahman, 2019).

Sampling process of NAEP. NCES uses multistage sampling for the selection of the public school sample in NAEP (NCES, 2017a). Table 5 shows the sample design of NAEP.

Table 5

Sample design of NAEP (NCES, 2018a)

Steps to select a student for the sample of NAEP

1. Identify all potential schools in each state.
2. Classify schools into groups.
3. Within each group, order schools by student achievement.
4. Develop an ordered list for sampling.
5. Select the school sample.
6. Confirm school eligibility.
7. Within sampled schools, select students to participate in NAEP.

The sampling frame is the list of public schools provided by the Department of Education. After obtaining this sampling frame, the first step is to classify every school per state. The second step is to classify schools in groups that represent their location (such as rural or urban); inside those groups each school is subcategorized according to their racial/ethnic composition. This process creates subgroups of schools with similar racial/ethnic compositions in each type of location. The third step is to list all students in the grade of interest per school, and classify them by achievement. The fourth step creates a comprehensive list of all schools according to the previous three characteristics of location, racial diversity, and student achievement. The probability of selecting these schools on the list is calculated by considering the size of its enrollment with respect to the size of the state's student population at the selected grade level. After this comprehensive list is created and each school has a calculated probability of being selected, a school sample is selected using systematic sampling with probability proportional to size. The sixth step is to verify that the school is eligible, which means that the school will still be open and will have students in the grade level that would be assessed. After a school is confirmed to be eligible, the school sample is complete. Then NCES randomly selects about 60 students per school. After being selected, they are randomly allocated to take one assessment: mathematics or reading (NCES, 2017a).

The process of sampling private schools is similar but schools are not classified by states. This limits inferences on student achievement to national level analysis (NCES, 2018a). The NCES classifies private schools by type (e.g., Catholic, Baptist), and then schools are grouped by the census division (Pacific, Mountain, West South Central, West North Central, etc.), the degree of urbanization of location (rural, suburban, urban), and minority enrollment (race/ethnicity). A sample of schools is taken by considering these characteristics. The random sample consists of approximately 60 students in each school selected (NCES, 2017a).

This multistage random sampling process results in a nested structure of students within schools. Students in the same school tend to share certain characteristics such as curriculum, educational experiences, and teachers. Their experiences are more similar to those students in the same school compared to other schools. Having these similar characteristics and being selected in a NAEP nested multistage sampling implies that NAEP data on student achievement is *not independent* within school. This assumption is foundational in standard parametric statistical analysis. The analysis through multilevel modeling considers the nested structure of the data, which produces models with unbiased estimates for population characteristics and corrected standard errors (Hox, 2010).

Statistical considerations for NAEP. The use of a small subset of all cognitive mathematical questions and the use of a small sample by using multistage random sampling allows NAEP to produce estimates for population groups, although not for individual students.

The American Institute for Research (AIR) is the world's largest behavioral and social science research and evaluation organization. The AIR's NAEP Education Statistics Services Institute (ESSI) provides technical assistance, research and developmental support, and project management services to the NCES on NAEP. To address the issues of NAEP using a small subset of cognitive items and a sample of students through multistage sampling, the AIR's NAEP ESSI provides statistical tools to support valid analysis of NAEP. This institute developed a unique combination of three areas of statistics: psychometrics such as the use of Item Response Theory (IRT), imputation for imputation of missing data, and survey sampling methodology such as the use of weighting.

IRT is used to create a probability model that measures the probability that a participant will respond correctly to a test question, given some individual characteristics such as participant's mathematical ability or the possibility of guessing the answer on an item. This probability model also considers parameters that measure item difficulty and item discrimination (efficiency of the item to differentiate). This probability model is then used in a likelihood function to visualize the patterns of answers from a participant.

By the technique of imputation of missing data, NCES creates 20 potential values that represent each student's score. Different from other testing programs, participants in NAEP are tested on a small portion of items, approximately 40 out of 150 total items, which is about 27% of the items. This reduces testing time and ensures school cooperation. On the other hand, each student is tested on too few questions to allow individual analysis. As a solution for analyzing these data, NCES treats the scale score as missing data by using missing data imputation. This process is done to fill in values for the questions that an individual student was not given on the test. To do this, NCES creates 20 potential values from a posterior distribution of the *latent traits* given the *observed responses* to both the assessment items and the survey questionnaires. The latent traits are individual characteristics of the student that are usually measured indirectly such as student ability or intelligence. These 20 potential values are called *plausible values*. Plausible values enable variance estimates considering the sampling variation and the measurement error. For this data analysis, the researcher accounts for these 20 plausible values in the data analysis to measure students' mathematical scores.

When conducting data analysis, the researcher also needs to use appropriate *weights*. An illustrative example to understand how weighting is performed is the following: if ten students are selected from two different schools, one with 50 students, and another school of 100 students; then the ten students from the first school are given twice the weight as the ten students from the second school. For the NAEP analysis, the researcher considers the use of weights for students and schools, known as ORIGWT and SKSRSWT variables, respectively, in the NAEP data set. These weights acknowledge the sampling process of schools and students. Specifically, these weights reflect the characteristics considered in the sampling process, for example, the average household income of the schools, or the ethnic group of the students.

NAEP data tools. The NCES provides two ways of analyzing data: online NAEP Data Explorer (NDE), and NAEP Restricted Data.

Users of NDE can examine performance data such as differences in scale scores, achievement level percentages, and percentiles across student groups. It also provides tools for examining contextual data such as parental education or race/ethnicity. Through this tool, researchers can perform significance testing, gap analysis and regression analysis. For example, through the NDE, a researcher can access averages and confidence intervals of the mathematics scores of students by frequency of using spreadsheets to do mathematics. However, the use of these tools is limited. For example, the regression analysis only allows a maximum of three variables and does not allow the examination of interactions.

The restricted data analysis provides more freedom to qualified researchers to examine NAEP data for secondary analysis (NCES, 2013). Using restricted data, the researcher can acknowledge, for example, the nested structure of students within schools and provide more accurate models. These restricted data contain individually identifiable information, which is confidential and protected by the federal law. NCES issues licenses to researchers to have access to restricted data. To apply for a license, the researcher needs to fill out an application, and meet security requirements to protect the data. For example, an applicant needs to fill a formal request, and sign an affidavit of non-disclosure. In addition, the restricted NAEP data users need permission to share their data analysis and publications (see Appendix H).

NAEP in Puerto Rico: sample and validation. NAEP was first implemented in Puerto Rico in 2003. The P.R. NAEP is in Spanish and it only includes the area of Mathematics.

Baxter et al. (2007) investigated the validation of NAEP exams in Puerto Rico for the years 2003 and 2005. Given that scores for students in Puerto Rico were very low, they were concerned about the use of the same scale in Puerto Rico as the states. They concluded that the scores in Puerto Rico *could* also use the 0-500 NAEP scale. However, the National Center of Education Statistics (NCES, 2007a) highlighted that the items in the P.R. NAEP Mathematics had a high percentage of missing data for 2003 and 2005. This issue questions the validity of results of NAEP to make inferences about the population of students in Puerto Rico for those years.

The 2011, 2013, 2015, 2017 NAEP was modified for both Puerto Rico and the United States. This modification consisted of including special sections of mathematics questions in the assessment to increase the precision and reliability of the scale. These sections allowed researchers to appropriately analyze results from NAEP in Puerto Rico using the same NAEP data scale as the rest of the states with small margins of errors (NCES, 2016a). As of the time this study was conducted, 2017 NAEP data were not available for secondary analysis to licensed researchers yet. So the 2015 P.R. NAEP data are the most recent data available, and is valid for analyzing the mathematics achievement of students in Puerto Rico (Daro, Hughes, & Stancavage, 2015).

Variable Selection

The selection of NAEP variables of interest to answer the research question relies on previous research on CSAPs and the mathematics achievement of students in Puerto Rico.

The dependent variable is the mathematics composite score of eighth grade students in NAEP; sub-content areas are not available for Puerto Rico. The mathematics composite scores, scaled from 0-500, are represented by a set of 20 plausible values. The output variable selected was *plausible value 1*, however the Hierarchical Linear Model (HLM) software considers the 20 plausible values for the dependent variable by producing estimated parameters for each plausible value. These estimated parameters are then averaged to produce the output model used in this study.

The explanatory variables of interest reflect the use of CASPs; the model also includes control variables based on the literature review.

Variables of interest: use of CSAPs to do mathematics. Table 6 contains the list of the group of questions of interest on the use of CSAPs to do mathematics. These survey questions measure the frequency of the use of CSAPs to do mathematics, specifically the use of spreadsheet, graphing, statistics, and geometric CSAPs. The questions are reported by students.

Table 6

The four variables of interest for eighth grade students in the 2015 P.R. NAEP Mathematics data and the possible responses from students

When you are doing math for school or homework, how often do you use these different types of computer programs? [M816001] A spreadsheet program for math class assignments. [M816501] A graphing program on the computer to make charts or graphs for math class. [M816601] A statistical program to calculate patterns such as correlations or cross tabulations. [M825001] A program to work with geometric shapes for math class.
The five options for students in each question 1. Never or hardly ever 2. Once every few weeks 3. About once a week 4. 2-3 times a week 5. Every day or almost every day

Controlling predictors. To explain the variation in the prediction model, the researcher considers control variables that, according to the literature, are expected to be related to the mathematics achievement of students in Puerto Rico.

Researchers found that the factors, identified by teachers, that possibly affect mathematics achievement of students in Puerto Rico are: parental support, missed school days, *absenteeism*, school disciplinary climate, failing grades in previous classes, and *students' attitude toward mathematics*. Research in the 1980s indicated that *students' attitude toward mathematics* and teaching practices such as planning, mode of instruction, and classroom management are important considerations for explaining mathematics achievement. The NAEP Report also indicated that eighth grade students' mathematics achievement in Puerto Rico could be affected by the *days absent from school*. Thus, researchers have consistently found that *absenteeism* and *student attitude toward mathematics* impacts mathematics achievement.

Studies on ethnic minorities in the United States also confirm *parental support* as an important factor to consider (e.g., Harrison, 2015). In addition, studies about the specific population of Puerto Ricans in the United States also found that absenteeism and parental support are important considerations that could explain the variation in mathematics achievement of these students. Other studies about ethnic minorities strongly rely on *socioeconomic status* (SES) as an important consideration when studying mathematics achievement (e.g., Byrnes, 2003).

Based on these findings, the researcher selected the following variables as possible control predictors for the model: absenteeism, parental support, attitude toward mathematics, and SES. Measuring student attitude toward mathematics is not a trivial task and cannot be effectively done with the NAEP data. So the researcher only considered the three control predictors: absenteeism, parental support, and SES.

Measuring absenteeism is possible through NAEP. This assessment has a variable that measures the number of days absent in the last month, shown in Table 7. This variable provides a range of options for the students between zero days, and more than ten days.

Parental support cannot be measured at the student level; instead this is measured as a percentage of parents that are volunteering at the school, or a percentage of parents that are attending teacher-parent conferences. NAEP does not provide information to know who these parents are or who their children are. Since the interest in including parental support is to measure the effect of parents on their own children, the researcher decided not to include this variable as a control predictor.

Measuring SES is also not a trivial task, however there are three main components that are usually used to measure SES: family income, parental educational attainment and parental occupational status (Cowan et al., 2012). NAEP uses the measure of SES through eligibility for the Department of Agriculture's National School Lunch Program (Cowan et al., 2012). This variable will indicate if a student is eligible for free lunch or reduced price lunch, which is objectively reflecting the family income of a student. However, the use of eligibility for the NSLP variable is not appropriate in Puerto Rico because all students are declared eligible for the NSLP regardless of their family income level (Cowan et al., 2012). Instead family income can be estimated from their home possessions. Table 7 also shows the information about home possessions collected by NAEP that indicates if the student has Internet access, a clothes dryer, a dishwasher, more than one bathroom, or their own bedroom at home. In addition, SES can be measured using parental educational attainment as shown in Table 7. The parental occupational status is not reported by NAEP, so it is not included in this study.

In summary, the control predictors included in the model are absenteeism and SES. The first control variable was measured through the days absents from school in the last month. While the second control variable is measured through home possessions and parental education attainment.

Table 7

Possible control variables for explaining the mathematics achievement of students in Puerto Rico, available in 2015 P.R. NAEP Mathematics

Variable to measure absenteeism, reported by students
[B018101] How many days were you absent from school in the last month? ● None ● 1-2 days ● 3-4 days ● 5-10 days ● More than 10 days
Variables to measure the Socioeconomic Status of students, reported by students
[PARED] Highest level achieved by either parent (based on student responses to two background questions) ● Did not finish high school ● Graduated high school ● Some education after high school ● Graduated college ● Unknown Do you have the following in your home? (Yes/No response) ● [B0267a1] Access to the Internet ● [B0267b1] Clothes dryer just for your family ● [B0267c1] Dishwasher ● [B0267d1] More than one bathroom ● [B0267e1] Your own bedroom

Data Analysis

In preparation to analyze the data, the researcher first conducted analysis to detect patterns, for example on demographic information in the sample. This analysis included percentages of students by school location category, race/ethnicity, sex, and disability status. The researcher also examined the percentages of the population of Latinxs on the island that are Puerto Rican, Cuban, Mexican or other. NAEP does not provide information on the population of Dominicans, which is the second largest population of Latinxs on the island.

The data analysis consisted of two main parts. First, the researcher conducted descriptive analysis for the variables of interest and the control variables. Then the researcher used multilevel modeling to model the mathematics achievement of eighth grade students in Puerto Rico. The diagram in Figure 3 summarizes the steps for data analysis.

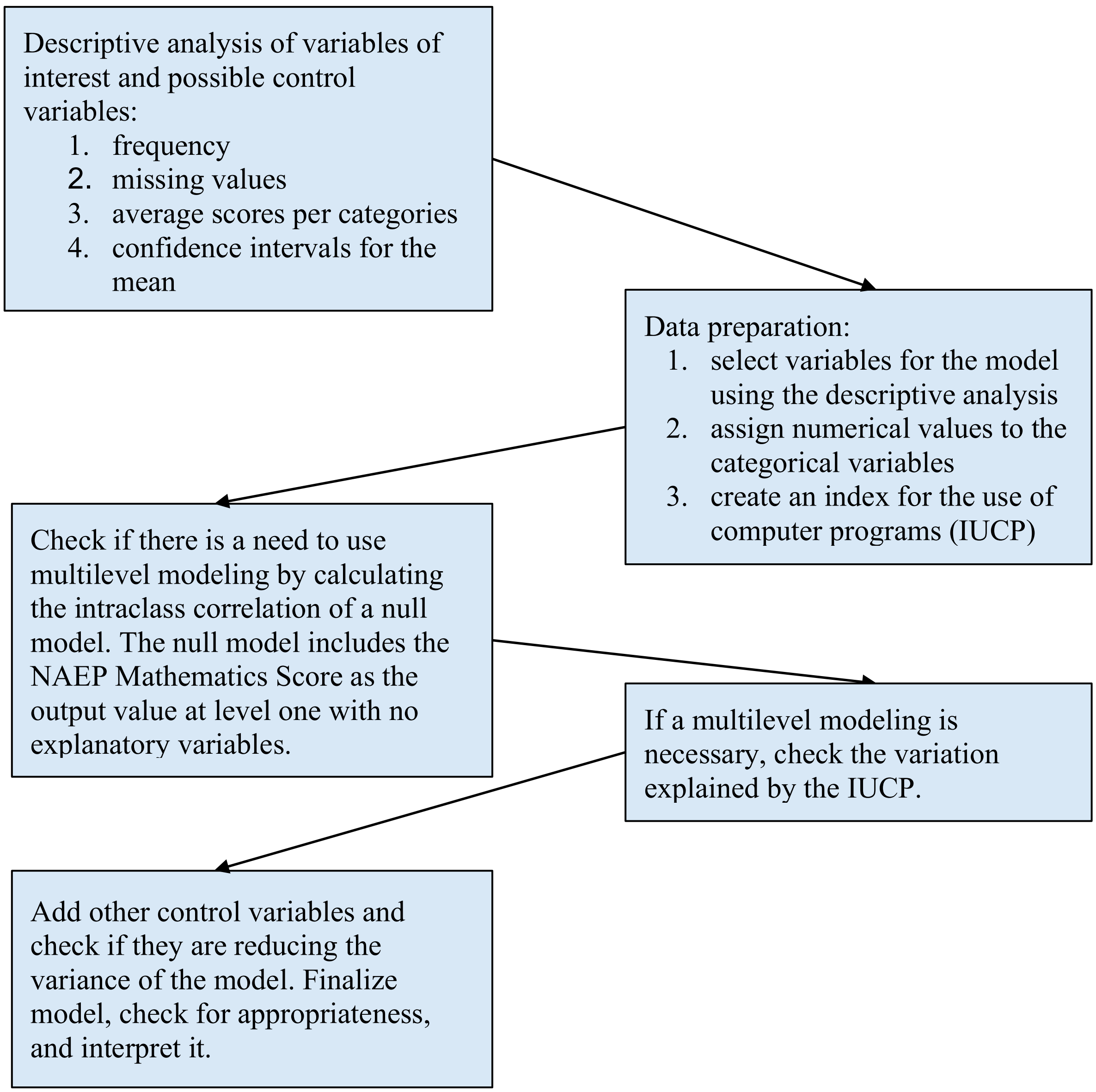

Figure 3. Data analysis procedures.

Descriptive analysis. This analysis included a frequency summary of each of the variable categories, the calculation of the average mathematics scores per category, and the number of missing values for each variable. The researcher conducted this analysis using the *EdSurvey* package (Bailey et al., 2019) in the *R* software.

For the variable that reflects the frequency of using CSAPs to do mathematics, the researcher used 95% confidence intervals for the means of NAEP mathematics scores for each category. A graph of these confidence intervals shows a preliminary explanation of the relationship between the use of each CSAP to do mathematics and the NAEP mathematics scores of students in Puerto Rico.

The researcher also created 95% confidence intervals for the means of mathematics scores for each of the categories of the possible control variables. This information helps to explain if there is an expected relationship between these variables and the mathematics achievement of students in Puerto Rico.

After conducting this analysis, the researcher made decisions to select appropriate control variables for the multilevel regression model. This means, for example, the exclusion of control variables that are not reflecting a relationship with the mathematics achievement of students in Puerto Rico.

Then the researcher set the variables as numeric. This facilitated the use of the variables in the multilevel model.

The variables that measure the frequency of the use of CSAPs have five possible answers or categories as shown in Table 6. The researcher assumed equal differences between each category. Thus, the five categories were coded in the following way: never or hardly ever as *1*, once every few weeks as *2*, about once a week as *3*, two to three times a week as *4*, and every day or almost every day as *5*. Missing data were omitted from the analysis.

The parental education variable was also coded as numeric. Because parental education contains an unknown category, students who selected this category were treated as missing data. The other categories were coded as follows: did not finish high school coded as 1, graduated high school coded as 2, some education after high school coded as 3, and graduated college coded as 4. All missing values were ignored, which included the unknown category.

The home possession variables were coded using a binary code of zeros and ones. The researcher used a one for all students who indicated “Yes” - to have the possession described-, and a zero otherwise. For example, if a student reported “Yes” when asked about having more than one bathroom at home, then this response was coded as a one. If not, it was coded as a zero. Home possessions, when used, were represented as an index, which had values between zero and one. The value of the index represents the percentage of items that the student reported to have.

Exhibiting absenteeism is considered as missing about 20% of the school days (Robins & Ratchiff, 1980). The variable provided by NAEP measures the number of days absent from school during the last month. A month could have about 20 to 23 school days, so an absenteeism problem could be identified when a student is absent for about 4.0 to 4.6 days during the last month. The scale provided by NAEP has a category of none, followed by one to two days, and three to four days. Given the categories for the variable, it is not possible to know if a student was absent three or four days, so the researcher re-coded the days absent from school as an indicator variable for absenteeism in the following way. Zero indicates that a student was absent from school for three or more days, and one indicates that a student was absent from school for two or less days during the last month. Given that this variable was transformed to a binary absenteeism variable, the researcher included a new descriptive data analysis for this variable.

After re-coding these variables, the researcher created an index to measure the use of CSAPs. Each variable already had a created scale-value from one to five for each of the responses. The index was then an average of the scale-value. Table 8 provides an example on how to compute the index that measures the frequency of the use of CSAPs to do mathematics. This example provides an illustration of the values reported by a student in each variable and reflects an index of 2 for the frequency of using CSAPs to do mathematics. This index of 2 is obtained by calculating an average of the frequency of using spreadsheet, graphing, statistics, and geometric CSAPs. In other words, the index is obtained by adding all the values that represent the student reported answers, $1 + 2 + 3 + 2$, which is 8, and divide this by a total of four items. An index of 2 means that the student, on average, used CSAPs to do mathematics once every few weeks. Observe that the Index of the frequency of Using Computer Programs $(IUCP)_{ij}$, is a variable representing student i in school j in the first level of the multilevel model.

Table 8

Example of computing an Index IUCP of 2 for an eighth grade student in 2015

Variable	Student reported value
[M825001] A program to work with geometric shapes for math class.	2: Once every few weeks
[M816001] A spreadsheet program for math class assignments.	1: Never or hardly ever
[M816501] A graphing program on the computer to make charts or graphs for math class.	2: Once every few weeks
[M816601] A statistical program to calculate patterns such as correlations or cross tabulations.	3: About once a week

Multilevel modeling. Multilevel modeling is a statistical model that has become popular in psychology and educational research (Jackson, 2010). It estimates a set of fixed and random effects that capture relationships among variables at different levels. In particular, two-level cross-sectional multilevel modeling is used to analyze data structured as observations at one level that are nested within observations at another level (Nezlek, 2012). This *nestedness* causes violations of the independence assumption in regression analysis. Multilevel modeling addresses this lack of independence by partitioning within and between group variances and accounting for the between group variance in the hierarchically structured data for the purpose of estimation (Woltman, Feldstain, MacKay, & Rocchi, 2012).

Two-level cross-sectional multilevel modeling. The nesting of students within a school created by NAEP sampling methods suggests the use of multilevel modeling. This study used two-level cross-sectional multilevel modeling: level one considered the observation of students, and level two considered schools. This study is cross-sectional because it analyses data from 2015, which is a specific point in time. In the 2015 P.R. NAEP, the sample size of students was 5,150, and the sample size of schools was 120. Each school sample had about 40 students.

Null model. In addition to incorporating the NAEP sampling design into the analysis, the need to use multilevel modeling is supported when the amount of variation in student scores is, in part, explained by their school. The null model allows estimation of the total variation in the model response as well as the variation explained by school. The null model for this study is:

student level: $NMS_{ij} = \beta_{0j} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + u_{0j}$,

where NMS_{ij} is the NAEP *mathematics score* of the eighth grade student i in the school j in Puerto Rico, e_{ij} is the level one residual error for student i in school j and the parameter β_{0j}, the mean NMS score random effect for school j, is explained by parameter γ_{00} with the level two residual error u_{0j} for school j. The purpose of using this model is to estimate the variances $\sigma^2_{(m0)}$

and $\tau^2_{00(m0)}$ of e_{ij} and u_{0j}, respectively to compute the intraclass correlation coefficient (ICC). The ICC is calculated using

$$ICC = \frac{\tau^2_{00(m0)}}{\sigma^2_{(m0)} + \tau^2_{00(m0)}} \, .$$

The ICC measures the proportion of variance in the NAEP mathematics composite scores that is accounted for by the school level. Larger ICC values are indicative of a greater impact of the school effect on the mathematics composite scores of students. The ICC typically ranges from 0.10 to 0.25 based on a large variety of studies of student achievement in the United States (Hedges & Hedberg, 2007).

As a first step for the multilevel modeling analysis, the researcher calculated the ICC to measure the proportion of the variance of the NAEP mathematics scores (NMS) accounted for by the school level. This ensured that it was important to consider the two levels selected for the multilevel modeling.

Fixed and random effects. When using a multilevel model, the parameters at level one, such as the coefficients and the slope, can be modeled at level two by random or fixed parameter effects. For example, the random effect of the slope, β_{0j}, is given by u_{0j} and the fixed effect is given by γ_{00}. Decisions to include a random effect at level two were made based on statistically significance of the variance component that the random effect contributes to the model (Hox, 2010).

Method of estimation. The maximum likelihood (ML) estimation method has many advantages and is the most commonly used for multilevel modeling (Hox, 2010). One of the advantages of using this method is that ML produces estimates that are generally robust to the non-normality of the errors. In addition, ML produces asymptotically efficient and consistent estimates. When using large samples such as NAEP, this method is robust against the violation of non-normal errors.

The ML method estimates parameters by maximizing a function called the likelihood function. This likelihood function is a function of the model parameters given the data that were, in fact, observed (Hox, 2010). The two likelihood functions that can be used for multilevel modeling are the full ML and the restricted ML. The full ML includes the regression coefficients and the variance components in the likelihood function, while the restricted ML only includes the variance components. Since the regression coefficients are included in the likelihood function of the full ML estimation, an overall chi-square test based on differences in the log-likelihood can be used to compare models with different fixed effects, if needed. For this dissertation, the researcher used the full ML to estimate and compare the models.

Centering. The purpose of centering the independent variables is to make clearer interpretations of the models. There are three types of centering of variables that are used in multilevel modeling: uncentered, group centered, and grand centered (Hox, 2010). These are linear transformations of the variables in the model that consist of shifting the location of the variable by adding or subtracting a constant. Uncentered variables, also known as zero-centered, are variables that are not changed. Group centered variables at level one subtract the corresponding group mean of the variable. Grand centered variables are obtained by subtracting the mean of the variable across all the observations.

The level one predictors can use any centering method, and the level two predictors can use uncentering or grand centering (Nezlek, 2012). Since level two uses group predictors, group centering will produce a meaningless value of zero. The level one predictor variables are usually centered as group centered variables due to its statistical and interpretive advantages (Jackson, 2010). For this study, the researcher used group centered variables at level one, and grand centered variables at level two. This helped the researcher to analyze the model in a meaningful way.

By using group centering at level one, the researcher could interpret the intercept as the expected value for a student whose value on the variable is the same value of their school mean; and the coefficients as the magnitudes of the difference between the student and the school average for a specific predictor value. The variance of the intercept, τ_{00}^2, measured the variability among the school level units.

By using grand centering for the level two variables, the researcher could interpret the intercepts at level two as the expected value for a school when the school value on the variable is the same as the grand mean of the variable across all the observations. The coefficients were the magnitudes of the difference between the school and the average for a specific predictor.

Aggregation. The aggregation of variables allowed the researcher to analyze the relation of the variables at a higher level (Woltman et al., 2012). For example, the IUCP is the index that measures the frequency of using CSAPs to do mathematics at level one, because it is describing the frequency of using CSAPs to do mathematics for each student. This variable was aggregated by calculating the average of IUCP by school, and assigning this average to each of the schools. Thus, the average of IUCP became a variable at level two.

In this dissertation, the researcher used aggregation of level one variables by calculating the average of variables per school. The aggregated variables were the averages of IUCP, parental education, home possession index, and days absent calculated for each school.

Constructing the model. The data analysis considered two levels (student and school) using multilevel modeling. First, the researcher conducted an analysis with the IUCP to estimate the variation in the relationship with NMS across schools. The model for student i in school j is:

student level: $NMS_{ij} = \beta_{0j} + \beta_{ij} IUCP_{ij} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + u_{0j}$,

$$\beta_{1j} = \gamma_{10} + u_{1j},$$

where $IUCP_{ij}$ captures the calculated index for the frequency to use CSAP to do mathematics for student i in school j, e_{ij} is the residual error for student i in school j, and the residual errors for school j are u_{0j} and u_{1j}. The analysis of this model helped the researcher evaluate the strength of the relationship between IUCP and NMS. The results for the model also provided information about the random effects of this variable, and the total within group variance of the model explained by the variance of the IUCP.

After analyzing this model, the researcher added the first level control variables to estimate the conditional variation in their relationship with the NMS across schools. The variables included were: index for home possessions (IHP), parental education (PARED), and the indicator variable for the days absent from school (IDAS). The decision to include each variable in the final model was made by considering their statistical and practical significance. A random effect was retained depending on the significance of its variance component. In other words, if the variance of the intercept or slopes was significantly large based on a likelihood ratio test, then the random effect was included.

After analyzing this model with all the level one variables, the researcher added variables at the second level. The second level variables are the aggregated school averages of the variables at level one. A variable at level two was selected in the final model if the researcher is interested in the interpretation of the variables on the model. Thus, the finalized model included the school average of each variable to explain the variance in the intercept among schools, which gave information about the school average effect on NMS. The model also included the average of IUCP to explain the coefficient β_{1j}, which gave information about the contextual effect of the variable IUCP. Thus, the finalized model for student i in school j is:

student level: NMS $= \beta_{0j} + \beta_{1j}IUCP_{ij} + \beta_{2j}IHP_{ij} + \beta_{3j}PARED_{ij} + \beta_{4j}IDAS_{ij} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + \gamma_{01}\overline{IUCP_j} + \gamma_{02}\overline{IHP_j} + \gamma_{03}\overline{PARED_j} + \gamma_{04}\overline{IDAS_j} + u_{0j}$,

$$\beta_{1j} = \gamma_{10} + \gamma_{11}\overline{IUCP_j} + u_{1j},$$

$$\beta_{2j} = \gamma_{20} + u_{2j},$$

$$\beta_{3j} = \gamma_{30} + u_{3j},$$

$$\beta_{4j} = \gamma_{40} + u_{4j},$$

where $\overline{IUCP_j}$, $\overline{IHP_j}$, $\overline{PARED_j}$, and $\overline{IDAS_j}$ represent the mean of IUCP, IHP, PARED, and IDAS for the school j (i.e., the contextual effect of the level one variables); and the residual errors for the schools are u_{0j}, u_{1j}, u_{2j}, u_{3j} and u_{4j}; e_{ij} is the residual error for student i in school j. This model was interpreted.

Sensitivity analysis for the multilevel model. To check the appropriateness of the model, the researcher tested the multilevel modeling assumptions. These assumptions are classified in two different groups, the first group includes three assumptions about the relationship between predictors and error terms, and the second group includes three assumptions about the distribution of random terms and relationships among random terms. These assumptions are shown in Table 9.

Table 9

Assumptions for the multilevel model

Relationship between predictors and error terms

- The student level predictors are independent of the student level residuals, e_{ij}.
- The school level predictors are independent of the school level residuals u_{kj}, where k is the number of random errors at level two.
- Predictors at each level are uncorrelated with the residuals at another level.

Distribution of random terms and relation among random terms

- Student level residuals, e_{ij}, must be independently and normally distributed with a common variance.
- School level residual vectors $(u_{0j}, u_{1j}, \cdots, u_{kj})$, have a multivariate normal distribution with a constant covariance matrix.
- Student level residuals, e_{ij}, are independent of any of the school level residual errors u_{kj}.

Statistical software used for the data analysis. The researcher used the statistical tools: *SPSS*, *Hierarchical Linear Model (HLM)*, and *R* software.

The AIR-NCES developed the *EdSurvey* package (Bailey et al., 2019) to analyze NAEP data in the *R* software. The use of this package ensures the use of appropriate methods by using default weights and plausible values for the analysis. Descriptive analysis for the selected variables was conducted using the *EdSurvey* package. The researcher used the function *edsurveyTable* to obtain descriptive analysis of the variables and create confidence intervals.

In addition, the use of *EdSurvey* provides an option for multilevel modeling analysis, but this is currently under development. Another package for *R*, *WeMix*, was recently developed, but was not available at the time the researcher conducted the multilevel modeling analyses for the study. So the researcher used the *HLM* software for conducting the multilevel modeling analysis.

The *HLM* software is specialized for conducting multilevel modeling analysis. The researcher used the *HLM 7.03* student edition for *Windows*. *HLM* fits models for outcome variables with explanatory variables that account for variations at each level utilizing variables specified at each level. Because the *HLM* software does not allow data manipulation, such as re-coding variables or identifying missing values, the researcher used the SPSS software for any data manipulation in preparation for using HLM.

Chapter 4

Results

This chapter includes the results of the data analysis for this investigation to answer the research question. In the first part of this chapter, the researcher presents the demographic information of the sample. The second part of this chapter includes results of descriptive data analysis on the variables for the use of computer software application programs (CSAP) to do mathematics, and selected control predictors. After the descriptive data analysis, the researcher presents results from the multilevel modeling analysis to illustrate the relationship between the frequency of using CSAP to do mathematics and the mathematics achievement of eighth grade students in Puerto Rico for NAEP 2015.

Demographic Information of the Sample

The demographic information of the sample includes student sample information such as race/ethnicity, sex, disability status, and eligibility for the National School Lunch Program (NSLP). The researcher also included information about the school type and location for the sample.

NAEP collected race and ethnicity information for students in Puerto Rico reported by school or by students. Students self-reported 95.3% Puerto Rican, 0.6% Mexican, 0.5% Cuban, 2.5% other Hispanic or Latinx, and 4.6% not Hispanic. The total of these percentages is not 100% because students could select two choices for the question. On the other hand, schools reported 99.96% of students as Hispanic or Latinx.

In Puerto Rico, all students (100%) were eligible for NSLP and the sex distribution was 48.5% females and 51.5% males. In addition, 23.6% of the students were identified as students with disabilities, and 76.4% were not identified as students with disabilities. Students with disabilities include those with a specific learning disability, visual impairment, hard of hearing, deafness, speech impairment, orthopedic impairment, or health impairment. This variable also includes those students with a 504 plan, which means that students have accommodations that will ensure their academic success in a regular education environment.

All schools in this sample are public, because private schools were not considered in the sample. The location of the schools is shown in Figure 4. Whereas 23.1% of the students were from schools located in cities, a majority of 67.2% from suburbs, 4.7% from towns, and 5.0% from rural areas.

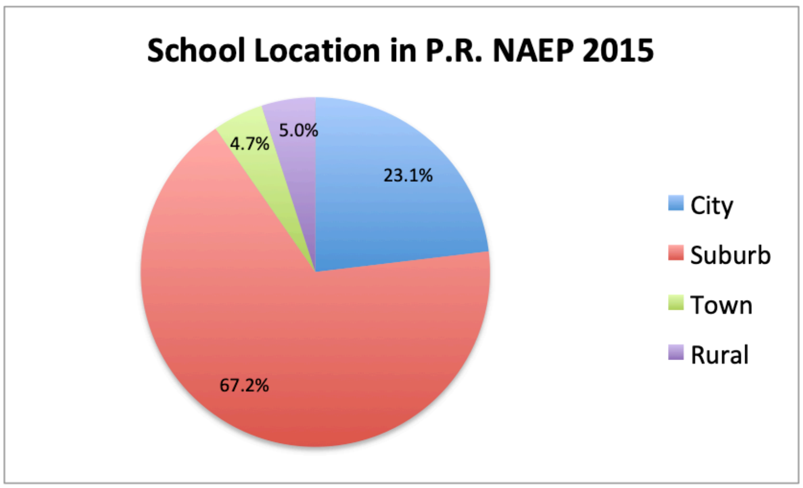

Figure 4. Percentages of students by school location in 2015 P.R. NAEP

Results from Descriptive Analysis

A descriptive analysis was conducted for the variables of interest and the possible control variables: the frequency of using CSAPs to do mathematics, parental education, absenteeism, and home possessions. For each of the variables, the researcher included the missing values, frequency, and the average mathematics score per category.

Variables for the use of CSAPs to do mathematics. The use of CSAPs to do mathematics for school or homework is measured through four questions. Eighth grade students in Puerto Rico reported the frequency of the use of geometric, spreadsheet, graphing, and statistics CSAPs. The reporting of the responses includes five categories: (1) Never or hardly ever, (2) Once every few weeks, (3) About once a week, (4) Two to three times a week, and (5) Every day or almost every day. Table 10 contains the frequency of each category for each variable and the mean NAEP mathematics composite score with the mean standard error per category.

Table 10

Descriptive statistics for the use of geometric, spreadsheet, graphing, and statistics, and computer software from a sample of 5,150 students

	[m825001] geometric			[m816001] spreadsheet			[m816501] graphing			[m816601] statistics		
Category	N[b]	mean	SE mean	N[b]	mean	SE mean	N[b]	mean	SE mean	N[b]	mean	SE mean
1	2500	229.83	1.17	2370	228.95	1.10	2670	229.04	1.25	2860	230.29	1.18
2	860	219.44	1.41	960	218.80	1.52	860	219.68	1.41	780	214.99	1.16
3	570	216.38	1.43	670	216.22	1.30	560	214.10	1.60	560	213.20	1.52
4	470	214.57	1.62	500	216.82	1.57	440	211.92	1.67	330	210.09	1.70
5	380	206.90	1.83	420	213.43	1.86	280	210.54	2.10	250	205.14	1.84
Total	4780 370[a]			4920 230[a]			4810 340[a]			4780 360[a]		

[a]Missing values
[b]Rounded to the nearest ten

The total number of eighth grade students sampled in the 2015 Mathematics NAEP in Puerto Rico was 5,150. Missing values for the use of these CSAPs ranged from 230 to 370, which is between 4.5% to 7.2% of the total number of students sampled. This means that using these variables as predictors of mathematics composite scores will reduce the sample size by at least 7.2 percentage points.

Each of the CSAP variables presents similar patterns. Students most commonly reported using these CSAPs never or hardly ever (category 1). Frequencies were between 2,370 (48.2%)

and 2,860 (59.8%). Students who reported never or hardly ever using these CSAPs had higher averages of mathematics composite scores with smaller mean standard errors. On the opposite end, the least common response was the use of these CSAPs to do mathematics every day or almost every day (category 5). The mathematics average scores of students who reported using these mathematics CSAPs every day or almost every day was lower than for the other categories and the standard error was higher. In general, the average NAEP mathematics composite scores decreased as the frequency of using a CSAP increased.

Figure 5 shows 95% confidence intervals for the mean score per category of using geometric, spreadsheet, graphing, and statistics CSAPs to do mathematics. This graph shows the patterns in each of the variables related to the use of CSAPs to do mathematics, especially between the mean and variability for each of the variables.

The confidence intervals for all variables in the category of never or hardly ever using a CSAP, are all between 226.6 and 232.6 points, and each confidence interval has a range of 4.3-4.9 points. There is a notable gap between the confidence intervals for NAEP mathematics scores (NMS) of students that never or hardly ever used each of these CSAPs and the other categories (once every few weeks, about once a week, etc.). Confidence intervals for the mean NMS for students who reported using CSAPs once every few weeks are between 212.7 and 222.4 points, and each of them have a range of 4.6-6.0 points. These intervals overlap with those in the category of students using CSAPs once a week. The category of using CSAPs once a week yields confidence intervals of 210.2-219.2 points for the mean NMS with ranges of 5.1-6.3 points. The category of two to three times a week gives confidence intervals of 206.8-219.9 points with a range of 6.1-6.7 points. The last category, every day or almost every day, has confidence intervals for the mean NMS of 201.5-217.1 points with ranges of 7.2-8.2 points.

This category overlaps with the one that describes using spreadsheet, graphing, and statistics CSAPs two to three times a week, but not for using geometric CSAPs two to three times a week.

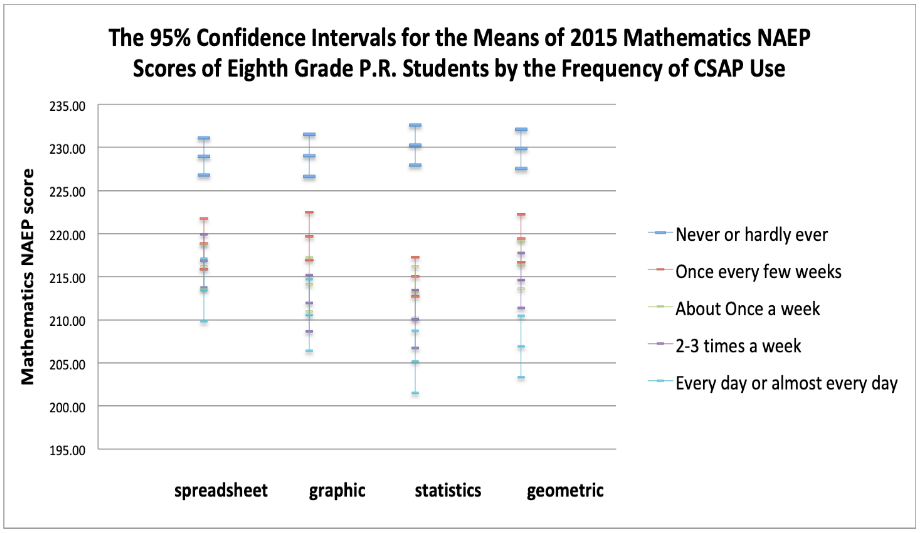

Figure 5. The 95% confidence intervals for the mean 2015 NAEP mathematics score of eighth grade students per CSAP category.

These variables were used to create an index that reflects the frequency of using these CSAPs to do mathematics. The index has values between one and five. The 25th percentile for this index has a value of 1, the median is 1.75, and the 75th percentile is 2.5. In other words, 75% of the students reported an average CSAP use to do mathematics with a frequency of less than once a week.

Selected control predictor variables. Possible control predictors for this study are socioeconomic status (SES) and absenteeism. The selected 2015 NAEP variables for measuring the SES of students in Puerto Rico are home possessions and parental educational attainment. Absenteeism is measured as the days absent from school in the last month.

Home possessions of students are measured in a survey question, where students can choose one or more options. The question is: *Do you have the following in your home?,* followed by a list of five items: access to the Internet, clothes dryer just for family, dishwasher, more than one bathroom, and own bedroom at home. Student can either select or not select the option.

Thus, this variable does not show missing data. In other words, not selecting an option could either mean that the student does not have the item or that the student skipped the question. However, in the data analysis tool provided by NCES in the NAEP Data Explorer, students who did not select the option are analyzed as students who did not have the item. The researcher also used this assumption for the analysis.

The percentage of students having each of the home possessions are summarized as follows: 83.2% had access to Internet, 54.7% had a clothes dryer, 40.3% had a dishwasher, 44.3% had more than one bathroom, and 79.9% had their own bedroom. The confidence intervals for the mean NMS of eighth grade students broken down by the five home possessions are shown in Figure 6. If repeated samples were taken on this population and the 95% confidence intervals were computed for these home possession variables, 95% of these intervals will contain the population mean NMS. The primary pattern observed is that confidence intervals for those students who reported having the items are greater than for those who did not report having these items. However, this observed difference is smaller for the dishwasher variable. The researcher did not include this variable in the calculation of the index of home possession (IHP).

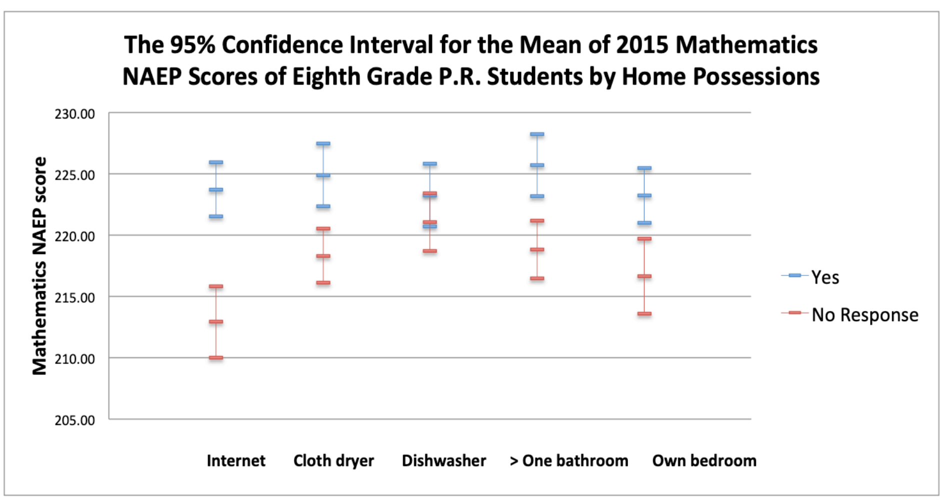

Figure 6. The 95% confidence intervals for the mean 2015 NAEP mathematics score of eighth grade students by home possessions.

Using this descriptive analysis, the researcher in this study used the variables of having *Internet access, clothes dryer, more than one bedroom,* and *their own bedroom* for explaining the students' home possessions. For each variable, if a student reported to have a home possession then it was coded with a one, and zero otherwise. For reducing the complexity of the model, the researcher uses the index IHP to reflect the home possessions of a student.

The IHP is a number between zero and one, and the number represents the percentage of home possession items (out of four items) that a student has. The 25th percentile for this index has a value of 0.5, the median is 0.75, and the 75th percentile is 1. In other words, half of the students reported having at least three of the home possessions.

Descriptive analysis on the parental education (PARED) variable was also conducted. This variable considers the highest degree achieved by parents reported by students from two questions asking about each parent's educational attainment. PARED has four categories shown in Figure 7. Patterns of confidence intervals suggest that students had higher mathematics achievement when their parents had a higher level of education. This variable has 19.8% of its values missing, reducing the sample size to 80% of the total.

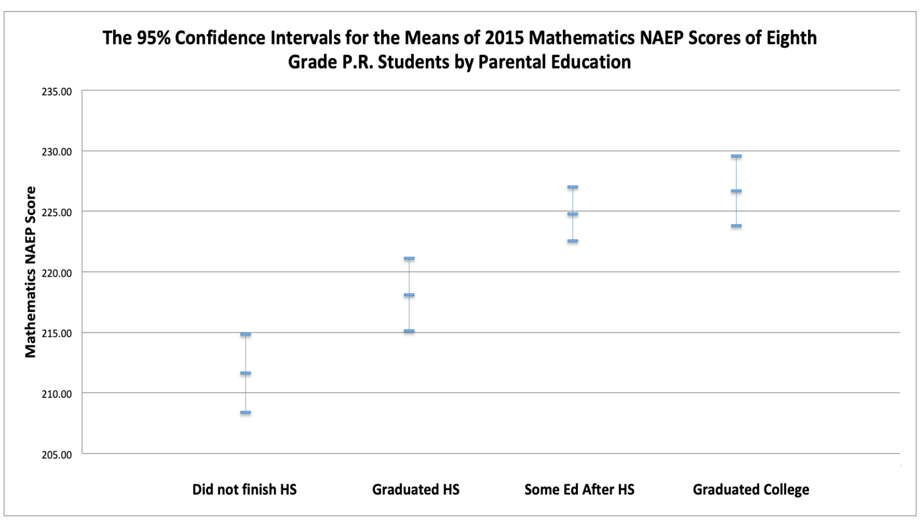

Figure 7. The 95% confidence intervals for the mean 2015 NAEP mathematics score of eighth grade students by parental education.

The variable of days absent from school in the last month has five categories, and is also reported by students. The 35.1% of the students reported none days absent from school, 39.3% one to two days absent from school, 17.9% three to four days absent from school, 5.4% five to ten days absent from school, and 2.1% more than ten days absent from school. This variable has 2.3% of its data values missing. Confidence intervals for the variable reflecting the absenteeism of students are shown in Figure 8. If repeated samples were taken on this population and the 95% confidence intervals were computed for the categories of parental education, 95% of these intervals will contain the population mean of the NMS. This suggests a pattern of lower NMS when the number of absences is higher.

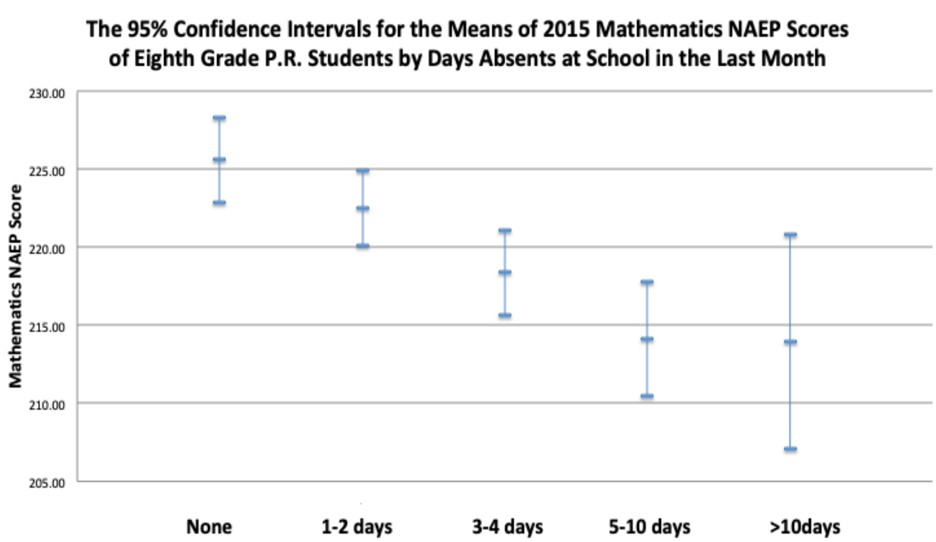

Figure 8. The 95% confidence intervals for the mean 2015 NAEP mathematics score of eighth grade students by absenteeism.

This variable was also used as an indicator variable with values of zero or one. A value of one represents a student who was absent for two or less days, which were about 74.5% of the students. A value of zero represents students who were absent for three or more days, which were 25.5% of the students.

Results from the Two-level Cross-sectional Multilevel Modeling

A two-level cross-sectional multilevel model was created by using the variables indicated above. First, the researcher conducted an analysis with a null model. Then the researcher ran the analyses with the variables at level one and at level two. The model was finalized and the results were interpreted.

Null model. The researcher ran a null model (m_0) to check the variation in the NMS that is explained by the school of the student. The null model is:

student level: $NMS_{ij} = \beta_{0j} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + u_{0j}$,

where NMS_{ij} is the composite mathematics score of student i in school j, e_{ij} is the residual error for student i in school j, and u_{0j} is the residual error for school j. The estimated value of γ_{00} is $\hat{\gamma}_{00} = 221.57$, which is just the average NMS for eighth grade students.

The residual errors, e_{ij}, have an estimated variance of $\hat{\sigma}^2_{(m0)} = 598.399$, and the u_{0j}'s for the schools have an estimated variance of $\hat{\tau}^2_{00(m0)} = 96.825$. A Chi-squared test indicates that the random effect variance is different from zero ($\chi^2 = 1014.72, p < 0.001$). Since the p-value is so small, this indicates that the variance of this random effect is statistically significant not equal to zero. The null model, m_0, has a total variance of 695.224, and the intraclass correlation coefficient (ICC) is:

$$ICC = \frac{\hat{\tau}^2_{00(m0)}}{\hat{\sigma}^2_{(m0)} + \hat{\tau}^2_{00(m0)}} = \frac{96.825}{598.933 + 96.825} = 0.139.$$

This means that 13.9% of the variance in eighth grade students' scores is explained by their school difference, while 86.1% of the variance is explained individually by students. According to Hedges and Hedberg (2007), ICC values between 0.10-0.25 are typical in nested data in educational research. A value of 13.9% is practically significant and supports the use of multilevel modeling to explain the effect of school on the variability in mathematics scores of eighth grade students in Puerto Rico.

Conditional models. The selected variables were included in the following models to assess their importance as predictors of the 2015 NMS of eighth grade students in Puerto Rico. The random intercept model (m_1) tests the effect of IUCP on the mathematics achievement of eighth grade students in Puerto Rico when the nestedness of the school is considered. The m_1 is:

student level: $NMS_{ij} = \beta_{0j} + \beta_{1j} IUCP_{ij} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + u_{0j}$,

$\beta_{1j} = \gamma_{10} + u_{1j}$,

where e_{ij} has an estimated variance of $\hat{\sigma}^2_{(m1)} = 519.504$, the u_{0j}'s have an estimated variance of $\hat{\tau}^2_{00(m1)} = 95.250$, and u_{1j}'s have an estimated variance of $\hat{\tau}^2_{11(m1)} = 2.737$. The Chi-squared tests indicate that the variance of these random effects is statistically significant ($\chi^2 = 1050.43$, $p < 0.001$ for u_{0j}; $\chi^2 = 152.50, p = 0.021$ for u_{1j}). This means that the effect of IUCP on NMS varies across schools, so the random effect u_{1j} will be retained in the model.

To calculate the practical significance of the random effect IUCP in the model, the researcher used the value of the level one variance for the random error of this model $\hat{\sigma}^2_{(m1)} = 519.504$ and the variance for the null model $\hat{\sigma}^2_{(m0)} = 598.399$. This practical significance is the effect size of IUCP in the model that calculates the portion of the total variance that the IUCP is contributing to the model:

$$Effect\ size\ of\ IUCP = \frac{\hat{\sigma}^2_{(m0)} - \hat{\sigma}^2_{(m1)}}{\hat{\sigma}^2_{(m0)}} = \frac{598.399 - 519.504}{598.399} = 0.132.$$

This value suggests that 13.2% of the variation in student differences in NMS is accounted for by IUCP in the random effects model.

The estimated value of γ_{00} is $\hat{\gamma}_{00} = 222.57$ and the estimated value of γ_{10} is $\hat{\gamma}_{10} = -7.99$. The Wald ratio tests indicate that these values are significantly different from zero for the population (*t-ratio*= 210.00, $p < 0.001$ for $\hat{\gamma}_{00}$ and *t-ratio*= -16.70, $p < 0.001$ for $\hat{\gamma}_{10}$). Since this model, m_1, considered the IUCP to be group centered, 222.57 is the predicted NMS when the IUCP for a student matches the average of IUCP for his or her school. The value $\hat{\gamma}_{10} = -7.99$ ($p < 0.001$) suggests that, on average, there is a decrease of eighth points in NMS for each point a student falls above the average IUCP of their school. For example, if an eighth grade student in Puerto Rico has an IUCP of 2 (once every few weeks), and is in a school with an average IUCP of 1 (never or hardly ever), then the NMS of the student is predicted to be eight points less than the average student who had an IUCP of 1. The random effect u_{1j} is significant, so the average of the effect for each school would vary significantly across schools. Thus, not every school will have the same effect on their students' NMS.

The next model, m_2, includes the level one predictor variables. This helped the researcher decide on the inclusion of their random effect in the model. The conditional model with all level one predictors is the following:

student level: $NMS_{ij} = \beta_{0j} + \beta_{1j}IUCP_{ij} + \beta_{2j}IHP_{ij} + \beta_{3j}PARED_{ij} + \beta_{4j}IDA_{ij} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + u_{0j}$,

$\beta_{1j} = \gamma_{10} + u_{1j}$,

$\beta_{2j} = \gamma_{20} + u_{2j}$,

$\beta_{3j} = \gamma_{30} + u_{3j}$,

$\beta_{4j} = \gamma_{40} + u_{4j}$,

where the slopes and intercepts at the student level are explained by the level two fixed estimated parameters $\hat{\gamma}_{00} = 224.02$ (Wald ratio test *t-ratio*$= 205.15$, $p < 0.001$), $\hat{\gamma}_{10} = -8.16$ (Wald ratio test *t-ratio*$= -15.23$, $p < 0.001$), $\hat{\gamma}_{20} = 10.66$ (Wald ratio test *t-ratio*$= 4.63$, $p < 0.001$), $\hat{\gamma}_{30} = 2.57$ (Wald ratio test *t-ratio*$= 5.07$, $p = 0.002$), and $\hat{\gamma}_{40} = 3.62$ (Wald ratio test *t-ratio*$= 3.19$, $p < 0.001$); as well as the residual errors for the schools $u_{0j}, u_{1j}, u_{2j}, u_{3j}$, and u_{4j}. The Wald ratio tests suggest that there is strong evidence that the variables IUCP, IHP, PARED, IDAS were significantly different from zero in the model.

The estimated variance components of $u_{0j}, u_{1j}, u_{2j}, u_{3j}$, and u_{4j} are $\hat{\tau}^2_{00(m2)} = 96.147$ (Chi-square test $\chi^2 = 963.53$, $p < 0.001$), $\hat{\tau}^2_{11(m2)} = 5.449$ (Chi-square test $\chi^2 = 150.41$, $p < 0.027$), $\hat{\tau}^2_{22(m2)} = 48.898$ (Chi-square test $\chi^2 = 154.91$, $p = 0.015$), $\hat{\tau}^2_{33(m2)} = 2.550$ (Chi-square test $\chi^2 = 131.62$, $p = 0.202$), and $\hat{\tau}^2_{44(m2)} = 12.477$(Chi-square test $\chi^2 = 126.77$, $p = 0.296$) respectively. The estimated variance of e_{ij} is $\hat{\sigma}^2_{(m2)} = 483.258$. These Chi-square tests indicate that there is statistical evidence for including the random effect for IHP and IUCP. The random effect for PARED and IDAS were not included based on their levels of significance. Practically speaking, this means that the effect of PARED and IDAS do not vary across schools, but IUCP and IHP are varying across schools.

To calculate the portion of the total variance that the control predictors are adding to the previous model, the researcher used the value of the level one variance for the random error of this model, estimated as $\hat{\sigma}^2{}_{(m2)} = 483.258$ and the estimated variance for the random intercept model, m_1, $\hat{\sigma}^2{}_{(m1)} = 519.504$. This ratio measures the practical significance of the control predictors to the previous model:

$$\frac{\hat{\sigma}^2{}_{(m1)} - \hat{\sigma}^2{}_{(m2)}}{\hat{\sigma}^2{}_{(m1)}} = \frac{519.504 - 483.258}{519.504} = 0.070$$

Interpreting this, adding the control predictors contributes to a reduction of unexplained variance of approximately 7.0%.

Finalized model. The final step in this multilevel model process is to include the school level variables selected for the finalized model (m_3). The m_3 model will not include the random effects for the variables $PARED$ and $IDAS$, because they were not significant. The school level variables that the researcher used in the finalized model m_3 are the averages of the level one variables $IUCP$, IHP, $PARED$, and $IDAS$. The model m_3 for student i in school j is given by the following:

student level: $NMS_{ij} = \beta_{0j} + \beta_{1j}IUCP_{ij} + \beta_{2j}IHP_{ij} + \beta_{3j}PARED_{ij} + \beta_{4j}IDAS_{ij} + e_{ij}$,

school level: $\beta_{0j} = \gamma_{00} + \gamma_{01}\,\overline{IUCP_j} + \gamma_{02}\,\overline{IHP_j} + \gamma_{03}\,\overline{PARED_j} + \gamma_{04}\,\overline{IDAS_j} + u_{0j}$,

$\beta_{1j} = \gamma_{10} + \gamma_{11}\,\overline{IUCP_j} + u_{1j}$,

$\beta_{2j} = \gamma_{20} + u_{2j}$,

$\beta_{3j} = \gamma_{30}$,

$\beta_{4j} = \gamma_{40}$,

where NMS_{ij} is the NAEP mathematics score for student i in school j; the variables at level one are $IUCP$, IHP, $PARED$, and $IDAS$; the variables at level two are the means $IUCP$, IHP, $PARED$, and $IDAS$ given by $\overline{IUCP_j}$, $\overline{IHP_j}$, $\overline{PARED_j}$, and $\overline{IDAS_j}$, which are the contextual effects of the level one variables; the residual errors for the schools are u_{0j}, u_{1j}, and u_{2j}; and e_{ij} is the residual error for student i in school j.

The slopes and the intercept at the student level are explained by the level two fixed parameters. Table 11 presents the estimated value of each fixed parameter and the corresponding interpretation.

Table 11

Interpretation of the fixed effect values in the final model

Level one coefficient	Estimated fixed effect. Wald-ratio test (p-value)	Interpretation
β_{0j}	$\hat{\gamma}_{00} = 223.79$[a] (*p<0.001*)	A student who has average school values for the variables is predicted to have a score of 223.79 NMS.
	$\hat{\gamma}_{01} = -13.72$[a] (*p<0.001*)	The average score of the school that has about one point of average IUCP above the overall average IUCP is predicted to have an average NMS of about 14 points less than schools meeting the average overall IUCP.
	$\hat{\gamma}_{02} = 43.41$[a] (*p<0.001*)	The average score of the school that has about one point of average IHP above the overall average of IHP is predicted to have an average NMS about 43 points more than schools meeting the average overall IHP.
	$\hat{\gamma}_{03} = 9.84$[a] (*p=0.009*)	The average score of the school that has about one point of average PARED above the overall average of PARED is predicted to have an average NMS about 10 points more than schools meeting the average overall PARED.
	$\hat{\gamma}_{04} = 20.98$[a] (*p=0.038*)	The average score of the school that has about one point of average IDAS above the overall average IDAS is predicted to have an average NMS about 21 points more than schools meeting the average overall IDAS.
β_{1j}	$\hat{\gamma}_{10} = -8.20$[a] (*p<0.001*)	A student who has about one point above IUCP average of the school with an average IUCP, is predicted to have about 8 points on NMS less than students who are in the same school and have the school average IUCP.
	$\hat{\gamma}_{11} = 1.84$ (*p=0.445*)	This measures the contextual effect of the variable IUCP. In other words, this measures the strength of the effect of the average IUCP of the school that moderates the effect of each student IUCP. For example, if a school average IUCP is one point higher than the overall value of IUCP, the effect of IUCP of a particular student in that school is stronger. On the other hand, if a school average IUCP is one point lower than the overall value of IUCP, the effect of the IUCP of a particular student in that school is weaker. This effect is not statistically significant, which means that there is no contextual effect by average IUCP.

β_{2j}	$\hat{\gamma}_{20} = 10.52$[a] (*p<0.001*)	A student who has about one point above the IHP average of the school with an average IHP, is predicted to have an NMS about 11 points more than students who are in the same school and have the school average IHP.
β_{3j}	$\hat{\gamma}_{30} = 2.53$[a] (*p<0.001*)	A student who has about one point above the PARED average of the school with an average PARED, is predicted to have an NMS about 2.5 points more than students who are in the same school and have the school average PARED. Since β_{3j} does not have a significant random effect, this effect does not vary across schools.
β_{4j}	$\hat{\gamma}_{40} = 3.80$[a] (*p<0.001*)	A student who has about one point above the IDAS average of the school with an average IDAS, is predicted to have an NMS about 4 points more than students who are in the same school and have the school average IDAS. Since β_{4j} does not have a significant random effect, this effect does not vary across schools.

[a]Value statistically significant different than zero at an alpha of 0.05.

Table 12 includes the estimated variance components of u_{0j}, u_{1j}, and u_{2j}; the proportional reduction of the estimated variances $\hat{\tau}^2_{00(m3)}$ and $\hat{\tau}^2_{11(m3)}$; and their statistical significance. The estimated variance, $\sigma^2_{(m3)}$, of e_{ij} is also included. Interpretations for each of the variances are provided.

Table 12

Variances and interpretations for m_3

Random effect	Estimated variance Chi-squared test for random effect variance significantly ≠ 0, (d.f. and p-value)	Proportional reduction of variance compared to m_2.	Interpretation
u_{0j}	$\hat{\tau}^2_{00(m3)} = 38.117$ ($\chi^2 = 432.50$ and *p<0.001*)	60.36%	There is strong statistical evidence of variation in the school averages across schools. When adding the four second level predictors to this model, there was 60.36% reduction in the variance of β_{0j}.

u_{1j}	$\hat{\tau}^2_{11(m3)} = 5.048$ (χ^2 =162.58 and *p=0.004*)	7.36%	There is strong statistical evidence of variation across schools in the IUCP effect on mathematics achievement. When adding the average of IUCP to explain the β_{1j} slope in this final model, the variance of this random effect was reduced by 7.36%.
u_{2j}	$\hat{\tau}^2_{22(m3)} = 35.666$ ($\chi^2 = 143.77$ and *p=0.061*)		There is weak statistical evidence of variation across schools in the effect of IHP on mathematics achievement; 6.28% of the variance is explained by the differences across schools in the IUCP.
e_{ij}	$\hat{\sigma}^2_{(m3)} = 488.987$		About 86.1% of the total variance is explained by differences across students after including all level one predictors within the same school.
Total	567.818		

Changes in the models. Table 13 presents the estimates for each of the models from the multilevel modeling. In this table, the inclusion of variables at level one and level two are changing the predicted NMS score of students, γ_{00}, by one to three points. The estimation of the fixed parameters to explain the effect of IUCP, IHP, and IDAS on NMS scores of students was consistent across the models. The estimation of the variance of u_0 is given by $\hat{\tau}^2_{00}$. This value changed when the level two variables were included in the model, which means that part of the variance of u_0 is explained in the final model by the school averages variables added to the intercept β_{0j}. The estimated variance $\hat{\tau}^2_{11}$ of u_1 increased when other variables at level one were included in the model; this means that the random effect of IUCP explains more variance when other variables are included. The variance $\hat{\tau}^2_{22}$ of u_2 decreased when the school variables were included in the model with weak evidence of statistical significance ($p = 0.06$).

Table 13

Multilevel modeling estimates for m_0, m_1, m_2, and m_3

Level one	Parameter	m_0	m_1	m_2	m_3
β_{0j} (intercept)	$\hat{\gamma}_{00}$	221.57[a]	222.57[a]	224.02[a]	223.79[a]
	$\hat{\gamma}_{01}$				−13.72[a]
	$\hat{\gamma}_{02}$				43.41[a]
	$\hat{\gamma}_{03}$				9.84[a]
	$\hat{\gamma}_{04}$				20.98[a]
	$\hat{\tau}_{00}^2$	96.83[a]	95.25[a]	96.14[a]	38.12[a]
β_{1j} (coefficient of IUCP)	$\hat{\gamma}_{10}$		−7.99[a]	−8.16[a]	−8.20[a]
	$\hat{\gamma}_{11}$				1.84
	$\hat{\tau}_{11}^2$		2.74[a]	5.45[a]	5.04[a]
β_{2j} (coefficient of IHP)	$\hat{\gamma}_{20}$			10.66[a]	10.52[a]
	$\hat{\tau}_{22}^2$			48.90[a]	35.67
β_{3j} (coefficient of PARED)	$\hat{\gamma}_{30}$			2.57[a]	2.53[a]
	$\hat{\tau}_{33}^2$			2.55	
β_{4j} (coefficient of IDAS)	$\hat{\gamma}_{40}$			3.62[a]	3.80[a]
	$\hat{\tau}_{44}^2$			12.48	
e_{ij}	σ^2	598.40	519.50	483.26	488.99

[a]Value statistically significantly different from zero at an alpha of 0.05

Sensitivity analysis for the multilevel model. The researcher checked the six assumptions for the multilevel model. Details and graphs of these assumptions can be found in the Appendix.

The student level predictors IUCP, IHP, PARED, and IDAS are assumed to be independent of the level one residuals e_{ij}. To validate this, the researcher computed the Pearson correlation coefficients of the predictors with the level one residuals. All of them were very close to zero, with correlations of 0.002, -0.010, -0.010, and 0.009 respectively. So the level one predictors do not appear to be correlated to the level one residuals. The scatterplots in Appendix A also show that the residuals at level one are independent to the predictors at the same level.

The level two predictors $\overline{IUCP}$, $\overline{IHP}$, $\overline{PARED}$, and $\overline{IDAS}$ are also independent of the level two residuals u_{0j}, u_{1j}, and u_{2j}. A correlation analysis shows that all Pearson correlations are at absolute values of 0.20 or less. The scatterplots in Appendix B show that the level two predictors are independent of level two residuals.

The predictors at level one are not correlated to the residuals at level two (see Appendix C), and the predictors at level two are not correlated to the residuals at level one (see Appendix D). The Pearson correlation coefficients for the level one residuals with the averages of IUCP, IHP, PARED and IDAS are 0.004, -0.01, 0.01, and -0.01 respectively. For the level two residuals, the Pearson correlation coefficients are approximately zero for each of the level one predictors.

The level one residuals are independent and normally distributed with a constant variance. First, the researcher conducted a Chi-squared test of homogeneity of the level one variance. This test indicated that variances across groups are statistically different ($p<0.001$). In the case of heteroscedasticity, since the number of schools is large the researcher can use robust standard errors for parameter estimation. However, the Chi-squared test becomes more likely to be statistically significant when the sample size gets large. Thus the researcher created boxplots to visualize the differences in level one variances, if any. The boxplots did not show differences in the variances of the level one residuals by schools. Appendix E presents the variances of some of the residuals, and suggests that the homogeneity of level one variances is not necessarily violated. The researcher used a normal Q-Q plot to justify the normality of the level one residuals (See Appendix E). Therefore the level one residuals appear to be normally distributed with equal variance. These residuals are also independent because they were randomly selected.

The multivariate normality test was not conducted because the data are weighted so the HLM software did not generate appropriate estimates for Mahalanobis distances. Failure to satisfy this assumption can affect the consistency of standard errors of the fixed effects and the precision of variance estimates. However, the researcher checked that each of the sets of level two residuals is approximately normal and uncorrelated to each other (see Appendix F).

The level one residuals are independent of the level two residuals (see Appendix G). The Pearson correlation coefficients are all between -0.002 and 0.100, so these pairs of sets of residuals are not correlated.

Chapter 5

Discussion and Conclusions

This chapter includes a discussion of the results from the descriptive analysis, and the two-level cross-sectional multilevel modeling. Then the researcher discusses the usefulness and limitations related to the variables and the NAEP policies, and the possible links of the association of the use of computer software application programs (CSAPs) and the mathematics achievement of students in Puerto Rico. In addition, the researcher presents the implications of this dissertation for administrators, teachers, and researchers. Lastly, the researcher highlights the primary findings and conclusions.

The researcher investigated the research question: *How does the use of computer software application programs to do mathematics by students relate to the 2015 NAEP Mathematics scores of eighth grade students in Puerto Rico?* To answer this research question, the researcher used the restricted P.R. NAEP Mathematics data. This data set includes variables that allow for the measurement of the relationship between the frequency of using geometric, spreadsheet, graphing, and statistics CSAPs to do mathematics and mathematics achievement reflected by NAEP score. NAEP Mathematics is a common assessment of students' mathematics achievement. It measures students' knowledge and skills in mathematics and students' ability to apply their knowledge in problem-solving situations.

Using multilevel modeling, the researcher found that the frequency of using geometric, spreadsheet, graphing, and statistics CSAPs to do mathematics is *negatively* associated with mathematics achievement of eighth grade students in Puerto Rico. This is somewhat surprising since previous studies in the literature have shown that the use of CSAP improves mathematics learning (e.g., Abrahamson & Wilensky, 2007; Carreira, Amado, & Canário, 2013; Friedlander, 1998; Saha, Ayub, & Tarmizi, 2010).

Discussion of Results

In this section, the researcher discusses interesting results from the descriptive analysis, as well as results from the two level cross-sectional multilevel modeling.

Discussion of interesting results from the descriptive analysis. The descriptive analysis included the frequency, missing values summary, and average mathematics score for the different uses of CSAPs to do mathematics and for selected predictors. The researcher discusses results from the descriptive analysis such as the frequency of using technology in Puerto Rico and how this makes sense with the eighth grade Mathematics Common Core State Standards (Math-CCSS). Other discussion includes the differences among the four CSAPs to do mathematics, and summaries of the SES variables.

In eighth grade, the Math-CCSS provides the expectation for the use of technology, but the researcher found that about half of the students never or hardly ever used geometric, spreadsheet, graphing, and statistics CSAPs to do mathematics. Specifically, the content standards 8.EE.A.4 and 8.G.A.6 on the Math-CCSS explicitly recommend the use of technology, and the MP5 standard of mathematical practices recommends the use of appropriate tools strategically in the content standards. The standard 8.EE.A.4 expects the use of technology to perform operations with numbers expressed in scientific notation, and standard 8.G.A.6 expects the use of a geometry CSAP for manipulating objects to learn rotations, reflections, and translations. In addition, the content standards for eighth grade, summarized in an overview in Table 14, also provide multiple opportunities to use technology as an appropriate tool to learn mathematics. For example, a graphing CSAP can be used for enhancing the understanding of functions, expressions and equations (e.g., Koştur & Yılmaz, 2017; Zulnaidi & Zakaira, 2012). Specifically, Zulnaidi and Zakaira (2012) recommend *GeoGebra* graphing software for understanding functions, and Koştur and Yılmaz (2017) recommend the use of *Desmos* for exponential functions. Another example is using spreadsheet CSAPs to understand functional relationships (Sutherland & Rojano, 1993). Statistics CSAPs can also provide tools for graphing scatter plots and regression lines to analyze bivariate data (Franklin et al., 2007; Lesh, Caylor, & Gupta, 2007). These are examples of the multiple opportunities to use CSAPs in eighth grade, but half of the students reported not being exposed to these tools.

Table 14

Eighth grade overview of the standards

The Number System
- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry
- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability
- Investigate patterns of association in bivariate data.

Because half of the eighth grade students in Puerto Rico are not using the geometric, spreadsheet, graphing, and statistics CSAPs, teachers are likely not using it in their classrooms even when it is recommended. There are possible explanations that could be explored about the lack of use of technology in the classroom. For example, there is potentially the need for more resources or professional development. It could be that teachers in Puerto Rico do not feel prepared to teach a class using these CSAPs to do mathematics. This question remains open for future investigations.

Among the four categories of CSAPs in this dissertation, students in Puerto Rico reported using spreadsheet CSAPs most frequently, and the statistics CSAPs least frequently. Spreadsheets have been used for improving the learning of algebra concepts, as well as statistical concepts (Pace & Barchard, 2006; Sutherland & Rojano, 1993). One reason for having higher numbers of students working on spreadsheets instead of with statistics CSAPs is that spreadsheets could be used for investigating bivariate data, which is included in the eighth grade statistics standards. Also, spreadsheet CSAPs are easy to access. For example, *Google Sheets* can be accessed online. In contrast, statistics CSAPs such as *Fathom*, *Tinkerplot*, *Minitab*, and *SPSS* require users to buy a license. Since Puerto Rico is having an economic recession, using spreadsheets instead of statistical packages might be a reflection of 2015 Puerto Rico economic issues.

The descriptive analysis of the socioeconomic status in Puerto Rico reflected expected results. In the case of home possessions, about 80% of the students reported having their own bedroom. This number makes sense because families in Puerto Rico are relatively small; the U.S. Census (2010a) found that the average household size was 2.68 people. The use of dishwashers is not common in Puerto Rico, because historically and culturally the people of Puerto Rico like to wash dishes with soap and let them dry naturally. Thus it is not surprising that this was the least frequent possession, and that it was not presenting a clear distinction in the mathematics achievement of those students who owned versus those who did not own a dishwasher. In the case of parental education, 18.3% of eighth grade students do not know the level of education of their parents. The parental education variable is only reported in the eighth and twelfth grade students questionnaires. Cowan et al. (2012) discussed the importance of this variable and suggested the inclusion of this variable in fourth grade. However, a self-reported questionnaire for fourth grade students would probably produce even more missing data. So Cowan et al. suggested that NAEP searches for ways to connect data from the American Community Survey with the students in the sample. In contrast, the researcher thinks that this approach could affect the confidentiality in NAEP questionnaires. Instead, the researcher proposes to ask some of the background questions a week prior to the assessment. In this way, students can ask their parents and provide more informed answers in the NAEP questionnaires. A matching identification to the assessment could keep confidentiality, and could reduce the amount of missing data.

Discussion of results from the two-level cross-sectional multilevel modeling. The two-level multilevel model was built based on three sequential models. The first model, m_0, was used to provide support for using multilevel modeling. The second model, m_1, was used to understand the effect of the Index of the frequency of Using Computer Programs (IUCP) on the NAEP mathematics scores (NMS). Then all the first level variables were added to the m_2 model to check the significance of random effects for explaining the coefficients. The fourth (and final) model, m_3, added second level variables.

The researcher found that schools explain about 14% of the variation in NMS, while 86% is explained by students. This means that the effect of school on students is important for student performance. This result is not surprising for the researcher because previous research in Puerto Rico and ethnic minorities in the United States support that school factors such as teaching practices (Kellermeier, 2012; Moschkovich, 1999; Rivera, 1987; Young, 2017), school disciplinary climate (Álvarez Suárez, 2014), and absenteeism (Álvarez Suárez, 2014) can affect mathematics achievement.

In the final model, the researcher found that the variable IUCP was significant. The IUCP of students and of schools have a negative relationship with students' mathematics achievement. This finding is unexpected, since the literature suggest that the use of CSAPs to do mathematics has a positive impact on students' mathematics achievement. However, a study in the United States using data from the standardized test *Early Childhood Longitudinal Survey Kindergarten Cohort* (ECLSKC) also found a negative relationship of frequently using computers for learning mathematics on the mathematics achievement of Hispanic or Black students whose first language is English (Kim & Chang, 2010). This study could be connected to students in Puerto Rico, because they speak Spanish as a first language, and they take the Spanish version of NAEP. In later sections, the researcher presents additional explanations of this negative effect and how the findings of this dissertation compare and contrast to the existing literature. These discussions look at the usefulness and limitations related to NAEP variables and NAEP policies on the use of technology for taking the assessment. Also, the researcher discusses some possible links for the negative association of mathematics achievement of students in Puerto Rico and their use of CSAPs to do mathematics.

The inclusion of control predictors was statistically significant for the model. The researcher found that the variables index for home possessions (IHP) and parental education (PARED) positively relate to mathematics achievement. The IHP is an index to represent the ratio of home possessions of the students from a total of four: Internet access, clothes dryer, more than one bedroom, and their own bedroom. This variable reflects household income, which is one component of the SES of a student. An IHP value close to one represents a high SES, while an IHP close to zero represents a low SES. The PARED variable represents the parental education of the student. Higher values of PARED indicate higher SES and are positively associated with students' mathematics' achievement. The significance of these variables is not surprising because researchers have found that SES is an important consideration for explaining mathematics achievement in the existing literature (e.g., Byrnes, 2003; Díaz, 1998). The positive association between SES and mathematics achievement is also not surprising because students' low socioeconomic status is related with students' underachievement (e.g., Díaz, 1998; Reyes & Stanic, 1988).

Another control variable, the indicator variable for the days absent from school (IDAS), measures absenteeism and also had a significant relationship with mathematics achievement. IDAS is an indicator variable where a value of zero indicates that students had an absenteeism problem reflected during the last month. Thus, the positive effect of this variable means that a student without an absenteeism problem tends to have better mathematics achievement. This matches the literature because researchers have found that students' school attendance is positively related to mathematics achievement (Alsace & Samora, 2008).

Usefulness and Limitations Related to the NAEP Variables

NAEP can be very useful for finding general patterns in mathematics achievement. The researcher used this assessment to explore the variability of mathematics achievement explained by the use of CSAPs to do mathematics by eighth grade students in Puerto Rico. Using the foundations of previous studies, the researcher assessed the hypothesis that the use of CSAPs is positively associated with mathematics achievement of students in Puerto Rico. Finding unexpected results on the association of the IUCP guided the researcher to discuss and explore this result by examining some of the possible reasons for this finding associated to NAEP. In this section, the researcher explores the availability and limitations of using NAEP variables to investigate the use of technology.

Variables for measuring technology use in NAEP (2003-2017). The use of technology is measured in NAEP using questionnaires for students, teachers, and schools. The researcher found over one-hundred variables about technology that have been used in NAEP since 2003. Some of these variables were used in multiple years without any modifications, in fact, a large number of the technology variables from the 2005-2015 questionnaires are the same. There are also some questions that were modified from previous years, and some variables that were new.

In 2003, the variables for measuring the use of technology were heavily focused on the use of calculators and all were reported by students. For example, the questions included information about the type of calculator used by students, and the frequency of using it for homework and for the mathematics class. In addition, variables about computers were included. Specifically, the 2003 NAEP questionnaire included two questions about owning an encyclopedia or a world atlas at home that could be in a computer format. The NAEP questionnaire also collected information on whether students had a computer at home for their use. The early versions of the questions used were (1) *When you do mathematics in school how often do you use computers?,* and (2) *Do you use a symbol manipulator (computer algebra system) for your mathematics schoolwork?* The first question gives options that measure frequency of use, but does not specify the type of program used on the computer. In contrast, the second question specified the computer algebra system, but the question does not request a frequency.

In 2005, the number of questions about technology increased. The NAEP questionnaire included questions for students and teachers, and thoroughly measured the technology use. The calculator variables were very similar as in 2003, but now they included questions for teachers such as their decisions about using technology in class allowing comparison of calculator policies for the least advanced courses versus the most advanced courses. The 2005 questionnaire also started to include variables to measure the frequency of using computers to do mathematics on specific CSAPs such as spreadsheet, graphing, word processing, geometric, and statistics programs. This was the first time the questionnaire included the variables used in this dissertation. In addition, the questionnaire had questions about the use of computers to play games, or talk about mathematics through online chat, instant messages, and e-mail. Teachers also reported on computer access at school, and their technology professional development.

The technology available variables on the NAEP 2005-2015 questionnaires were very similar. One of the differences between the questionnaires is the exclusion of the question about the type of calculator used by students in the 2007 questionnaire. However this question was re-incorporated in 2009-2015, and a modified version was included in 2017 with a different scale option. Other variables that changed between 2005 and 2015 were the variables about the teachers' calculator policy. In 2005, 2007, and 2011 teachers reported on the use of technology of their most advanced courses in comparison to their least advanced courses. These comparisons have not been used since the 2013 version.

In 2017, most of the questions were either new or modified from previous years. Some of the modifications incorporate specific digital devices to the questions. For example, questions specified that calculators are not included when mentioning a *computer or a digital device*, while the use of desktop or laptop does include *Chromebooks*. Questions that mentioned tablets include examples such as *Surface Pro*, *iPad*, and *Kindle Fire*. In addition, there were more questions about the professional development of teachers on the use of technology. These questions included specific timelines about the professional development to know if it occurred in the last two years, or in the current school year. This approach is useful for conducting research about professional development on technology, since technology is constantly changing. The questionnaire for schools was also more complete. For example, it included questions about the number of technology devices at school and the ratio of devices per student. Schools also reported where the desktops, laptops (including *Chromebooks*), and tablets (for example, *Surface Pro*, *iPad*, and *Kindle Fire*) were available for students. The questions about the use of geometric, spreadsheet, graphing, and statistics CSAPs were not included in the 2017 teacher questionnaire. Instead, there was a general question about the frequency of using a computer or other digital device (excluding handheld calculators) for mathematics at school.

Self-reporting on doing mathematics. The NAEP question available for the use of CSAPs in 2015 is: *When you are doing math for school or homework, how often do you use these different types of computer programs?* This question included the four categories of CSAPs considered in this dissertation: spreadsheet, geometric, graphing, and statistics CSAPs. It also included the use of programs to drill on mathematics facts, to see a new mathematics lesson with problems to solve, or to learn new things on the Internet. However, these three uses did not mention a specific CSAP; thus they were not considered in this dissertation. The NAEP question for the use of CSAPs also included the use of word processing programs and the use of calculator computer programs. The use of word processing programs is not of interest because this is not a mathematical tool. The researcher finds that the use of calculator programs *on a computer* is merged with the use of geometric, spreadsheet, graphing, and statistics CSAPs. Thus, the calculator CSAPs were not considered in this dissertation.

This question is self-reported by students. The researcher finds some limitations related to eighth grade students self-reporting the use of computer programs to do mathematics. First, the researcher acknowledges that self-reporting variables can be problematic, especially for eighth grade students who might not know the meaning of the question. For example, doing mathematics is a very broad and subjective concept. Through the data, the researcher cannot know how an eighth grade student defines *doing mathematics*, or if the student even considers this phrase in the question while answering it. In addition, the selected variables cannot be cross checked with teacher reported variables, because teachers were not asked about the use of any technology for *doing mathematics.* Some questions in the teachers' questionnaire address the use of technology, for example practicing or reviewing mathematics topics on the computer, or extending mathematics learning with activities on the computer. They can also report on the use of drawing programs for geometric shapes, or graphing programs. However, these questions do not specifically ask about using geometric, spreadsheet, graphing, or statistics CSAPs to *do mathematics*. Therefore, there is no way to know how the technological resource was used in the mathematics class.

Measuring frequency. There are limitations to examining the frequency of use of technology as a variable. The use of the question *When you are doing math for school or homework, how often do you use these different types of computer programs?* is limited to measuring the frequency of using this type of technology. However, in the literature, there is an emphasis on the process of using technology and how students are using it, not the frequency. The researcher presents a discussion about some of the factors that previous studies acknowledge, such as the use of interventions, classroom settings, available resources, teachers, and the motivation of students when using CSAPs to do mathematics. Future studies can explore the effect of the factors presented in the literature on the mathematics achievement of *students in Puerto Rico*.

One factor is the classroom and school environment when using CSAPs. Most of the existing studies were conducted on specific lessons with short term intervention activities or worksheets. For example, Zulnaidi and Zakarie (2012) used an intervention activity with graphing CSAPs to understand functions. Also some studies used informal classroom settings (Dettori, Garuti, & Lemut, 2001), constructivism (Li & Ma, 2010; Zengin, Furkan, & Kutluca, 2012), or games (Kazak, 2015). In addition, the available technological resources can improve mathematics achievement. Specifically, when having enough funding for ensuring appropriate and updated platforms, hardware and software (Bitner & Bitner, 2002). Another factor is the attitude of students for learning, such as their level of engagement (Dimitrova, Persell, & Maisel, 1993), or enthusiasm (Isiksal & Askar, 2005) to solve mathematics problems.

Researchers have found that another factor for the success of mathematics learning is the effect of teachers. Some of the teacher considerations are: skills and attitude of the teacher toward technology (Bitner & Bitner, 2002), professional development of teachers to use technology (Vannatta & Nancy, 2004), and planning by the teacher to ensure effective lessons (Arbain & Shukor, 2015; Bitner & Bitner, 2002; Ruthven, Deaney, & Hennessy, 2009). For example, Ruthen, Deaney, and Hennessy (2009) indicate that teachers should provide suitable pre-structured lesson tasks that support students to formulate mathematical interpretation of the results.

Usefulness and Limitations Related to the NAEP Policies for Students

Using a standardized assessment to measure the use of CSAPs to do mathematics has some limitations for students. These limitations could include the time constraint and the individual work constraint. The researcher also discusses issues with the assessment policies for the use of technology tools when taking the assessment, and provides background on the variables included by NAEP to measure the use of technology in mathematics classrooms.

Time and collaboration constraints with NAEP. The amount of time with NAEP for each set of questions is limited. Mathematical practices include, for example, making sense of problems and persevering in solving them (NGACBP & CCSSO, 2010). When time is constrained by the assessment, students cannot persevere in solving problems and have a limited amount of time to make sense of the problems. The researcher thinks that this factor might affect students' mathematical thinking process for answering the mathematical questions.

Another mathematical practice is to construct and critique the reasoning of others. The NAEP assessment does not allow students to collaborate with classmates in small groups. Since the action of doing mathematics involves participating in a social activity (Schoenfeld, 1994), this might also impact student's mathematics achievement when answering the mathematical questions. During this assessment, mathematics achievement is measured through the knowledge and skills in mathematics as well as the ability to apply their knowledge in problem solving situations.

Technology use with NAEP. Another limitation that the researcher discusses in detail is the limitation of technology use during NAEP.

The use of CSAPs is not allowed during the mathematics NAEP. Students can use calculators for NAEP for some portions of the assessment. The type of calculator depends on the grade level and the NAEP framework for the assessment. For example, in the 1990 NAEP the use of calculators was allowed for two out of seven blocks of the questions; NAEP provided basic calculators for fourth grade students, and scientific calculators for eighth and twelfth grade students (Mullis, Dossey, Owen, & Phillips, 1991).

The National Assessment Governing Board (NAGB) develops the NAEP framework. NAGB archived documents provide the mathematics framework assessments from 2005 to 2017 (NAGB, 2017). These frameworks describe policies about the use of tools during the test. The calculator policy for these years is the same. Students can use calculators on one-third of the NAEP questions. Calculators are provided by NAEP, and students receive appropriate training at the time of administration. For fourth grade students NAEP provides basic calculators with the four functions of addition, subtraction, multiplication, and division; for eighth grade students NAEP provides scientific calculators. Eighth and twelfth grade students can bring their own calculators, including graphing calculators, to the exam. Since 2017, the mathematics NAEP is on a digital platform. The calculator use policy for 2017 is still the same; however calculators were provided in a virtual form.

Early versions of NAEP mathematics assessment provided data on appropriate use of calculators by asking students how frequently they used it during the sections that allowed using calculators. Since these sections included exercises where calculators were allowed, but not always necessary, this variable compared the frequency of using calculators in these sections with the proportion of exercises where calculators were really needed. Students who appropriately used calculators were identified as those who used calculators at an expected frequency in these sections for at least 85% of the time. Results show that appropriate calculator users performed better on NAEP mathematics questions than those who did not show an appropriate use in the assessment (Mullis, Dossey, Owen, & Phillips, 1991, p. 203). This variable is not available in recent NAEP data.

Since the use of CSAPs is not allowed when students take the NAEP assessment, this might limit the approaches and tools that students have to solve the mathematics problems in NAEP. To examine this possible effect, NAEP could allow the use of CSAPs during the assessment. The new digital form of NAEP can facilitate the inclusion of geometric, spreadsheet, graphing, and statistical CSAPs. Moreover, allowing students to use CSAPs provide access to new variables for exploring the use of this technology. For example, using previous approaches to measure the appropriate use of calculators could help NAEP to create new variables to measure an appropriate use of CSAPs.

Possible Links of the Association

The frequency of using CSAPs to do mathematics is negatively associated with mathematics achievement of eighth grade students in Puerto Rico. In this section, the researcher highlights that an association does not imply causation. The negative association could be explained by possible links between the frequency of using CSAPs to do mathematics and the mathematics achievement. The researcher presents the hypothesis of two of the possible links that could explain this association: the computer professional development for teachers and the different ways that technology can be used in the classroom.

Technology professional development for teachers could be explaining the negative association between the frequency of use of CSAPs and mathematics achievement. Because technology is changing every day, teachers need guidance for seeing ways to update their knowledge and use computers in meaningful ways to teach mathematics. Computer professional development can be a key guidance for fulfilling this purpose. In addition, professional development can have a longitudinal effect to support teachers on the integration of technology to mathematics learning (Watson, 2006).

A second possible link to explain the association is the different ways that computers can be used in the classroom. Through NAEP data set we are limited to the student reported variable of frequency of using CSAPs to do mathematics. This information does not specify how the CSAPs were used. For example, there is no distinction between using a graphing CSAP to draw a graph as a response to a question or to obtain a graph that will help students to make sense of a mathematical problem. It is probable that a meaningful way of using CSAPs will help students improve their understanding of mathematics and therefore their mathematics achievement.

Limitations of Using Average for Calculating the IUCP

During this dissertation the researcher measured the use of CSAPs through the IUCP. This index is calculated by taking each of the responses reported by students on the use of the geometric, spreadsheet, graphing, and statistics CSAPs, and then calculating the average.

Each of the variables has five possible answers or categories and the researcher assumed equal differences between each category. This assumption presents limitations to the study because there is a possibility that the difference between two consecutive categories is not the same as the difference between other two consecutive categories. For example, category 1 *never or hardly ever* might not be at the same distance to the category 2 *once every few weeks* than category 3 *about once a week* to category 4 *two to three times a week*.

The IUCP is calculated by averaging the values of the frequency of using geometric, spreadsheet, graphing, and statistics CSAPs, based on the assumption that students have been provided opportunities and access to all these four CSAPs, as these technologies were listed in the Common Core State Mathematics Standards. The use of average might not capture appropriately the use of CSAPs in some cases such as when students only use one of the CSAPs. For example, if a student uses spreadsheets two to three times a week (category 4), but does not uses any of the other CSAPs (category 1) then the frequency of technology use is two to three times a week. However the IUCP calculated will indicate that the average use is 1.75, which indicates that, on average, students use IUCP about *once every few weeks*. The researcher was aware of the danger of such assumption. To address such concern, the researcher created another index using the highest frequency among the four CSAPs and used this index as the variable to capture IUCP in the multiple level models. The results such as the coefficients and whether or not a variable was significant were very similar with the results from the models using the average of CSAPs. However, we are aware of the limitation on interpretation based on this choice.

Implications and Recommendations

Findings from this dissertation suggest that increasing the use of CSAPs to do mathematics is negatively related to the mathematics achievement of students. The implications of these findings should be used cautiously.

Implications for administrators and teachers. Administrators and teachers should avoid focusing on increasing the frequency of using technology. Administrators can make decisions about whether to assign funds to increase the use of CSAPs to do mathematics, and might instead focus on other areas such as professional development on the appropriate use of technology to facilitate students' learning.

Increasing number of jobs require the use of technology and the ability to use CSAPs to solve problems. The need of technology in mathematics learning is highlighted in the Math-CCSS (NGACBP & CCSSO, 2010) and the Guidelines for Assessment and Instruction in Statistics Education Report (Franklin et al., 2007). Thus, administrators can search for ways to align the policies of the use of technology in classrooms and the use of technology in standardized assessments.

Implications and recommendations for researchers. The results of this dissertation have implications and recommendations for researchers interested in the mathematics achievement of students in Puerto Rico, mathematics achievement in general, and technology. Follow-up investigations to this dissertation could be done for a deeper understanding in multiple research areas such as mathematics achievement of students in Puerto Rico, mathematics achievement, and technology use for mathematics learning.

For researchers interested in the mathematics achievement of students in Puerto Rico, the researcher recommends the following investigations:

1. Use NAEP to examine additional factors beyond the use of CSAPs to do mathematics that could be related to the mathematics achievement of students in Puerto Rico. Some examples include the available technological resources and the teachers' professional development on computers.
2. Use a different standardized assessment or a small qualitative study to examine the use of CSAPs to do mathematics in Puerto Rico. For example, one might consider the use of constructivism, or teaching intervention activities.
3. Use NAEP to examine the use of CSAPs to do mathematics in fourth grade in Puerto Rico.
4. Use a theoretical framework different than the educational production function to answer the research question. This could help researchers to search for alternative ways that promotes more equitable learning environments (Fortune & O'Neil, 1994).

For researchers interested in mathematics achievement, the researcher recommends the use of NAEP to examine the effect of CSAPs to do mathematics in other populations such as the total population of the United States or specific racial or socioeconomic subgroups of the population.

For researchers interested in technology use for mathematics learning, the researcher suggests conducting the following investigations:

1. Use NAEP to examine additional factors related to technology other than CSAPs to do mathematics, such as other computer programs or the use of calculators.
2. Use a different standardized assessment or a small qualitative study to examine the use of CSAPs to do mathematics.
3. Study the use of CSAPs in other grade levels and populations of study.
4. Explore how CSAP are used in mathematics classrooms.
5. Explore whether the lack of using CSAPs to do mathematics is exclusive for the population of Puerto Rico, search for reasons, and explain this pattern.

Conclusions

In this dissertation, the researcher addressed the research question: *How does the use of computer software application programs to do mathematics by students relate to the 2015 NAEP Mathematics scores of eighth grade students in Puerto Rico?*

This question was answered by measuring the frequency of using geometric, spreadsheet, graphing, and statistics CSAPs to do mathematics. This variable, included in NAEP, was student reported. The researcher found that the frequency of using CSAPs by eighth grade students in Puerto Rico is negatively associated with mathematics achievement of eighth grade students in Puerto Rico. Specifically, students who had about one point above the average IUCP of the school with an average IUCP, are predicted to have 8.20 points less in their NMS score than students who are in the same schools and have the school average IUCP. The effect of the use of CSAPs varies across schools.

This dissertation shows that frequency of CSAP use is not associated with an improvement of mathematics learning of students. Therefore, the researcher recommends that researchers, school administrators, and teachers be cautious when trying to increase frequency of CSAP use to do mathematics for improving mathematics learning.

The control variables that were statistically significant in predicting students' mathematics achievement were SES and absenteeism of eighth grade students in Puerto Rico. The SES was measured as an index which addressed home possessions (an indicator of family income) and parental education attainment, while absenteeism was measured by the days absent from school during the last month. The effects of these variables were as expected: SES was positively related to mathematics achievement, while absenteeism was negatively related to mathematics achievement of these students. The researcher also found that the home possessions effect on mathematics achievement varied across schools, while the parental education attainment and absenteeism effect on mathematics achievement did not vary across schools.

In summary, this study was the first investigation that explored the relationship between the use of computer programs to do mathematics and the mathematics achievement of eighth grade students in Puerto Rico. This dissertation sheds light on understanding the relationship between the classroom technology policies and the mathematics achievement of eighth grade students in Puerto Rico. Importantly, it also indicates that more frequent use of CSAPs to do mathematics is negatively associated with mathematics achievement of eighth grade students in

Puerto Rico. Therefore, CSAPs to do mathematics should be used cautiously without a mere focus on increasing its frequency of use.

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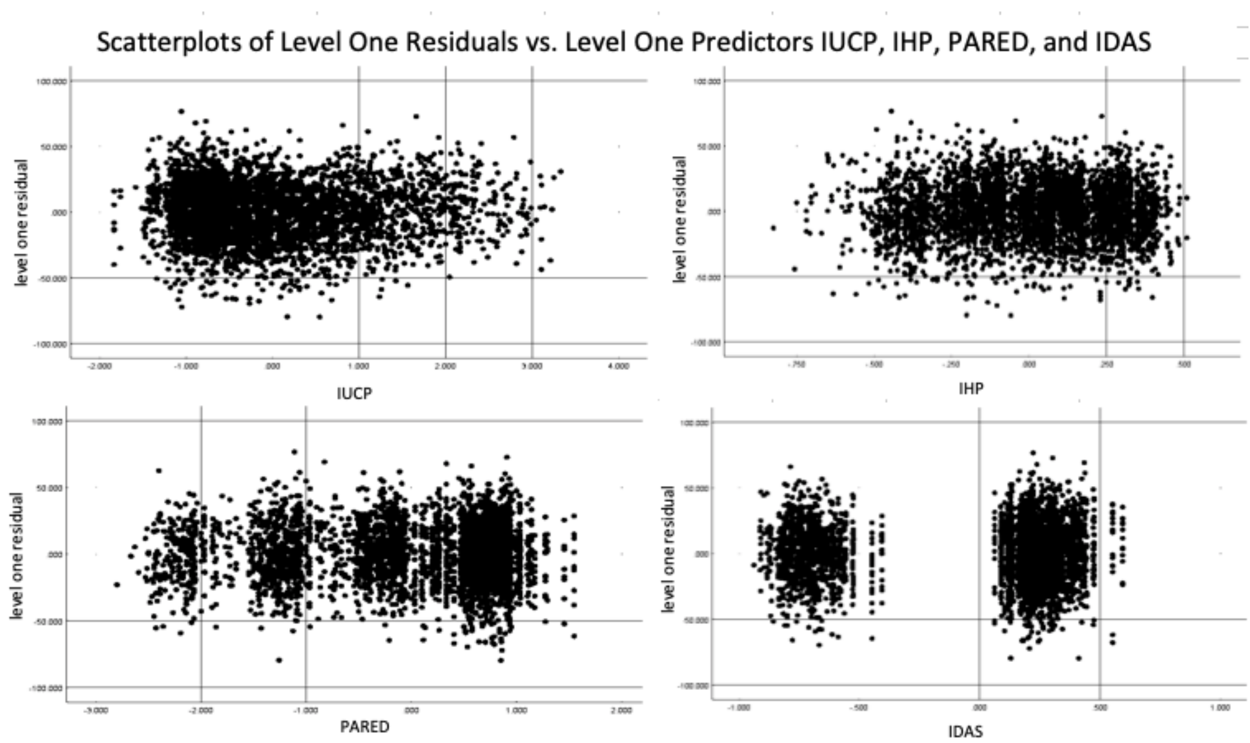
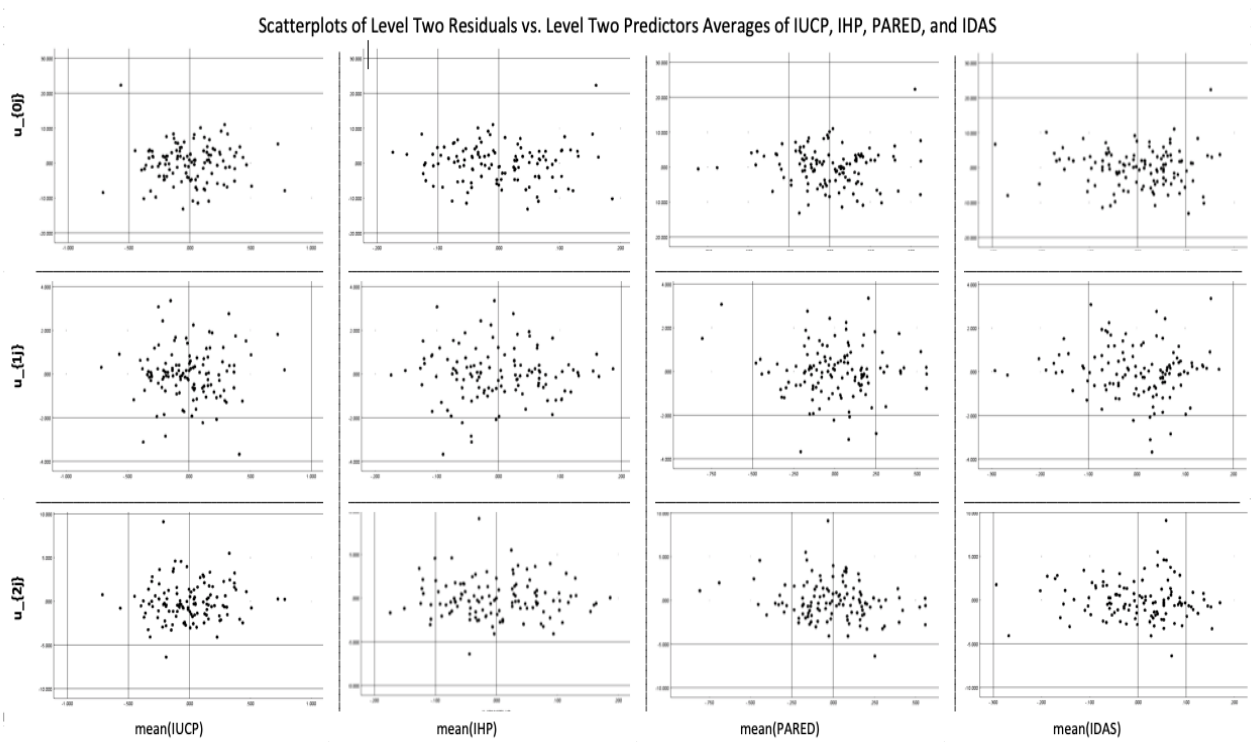
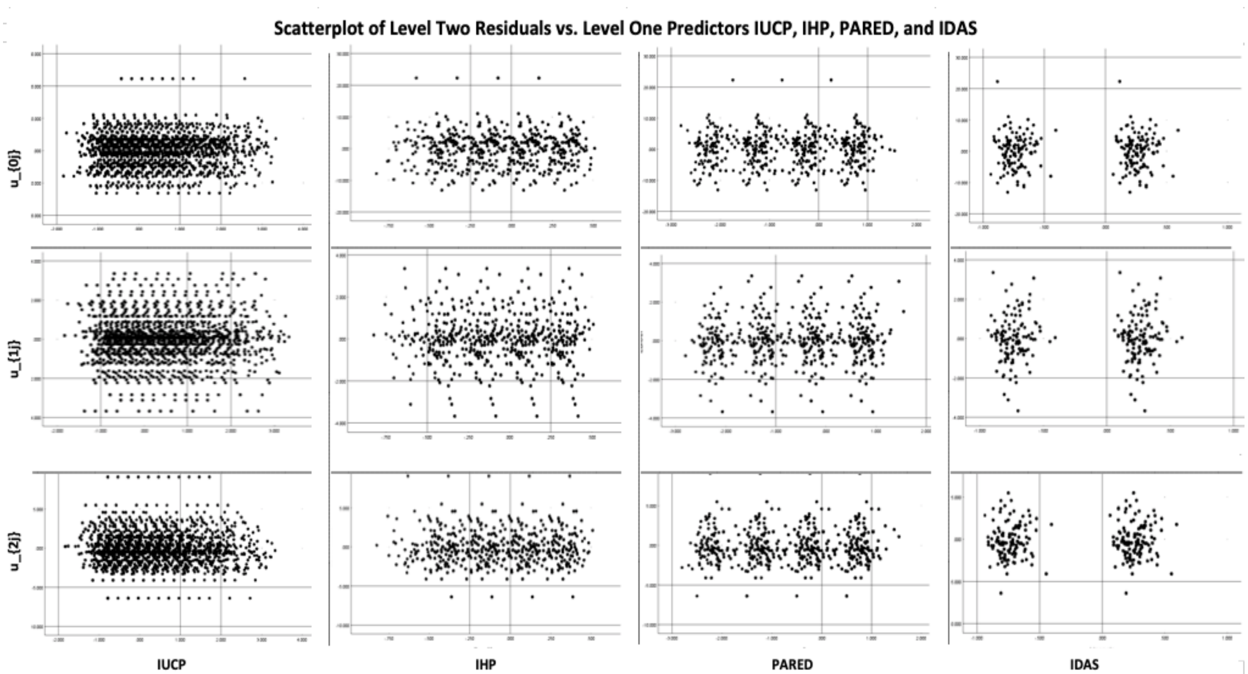
Appendix A. Independence of Level One Residuals and Level One Predictors



Appendix B. Independence of Level Two Residuals and Level Two Predictors

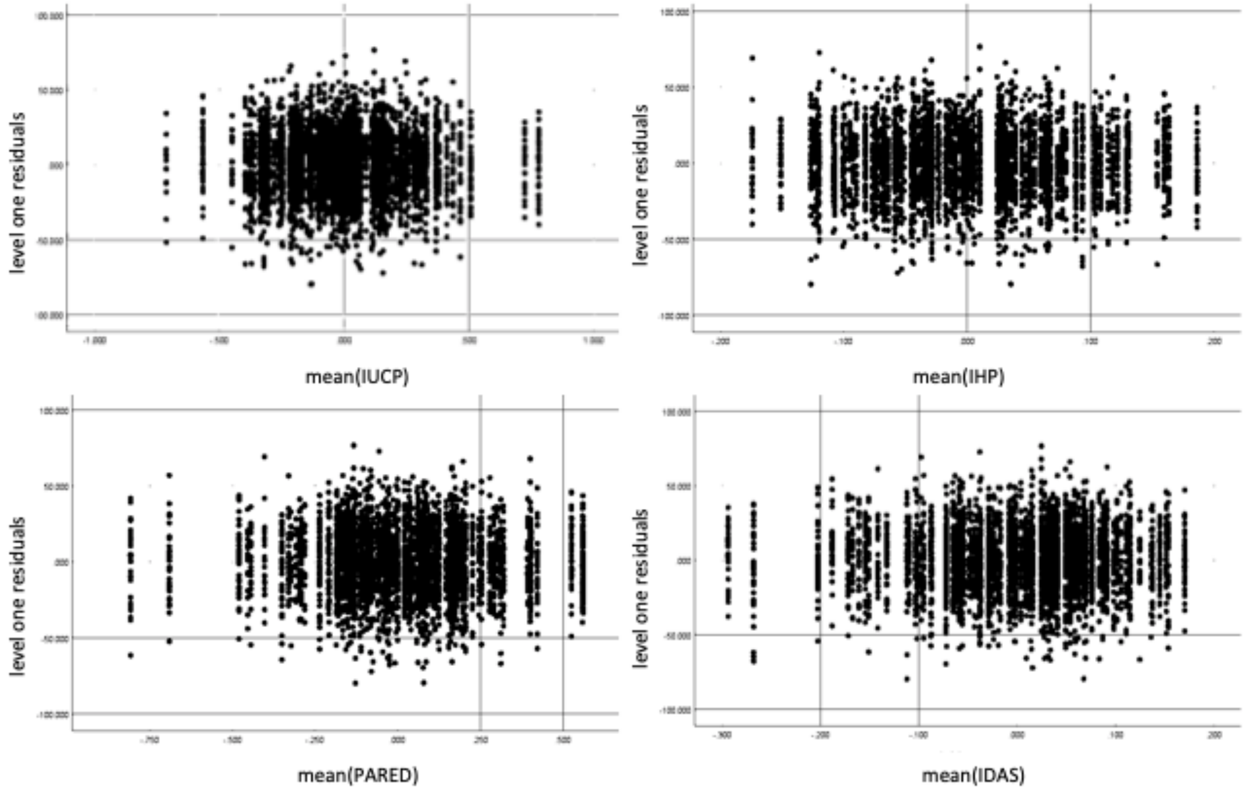


Appendix C. Non-correlation of Level Two Residuals and Level One Predictors

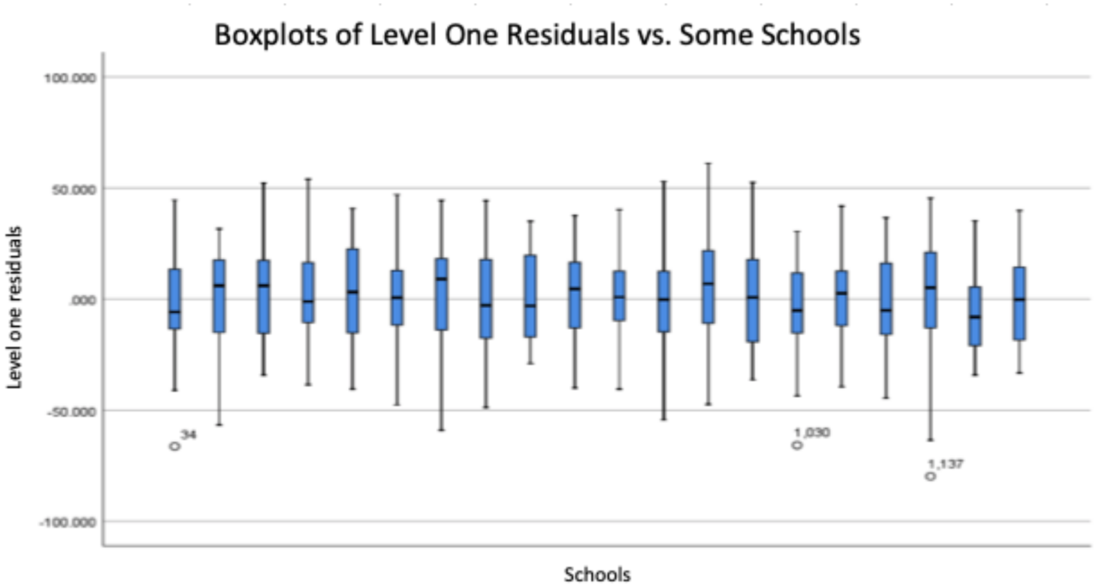


Appendix D. Non-Correlation of Level One Residuals and Level Two Predictors

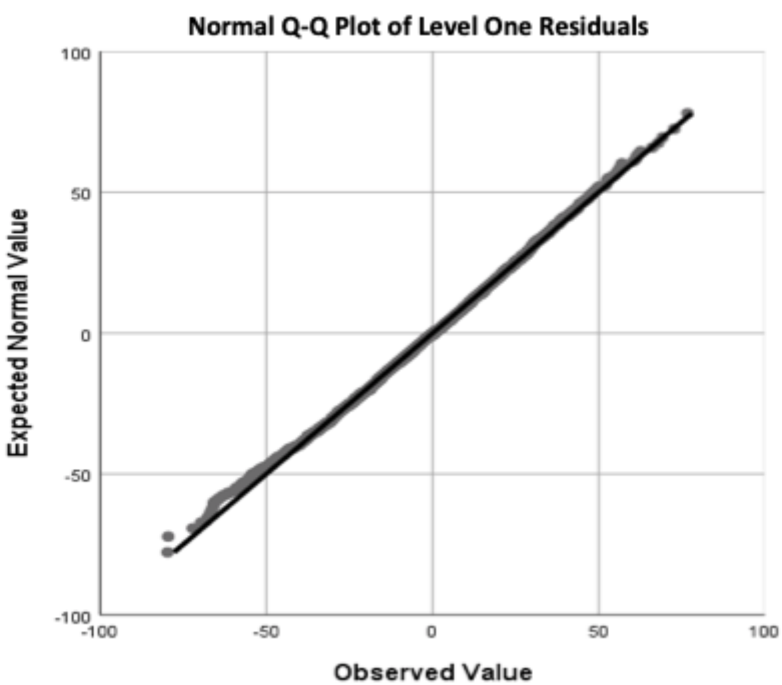
Scatterplots of Level One Residuals vs. Level Two Predictors Averages of IUCP, IHP, PARED, and IDAS



Appendix E. Level One Residuals are Independent and Normally Distributed with Constant Variance



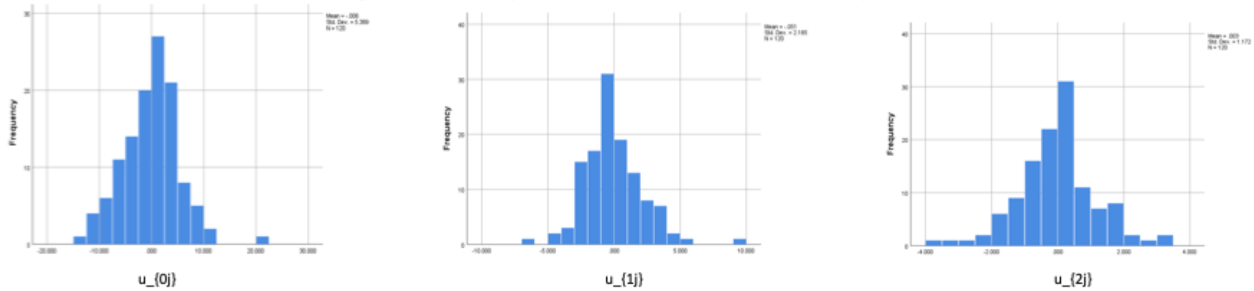
Boxplot of level one residuals and the variance that they have in some schools. This does not contain all the boxplots of the 120 schools, instead, this provide a visualization of twenty of the schools.



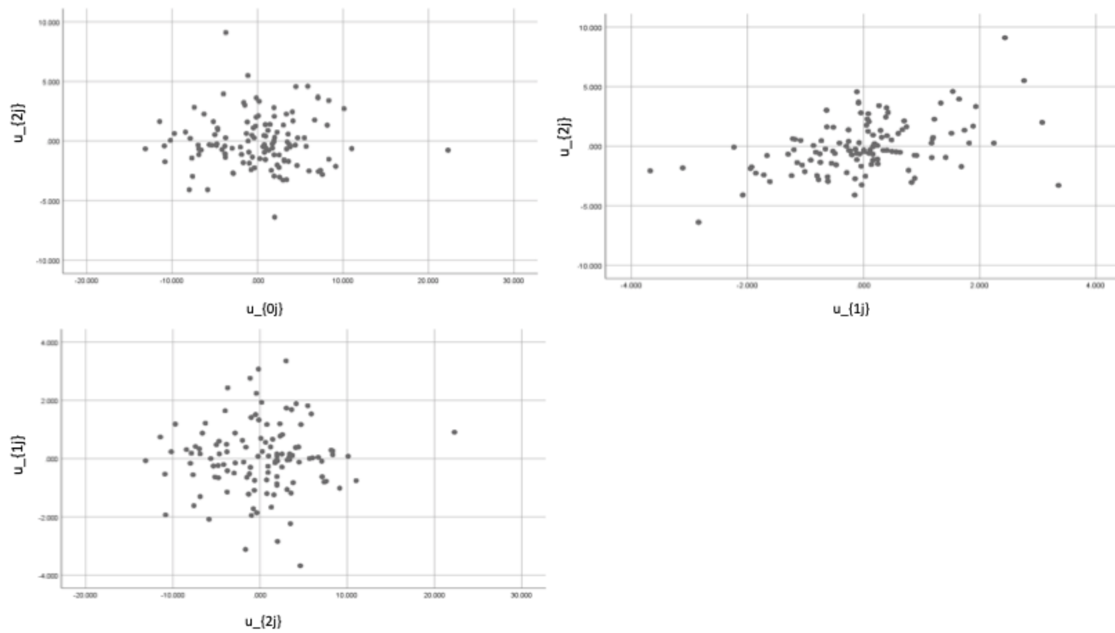
Normal Q-Q Plot of the level one residuals, this shows normality of the level one residuals.

Appendix F. Normality and Non-correlation of Level Two Residuals

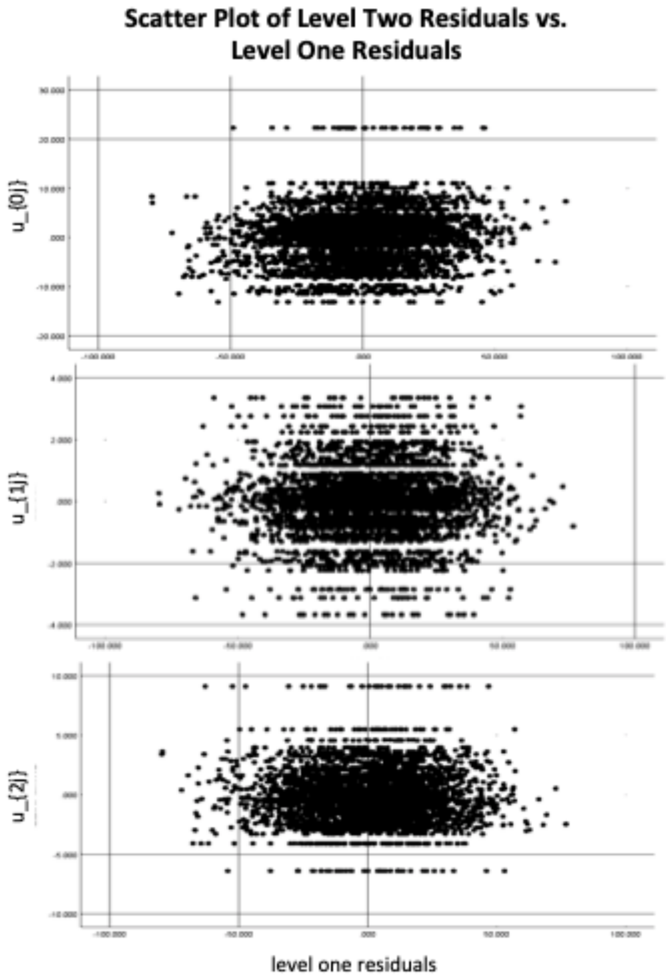
Histograms of Frequency of Level Two Residuals to Show Normal Approximation



Scatterplots of Level Two Residuals



Appendix G. Level One Residuals are Independent to Level Two Residuals



Appendix H. Disclosure Risk Review

The Institute of Education Sciences (IES) Data Security Office reviewed this dissertation. No disclosure risks were identified.

Disclosure Risk Review

Received	Due	Returned	Title	Author(s)	Dataset(s)
5/21/2019	5/24/2019		Feliciano	Ricela Feliciano-Semidei	NAEP

_X_ No disclosure risks identified.

OR

__ Risks identified. Please see below.

Page	Para	Line	Comment	Action