

University of Montana
ScholarWorks at University of Montana

University of Montana Conference on
Undergraduate Research (UMCUR)

2018 University of Montana Conference on
Undergraduate Research

Apr 27th, 11:00 AM - 12:00 PM

Building Prime Towers to Understand Prime Number

Alexis J. Feffer

University of Montana, alexisjulianna22@gmail.com

Let us know how access to this document benefits you.

Follow this and additional works at: <https://scholarworks.umt.edu/umcur>

Feffer, Alexis J, "Building Prime Towers to Understand Prime Number" (2018). *University of Montana Conference on Undergraduate Research (UMCUR)*. 1.

<https://scholarworks.umt.edu/umcur/2018/amposters/1>

This Poster is brought to you for free and open access by ScholarWorks at University of Montana. It has been accepted for inclusion in University of Montana Conference on Undergraduate Research (UMCUR) by an authorized administrator of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

Building Prime Towers to Understand Prime Number

A teaching experiment investigating the use of a manipulative to support student understanding of prime and composite numbers.

Introduction

There has been a fair amount of research over the past several decades on teachers' understanding of the multiplicative structure of integers. Yet, very little research has examined children's understanding of this mathematical idea. In this quasi-experimental study, we focus on the effects of the use of a manipulative, the prime towers, in a three-day teaching experiment carried out in fourth grade classrooms. Students "build" towers of blocks that represent each number 2-100 as a product of prime factors. Towers are studied, compared, and contrasted to build understanding of the significance of prime factorization in predicting a number's multiplicative structure.

Standards

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (Common Core Standard 4.OA.4)

Core Questions

1. To investigate the procedural learning gains associated with a three-day educational lesson incorporating the prime towers manipulative in a 4th grade educational setting.
2. To investigate the conceptual learning gains associated with a three-day educational lesson incorporating the prime towers manipulative in a 4th grade educational setting.
3. To identify and refine mathematical tasks and instructional practices that may promote procedural and conceptual understanding of the Fundamental Theorem of Arithmetic and its application to factors among 4th grade elementary school students.

Literature Review

Research has shown that many preservice elementary school teachers have difficulty applying a number's unique prime decomposition to the understanding of "factors" (i.e. Zazkis & Campbell, 1996a; Zazkis & Campbell, 1996b; Zazkis, 1999; Brown, Thomas & Tolia, 2002). Poor understanding of the Fundamental Theorem of Arithmetic has been demonstrated among preservice teachers, but no research has investigated the understandings of elementary school students.

Methods

- ❖ Pretest students understanding of prime numbers, prime factorization, and unique factorization
- ❖ Three days of instruction
- ❖ Posttest students understanding of prime numbers, prime factorization, and unique factorization

Day One: What Are Prime Numbers?

- ❖ Explore the difference between prime and composite numbers
- ❖ Identify primes and composite numbers 1-100 using the sieve of Eratosthenes

Prime versus Composite – What's the Difference?

Identify prime and composite numbers. Draw a circle around prime numbers. Write the number of factors you find. The numbers 2 and 1 are there for you as examples. Then answer the questions that follow.

Number	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

Sieve of Eratosthenes

Complete the sieve of Eratosthenes for all the primes < 100. To complete the sieve, identify the smallest prime for the 100 and cross out all its multiples. Repeat the process until you have a prime and all its multiples crossed out in a single column and only prime numbers left. These are the primes < 100.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

Prime Problems

1. How many numbers between 1 and 100 are prime?
25 number

2. What numbers in the range 60-70 are composite? List them below.
61, 67

3. How many prime numbers are there between 10 and 20?
4

4. How many prime numbers are there between 20 and 30?
3

5. How many prime numbers are there between 30 and 40?
4

6. How many prime numbers are there between 40 and 50?
3

7. How many prime numbers are there between 50 and 60?
3

8. How many prime numbers are there between 60 and 70?
3

9. How many prime numbers are there between 70 and 80?
3

10. How many prime numbers are there between 80 and 90?
3

11. How many prime numbers are there between 90 and 100?
3

Composite Problems

1. How many numbers between 1 and 100 are composite?
75 number

2. What numbers in the range 60-70 are prime? List them below.
61, 67

3. How many composite numbers are there between 10 and 20?
16

4. How many composite numbers are there between 20 and 30?
17

5. How many composite numbers are there between 30 and 40?
16

6. How many composite numbers are there between 40 and 50?
17

7. How many composite numbers are there between 50 and 60?
17

8. How many composite numbers are there between 60 and 70?
17

9. How many composite numbers are there between 70 and 80?
17

10. How many composite numbers are there between 80 and 90?
17

11. How many composite numbers are there between 90 and 100?
17

Pretest

1. Circle every number below that is a prime number.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

2. A number 5 has a prime factorization of 5 = 5 x 5 x 5. List as many divisors as you can and show how you found them.

3. A number 5 has a prime factorization of 5 = 5 x 5 x 5. List as many divisors as you can and show how you found them.

Posttest

1. Circle every number below that is a prime number.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

2. A number 5 has a prime factorization of 5 = 5 x 5 x 5. List as many divisors as you can and show how you found them.

3. A number 5 has a prime factorization of 5 = 5 x 5 x 5. List as many divisors as you can and show how you found them.

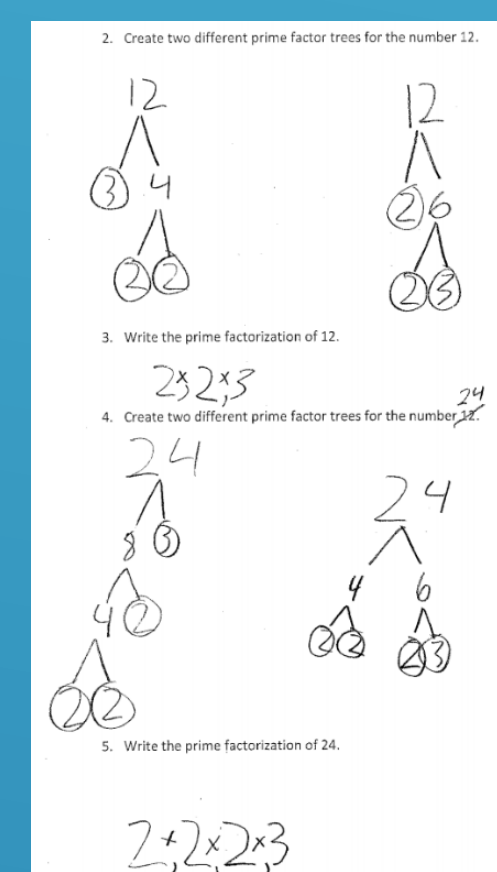
Day Two: What is a Number's Prime Factorization?

- ❖ Construct the prime factorization for any number 2-100

What is a Number's Prime Factorization?

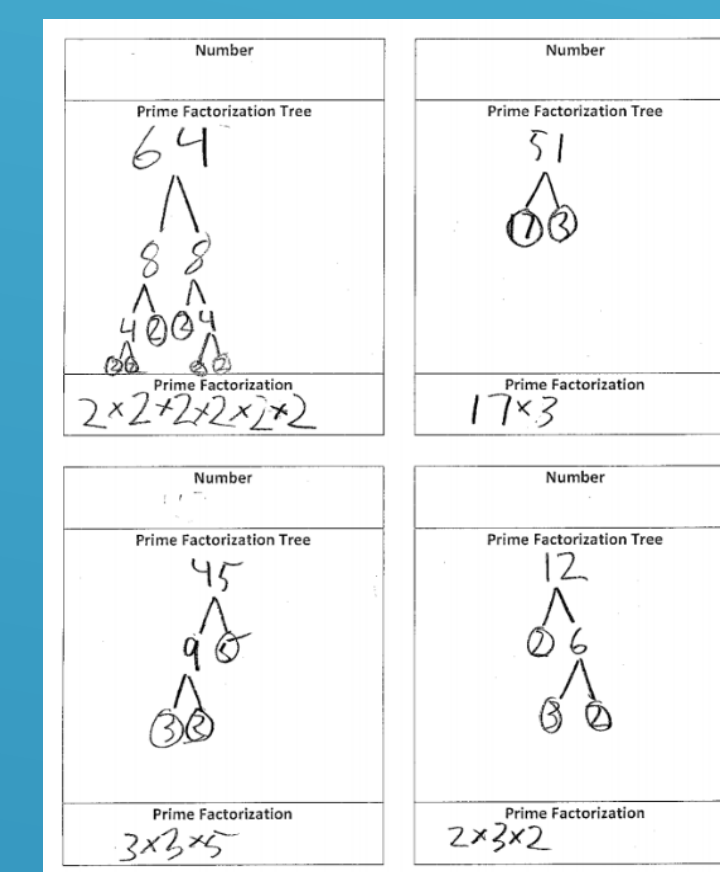
1. Circle prime, composite or neither for each number below. Be prepared to explain your choices. The numbers 2, 3, 24 and 25 are done for you as examples.

Number	Type	Number	Type
1	Prime (Composite/Neither)	33	Prime (Composite/Neither)
2	Prime (Composite/Neither)	34	Prime (Composite/Neither)
3	Prime (Composite/Neither)	35	Prime (Composite/Neither)
4	Prime (Composite/Neither)	36	Prime (Composite/Neither)
5	Prime (Composite/Neither)	37	Prime (Composite/Neither)
6	Prime (Composite/Neither)	38	Prime (Composite/Neither)
7	Prime (Composite/Neither)	39	Prime (Composite/Neither)
8	Prime (Composite/Neither)	40	Prime (Composite/Neither)
9	Prime (Composite/Neither)	41	Prime (Composite/Neither)
10	Prime (Composite/Neither)	42	Prime (Composite/Neither)
11	Prime (Composite/Neither)	43	Prime (Composite/Neither)
12	Prime (Composite/Neither)	44	Prime (Composite/Neither)



6. Find each number's prime factorization. The numbers 3, 12, 15, 18 and 24 are done for you as examples. Show any work done below the table.

Number	Prime Factorization	Number	Prime Factorization
1	None	13	13
2	2	14	2 x 7
3	3	15	3 x 5
4	2 x 2	16	2 x 2 x 2 x 2
5	5	17	17
6	2 x 3	18	2 x 3 x 3
7	7	19	19
8	2 x 2 x 2	20	2 x 2 x 5
9	3 x 3	21	3 x 7
10	2 x 5	22	2 x 11
11	11	23	23
12	2 x 2 x 3	24	2 x 2 x 2 x 3



24. Write the prime factorization of the number 28.

25. Write the prime factorization of the number 25.

26. Write the prime factorization of the number 30.

27. Write the prime factorization of the number 35.

28. Write the prime factorization of the number 40.

29. Write the prime factorization of the number 45.

30. Write the prime factorization of the number 50.

31. Write the prime factorization of the number 55.

32. Write the prime factorization of the number 60.

33. Write the prime factorization of the number 65.

34. Write the prime factorization of the number 70.

35. Write the prime factorization of the number 75.

36. Write the prime factorization of the number 80.

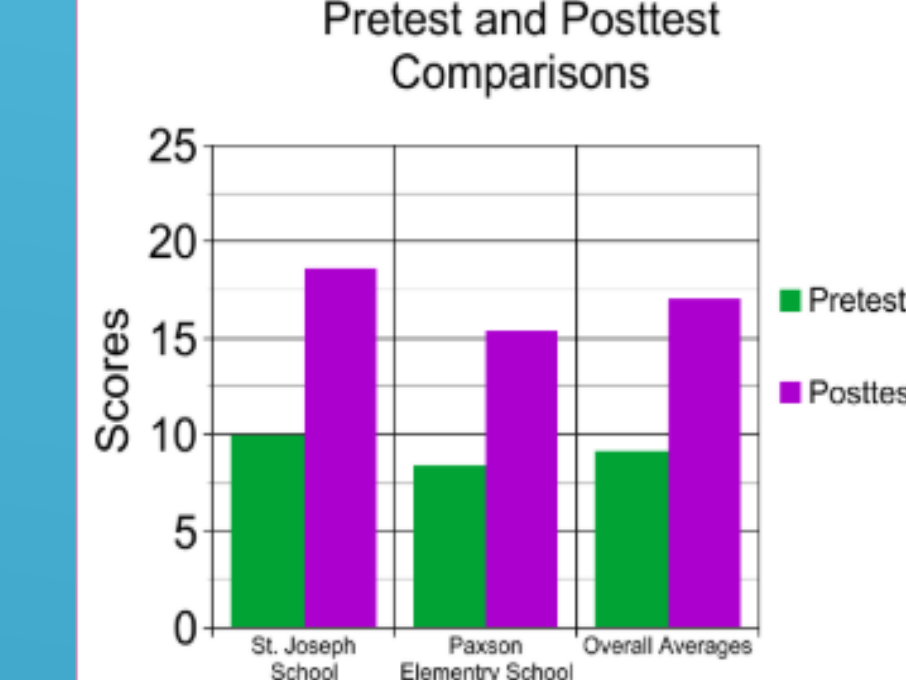
37. Write the prime factorization of the number 85.

38. Write the prime factorization of the number 90.

39. Write the prime factorization of the number 95.

40. Write the prime factorization of the number 100.

Data and Results



	Mean	SD
Pretest	3.43	0.8386
Posttest	16.15	4.0349
Pre-post	7.36	4.1590

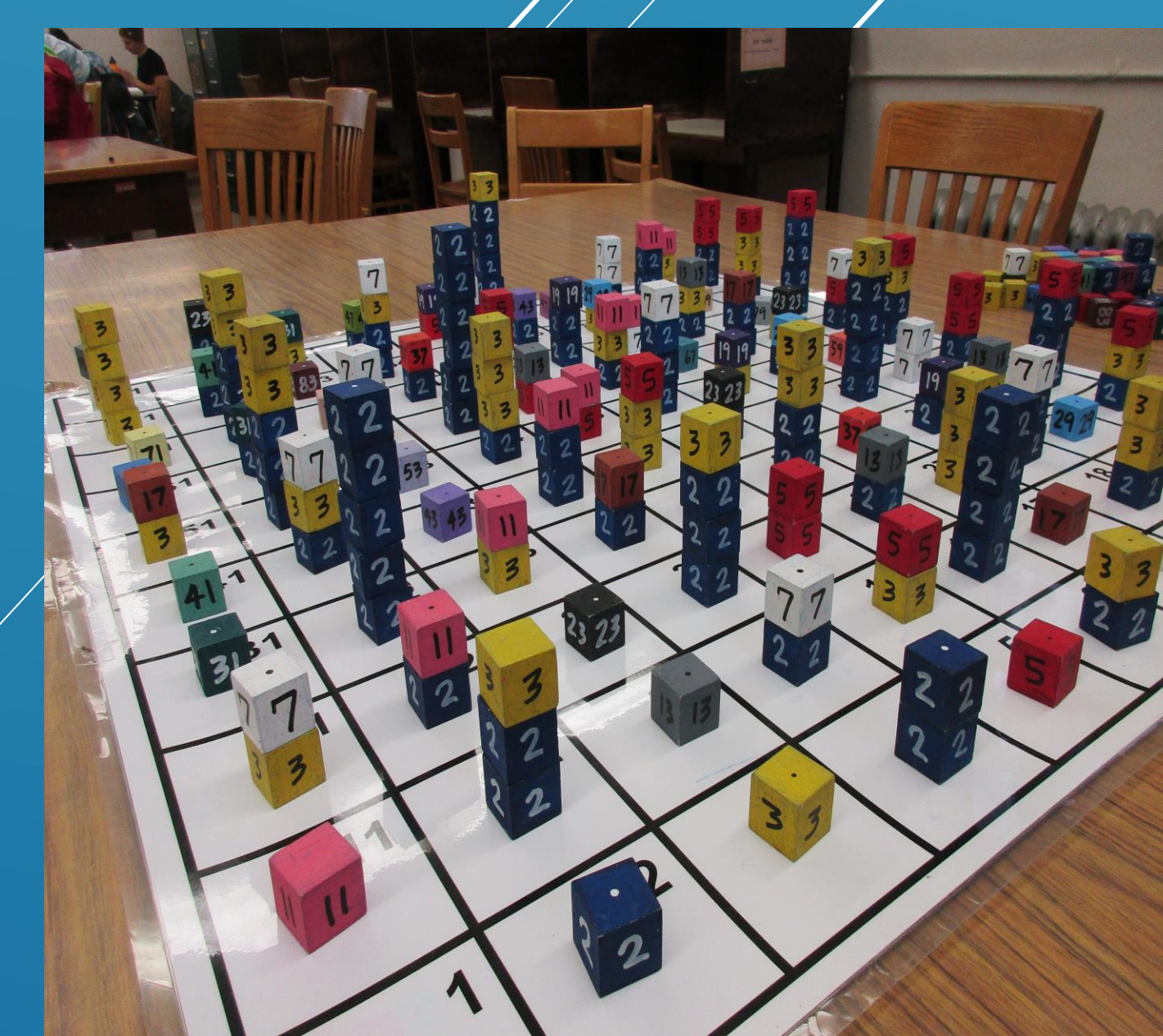
- ❖ Validity analysis resulted in 87.5 % agreement
- ❖ Students gained an average of 7.36 points between the pre and post tests, a statistically significant result ($p < 10^{-25}$). An effect size of $d = 1.77$ is associated with the result.
- ❖ We saw a 1.18 point gain in understanding of prime and composite numbers (Q1).
- ❖ There was a gain of 6.63 points in the understanding of prime factorization, factors and multiples (Q2-6).

Conclusion

- ❖ Students clearly showed a procedural understanding of the material represented though the assessment and data presented above.
- ❖ Conceptual understanding was represented though students ability to explain answers. It was clear that this was still emerging and fragile.
- ❖ Some elements worked better than others. We found that using a small subset of towers to compare was an incredibly successful moment.

Future Research

Question number seven on the pre and post tests where not used in this data as the results were inconclusive. More research should be done to determine how the manipulative can support reversibility of "finding factors given a number".



Alexis Feffer

College of Education and Human sciences

Special Thank You to Matt Rosco, Ph.D., Department of Mathematics Education at the University of Montana for his contributions and guidance throughout this project.

Students' understanding of the role of prime factorization was very evidently still fragile. Future research should investigate how the manipulative can support the solidification of students' emerging