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Building Prime Towers to Understand Prime Number

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A teaching experiment investigating the use of a manipulative to support student understanding of prime and composite numbers.

Introduction

There has been a fair amount of research over the past several decades on teachers' understanding of the multiplicative structure of integers. Yet, very little research has examined children's understanding of this mathematical idea. In this quasi-experimental study, we focus on the effects of the use of a manipulative, the prime towers, in a three-day teaching experiment carried out in fourth grade classrooms. Students "build" towers of blocks that represent each number 2-100 as a product of prime factors. Towers are studied, compared, and contrasted to build understanding of the significance of prime factorization in predicting a number's multiplicative structure.

Standards

Find all factor pairs for a whole number in the range 1– 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1– 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1– 100 is prime or composite. (Common Core Standard 4.OA.4)

Core Questions

1. To investigate the procedural learning gains associated with a three-day educational lesson incorporating the prime towers manipulative in a 4th grade educational setting.
2. To investigate the conceptual learning gains associated with a three-day educational lesson incorporating the prime towers manipulative in a 4th grade educational setting.
3. To identify and refine mathematical tasks and instructional practices that may promote procedural and conceptual understanding of the Fundamental Theorem of Arithmetic and its application to factors among 4th grade elementary school students.

Literature Review

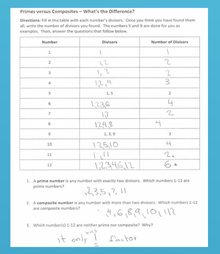
Research has shown that many preservice elementary school teachers have difficulty applying a number's unique prime decomposition to the understanding of "factors" (i.e. Zazkis & Campbell, 1996a; Zazkis & Campbell, 1996b; Zazkis, 1999; Brown, Thomas & Tolias, 2002). Poor understanding of the Fundamental Theorem of Arithmetic has been demonstrated among preservice teachers, but no research has investigated the understandings of elementary school students.

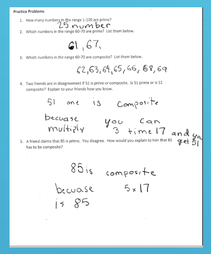
Methods

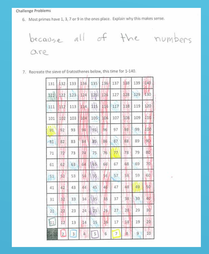
- Pretest students understanding of prime numbers, prime factorization, and unique factorization
- Three days of instruction
- Posttest students understanding of prime numbers, prime factorization, and unique factorization

Day One: What Are Prime Numbers?

- Explore the difference between prime and composite numbers
- Identify primes and composite numbers 1-100 using the sieve of Eratosthenes

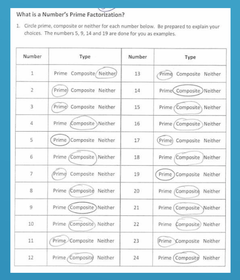


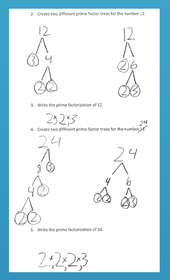


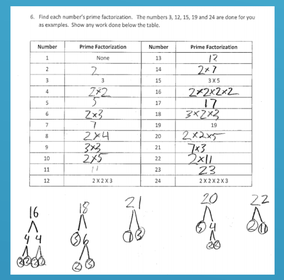


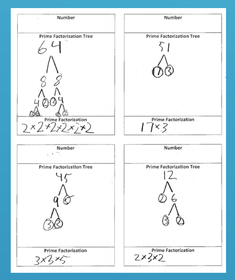
Day Two: What is a Number's Prime Factorization?

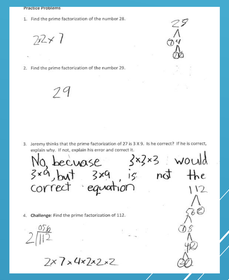
- Construct the prime factorization for any number 2-100





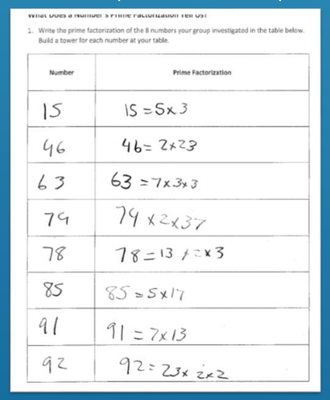


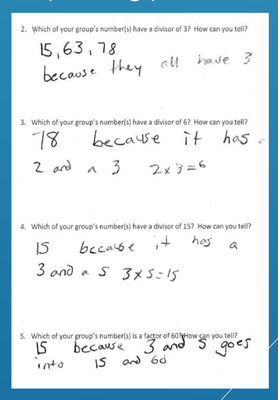


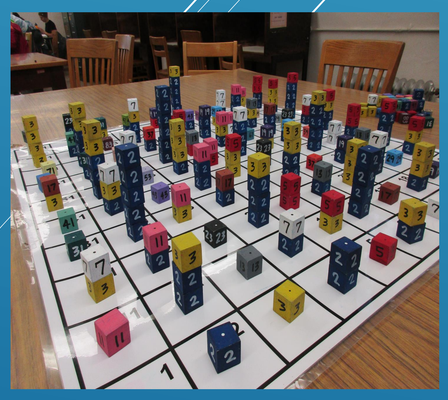


Day Three: What Does a Number's Prime Factorization Tell Us?

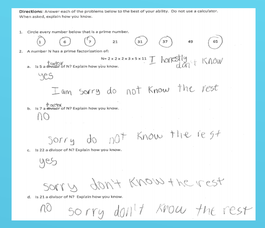
- Explore the difference between prime and composite factors
- Identify factors and multiples using prime factorizations
- Identify common factors of two numbers using prime factorizations
- Identify factor/multiple relationships using prime factorizations

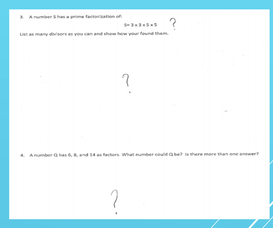


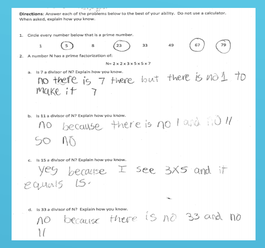


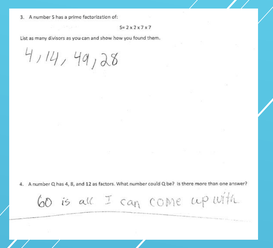


Pretest and Posttest

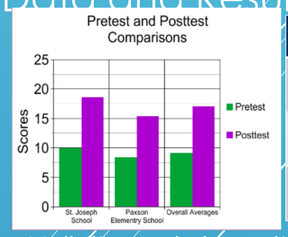








Data and Results



	Mean	SD
Pretest	**3.43**	**0.8386**
Posttest	**16.15**	**4.0349**
Pre-post	**7.36**	**4.1590**

- Validity analysis resulted in 87.5 % agreement
- Students gained an average of 7.36 points between the pre and post tests, a statistically significant result ($p<10^{-25}$). An effect size of $d=1.77$ is associated with the result.
- We saw a 1.18 point gain in understanding of prime and composite numbers (Q1).
- There was a gain of 6.63 points in the understanding of prime factorization, factors and multiples (Q2-6).

Conclusion

- Students clearly showed a procedural understanding of the material represented though the assessment and data presented above.
- Conceptual understanding was represented though students ability to explain answers. It was clear that this was still emerging and fragile.
- Some elements worked better than others. We found that using a small subset of towers to compare was an incredibly successful moment.

Future Research

Question number seven on the pre and post tests where not used in this data as the results were inconclusive. More research should be done to determine how the manipulative can support reversibility of "finding factors given a number".

Students' understanding of the role of prime factorization was very evidently still fragile. Future research should investigate how the manipulative can support the solidification of students' emerging

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