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Abstract: Educational institutions face increasing pressures to respond to the changing environment, including the rapid advances in technology. All curricula are subject to scrutiny, but arguably the sciences and mathematics curricula are under a special lens, given the impact of changing technology and the increasing importance accorded to the knowledge economy. This paper explores the continuing dialogue regarding the teaching and learning of mathematics including the transition from secondary to post-secondary and the emergent trends in the field of mathematics education. The recent evolution of mathematics teaching is described and the implications for the future of mathematics teaching are highlighted, including the need for an examination of the learning experiences afforded secondary and post-secondary mathematics students, as well as the need for dialogue between the secondary and post-secondary levels of mathematics education with the purpose of evidence-based development for mathematics teaching.

Keywords: Mathematics education, transitions, evolution in mathematics teaching, post-secondary education, emerging pedagogical trends

In 1968, Philip Coombs, an advocate for educational transformation, wrote a book entitled *The World Educational Crisis.* He presented the argument that education was experiencing a crisis on a global scale due to unprecedented expansion. In his book, Coombs set out to accomplish two things: to encourage education to use facts when creating strategic plans and to ensure that education would be analyzed as a system of wholes and not as a series of stand-alone facets. In more modern times, educators and other experts have used the term

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emerging crisis to describe the ability of educational institutions to respond to the evolving demands of a technologically driven future (Black & Hernandez-Martinez, 2016; Schmidt, Hardinge, & Rokutani, 2012). Regardless of root causes, "It is true that national educational systems have always seemed tied to a life of crisis" (Coombs, 1968, p. 3) and, irrespective of how profoundly different the causes may be, the ramifications for systems that are slow to adapt are often far-reaching.

Schmidt et al. (2012) and other researchers proposed that in order for societies to thrive and compete with new-order demands, students would need to develop capacities in science, technology, engineering, and mathematics at levels well beyond what were previously considered acceptable. Nations responded to this potential crisis in learning by increasing focus on what students were learning in mathematics and science. The results, however, were unimpressive and experts began to question what they had previously believed to be the most important factors for success in mathematics (Schmidt et al., 2012). Much like Coombs (1968), Goyder and Miller (2000) identified that trends in educational standards needed to be approached with an open mind and they emphasized the importance of responding to evidence rather than intuitive hypotheses. The contention that there had been a steady decline in skill among students, particularly in mathematics, was not a new premise but the current trend had seen an interesting change in how the problem was defined and how the key variables were understood (Goyder & Miller, 2000). At the post-secondary level, the frame of reference through which mathematical teaching and learning was being interpreted was shifting; the changing trends and newly emerging themes focused not only on what was being learned in mathematics but also how mathematics was being taught.

Angle of Vision

In 2013, researchers Chan and Wahl published a paper summarizing a decade of peer-reviewed journal articles that focused on teaching and learning in post-secondary mathematics. As part of their analysis, they were able to identify consistent topics of concern. The research showed that the majority of papers addressed one of the following areas of interest: student thinking, proof, technology, transition, and retention. They were further able to ascertain that a significant proportion of the papers (31 per cent) addressed specific content topics and that subject-based papers were consistently spread across the study period. These findings underscored the importance that post-secondary institutions were putting on content skills in mathematics. The fact that the emphasis on content was distributed evenly across the study period also suggested that it had been a continued area of interest for some time. The authors noted that the content emphasis was consistent with expectations based on the strong history in post-secondary mathematics of publishing subject- and topic-specific instructional ideas.

In 1996, Philip Swedosh reinforced the notion that concept attainment and the importance of mastering content, were critical to instructional and learning practices due to the sequential nature of mathematics. According to Swedosh, research identified that the content approach often focused on diagnosing and then correcting misconceptions of mathematical understanding. These concepts were viewed as essential skills to ensure that the students' existing mathematical constructs would not be incompatible with newly presented information (Swedosh, 1996). The emphasis on mastery learning being a key indicator of success in future mathematical courses played a central role in mathematic departments' instruction and development approaches. The perceived problem of content deficiencies in first year students troubled departments and, although the emphasis on content mastery was highlighted by these institutions, the problem appeared to be growing (Goyder & Miller, 2010).

Concerns about declining standards for secondary and postsecondary students had been a continual area of concern since the mid-1950s according to research released by Goyder & Miller in 2010. Despite considerable trepidation and concerted efforts, questions continued to circulate as to why standards in mathematics seemed to be declining and what could be done to stop the spiral descent. A study in 2009 by eight university professors (Hong et al., 2009) theorized that the introduction of home computers and the infusion of classroom computers in the late 1990s may have played a role in the perceived attrition of content ability. The notion that students were becoming over-reliant on machines to do calculations, and that their basic mathematical skills were suffering because of it, was a topic of debate in mathematical circles. There was no doubt, however, that the very questions of what mathematics was as an area of study, and how mathematics was being utilized and understood by students, was being revolutionized by technology (National Council of Teachers of Mathematics, 2017). In addition to the technological explosion, another reason proposed for a seeming decline in standards continued to be the rapid expansion of post-secondary enrollment in the 1970s. The enrollment surge had led some researchers to surmise that “universities no longer take only the best students and no longer give their students the best education” (Goyder & Miller, 2010, p. 58). In 1968, Coombs foreshadowed that “it is doubtful in the extreme … that there is any one way, one technique, one gadget, which all by itself could achieve better results than the generally unsatisfactory ones of the traditional teaching-learning process” (p. 169). It was apparent that more contemporary researchers were starting to agree.

The research by Chan and Wahl (2013) also revealed, not surprisingly, that a relatively high percentage of papers written between 2000 and 2010 addressed the transition from secondary to post-secondary education and the interest appeared to be maintained through the years of the study. What was surprising, however, was the changing dialogue about transition over the 10-year period. In the early 2000s, the transition-focused papers were consistently suggesting that secondary systems were underpreparing students, but the discourse began to change as the study period progressed (Goyder & Miller, 2000). Over the decade of time covered, it became more apparent that an "important variable in student achievement is the quality of instruction" (Modjeski, p. 1, 2017). In 2011, Norman, Medhanie, Harwell, Anderson, and Post studied students from over 300 public high schools and one of the key results of the study was that there was "an absence of significant relationship between high school mathematics curricula and the calculus placement recommendation" (p. 445). The study found that statistically there was no connection between what the student had learned in high school and the placement recommendations made by the university. The implications of this study were important: if "none of the high school mathematics curricula sufficiently or insufficiently prepared students for placement in university calculus courses" (Norman et al., 2011, p. 445), then what would prepare students to succeed at the college level in mathematics?

The research was beginning to shift focus away from *what* was being taught to *how* mathematical constructs were being taught. Additionally, some researchers such as Black and Hernandez-Martinez (2016) were suggesting that "achievement in mathematics in secondary school does not necessarily motivate students to … choose and succeed in mathematically demanding programmes at post compulsory level" (p. 131). This phenomenon perplexed mathematics departments, whose recruitment had thrived on the idea that students who were good in mathematics in secondary school would choose to continue to study mathematics at the post-secondary level. To better understand what was happening with recruitment, institutions needed to look beyond what typical mathematics departments understood were the variables that affected recruitment and retention (Fenwick-Sehl, Fioroni, & Lovric, 2009).

The mounting interest in mathematics success was being fueled by concerns that while overall enrollment at the post-secondary level was increasing, the number of students choosing to study mathematics at the university level was dropping. Hong et al. (2009) contemplated that the transition between secondary and post-secondary may have come under additional scrutiny in research as a result of institutions facing increasing demands of globalization and competition. The researchers proposed that the perceived decreasing level of competence in content understanding, and a decline in such concepts as analytical ability, could lead to a widening gap between secondary and tertiary education with regard to a lack of fundamental concepts of mathematics. They indicated that the transition disparity could be a major stumbling block not only in the learning of mathematics but also in the teaching of mathematics. Light, Cox, and Calkins (2009) suggested that the "discourse of excellence" conversations connected to the new social and economic realities were putting pressure on educational institutions as well. The insistent attention to excellence meant that the dialogue connected to transparency and accountability became more urgent. It also meant that learning was being seen as a commodity that needed to produce measurable outcomes (Muller, 2017). Mathematics departments responded with measurable data, content measurements, placement test results, and failure rates being amongst the most accessible data to obtain. Success, or lack thereof, in mathematics was beginning to be seen as a gateway or a roadblock to other fields, and institutions started to recognize that in order to be competitive in a modern world they would need to rethink the way they were approaching mathematics learning as well as mathematics teaching (Prendergast, Faulkner, Breen, & Carr, 2016).

The concern of eroding standards had been a matter that post-secondary institutions had grappled with for decades. The change in perspective began to emerge, however, when institutions started to challenge the way the research was being designed and how the questions were being asked (Ellett, Monsaas, Martin-Hansen, & Demir, 2012; Goyder & Miller, 2000). Changing the lens to consider that each incoming year's knowledge was different from its predecessor and that content understanding may be experiencing a "lateral shift" rather than just a "horizontal shift" raised intriguing questions that perhaps the standards "were neither higher nor lower … simply different" (Goyder & Miller, 2000, p. 59). A revolution was beginning to emerge in mathematical learning that was challenging not only the system but also the prejudices of the people within that system. Experts started to insinuate that it was not the math that was the problem but rather it was the system, a system that had relied heavily on the notion that in mathematics, students had one of two choices: either sink or swim within a system that had used conventional practices and thinking to produce the current experts in the field.

Changing Trends

Goyder and Miller (2000) acknowledged that perceived changing trends in educational standards would need to be approached with the best evidence obtainable rather than suppositions based on traditional practices. Chan and Wahl's (2013) research identified two notable trends in mathematical learning and teaching that had emerged in the late portion of their study timeline. It was observed that 16 per cent of papers addressed the use of technology in mathematics but interestingly, the papers were clustered in the last two years of the study time period. The other trend that they noted in the latter portion of the study was the increased interest in student recruitment and retention. Both issues appeared to have a direct correlation with the changing standards conversations, so educators and institutions had vested interests in exploring these trends. With larger and more varied classes, post-secondary institutions had a difficult time managing the scale and capacity factors linked to both changing technology and changing student demographics.

Strategic Use of Technology

Institutions were not only struggling with the logistics of the changing composition of their environment, but also with the rationale of using technology in mathematics courses that was being disputed on a multitude of fronts. One post-secondary level instructor stated, “My belief is that calculators are undermining what I’m trying to get through in a subject like calculus” (Hong et al., 2009, p. 883). Another instructor said, “I haven’t personally used a lot of technology apart from calculators, partly because of difficulty in getting time on computers” (Hong et al., 2009, p. 883). Pedagogical, pragmatic, vocational, and instructor abilities were all important considerations that were being examined, while lack of resources, ineffective infrastructure, and difficult access to software were proving to be considerable barriers to be overcome (Demana &Waits, 1990). Authors Bartolini Bussi and Borba (2010) identified that technology had been a theme of considerable debate within the mathematics education community for a long time and that deeper understanding may become clearer when viewed through a historical perspective. They suggested that the current technologies being used in mathematics were “the heirs of a long tradition of mathematical instruments” (Bartolini Bussi & Borba, 2010, p. 2) that dated back to the ancient abacus and other pre-modern positional systems used in navigation calculations.

The National Council of Teachers of Mathematics (NCTM) were early adopters of the strategic use of technology in teaching and learning in mathematics and formally drew a line in the sand when they championed the position:

> It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students (NCTM, 2017).

According to the NCTM, the strategic use of technology enhanced student interest in mathematics, further developed their understandings of the concepts, and facilitated increased competency and achievement in mathematics.

The NCTM (2017) continued to disseminate the conviction that technology had radically changed two aspects of learning in mathematics. One aspect was that content-specific applications of mathematics supported student exploration that significantly improved the students' abilities to identify mathematical concepts and relationships. The second function was deemed "content-neutral" and included the use of technologies that drastically changed how students communicated and collaborated. Demana and Waits (1990) claimed that there should be a strong expectation that educational institutions mirror the surrounding society with respect to technology usage even though "the technology…does not always meet appreciation from teachers, who probably are far too busy with issues involved in their everyday teaching" (p. 29). Norm-referenced studies in the 1990s, which focused on technology integration, revealed that students who used technology had significantly better results on tasks that required conceptual understanding and problem-solving abilities while no difference could be found with routine calculation skills. These early findings were met with some skepticism but proponents of technology continued to research areas such as mathematical modeling in order to identify conceptions as well as misconceptions of mathematics (Demana & Waits, 1990). The use of technology was becoming a vital research component of mathematical learning and understanding in classrooms at all levels. The results were significant enough that the need for further examination was irrefutable.

The attention on technologies further emphasized the importance of acknowledging the complex relationship between mathematics as a "pure" discipline and mathematics as an active part of real-life experiences. Technology had exploded students' mathematical modeling potential, which had previously been limited to physical representation of mathematical concepts. The educational trend in modeling was compatible with the development of active learning that was supported by educational and pedagogical leaders such as John Dewey (Bartolini Bussi & Borba, 2010). Leaders in the mathematical reform movement were challenging established mathematical practices that included or excluded the use of technology and its learning potential. Pressure was being placed on preparation programs and mathematical departments to model learning environments that reflected a world that was being directed more and more by technology, and yet the reality for many post-secondary courses was that traditional approaches were not only still being used but were being protected (Kajander, 2006).

Instructional Reform

Researchers noted “there was a substantial need of development concerning different ways to train and educate all those who are involved in decisions about and realization of education in mathematics" (Demana & Waits, 1990, p. 31). The explosive momentum to thrive in a technologically driven society posed a myriad of challenges for post-secondary institutions. Institutions recognized that students needed to be exposed to more science, technology, engineering, and mathematics (STEM) activities but there were serious obstacles that would need to be overcome. Personnel at all levels required additional training in order to create explicit and meaningful connections while reform-based pedagogy such as hands-on learning and problem-solving activities was difficult with the realities of large class sizes. Ever-increasing diversity in student population was also making it difficult to implement program changes that involved facilitation rather than direct instruction. Institutions were recognizing that "evidence existed regarding the limitations of traditional mathematics teaching practices at the post-secondary level," (Kajander, 2006, p. 234) but the reality of being able to transform towards new directions of constructivist pedagogy would require considerable determination and resources.

Visionaries like Coombs (1968) asserted that while raising standards of excellence would take on many different forms among institutions and nations, the common criteria for excellence would need to include an education that would "fit the real needs and values, currently and prospectively, of a given country" (p. 106). An incipient conceptual shift that practices of teaching were different from practices for learning was beginning to form. The idea was emerging that knowledge would be used to foster the ability of students to discover, develop, and reconstruct knowledge for themselves in a way that would make sense in their lives. This understanding was reshaping the professional challenge for instructors based on the changing nature of higher education as well as the changing role of the teacher within the higher education framework (Freeman et al., 2014; Light et al., 2009). The academic relationship between knowledge, student, and teacher were converging and diverging in different patterns and those relationships appeared to have different purposes.

Learning was no longer considered just in terms of understanding particular disciplines or areas of knowledge but was becoming a way to address and engage the wider multiple discourses of the "life-world." Researchers such as Prendergast et al. (2016) were asserting that while post-secondary should not be a mere extension of secondary level education, there was a need for instructors to be at least aware of what was happening in each other's sectors. Institutions at all levels, including post-secondary mathematics departments, were gaining understanding that they were being faced with the complexity of integrating real-life transferable skills, with "apparent limitless potential for change" (Light et al., 2009, p. 55), with strategies that would effectively facilitate, assist, and support that type of learning. The hypothesis was taking hold that a student who takes a deep or transformational approach to learning, one that emphasizes meaning making and has a connection to personal experiences, should be better able to transform their learning in meaningful ways. Educational giants like Kolb and Dewey had made compelling arguments that the experiential learning cycle was necessary in order to close the gap between understanding theory and having the ability to practice effectively (Light et al., 2009). Early research at the university level was also indicating that students in classes with traditional lecturing were more likely to be unsuccessful than in classes that used a more active learning approach (Freeman et al., 2014). If universities, departments, and programs wanted to prove that they were "excellent," then they would need to demonstrate that the students they were producing were also "excellent" beyond the walls of the classroom and beyond the scope of the institution. The need to quantify evidence of learning was no longer just a matter of recording grades. Experts were beginning to question the long-held belief that teachers were the ultimate determiners of student learning

(Goyder & Miller, 2000). For better or worse the shift in focus and in a sense empowerment, was clearing a path for a new way of thinking that students needed to see themselves as independent learners and ultimately to be in charge of their own education. Some experts were casting a caveat - if students could not apply what they had been learning, then had true learning even occurred?

Emerging Themes

Educators, schools, and other institutions were beginning to understand that a professional paradigm shift was emerging connected to teaching and learning in mathematics (Fenwick-Sehl et al., 2009). Engagement, problem solving, communication, and critical thinking were being recognized as integral parts of the reform movement in mathematics education Project-based learning, design thinking management, and other active learning models accentuated independence and the ability of an individual and a group to analyze context in a different way(Bain, 2004; Freeman et al., 2014). The new perspective of engaging in mathematics allowed for development of teamwork and group skills as well as giving students exposure to authentic inquiry opportunities. Bressoud, Friedlander, & Levermore (2017), Muller (2009), Korey (2000) and other researchers were recognizing that there was a capacity need for students to be able to communicate, to visualize, and to make decisions in a change-driven society. This realization was a catalyst for many institutions to begin building different kinds of experiences. The practice of bringing real-life experiences into the mathematics classroom was also appealing as it recognized that students had a broader range of capabilities than what may be displayable in a more traditional math classroom. In a study at Dartmouth College, it was found that "offering new mathematics in different pedagogical format attracted both the mathematically sophisticated and the mathematically timid" (Korey, 2000, p. 1). The researchers found that despite the challenges of a less structured learning environment, these types of courses brought fresh insights and created new interest that broadened understanding of mathematics as a discipline as well as its role in other areas.

Restructuring Understanding

The belief that students should be actively working to develop deeper conceptual understanding in practical applications was embraced at the primary and secondary levels, but taking the model to the post-secondary level had proven to be a bigger challenge (Kajander, 2006). In Hong et al.'s (2009) study, one instructor spoke out against the more practical context of mathematics:

> …students who are less capable struggle to understand math unless they can fit into a practical situation then everything we do needs to have a direct link to a practical situation, and that isn't mathematics in my view…If we keep going this way…then we'll actually lose what mathematics offers, because it will become so simplistic…if we keep going this way then we're getting out of theoretical mathematics. (p. 880)

An emphasis on practical applications shifted the instruction of mathematics away from the exploratory phases of mathematics learning, including the examination of underlying mathematical theories. The perception that some realistic mathematical applications may lead to the over simplification or under appreciation of theoretical mathematics appeared to contradict what the transformation movement suggested was one of the central tenets of mathematical understanding. Reformists were suggesting that theoretical math, without context, had limited use, but pure mathematicians believed in the inherent value of the theories. The gap between theory and practice was not a new one but for mathematicians such as Devlin (2008), the belief that a new form of reasoning, one that supported a bottom-up approach of making human experiences explainable by math rather than a top-down approach of using math to explain human experiences, was the future of mathematical understanding. Devlin and others believed that "shocking as it may be to a conservative logician, the day will come when currently vague concepts … will be allotted the equal status they deserve, side-by-side with axioms and theorems" (Devlin, 2008, p. 312). Restructuring an understanding of what mathematics really was meant that theory-based courses would need to include the ability to respond to the increasingly complex world beyond that of pure academics, a world that relied more and more on big data, analytics, problem-solving skills, and informed citizenry (Clark & Lovric, 2008).

Higher educational institutions were feeling increased pressure by outside forces to modernize undergraduate programs and Holm (2016) identified that a concerted effort to drive "constructive change in education in the mathematical sciences" (p. 363) was beginning to take hold. In the project report, *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*, researchers named areas that required concerted, coordinated efforts in order to improve the prospects of mathematical departments' relevance leading into the 21st century (Holm, 2016). Curriculum, course structure, workforce preparation, and faculty development were recognized focal areas of importance for the mathematical science community as a whole, at all levels including higher educational environments. The field of mathematical study was being claimed as a foundational science, touted as having a critical role in the substantial advancements being made in medicine, technology, bioscience, chemistry, computer science, social sciences, and beyond. Mathematics departments were being encouraged to expand partnerships beyond traditional lines into disciplines not necessarily considered purely mathematical (Bressoud et al., 2017; Korey, 2000; Schmidt, Hardinge, & Rokutani, 2012).

Changing Perceptions

As opinions of what constituted the definition of "mathematics" shifted, the change in perceptions drew attention for a need to rethink the role of both investment and identity when examining mathematically connected programs at the university level. Black & Hernandez-Martinez (2016) suggested that when a student accessed science or math as a means to accumulate qualification for its own sake, the learning was simply a form of exchange currency. Recognizing that mathematical understanding had value beyond just exchange capital transformed mathematics as a way to engage with not only science and mathematics but as capital that could be used for many other purposes. Black & Hernandez-Martinez (2016) highlighted that practices that focused on "teaching to the test" (exchange value), which they presented as still dominant in too many schools and universities, could actually have a negative influence on students' opinions of mathematics which may alienate them from future careers both within STEM areas and outside STEM areas. Establishing a new understanding of what mathematics was, beyond the pure base, could make mathematics accessible to a much wider segment of disciplines and fields (Korey, 2000). If mathematical understanding was a true gateway to other discipline areas, then mathematicians and institutions would have to define mathematics as more than disconnected theorems. Black & Hernandez-Martinez (2016) and other reformers were suggesting that mathematical theory should not be limited to a currency of exchange, but rather math should be approached as a practice that would then become a part of the learners' identity, and that identity could be the ticket to all of the science, technology, engineering and mathematics environments and then some.

Environmental Impact

In order to thrive in STEM-enriched environments however, post-secondary institutions recognized the need to address the nature of mathematics as well as what was happening with programs that existed the mathematics departments. Clark and Lovric (2008) emphasized the need to develop and to promote reform in upper-division mathematics programs in response to evolving career opportunities and technological developments, but those were not the only driving forces for change. Institutions were realizing the potential of learning mathematics across other curricula as a way to provide meaningful and relevant connections to otherwise seemingly disconnected events (Korey, 2000). Many educational and non-educational sectors recognized that it was time for broad-sweeping reform in mathematics teaching and learning. The real challenge however came from providing useable frameworks for the implementation of effective changes (Clark & Lovric, 2008). Programs were needed that would provide a sustained proficiency in mathematical teaching and learning as a way to better align coursework with areas of study beyond pure mathematics or statistical analysis. Courses that would provide undergraduate students with much broader training opportunities in mathematics as a way to combining quantitative literacy with analytical skills, irrespective of the student's major, were being seen as a way to entice more students into mathematics classes and other complementary areas of study (Fenwick-Sehl et al., 2009).

The ongoing conversations about how to best prepare students for real-world careers had created opportunities for departments and institutions to examine a new culture of classroom innovation and practice. Holm (2016) and others saw that the moment was at hand to seize a "once-in-a-generation" opportunity to engage powerful collective action towards transforming post-secondary mathematical sciences. Departments needed to be able to explore the potential of engaging learners in meaningful ways (Fenwick-Sehl et al., 2009). More and more of the discussions were focused not just on the need to redesign mathematical classrooms but to actually re-evaluate the importance of student-centered pedagogy and active-learning strategies when creating those new environments. The underlying belief that if students had a strong faith in their own abilities in mathematics then they would be more likely to pursue further studies in other areas gave credence to the philosophical shift towards student-centered pedagogy (Bain, 2004).

In 2013, a group of science and mathematics university instructors' meta-analyzed 225 studies that compared student performance in undergraduate STEM courses (Freeman et al., 2013). The study specifically compared traditional lecturing versus active learning environments. The overall data determined that the improvement experienced by students in the active learning environment was so statistically significant that the authors proposed that if the same results were obtained in a medically based experiment, the experiment would be stopped for benefit. The researchers were declaring that to continue to use lecturing as a control group would be so non-beneficial to students that it would be wrong ethically for them to continue teaching using that method. This finding reinforced what other researchers had been identifying - that teaching needed to be based on evidence rather than tradition (Ellett et al., 2012). The results of the analysis by Freeman et al. clearly indicated that the traditionally accepted lecture style approach was not as effective as lecturers may have imagined. The meta-analysis suggested that a change in practice could have a significant impact on the "pipeline problem" that low grades and high failure rates were having in other science and technology fields of study as well. A new focus was evolving across the higher education landscape and it was a learner-focused environment.

Challenging Demands

There continued to be a number of obstacles, however, confronting learning and teaching in higher education. Increasing numbers and diversity of students, tougher requirements for professional accountability and educational relevance, continued strains on resources, and the changing environment of the global market were all factors that affected decision making at the post-secondary level (Light et al., 2009). The rapidly changing higher education sector made it even more difficult to identify a comprehensive process to approach the challenges faced by institutions and in particular the challenges faced by the teaching faculty. The demands on faculty time and the complexity of those demands had grown, while at the same time there appeared to be mounting criticism of established practices (Anderson, 2016). Addressing the challenges of teaching and learning in mathematics at the post-secondary level would open the door for institutions to support their instructors to be more than just lecturers, but that support would need to be backed up with resources that would inevitably have a cost-benefit analysis attached to them (Bain, 2004).

Bain (2004) surmised that teaching was “anything [that] we might do that helps and encourages students to learn” and that this “demands a fundamental conceptual shift in what we mean by teaching,” (p. 173) but the message was not always received at the strategic level. The additional challenge confronting reformists then was that there was a traditional supposition at the higher institution level that the development of teaching was an add-on process with an underlying assumption that teaching was something that an instructor would just "pick up" (Anderson et al., 2016; Light et al., 2009). The researchers noted the irony that “the very teaching and learning challenge which excellence had articulated had often failed to address the substance and complexity of the challenge itself” (Light et al., 2009, p. 12). It was identified by Hong et al. (2009) as well as Anderson et al. (2016) that while secondary teachers had ample support in professional development to engage with new teaching methods, lecturers at post-secondary institutions were often left to their own devices to find professional development. While tertiary lecturers were perceived to be better prepared in terms of content mastery than their secondary counterparts, they were less well-prepared for the realities facing them in a classroom, and especially in a classroom that should not be focused on non-traditional lecture style formats.

Mathematics instructors were being asked to facilitate reform-based learning using social constructionists' models that included experiential problem solving and real-life exploration. The reality at the university level, however, was that in many mathematics courses, particularly entry-level courses, enrollment numbers were often very high and some instructors were regrettably unprepared for the demands of teaching. Once again technology became a driver for change. Mikropoulos and Natsis (2010) proposed that information and communication technologies were proving to be powerful tools to support learning processes. Higher learning institutions were noticing the contribution of these technologies as a way to manage data and to communicate information. It was recognized that "technologies themselves do not directly cause learning to occur but can afford certain tasks that themselves may result in learning" (Mikropoulos & Natsis, 2010, p. 769). Bartolini Bussi & Borba (2010) and Mikropoulos & Natsis (2010) identified that technologies had the ability to create highly interactive environments that could not only support contextual learning in mathematics but could also support the content-neutral environment that was needed for the shift in the pedagogical approach of constructivism. Technology was giving the institutions the opportunity to expand content knowledge and build real-life connections to learning as well as providing a systematic communication and management system for large class sizes.

Technology was not only changing the face of learning in mathematics; it was changing the needs of entire industries (Kassicieh, 2010). With the inception of Massive Open Online Courses, online degree programs, and other distance learning opportunities, the function of universities and traditional learning environments was becoming contentious. These "newer" educational environments were driven by the realities of globalization and economies that were no longer isolated by geography. Institutions had to consider the increasingly complex expectations that they faced due to claims by some partners that “the current structures and practices [were] no longer appropriate for the new, rapidly changing higher education environment” (Austin & Jones, 2016, p. 63). Technology, as an unsuspecting catalyst for significant change, was highlighting the limits of the traditional mathematical curricula not only in terms of content but also in terms of delivery.

The final stages of a profound evolution enabled by technology learning environments may be the rapid growth and emergence of web-based personalized learning systems (Rae & Samuels, 2011). Perhaps the backbone of the entire paradigm shift from teacher-centered to learner-centered approaches has been the aspiration that instructors would be able to meet the learning needs of each individual student exactly where the learners were at, with content that was engaging and interesting to each individual student (Bain, 2004). Realizing a culture of curious, enthusiastic, and independent math students may embody the essence of learning excellence in any program, the precondition for attaining that level of innovation requires significant efforts of a coalition of participants and stakeholders. For environments such as mathematical education, where uncertainty was certain, the movement from static to dynamic practices was becoming more normative and the pressure to become more relevant prevailed (Moreno-Armella, Hegedus, & Kaput, 2008).

Implications for Future Research

In 2008, Keith Devlin asked the question: "What will count as mathematics in 2100?" From a pragmatic viewpoint, mathematics is developed as the needs of society develop. Arithmetic was developed for commerce and trade, geometry and trigonometry in response to navigation and architecture, and the invention of calculus was motivated in large part as a way to more precisely define planetary movement (Devlin, 2008). So when we look ahead, we need to ask the question: what will society need of mathematics in the future? Some suggest that while the fundamental nature of mathematics will remain unchanged, the way we approach and use mathematics will look very different. Muller (2009) and other leading experts in the teaching and learning of mathematics seem to agree. While future mathematics will continue to require the rigour of arithmetic, logic, and measurement, the key difference will be that it will all depend on analysis of environments that are dynamic and constantly transforming (Moreno-Armella et al., 2008). It is highly probable that the future of what is classified as mathematics will not be defined by a particular organization or group, but rather definition of mathematics will be determined by what society expects from those who act as the mathematicians.

Eight years later, in 2016, Holm described the work of mathematicians and educators in mathematics through the group *Transforming Post-Secondary Education in Mathematics* (TPSE). The manuscript reinforced that the “education landscape has changed dramatically in the last half century (Holm, 2016, p. 363)” and that the new pedagogical and technological applications had made it possible to not only reach students in more ways, but these advancements had also equipped student learning in ways that previously were not even a consideration. Regardless of advancement in practice and mobility, however, two quintessential questions keep arising connected to mathematical learning: Why are students underperforming as they transition from secondary to post-secondary mathematics, and exactly what mathematics should students be learning and why? Accepting that both *the what* and *the how* of mathematics will have significant impacts on the teaching and learning practices of our educational institutions is fundamental to understanding the role that mathematics will play in the very nature and structure of our intellectual development (Holm, 2016). Stakeholders at all levels will need to continue to build capacities that will not only close the gap between theoretical and practical mathematics, but all institutions need to ensure that the chasm between secondary and post-secondary learning does not continue to swell. Changes in the culture of mathematical and science education have been evolving at the primary and secondary levels for more than two decades but changes in the culture have only just become a priority for research universities as the importance of learning connected to the “rapidly changing, technologically dependent world" (Anderson et al., 2016, p. 152) becomes more apparent. The repercussions of resistance to transformational change, at any level, are significant and the mandate to gain skills and competencies in a technologically mobilized world are becoming even more apparent and ominous.

The implications for further research are multi-faceted. From a learning perspective, it is important for departments and institutions to consider forms of pedagogy that will best support the development of critical thinking and problem-solving skills in order to maximize concept attainment and maintain rigorous academic standards. Aligning curriculum and supports, while acknowledging the need for better engagement and articulation of diversified understanding, means that more work needs to be done to understand how to enable learners whose interests may not be purely mathematical. Departmental and university cultures, however, do not always reward effective pedagogy (Anderson et al., 2011) as faculty often receive accolades for contributions to research to a greater extent than for the quality of their teaching. From the economic perspective, educational institutions need to critically collect and assess data in order to best utilize and develop future competencies that advocate for innovation in both research and instruction (Ellett et al., 2012). With global economies expanding, universities are under increasing external pressure to create cultures where mathematics departments can continue to generate new knowledge while effectively educating students for the world beyond the walls of a classroom.

The educational perspectives need to further include whether or not the constructionist forms of pedagogies are most appropriate to support the development of mathematical skills that will support both research and learning at the post-secondary level. Norman et al. (2011) identified that while students who experienced a constructionist form of learning style at the secondary level were at a disadvantage at the post-secondary level due to less content alignment, there was no data to explore how those students would perform if given similar constructionist environments at the post-secondary level. The researchers further suggested that the contrast in teaching style, as opposed to the actual content material itself, might be the significant factor in performance outcomes of first year post-secondary students. Anderson et al. (2011) acknowledged that “no scientist would engage in research without exploring previous work in the field, yet few university educators read education research" (p. 152). The issues raised by the lack of research at the post-secondary level are significant. Higher education institutions recognize that a critical ingredient in creating an effective learning environment is to ensure that instructors are teaching based on research that supports best practice. But what happens when there is a gap in the research? If the disconnect cannot be attributed to *the what* and there is no data to support what happens if *the how* is different, then the implications for future research are significant.

In 1968, Coombs already knew that in order for education in all of its capacities to be viable and innovative, two things would need to happen. One was that there would need to be a systematic plan in place to ensure that practices would be based on sound research. The second requirement for successful progress is founded on the necessity of departments to not exist in independent learning silos. Mathematicians and mathematical educators today have echoed Coombs's sentiments, recognizing the importance of coordinated and constructive change in order to build on the successes of the entire mathematical science community (Holm, 2016). In the world of driving change in mathematical education, shifts in pedagogical understanding and technology have been game changers but how those practices and technologies are allowed to co-evolve will be determined by the express demands of society (Moreno-Armella et al., 2008). Teachers and learners in mathematics need to prepare themselves that "sometime, in a future that is knocking at our door, we shall have to retrain ourselves or our children properly to tell the truth. The exercise will be particularly painful in mathematics"(Devlin, 2008, p. 312), unless mathematics retrains itself first.

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