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Financial leverage: Measures and effects

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FINANCIAL LEVERAGE: MEASURES AND EFFECTS

by

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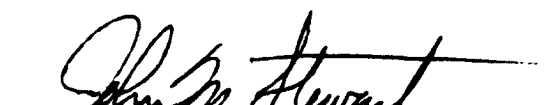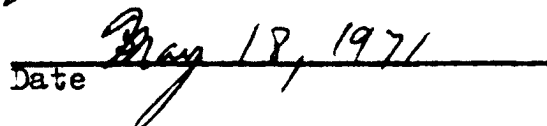Master of Business Administration

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TABLE OF SYMBOLS USED WITHIN PAPER*

BR   – business risk
CV   – coeficient of variation
D   – debt capital
EBIT – earnings before interest and taxes
FR   – financial risk
K_d – cost of debt capital
K_e – cost of equity capital or rate of return on common stock
K_o – weighted average cost of capital
K_t^o – after tax capitalization rate
L   – leverage
NI   – net income
NOI   – net operating income
S   – common stock
T   – tax rate
V   – total capital (V = D + S)
$\bar{X}$ – expected income (expected mean of distribution)
Z_p – random variable from a normal distribution
σ – standard deviation of a distribution

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*Throughout this paper in order to provide some degree of consistency these symbols will be substituted for quoted authors' symbols when they refer to the same variable.

Chapter 1

INTRODUCTION

Financial leverage is an integral part of financial management and has some degree of influence, either directly or indirectly, upon nearly any financial decision facing a firm. Because of this important role, effective mensuration of and accurate effectual relations relevant to financial leverage must be developed.

This paper offers a first step toward these objectives. Chapter 2 deals with different measures that have been suggested for leverage and Chapter 3 concerns itself with theories about the effect of leverage on the cost of capital.

The discussions regarding both leverage measurements and cost of capital theories will be directed toward the objective of this paper: comparison between views.

> A firm may be said to be levered when there are securities of ownership outstanding, which have different priorities of payment, and where some of the promised payments for the use of funds are of limited amount (so that if more than the limited amount is earned the holder of a different type of security benefits).[1]

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[1]Harold Bierman, Jr., *Financial Policy Decisions* (London: The Macmillan Company, 1970), p. 87.

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Or more simply,

> Leverage may be defined as the employment of an asset
> or funds for which the firm pays a fixed cost or
> fixed return.

The case of farmer Jones represents a simple analogy exercising the underlying principles of financial leverage. If farmer Jones needs another horse and can get more work out of his neighbor Joe's horse than Joe can, both of them may be able to benefit. For example, if Joe can only make Trigger plow 50 acres a day and farmer Jones can get him to turn under 60, it would be possible for neighbor Joe to rent Trigger to farmer Jones at a set charge equal to the profits of 55 acres a day, giving both of them an extra 5 acres profit.

From farmer Jones point of view, leverage would be the employment of Trigger for a fixed cost of 55 acres profit a day. As long as Trigger kept up the good work and plowed more than 55 acres a day the leverage would be called positive; below 55, however, it would be negative and farmer Jones had better hope that Joe is a friendly neighbor. The latter situation points out one of the main dangers of using leverage; although there is a chance of earning more through the utilization of leverage there also exists the chance of being worse off if the return on borrowed funds does not exceed the fixed charges for their use.

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[1] James C. VanHorne, **Financial Management and Policy** (Englewood Cliffs: Prentice-Hall Inc., 1968), p. 547

Chapter 2

LEVERAGE MEASUREMENT

Whether or not a firm is leveraged is not the most important aspect of financial leverage, rather it is the effects upon the firm caused by leverage. Before any such effects can be dealt with, however, leverage must be quantified. This is not a simple task as will be made apparent in this chapter.

PROBLEMS AND OBJECTIVES

There have been many different approaches suggested for measuring leverage but none has been proven fully superior and up to the task of providing an adequate measurement of leverage for all situations. Appendix 1 contains a list of 27 formulas which have been used by financial writers to measure leverage. The reason for this diversity is the complexity of the object measured and the lack of a posited definition for it. Since a definition is closely related to the defining equation it might be said that there are at least 27 definitions of leverage. These variations are made possible by numerous factors affecting leverage measurements such as taxes and whether the figures are based on book or market values, as well as the use of

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definitions designed to measure specific aspects of
leverage or viewpoints of the measurer. Such a situation
exists because no single measure of leverage, in a realistic
or practical sense, can measure all aspects associated with
leverage. Still such a definition of leverage, as demonstrated
on pages 1 and 2, is possible if it can be made general enough
to apply to most situations. By their very nature, however,
such definitions prove of little or no use in the detailed
objective world of actual physical mensuration.

As a result what is needed is either, (1) a workable
leverage measure applicable in all situations, which seems
impossible at this time, (2) a proven leverage measure for
each different and definable circumstance demanding one, or
(3) some knowledge of the interrelationships between measures
so that consistencies and inconsistencies can be identified.
The remaining portion of this chapter will discuss suggested
leverage equations and several studies that have been applied
to problems of their usage in an attempt to decide upon the
best means of developing an acceptable and consistent lever-
age measurement system represented by one of the three courses
of action just stated.

STUDIES CONDUCTED & MEASURES SUGGESTED

The most basic measure of financial leverage is a
simple ratio of debt (D) to equity or common stock (S):

$$L_1 = \frac{D}{S}$$

The higher the value of L_1 the higher the degree of leverage within the firm.

Another basic measure of leverage, consistent with L_1, is the ratio of D to total capital (V) where V =D+ S:

$$L_2 = \frac{D}{V} = \frac{D}{D + S}$$

This measure, unlike L_1, will always satisfy the condition: $0 < L_2 < 1$. As a result it is often preferred for reasons of ease of handling. The relationship between these two equations would be:

$$L_2 = \frac{L_1}{1 + L_1}$$

Although L_1 and L_2 are used by many financial writers in many situations they are very basic and fail to include many variables and events associated with the degree of leverage. For this reason numerous other equations for measuring leverage have been suggested and studied in an attempt to develop one more suitable for real world situations.

Pearson Hunt has addressed himself to a portion of this problem by recognizing that there are two different, yet often confused, aspects of leverage.[1] One,

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[1]Pearson Hunt, "A Proposal for Precise Definitions of 'Trading on the Equity' and 'Leverage'," The Journal of Finance, Vol. XIV, No. 3, (September, 1961), pp. 377-386.

termed trading on the equity by Hunt, concerns itself
with the optimum capital structure of a firm through proper
analysis of financial risk.[1] The remaining, called lever-
age, deals with the effects of fixed charge securities on
the earnings of a firm under conditions of a constant
capitalization rate. According to Hunt these are really
different aspects of the same thing, financial risk,
resulting from different viewpoints of the measurer.
Trading on the equity is that seen by the financial manager
as he attempts to determine the correct or optimum pro-
portions of debt and equity in his firm's capital structure
and leverage, as defined above, is that viewed by the
investment analyst as he tries to determine the resulting
earnings produced from the capital structure of a firm
that has already been decided upon. Hunt's equations for
each view are as follows:

$$\text{Trading on the equity} = \frac{EBIT - DK_d}{S}$$ (as seen by
(balance sheet leverage) the fin. manager)

$$\text{leverage} = \frac{EBIT}{EBIT - DK_d}$$ (as seen by
(income statement lev.) the fin. analyst)

Where: EBIT = earnings before interest and taxes
         D = debt capital
         S = common stock outstanding
         K_d = cost of debt capital

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[1]Briefly financial risk is the risk caused by the
presence of fixed charges resulting from the use of debt in
a firm's capital structure. It is discussed in more detail
on page 30.

In a later article commenting on Hunt's efforts, Harold Dilbeck states his general agreement but offers several criticisms and modifications.[1] He finds fault in Hunt's measure of trading on the equity in that it attempts to measure the effect of trading on the equity rather than the act being defined. As a result he feels that trading on the equity should be measured by the proportion of specified charge securities in a firm's capital structure since it is defined in this manner. Dilbeck also criticizes Hunt for not giving a complete analysis of the effects of changes in the cost of debt and equity, both held constant in the latter's trading on the equity measure. Finally he charges Hunt with the failure of stating the assumption that tax effects will only cancel out of the leverage ratio when marginal and average tax rates are equal; an assumption that must be present for his analysis to be complete.

In reply to Dilbeck's first criticism Hunt says,

> I cannot accept Dilbeck's suggestion that trading on equity be measured solely by the "proportion of specific charge securities in a firm's capital structure" . . . debt ratios . . . are inadequate measures of the consequences of using fixed-charge securities, since the quantity of the fixed charges do not enter into the definition.[2]

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[1] Harold Dilbeck, "A Proposal for Precise Definitions of 'Trading on the Equity' and 'Leverage': Comment," The Journal of Finance, Vol. XVII, No. 1, (March, 1962), pp. 127-130.

[2] Hunt, op. cit., p. 131.

Regarding his constant cost of debt and equity Hunt explained that his model was a simple one and, even though not perfectly correct theoretically, it was so in regard to his conclusions. He did, however, recognize his error in not stating the assumption equating marginal with average tax rate but at the same time declared that, "In actual practice, I believe, it would be very rare for the marginal tax rate to change significantly from the average."[1] Nevertheless Dilbeck's criticisms have generally been accepted as valid and Hunt's measures have not acquired widespread approval. Still, this discussion has served to demonstrate that a leverage definition as well as the resulting measurement is often determined in part by the user and his purpose of measuring leverage.

Another writer differentiating between aspects of leverage is Woods who makes a distinction between capital gearing and leverage. "Gearing is a prerequisite for, and in most cases actually results in, leverage; but a geared firm might still experience no leverage at all."[2] Basically this situation can occur due to the definitions given for capital gearing and leverage. For leverage to be present Woods states two requirements:

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[1]Ibid., p. 132.

[2]Ivan R. Woods, "Financial 'Leverage' and 'Gearing' in Perspective," The South African Journal of Economics, Vol. 32, (March, 1964), p. 28.

(1) A source of funds on which there is a fixed or limited charge paid for use.

(2) The borrower must earn a return on total capital not equal to the fixed charges on borrowed funds.

Capital gearing is in reality (1) above, simply the use of fixed charge funds in the financial structure, and therefore if the return on total capital equals the return required on borrowed funds gearing is present but leverage is not.

There are two measures of capital gearing recognized by Woods:

(1) $\frac{\text{prior charge capital}}{\text{total capital}}$

(2) $\frac{\text{annual amount payable in preferred charges}}{\text{expected annual distributable profit}}$

He goes on to show in an example, however, that neither is fully adequate in relation to leverage (i.e., the first can show identical gearing between two firms when they experience different leverage and the second can have the highest leveraged firm obtaining the lowest degree of gearing).

Next Woods compared five measures of leverage in regard to how well they were able to determine the degree of leverage in three hypothetical firms. The five measures were:

(1) The ratio of net rate of return on equity capital to rate of return (before interest, after tax) on

total capital:

$$L_1 = \frac{\text{net equity earnings/equity}}{\text{earnings before interest after tax/total capital}}$$

(2) The ratio of the net rate of return on equity capital to rate of return (before interest after deducting tax of 1-T) on total capital giving the rate of return on equity as if all capital were equity capital:

$$L_2 = \frac{\text{net equity earnings/equity}}{\text{(earnings before interest \& tax)(1-T)/total cap.}}$$

$L_1 = L_2$ where a firm has prior charge capital consisting of preference shares only.

(3) To determine the effects on equity earnings if total capital were increased by the issuance of more prior charge capital, use the ratio of the rate of return on new equity to the rate of return on old equity:

$$L_3 = \frac{\text{new equity earnings/equity}}{\text{old equity earnings/equity}}$$

(4) Ratio of the proportionate change in net equity earnings to the proportionate change in earnings before interest and after tax:

$$L_4 = \frac{\dfrac{\text{change in net equity earnings caused by change in earnings before interest after tax}}{\text{net equity earnings}}}{\dfrac{\text{change in earnings before interest after taxes}}{\text{earnings before interest after taxes}}}$$

Since change in net equity earnings caused by change in earnings before interest after tax equals change in earnings before interest and after tax,

$$L_4 = \frac{\text{earnings before interest after tax}}{\text{net equity earnings}} \quad .$$

(5) Ratio of the proportionate change in earnings after interest before tax to the proportionate change in earnings before both interest and tax:

$$L_5 = \frac{\dfrac{\text{change in earnings after interest before tax caused by change in earnings before interest \& tax}}{\text{earnings after interest before tax}}}{\dfrac{\text{change in earnings before interest \& taxes}}{\text{earnings before interest and tax}}}$$

Since the change in earnings after interest and before tax, caused by the change in earnings before interest and tax, equals the change in earnings before interest and tax if the tax rate is proportional,

$$L_5 = \frac{\text{earnings before interest and taxes}}{\text{earnings after interest before tax}}.$$

If the tax rate changes with income this simplification will only hold if there is no preference capital.

Woods in deciding upon the most effective measure of leverage made a distinction similar to that of Hunt. That is L_2 and L_3 would be favored by the management of a firm since they measure the effect of changes in the capital structure upon the rate of return on equity capital. On the other hand, an investor or financial analyst would prefer L_1, L_4 or L_5 in that the capital structure is fixed. Thus according to both Hunt and Woods the appropriate leverage measure is decided in part by the viewpoint of its user.

Each of the five measures provided the same ranking of firms according to leverage for the three hypothetical firms with the exception of L_3 concerned mainly with the effects of new equity earnings rather than"normal leverage". Therefore as far as ranking of firms according to degree of leverage, measures L_1 , L_2, L_4 and L_5 would serve equally well.

"Factors which affect the degree of leverage include debt-equity ratios, net earnings for capitalization, interest charges and allowable deductions for tax purposes."[1] Walters manages to get all of these variables into his measure of leverage represented by the equation:

$$L = 1 + \frac{equity}{debt} \left(\frac{1 - (1 - tax\ rate)\ (debt\ charges)}{e_1} \right)$$

Where e_1 = pro rata share of net earnings before interest assignable to the debt component of the capital structure

Using this measure the tax rate can or cannot have a large influence upon L, depending upon e_1. If the share of earnings available to the debt component is a great deal larger than debt charges the tax rate will have little effect.

In the case of measuring the effect of new financing

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[1]James E. Walter, "The Use of Borrowed Funds," The Journal of Finance, Vol. XXVIII, No. 2, (April, 1955), pp. 140.

on L the equation becomes:

$$L = 1 + \frac{(1 - \text{tax rate})(e_2 - K_{d2})}{e}$$

where e = net earnings available for equity
prior to new financing

e_2 = additional charges before interest
and taxes after new financing

K_{d_2} = additional interest charges with new
financing

Walter ends by making a distinction between the measurement of leverage and that leverage which is correct for an individual firm. Even though the equations may give an accurate measure of L, "Whether the degree of leverage (L) exhibited by a given company is reasonable or unreasonable is, within wide limits, largely a matter of individual judgement."[1] Or in other words the degree of leverage will be determined by a management decision deciding upon the desired trade-off between risk and profit.

(1) Stock or balance sheet, (2) flow or income, and (3) conjoint or rate of return are three categories into which Ghandhi classifies all measures of leverage.[2] These three categories encompass most of the previous measurements presented by Hunt, Woods and Walter.

The first class, including measures based on both

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[1] Ibid., p. 145.

[2] J. K. S. Ghandhi, "On the Measurement of Leverage," The Journal of Finance, Vol. XXI, No. 4, (December, 1966), pp. 715-726.

book and market values, acquires its figures either directly from the balance sheet or from recorded values determined in the market place. Measures in this category are characterized by a fixed capital structure and consist mainly of simple debt to equity ratios.

Flow or income measures, making up the second class, recognize the fact that debt leverage imposes prior claims on income and strive to measure the ability of a firm's income flow to meet these obligations. Thus the typical measure is the ratio of debt charges (bond interest, preferred dividends, etc.) to the total income or excepted income before such charges or taxes are subtracted. Hunt's leverage measure, as well as Dilbeck's modifications of it, are in this category.

The third group, which would include Walter's measures and most of Woods' encompasses all conjoint or rate of return measures. Measurements of this type consist mainly of the ratio of rate of return on equity (income - debt charges/equity capital) to rate of return on total capitalization (income/equity capital plus all debt capital). By considering different implications caused by tax laws several variations of this theme are possible.

Ghandhi narrowed down from 13 to 9 the number of leverage measures used in his test by eliminating those that by their very nature would provide consistent results with another measure already represented. Following this

the nine separate leverage measurements were used to deter-
mine the degree of leverage in two actual industries; (1)
light engineering, machine tools, etc. (fairly stable) and
(2) beers, wines, etc. (complex and unstable). By performing
a Spearman's coeficients of rank correlation test among the
results of each measure within the two industry groups
Ghandhi found a significant correlation between nearly all
measured results in both cases, although those correlations
for the stable industry were somewhat higher. From these
results it is concluded,

> . . . that in most "normal" cases there is a suf-
> ficient degree of stability in the critical variables
> so as not to cause substantial and significant incon-
> sistencies among the various measures of leverage . . .
> Thus in practice in circumstances other than the most
> extreme it would appear to be a matter of relative
> indifference which measure were adopted.[1]

Even if this conclusion can be accepted the measure-
ment of leverage is far from being home free. The reason
for this is the many variations of figures available to plug
into leverage equations and the lack of any real agreement
upon which one is the most appropriate. Several of these
decisions areas follow:

(1) One of them is in relation to what values are
to be used to represent debt and equity. Such elements
can be portrayed by book value, market value or flow value.

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[1]*Ibid.*, pp. 724-725.

Book value figures come right out of accounting reports and resultingly are subject to the effects of various accounting methods such as the practice of conservatism which may result in valuations much different from what many would feel is the real worth. On the other hand book values are objective and most likely to be used by practical men.[1]

Market values are just what the name implies and are obtained from the public's opinion, via market activities, of elements related to the firm. Usually, due to variations, these figures are represented by a calculated average of several values recorded over a certain time period. Market values have the advantages of giving some indication of the future value of elements as seen through the eyes of the general public and of eliminating problems of conservative evaluation present with book value. Still market values are not always as readily obtainable as book values and for some figures impossible Also, as will be recognized in the next chapter, the use of market value debt-equity ratios may be inappropriate as a leverage measurement relating to financial risk due to the possible introduction of a bias.

Flow measures refer to either the cash or income

[1]Bierman, Financial Policy Decisions, p. 90.

flows of a firm over a period of time and hence are classi-
fied as dynamic rather than static. A flow measure of
leverage compares that portion of income or cash flow
relating to debt with that associated with equity. Such
measures are valuable in that they do not require, as is
the case with book and market values, a rate of interest
in computing the present value of debt or equity. Flow
measures, however, provide a much better measure of the
ability of a firm to meet fixed debt obligations than they
do of other aspects of leverage. As a result they are
used for this purpose much more than they are as determiners
of the overall leverage in a firm.

(2) The classification of preferred stock as debt
rather than equity and its effect on leverage needs some
discussion. How preferred stock effects leverage is deter-
mined by whether or not it is cummulative. If it is non-
cummulative and dividends are not issued in year one,
leverage in year two will be increased ceteris paribus
since the debt from preferred in year one has been free.[1]
Preferred stock can also affect leverage comparisons
between firms because of tax laws. Preferred dividends
are not tax deductable while ordinary debt interest pay-
ments are. Therefore if two firms have the same expected

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[1]Woods, "Financial 'Leverage' and 'Gearing' in
Perspective," p. 28.

income ($\bar{X}$), and total debt (D), consisting of preferred stock and bonds, their actual leverage may be different. This will be the case whenever the percentage of preferred stock in D differs from one firm to the next.[1]

For purposes of measuring leverage, preferred stock can be classified as debt, but if leverage is to be used as an indicator of financial risk it cannot since ordinary debt and preferred stock represent different risks in that one always has to be paid for and the other doesn't under certain conditions.[2]

(3) Another decision area is whether or not to include short-term loans in the total debt when measuring leverage; the trend has been not to. "One of the most widely used classical definitions of leverage is long-term debt divided by long-term debt plus stockholders' equity."[3] According to Woods many feel that the use of only long-term debt is adequate since short-term debt often has no interest charges, fluctuates widely over time and represents such a small portion of all debt. He favors inclusion of short term debt and discards such arguments because they imply

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[1]Ghandhi, "On the Measurement of Leverage," pp. 717-718.

[2]Alexander Barges, The Effect of Capital Structure on the Cost of Capital (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), p. 35.

[3]Bierman, Financial Policy Decisions, p. 87.

interest charges are necessary for leverage (actually debt with no interest or free debt gives greater leverage) and are only excuses for eliminating problems in the measurement of short-term debt.[1]

Besides these problems there are other less frequent difficulties which must be faced such as how to compare leverage between two firms if one leases and the other buys[2] and how to account for possible accelerated effects on leverage from subsidiaries.[3] For example, what is the real leverage of firm A who holds shares of levered firm B who in turn holds shares in . . .

CONCLUSIONS

Due to all of these measurement problems and the complex and varied situations they attempt to measure, it is apparent that the art of measuring financial leverage has not yet satisfied any of the three possible objectives stated on page 3. At present it appears that some combination of objectives 1 and 2 will provide the best chances for obtaining an accurate index of leverage. That is, several measures or groups of measures for different purposes and a knowledge of their inter-relationships in the form of

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[1] Woods, "Financial 'Leverage' and 'Gearing' in Perspective," p. 35.

[2] Ibid., p. 34.

[3] Ibid., p. 27.

advantages and disadvantages of each measure. Several measures should be used in conjuntion, especially if there are unstable or extreme values present, and such measures should be chosen so as they are not mathematically consistent with one another by form. In elementary, uninvolved leverage situations, however, there is enough correlation between measures of leverage that almost anyone of them will suffice.

Through this practice and further research and study, it is hoped that one of the three objectives can be fully met and the problem of measuring leverage settled. However, even if some solution is reached to all the problems of leverage mensuration, there will still be numerous unsettled questions regarding leverage, particularly as it effects financial management. Some of these are considered in Chapter 3.

Chapter 3

LEVERAGE EFFECTS

Leverage can be charged with having some effect,
either direct or indirect, upon nearly every aspect of
financial management. This paper will deal only with the
direct effect upon the cost of capital in a static sense.

COST OF CAPITAL

When reading the views of financial theorists, it
seems as if the effects of leverage on a firm's cost of
capital has developed into the crux criticorum of modern
finance. The reason for this puzzlement and lack of a
posited theory lies with the yet impossible task of deter-
mining the interrelationships of many both objective and
subjective and significant and picayune variables. Still,
even with these problems, and perhaps because of them,
financial theorists have generally narrowed the problem
down to three major theories describing the effect of
leverage on the cost of capital.

Before studying these theories it is proper to dwell
briefly on the term, cost of capital. If capital is,
". . . the total of net worth, plus the long-term sources

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funds such as bonds and intermediate-term loans."[1] the cost of capital becomes the sum total of the costs of these separate sources. Included would be bonds, preferred stock, common stock, retained earnings and convertible securities, all possibly having a different cost and contribution to the total cost of capital. Each of these separate components have problems and controversies of their own concerning which method gives the best estimate or measure of their individual cost. Since it is not the purpose of this paper, thankfully, to study measurement problems relating to cost of capital elements,it is only noted at this time that disagreements and hence possible inaccuracies do exist. Therefore, the same can be said of the total cost of capital figure obtained by combining these components.

The most common method of arriving at a cost of capital value when a condition such as this exists is to calculate a weighted-average cost of capital (K_o). Since this measure and limitations inherent with it are part of the controversy in regard to leverage's effect on K_o, it will be explained in order that it may be included in a more meaningful fashion.

The weighted-average cost of capital is simply the

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[1]Gilbert W. Cooke and Edwin C. Bomeli, Business Financial Management (Boston: Houghton Mifflin Company, 1967), p. 8.

sum of products obtained when the percentage amounts of
each capital component of total capital are multiplied
by their respective costs. To demonstrate, assume that
a given firm had the following capital structure occurring
in the proportions given and at the cost stated for each
component:

	% amt.	% cost	weighted cost
convertible securities	20	13.3	2.60
bonds	20	6.5	1.30
preferred stock	10	7.5	.75
common stock	30	12.0	3.60
retained earnings	20	15.0	3.00

$$K_o = 11.25$$

Cost of capital figured for the firm as a weighted-average
of the five individual capital elements equals 11.25%.

One assumption of this method, limiting its use, is
that future financing occurs according to the same pro-
portions of capital components in the capital structure
used to measure cost of capital. If not, the actual cost
of capital after such financing, unless it is recalculated,
will differ from that measured by the former weighted-aver-
age.

The question of book value versus market value
figures is present with the weighted-average cost of capital
measure just as it was in the previous chapter with leverage
measures. The use of both for measuring the cost of capital
will understandably produce unequal results if market values

differ from book values.

It is known that leverage can directly influence individual cost of capital components but the real issue, which as yet has not been proven, is whether or not leverage influences the weighted-average cost of capital

As recognized by Solomon, there are two ways of studying the problems of financial structure, dynamic and static.[1] The former concerns itself with efforts to achieve optimal individual financing decisions throughout time and the financial structure existing at any one time is simply a result of these prior decisions. The static view, used in this paper, places the capital structure decision at a point of time in the drivers seat with the goal of determining the optimal capital structure, given the prevailing market conditions. The major decision involved in the search for this optimum concerns the degree of leverage and resulting risk and how they relate to the cost of capital. There is much disagreement, as will be seen shortly, as to the exact nature of this relationship, and in fact some interested parties claim that there is no relationship at all.

The next question that must be answered is why the

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[1]Ezra Solomon, The Theory of Financial Management (New York: Columbia University Press, 1963), p. 91.

leverage-cost of capital controversy is important. It is essential because of the role it plays in major fields of finance such as capital budgeting, dividend policy and market valuation.

Capital budgeting decisions often depend to a large extent upon a firm's cost of capital and any factor which can influence this figure becomes important in the capital budgeting decision process. Such a decision, for example, could be strongly affected by whether one felt that increased leverage, caused by a new investment financed by debt, would increase a firm's cost of capital or have no effect upon it.

Dividend policy is related to the cost of capital because of its relationship with two major sources of funds, debt and retained earnings. By restricting dividends and using retained earnings for financing, a firm can get by with a lower degree of leverage. The question is whether this lower leverage results in a lower cost of capital or higher market value than would be the case if dividends were paid and debt incurred.

The market value of a firm is very closely related to its cost of capital. The cheaper it can acquire its capital the more profits it can accumulate and the higher should be its market value.

Analysis in this paper, however, will concern itself

only with the basic relationship between leverage and the cost of capital and not with how this affects additional variables.

COST OF EQUITY

During most of the remaining portions of this paper for the purpose of simplicity the cost of capital will be thought of as the weighted-average of only two components, cost of equity (K_e) and cost of debt (K_d). How leverage effects K_e, which can also be viewed as the rate of return required by common stockholders on their investments, is therefore an important factor in the leverage-cost of capital relationship.

To study this connection the following equation measuring the cost of equity for a levered firm will be employed:

$$K_e = \frac{\text{earnings available to common stockholders}}{\text{outstanding stock}}$$

$$K_e = \frac{(EBIT - K_d \times D)(1 - T) + K_d(D)}{V - D}$$

Using this equation it can be demonstrated how the degree of leverage, coupled with the earnings before interest and taxes (EBIT), effects the cost of equity or return required by stockholders.

Considered in the first situation will be a moderately leveraged firm having total capital (V) of $100,000

consisting of $70,000 in common stock (S) and $30,000 in debt (D). Cost of debt and the tax rate (T) for this and the following examples in this section will be given as 6% and 50% respectively. Income before interest and taxes is $10,000. Substituting into the above equation:

(1) $K_e = \dfrac{(10,000 - .06 \times 30,000)(1-.5) + (.06 \times 30,000)}{100,000 - 30,000}$

$K_e = 8.4\%$

Now by changing the debt-equity ratio to 75% debt the effect of higher leverage on K_e will be shown. To accomplish this it is assumed that the firm purchases $45,000 of its own stock on the market and increases its debt by an equal amount, keeping total capital the same.

(2) $K_e = \dfrac{(10,000 - .06 \times 75,000)(1-.5) + (.06 \times 75,000)}{100,000 - 75,000}$

$K_e = 29.0\%$

Due to the increased amount of leverage the stockholders, although there are fewer of them, have demanded a larger rate of return on their investment. Before the leverage effects on K_e can be absolutely determined, however, another factor must be considered. This is the earnings variable. To view the effects of EBIT on K_e the same equation and leverage situations as used just previously will be utilized. For the first case the moderate or 30% debt level is assumed, but EBIT is

given as $5,000 rather than $10,000.

$$(3) \quad K_e = \frac{(5,000 - .06 \times 30,000)(1 - .5) + (.06 \times 30,000)}{100,000 - 30,000}$$

$$K_e = 4.9\%$$

So in comparison with example (1) with the same debt-equity ratio, the cost of equity has decreased as EBIT decreased.

To see how a highly leveraged firm is affected under similar circumstances the debt-equity ratio is changed to 75% D in the same manner as it was accomplished between examples (1) and (2), keeping all other variables the same.

$$(4) \quad K_e = \frac{(5,000 - .06 \times 75,000)(1 - .5) + (.06 \times 75,000)}{100,000 - 75,000}$$

$$K_e = 19.0\%$$

Again as leverage increases so does K_e.

The results of these four examples are summarized in the table below:

	low EBIT ($5,000)	high EBIT ($10,000)
low leverage (V is 30% D)	$K_e = 4.9\%$	$K_e = 8.4\%$
high leverage (V is 75% D)	$K_e = 19.0\%$	$K_e = 29.0\%$

Thus as leverage is increased so is cost of equity. The reason for this is that stockholders demand a higher rate of return on their investment as the financial risk of the firm is increased. The EBIT factor also has a

direct effect upon K_e as would be expected. The higher the earnings the more return stockholders will demand. Therefore the higher the leverage and EBIT of a firm the higher will be its cost of equity.

Throughout this analysis the cost of debt capital has remained constant for each degree of leverage. Many theorists, however, picture K_d as an increasing function of leverage and this will be discussed later in conjunction with the cost of capital theories.

RISK

As mentioned previously, along with additional chances of gains increased debt leverage also brings additional chances of losses. This risk can be statistically calculated.[1] To demonstrate, the probability of a loss occurring for situation (1) in the section just prior can be figured if the mean and standard deviation (σ) of expected EBIT are known. Assuming they are $10,000 and $3,000 respectively the following data exists:

EBIT (mean of probability dist.)	= $10,000
σ standard deviation	= $ 3,000
K_d cost of debt	= 6%
D debt	= $30,000
Z_p random var. from std. norm. dist.	

Given the following equation:

[1] Bierman, Financial Policy Decisions, p. 95.

$$(1) \quad Z_p = \frac{(EBIT - K_d \times D)}{\sigma}$$

$$Z_p = \frac{(10,000 - .06 \times 30,000)}{3,000}$$

$$Z_p = 2.73$$

From a random normal table it is determined that with a Z_p of 2.73 and D amounting to $30,000 the firm has a .003 probability of loss.

Moving on to example (2), however, with D increasing to $75,000 and all other factors remaining constant, the probability is greatly affected.

$$(2) \quad Z_p = \frac{(10,000 - .06 \times 75,000)}{3,000}$$

$$Z_p = 1.83$$

Consulting the table once again, the probability of loss under the increased leverage has jumped to .034, or nearly twelve times what it was under lower leverage conditions. It is because of this increase in the chances of suffering a loss as leverage is increased that most theorists picture the yield on stockholders' equity as a positive function of the amount of leverage.

At this point it is appropriate to mention characteristics of the types of risk that the employment of leverage entails. There are two basic types of risk in the financial world relating to leverage, business risk (BR) involving the basic operational activities of the firm, and financial risk (FR) having to do with the firm's financing

decisions. All firms have some degree of business risk
but all need not have financial risk. Why? No firm, no
matter what its business, is absolutely certain of its
future income and without this indubitable degree of
reliability some business risk is present, since it connotes
dispersion in the firm's expected net income. And since
business risk, is defined as ". . . the relative dispersion
of the net operating income . . ."[1] there is business risk
in all firms (i.e., all those not having 100% certain
incomes is assumed to encompass all firms). If some case
where such a high level of certainty did exist, there would
no longer be a problem concerning financing. "If returns
are completely certain then it is a matter of indifference
to stockholders how the firm is financed."[2]

For purposes of this paper financial risk will be
determined by the degree of leverage,so conceivably a firm
with no debt will have no financial risk. "Financial risk
is the additional risk to stockholders which arises through
the use of borrowed funds for the financing of a project
and is usually measured by leverage."[3]

---

[1]VanHorne, Financial Management and Policy, p. 18.

[2]Stephen J. Turnovsky, "Financial Structure and the
Theory of Production," The Journal of Finance, Vol. XXV,
No. 5, (December, 1970), p. 1064.

[3]Ibid., p. 1064.

The justification of this view is presented in the following discussion differentiating between BR and FR. Risk can be measured by the relative distribution of expected income and in relation to financial risk this income is the return to shareholders. The statistical measure for relative distribution is the coefficient of variation (CV) which is equal to the standard deviation of a distribution divided by the expected value of the same distribution $(\bar{X})$.[1,2] Using the CV in an example will demonstrate how business risk differs from financial risk and how debt leverage causes finahcial risk.

For this example three firms X, Y and Z are considered, all having expected net income earnings on assets equal to $10,000 with a standard deviation of $3,000. These firms are assumed to be identical in all aspects except for the degree of debt; X having 0, Y having $3,000 and Z $7,000.

Since business risk is the relative distribution of income which is measured by the CV, it can be represented by the equation

$$BR = \frac{\sigma}{\bar{X}}$$

and would equal for firms X, Y and Z:

---

[1]VanHorne, Financial Management and Policy, p. 146.

[2]Barges, The Effect of Capital Structure on the Cost of Capital, p. 17.

$$BR = \frac{3,000}{10,000} = .3000$$

Though the business risk for all three firms is equal, their financial risks are different due to varying fixed charges on debt which must be subtracted from net income from assets. Financial risk is measured by the relative dispersion of income belonging to shareholders. If debt charges (K_d) are fixed at 4% for all outstanding debt (D), the equations representing financial risk for the three firms are as follows.

Firm X -

$$FR_x = \frac{\sigma}{\overline{X} - (K_d)(D)}$$

$$FR_x = \frac{3,000}{10,000 - 0}$$

$$FR_x = .3000$$

Thus in the case of a debt free firm having no leverage, FR seems to equal BR. This, however true, is not the real case, rather the FR equation above is really a measure of total risk (BR + FR) and when FR = 0 the total risk of a firm ceteris paribus will equal business risk. In fact FR can be defined as BR plus any additional risk of insolvency caused by debt charges.

Firm Y -

$$FR_y = \frac{3,000}{10,000 - (.04)(3,000)}$$

$$FR_y = .3036$$

Firm Z –

$$FR_z = \frac{3,000}{10,000 - (.04)(7,000)}$$

$$FR_z = .30386$$

As expected the financial risk and leverage of a firm are directly related; as one increases so does the other. This relationship between financial risk and leverage is represented more clearly by Graph 1 on page 35 for two values of σ and K_d. As can be seen, the slope of the FR line is determined by K_d and its height above the 0 axis by σ. Also in either case the FR line represents the total sum of business and financial risk and is in a direct linear relationship with leverage measured by debt/common stock. The fact that there is this relationship between financial risk and leverage and none between leverage and business risk will play an important part in the next section where leverage and its effects, if any, on the cost of capital are studied.

It must be noted that it is only in the static sense that business risk and leverage have no relation. It is a well accepted fact that firms having low levels of business risk can carry financial structures much more leveraged than can firms associated with higher degrees of business risk.

> . . . the reaction of both borrowers and lenders of
> funds to the circumstance of low external risks
> [e.g. utilities] brings about an optimum capital

Graph 1
FR and BR as a Function of Leverage*
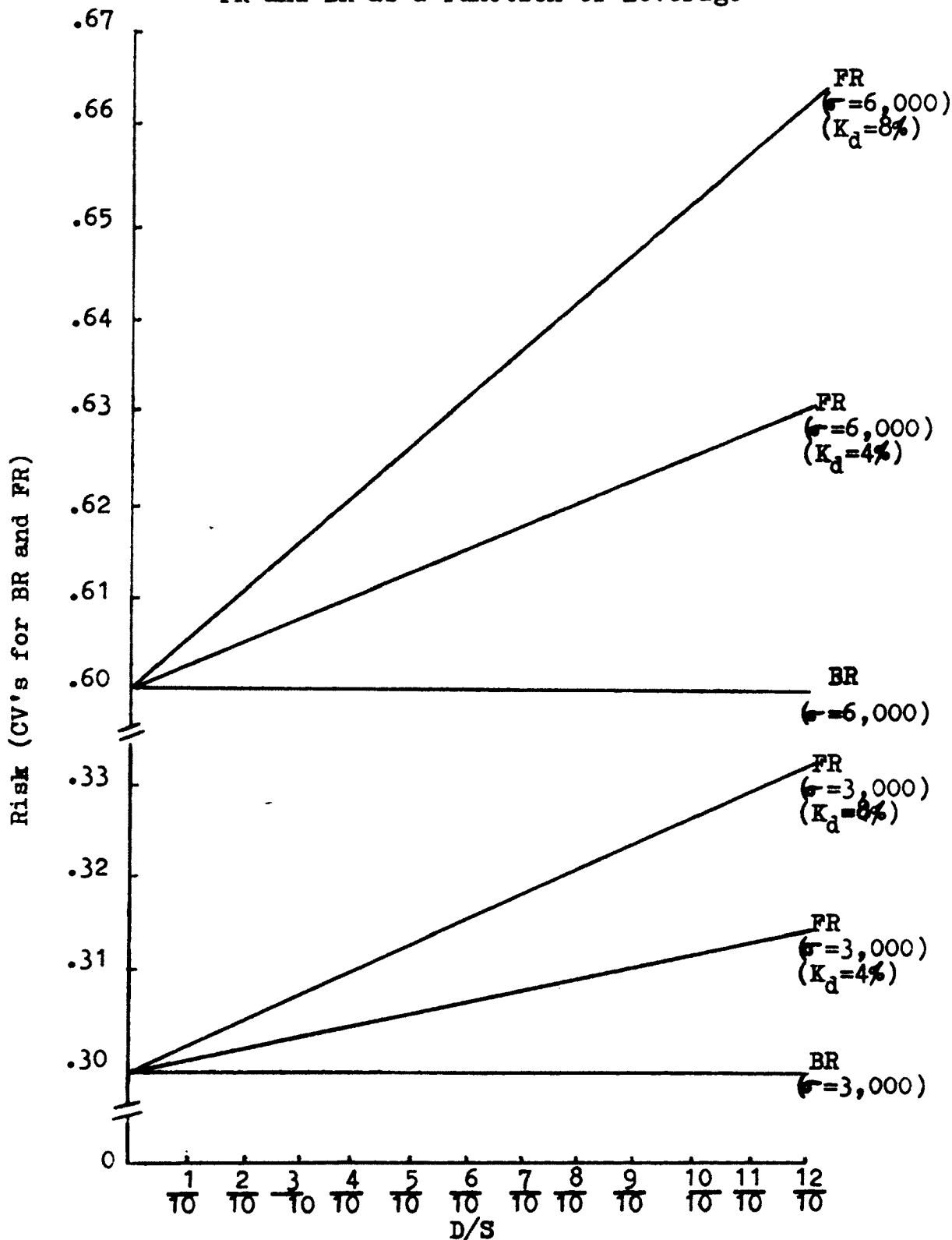
*Calculations in Appendix 2.

structure which is relatively heavily leveraged . . .
On the other hand, an industrial firm—facing heavier
external risks—is likely to experience a relatively
sharp rise in financial charges if its debt-to-equity
ratio (leverage) exceeds certain proportions . . .
Thus the typical industrial financial structure would
tend to contain a smaller proportion of debt to total
financing than either the utility or the bank.[1]

PRESENT THEORIES

Theories regarding the effect of leverage on the

cost of capital can generally be classified into three

categories: (1) net income (NI), (2) net operating income

(NOI), and (3) traditional. The NI theory has few pro-

ponents but the remaining two have strong and persuasive

supporters. In a sort of capsule description NI and NOI

theories seem to occupy the end locations in a range of

possibilities with the traditional theory situated some-

where between, in a somewhat compromising position.

Each of these theories will be presented in turn

before dealing with criticisms of each and their differences

and similarities.

NI Theory

The net income approach (named by Durand in 1952

and again in 1967 by Weston as, unfortunately, the theory

of traditional business finance) professes belief in a

_____

[1]Eli Schwartz, "Theory of Capital Structure," The
Journal of Finance, Vol. XIV, No. 1, (March, 1959), p. 35.

declining overall cost of capital as leverage is in-

creased.[1,2] The crux of this theory is that cost of debt

(K_d) and cost of equity (K_e) remain constant throughout

all levels of leverage. This and the following theories

will be demonstrated by using a numerical example followed

by a graphic representation of the relationship between

leverage and cost of capital. Leverage will be measured

by a simple debt to equity (D/S) ratio. Given a firm with

net operating income (NOI) of $1,000, outstanding debt (D)

of $4,000 and a K_d of 5% and K_e equal to 10%.

NOI	1,000
less $(K_d)(D)$	200
earnings available to stockholders	800
K_e	.10
market value of stock	8,000
D added	4,000
market value of firm	12,000

Calculating cost of capital by the equation:

$$K_o = \frac{NOI}{market\ value}$$

$$K_o = \frac{1,000}{12,000}$$

$$K_o = 8.33\%$$

---

[1]David Durand, "The Cost of Debt and Equity Funds
for Business: Trends and Problems of Measurement," Con-
ference on Research in Business Finance (New York: National
Bureau of Economic Research, 1952), pp. 215-217, reprinted
in Readings in Contemporary Financial Management, ed.
Keith B. Johnson, Donald E. Fischer, (Illinois: Scott,
Foresman and Company, 1969), p. 385.

[2]Fred J. Weston, "Valuation of the Firm and its
Relation to Financial Management," (McKinsey Foundation for
Management Research) printed in Financial Research and
Management Decisions, ed. Alexander A. Robichek, (New
York: John Wiley & Sons, Inc., 1967), pp. 14-15.

Next in order to view the effect of a change in leverage assume that D is increased to \$7,0C0 accompanied by the repurchase of \$3,000 worth of outstanding shares keeping total capital (V) the same as in the previous example.

NOI	1,000
less $(K_d)(D)$	350
earnings available to stockholders	650
K_e	.10
market value of stock	6,500
D added	7,000
market value of firm	13,500

Calculating K_o:

$$K_o = \frac{1,000}{13,500}$$

$$K_o = 7.41\%$$

Thus under NI theory the cost of capital decrease as the amount of leverage is increased. A graphic representation of this theory appears below.

As can be seen K_o continues to decrease, as leverage increases, until the point of optimum financial structure is reached where leverage is as high as possible. The reason for K_o's decline is that K_d is viewed as less than K_e and as D/S increases more of the lower priced D funds are substituted for higher priced S funds resulting in a decreasing K_o function.

The implications of this theory are that K_o will continue to decline with leverage on into infinity. Most theorists, however, realize this is not possible as stated by Durand,

NI Theory
Leverage and Cost of Capital
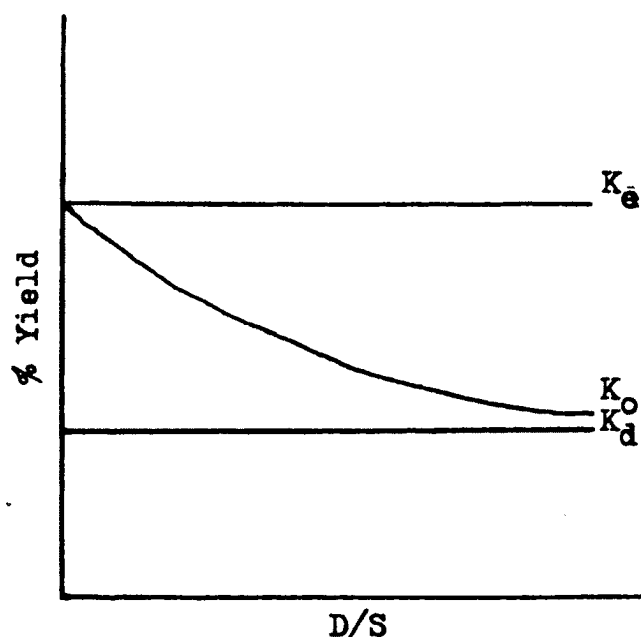
. . . such a relationship cannot continue indefinately,
. . . As the debt burden becomes substantial, the bonds
will slip below par, and the stock will cease to be
worth ten times earnings.[1]

If this should happen, however, the risk as viewed by

investors would be greater and K_e and K_d would increase,

driving up K_o. But as noted by VanHorne,

. . . the critical assumption for NI theory is that
the firm does not become increasingly more risky in
the minds of investors and creditors as the degree
of leverage is increased.[2]

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[1]Durand, "Costs of Debt and Equity Funds for Business:
Trends and Problems of Measurement," p. 386.

[2]VanHorne, Financial Management and Policy, p. 150.

This assumption understandably is one that cannot be generally accepted and as a result supporters of this view are nearly nonexistent.

NOI Theory

Moving to the opposite extreme, the next position viewed will be the net operating income theory. This method capitalizes a firm's net operating income with the use of an overall capitalization rate or cost of capital which is stated as constant throughout all levels of leverage. .

If K_o is given as 10%, this approach can be demonstrated mathematically using the same example as with the NI method just prior. Starting with debt equal to $4,000.

NOI	1,000
K_o	.10
market value of the firm	10,000
less market value of D	4,000
market value of stock	6,000

Next figuring for the missing statistic, K_e:

$$K_e = \frac{\text{earnings available to stockholders}}{\text{outstanding stock}}$$

$$K_e = \frac{NOI - K_d(D)}{S}$$

$$K_e = \frac{1,000 - 200}{6,000}$$

$$K_e = 13.3\%$$

To view the effects of increased leverage debt is increased to $7,000 by retiring the appropriate amount of S ($3,000) in order to keep total capital constant.

```
NOI                                    1,000
K                                        .10
market value of the firm             10,000
less market value of D                7,000
market value of stock                 3,000
```

Calculating K_e:

$$K_e = \frac{1,000 - 350}{3,000}$$

$$K_e = 21.7\%$$

Thus under the NOI method as leverage is increased K_e increases accordingly and K_o and K_d remain constant. Graphically these results are represented below.

NOI Theory
Leverage and Cost of Capital
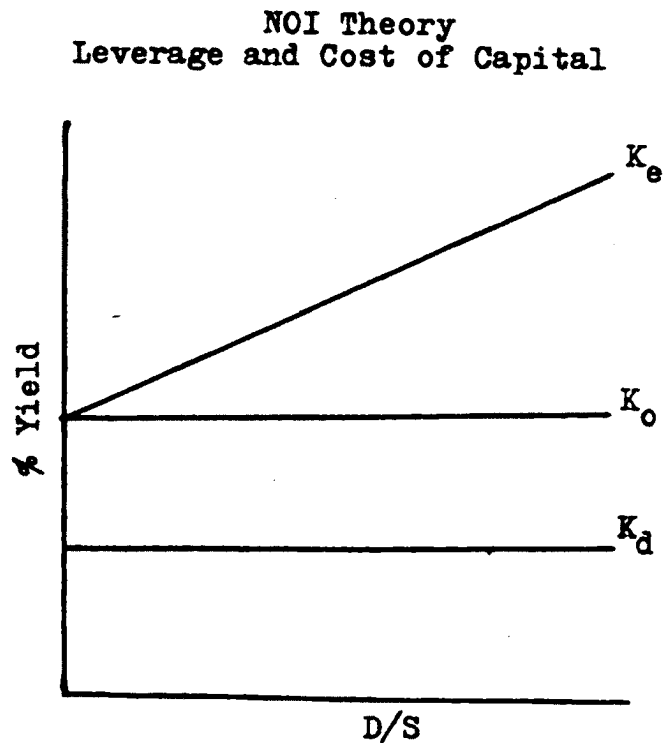
K_o is able to remain constant since it is assumed any increase in K_e is equally offset by the substitution of

lower priced D for S as a means of financing.

Leading backers of the NOI method are Franco Modigliani and Merton H. Miller who expounded their theory in 1958 and have vigorously defended it since.[1,2] Since their presentation is used for target practice and comparison purposes by most financial theorists writing on the effects of capital structure on cost of capital, it will be described in detail.

Modigliani and Miller (hereafter MM) have several assumptions or restrictions upon which their theory is at least partically dependent and these must be recognized before discussion of their actual theory begins. These assumptions are:

1. A free and perfect competitive market exists exclusive of such things as transfer costs and irrational investors.

2. Although it is recognized that the stream of profits or income accruing to stockholders extends

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[1] Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," The American Economic Review, Vol. XLVIII, No. 3, (June, 1958) pp. 261-297.

[2] For the remainder of this paper any references to this basic article by Modigliani and Miller will simply be recognized by the appropriate page(s) in brackets (e.g., 1 above [261-297]).

indefinitely into the future, it is assumed, " . . . that the mean value of the stream over time, or average profit per unit of time, is finite and represents a random variable subject to a (subjective) probability distribution." [265]

3. "We shall assume that firms can be divided into 'equivalent return' classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class." [266] This means that firms can be separated according to their business risk, as discussed previously on page 30, with each member of a specific class having the same degree of business risk.

4. It is assumed that no tax effects are present. This assumption is lifted later when tax effects are handled specifically.

After making the preceding assumptions, MM set forth three propositions, the first stating that, ". . . the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate pk appropriate to its class."[1] [268] Proposition I is represented by the equation:

---

[1]MM define pk as the market rate of capitalization for the expected value of uncertain streams occurring in the kth class or simply the average cost of capital, K_o.

$$V_k = D_j + S_j = \bar{X}/K_{oj} \quad \text{(for any firm in class } j \text{)}$$

Solving for K_{oj}:

$$K_{oj} = \frac{\bar{X}_j}{V_j} = \frac{\bar{X}_j}{D_j + S_j}$$

Thus, ". . . the average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class." [268-269]

Firms with the same business risk and expected income stream ($\bar{X}$) will have equal market values and stock prices in an equilibrium condition. According to MM, if the conditions of equilibrium are not present, that is if the total market value of two or more firms within the same business risk class differ due to capital structure, a process of arbitrage will occur to remedy the situation. In this process rational investors substitute personal debt for corporate debt (MM assume cost of borrowing to be the same for individuals and corporations) as is demonstrated in the following example. Given two companies, A and B, characterized by the figures below:

	Co. A	Co. B
$\bar{X}$	$1,000	$1,000
D	—	$4,000
K_d	—	5%
$K_d(D)$	—	$ 200
earnings on S	$1,000	$ 800
K_e	10%	12%
S	$10,000	$6,667
V	$10,000	$10,667

Since both A and B are in the same business risk class and
have equal expected incomes, their total values (V) should
be equal under conditions of equilibrium. To bring about
equilibrium MM predict that an investor in firm B, assumed
to have 1/100 of the outstanding shares, will arbitrage
according to the following procedure:

1. Sell shares in Co. B for $67.
2. Borrow $40 so that his personal debt-equity ratio
   is the same as Co. B's or 4/6.7.
3. Invest the $107 in Co. A.
4. Realizing profits -
   $107 x .10 (A's K_e)
   = $10.70
   less interest on $40 at 5% ($2.00)
   = $8.70 (profit from A)
   less profit he would have realized from B
   (.12 x $67) or $8.04
   giving a net gain of <u>$.66</u>

This procedure, given the laws of a perfect market, will
gradually drive down the value of Co. B and increase Co.
A until the two are equal as MM say they should be.

The arbitrage process will also work if the situa-
tion between the total values of firms A and B are reversed.
For example, if A's K_e = 9% giving a V of $11,111 an investor
in stock of A would sell his shares for $111 and use it
to purchase $67 worth of B's stock and $44 in 5% bonds. In
this case the arbitrager would realize a gain of $1.24
($10.24 - $9.00).

This process implies that investors will keep the
V of firms with the same BR equal. There is a great deal
of faultfinding related to this aspect of MM's theory which

will be covered later in the section dealing with **criticism**.

MM's Proposition II says that, ". . . the expected yield of a share of stock is equal to the appropriate capitalization rate pk $[K_o]$ for a pure stream in the class, plus a premium related to financial **risk** equal to the debt-to-equity ratio times the spread between pk $[K_o]$ and r $[K_d]$." [271] Represented by the equation:

$$K_{dj} = K_{oj} + (K_{oj} - K_{dj})(D_j/S_j) \text{ (for any firm in class } j)$$

As demonstrated below Proposition II can be derived from the combination between Proposition I and the formula for the expected rate of return on common stock.

Proposition I

$$K_{oj} = X_j/V_j$$

$$X_j = (K_{oj})(V_j)$$

since $V_j = D_j + S_j$

(a) $X_j = (K_{oj})(D_j + S_j)$

Given the following equation for expected rate of return on equity: (assume all equity exists of common stock, S)

$$K_{ej} = \frac{X_j - K_{dj}(D_j)^1}{S_j}$$

substituting (a) for X_j in the equation above

$$K_{ej} = \frac{(K_{oj})(D_j + S_j) - K_{dj}(D_j)}{S_j}$$

simplifying

$$K_{dj} = K_{oj} + (K_{oj} - K_{dj})(D_j/S_j)$$

---

[1]Use of this equation is questioned by Barges; see p. 68.

What this proposition really amounts to is that the increase in K_e as D/S increases is exactly offset by a decrease in K_o brought about by the use of lower priced D funds; lower by $K_o - K_d$.

Proposition III will be set forth here but not referred to again in this paper since it depends upon I and II and does not deal directly with leverage and the cost of capital. This proposition concerning investment policy says that, ". . . the cut-off point for investment in the firm will in all cases be pk $[K_o]$ and will be completely unaffected by the type of security used to finance the investment." [288]

Regarding the behavior of K_d, MM go one step further than is the case with the basic NOI method. Normal NOI theory has K_d constant throughout all levels of leverage implying that higher degrees of financial risk do not command higher interest charges on debt. MM recognize the falacy of this argument and grant that K_d may increase under high amounts of leverage. They claim, however, that such an increase in K_d is equally offset by a decrease in K_e keeping K_o constant. "If r $[K_d]$ increases with leverage, the yield i $[K_e]$ will still tend to rise as D/S increases, but at a decreasing rather than a constant rate. Beyond some high level of leverage, depending on the exact form of the interest function, the yield may even start to

fall." [274-275] The effects of such an increase in K_d is pictured graphically below.

MM Theory
Extreme Leverage and Cost of Capital
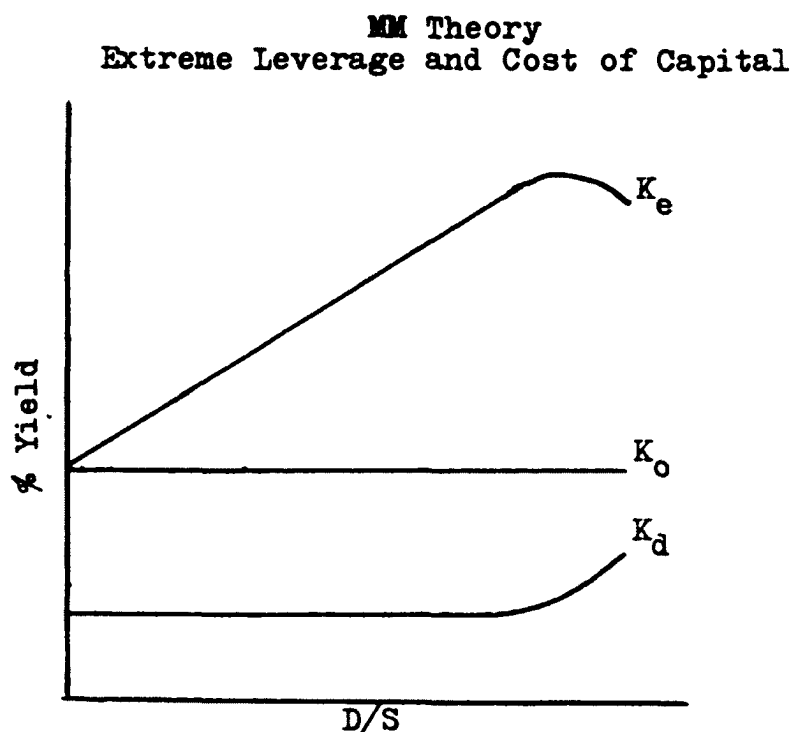
This prediction by MM concerning the effects of extreme leverage on K_e is another portion of their theory under strong attack from traditionalists and will be dealt with in a later section.

As presented up to this point the NOI or MM method has no optimal point in the capital structure, in fact capital structure is optimal at all points of leverage. When taxes are introduced MM are forced to revise their constant K_o position and accept an optimum capital structure

since under tax conditions the interest paid on debt is not taxable.

Using the same figures as in previous examples plus the addition of an after tax capitalization rate (K^t) equal to 7% and a corporate tax rate of 50% the following models can be constructed.

	L = D/S = .8	L = D/S = 2.7
NOI	1,000	1,000
less $K_d(D)$	200	350
taxable income	800	650
less T	400	325
after tax profit	400	325
add back $K_d(D)$	200	350
after tax income	600	675
K^t	.07	.07
V	⟶ 8,571	⟶ 9,643
D	4,000	7,000
S	4,571	2,643
K_o = NOI/V	⟶ 11.7%	⟶ 10.4%

Thus under the influence of taxes the K_o declines with increased leverage even under MM theory and in addition the total value (V) of the firm is increased. As seen by MM this decline is linear and would appear graphically as shown below.

According to MM the value of a leveraged firm under tax conditions is represented by the following equation:

$$V_1 = \frac{(1 - T)X}{p^t} + \frac{T(K_d)(D)}{r} = V_u + TD_1 \,^1$$

---

[1] Franco Modigliani and Merton H. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," The American Economic Review, Vol. LII, No. 3, (June, 1963), p. 436.

where: V_1, V_u - value of levered and unlevered
firm
p^r - market capitalization rate for X
net of taxes for an unlevered firm
r - market capitalization rate for a
sure debt stream

Continuing, MM show that V_1 must equal $V_u + TD_1$ at equilibrium
and if $V_1 \gtrless V_j + TD_1$ the arbitrage process will operate once
again. ". . . 'arbitrage' will make values within any class
a function not only of expected after tax returns, but of
the tax rate and degree of leverage."[1]

MM Theory
Taxes, Leverage and Cost of Capital
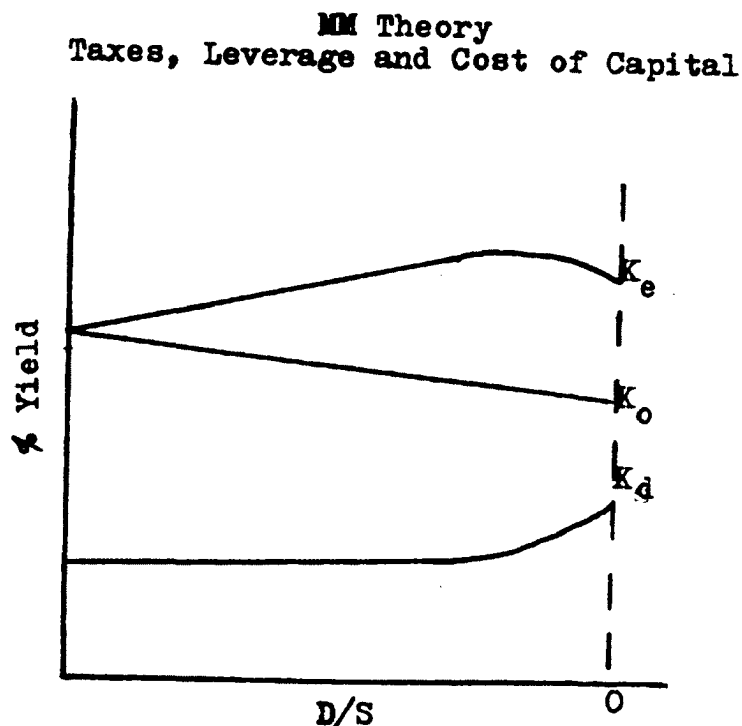
^{1}Ibid., p. 434.

From the equation at the top of the page preceding, MM derive an equation for a firm's after-tax cost of capital or earnings yield (X/V).

$$\frac{X}{V} = p^t - T(p^t - r)\frac{D}{V}^1$$

This equation conflicts with the view held by traditionalists and will be covered later in the section dealing with com - parisons.

Viewing the graph on page 50, it is apparent that due to the effects of income taxes there is now an optimum capital structure (0), located as far to the right and at the highest degree of leverage possible. Such a view implies that all firms should employ as much debt as possible in order to decrease K_o to its lowest value. How then do MM explain the fact that firms in the real world show no efforts in this direction? They feel that the actual firm does not increase debt unlimitedly due to cheaper means of financing and the need for a safety margin.

> . . . the existence of a tax advantage for debt
> financing . . . does not necessarily mean that cor-
> porations should at all times seek to use the maximum
> possible amount of debt in their capital structures.
> For one thing, other forms of financing, notably
> retained earnings, may in some circumstances be
> cheaper still when the tax status of investors under
> the personal income tax is taken into account. More
> important, there are . . . limitations imposed by

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[1]Ibid., p. 439.

lenders, as well as many other dimensions (and kinds of costs) in the real world problems of financial strategy which are not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety. These additional considerations . . . will normally imply the maintenance by the corporation of a substantial reserve of untapped borrowing power.[1]

A major threat involved with increased debt is that the carrying charges may not be met bringing about the danger or condition of bankruptcy. Robichek and Myers argue that these factors eliminate the tax advantage for higher degrees of leverage.

These contingent bankruptcy costs, to the extent that they exist in fact, constitute a disadvantage partially or wholly offsetting the tax advantage of leverage. It is our hypothesis that the present value of these costs will be an increasing function of leverage.[2]

As a result,

. . . we would expect the market value of the firm to be an increasing function of leverage for firms with little or no debt, but that the values of the firm ultimately declines if leverage is carried too far.[3]

They offer proof of this condition consistent with traditional theory even while assuming a perfect arbitrage process as under MM theory.

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[1]Ibid., p. 442

[2]Alexander A. Robichek and Stewart C. Myers, "Problems in the Theory of Optimal Capital Structure," Journal of Finance and Quantitative Analysis, (June, 1966), p. 16.

[3]Ibid., p. 19

Traditional Theory

Traditional theory is the name given to the third major opinion describing the relationship between leverage and the cost of capital. At present this approach seems to have the edge, as numbers of supporters are concerned, perhaps due to its somewhat compromising position and less idealistic assumptions. It is less idealistic specifically in the sense that traditional theory recognizes that there may be certain inequalities in the market such as transaction costs and higher rates of K_d for individuals than for large corporations.

Traditional theory separates degrees of leverage into three ranges, each having a different effect upon K_o. Throughout the three ranges of leverage K_e is increasing at an increasing rate due to investors' growing concern over financial risk. Subject to several variations among traditionalists, $\overline{K_d}$ is generally constant through the first two ranges up until the beginning of range three, termed the critical degree of leverage. At this point K_d begins to increase at an increasing rate in response to increased creditors' demands for more returns due to higher financial risks in the firm.

In the first range of leverage K_e tends to increase K_o but is more than offset by the substitution of D for S resulting in a decreasing K_o. During range two K_e

continues to increase but is equally offset by the sub-
stitution of D for S, now occurring at a slower rate than
in range one, all causing a level K_o function. Finally
in range three where K_e and K_d are both increasing K_o is
also forced upward.

To mathematically demonstrate the workings of
traditional theory the following series of examples is
presented. The first example is a debt free firm char-
acterized by the figures below.

NOI	$ 1,000
less K_d(D)	-
earnings available to stockholders	$ 1,000
K_e	.10
market value of stock	$10,000
add D	-
market value of firm (V)	$10,000

calculating K_o

$$K_o = \frac{NOI}{V}$$

$$K_o = \frac{1,000}{10,000}$$

$$K_o = 10\%$$

For the next example it is assumed that the firm
increases its debt from 0 to $4,000 and retires outstanding
stock of an equal amount. K_d is set at 5% and due to the
increased leverage K_e is now 11%.

---

[1]For a debt free firm $K_o = K_e$ since K_e is the only
element making up K_o.

```
NOI                                          $ 1,000
less K_d(D)                                      200
earnings available to stockholders              800
K_e                                              .11
market value of stock                        $ 7,273
add D                                        $ 4,000
market value of firm (V)                     $11,273
```

calculating K_o

$$K_o = \frac{1,000}{11,273}$$

$$K_o = 8.16\%$$

As can be seen, by increasing leverage the market value of the firm has increased and K_o has decreased.

Moving on to a still higher degree of leverage, assume that the firm increases its D to $7,000, again retiring outstanding stock of the same amount. Resulting from the increased financial risk K_e jumps to 14% and K_d to 6%.

```
NOI                                          $ 1,000
less K_d(D)                                      420
earnings available to stockholders              580
K_e -                                            .14
market value of stock                        $ 4,143
add D                                        $ 7,000
market value of firm (V)                     $11,143
```
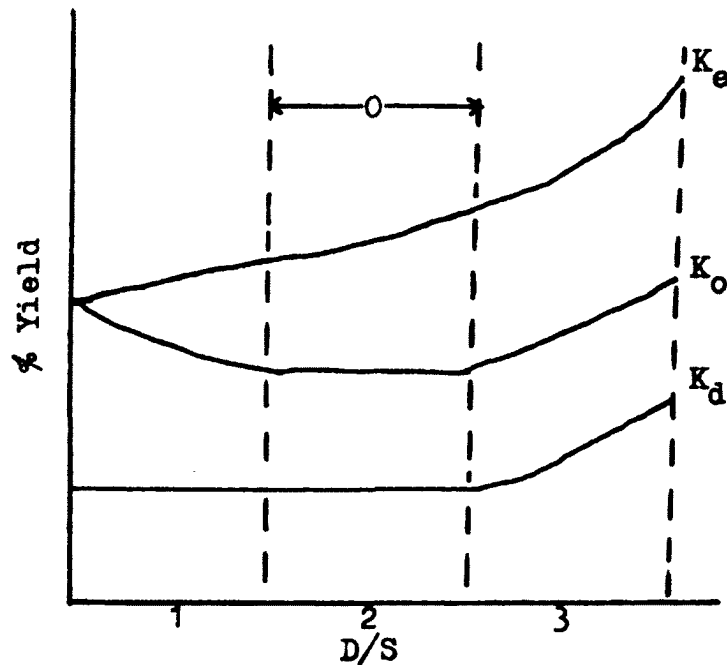
calculating K_o

$$K_o = \frac{1,000}{11,143}$$

$$K_o = 8.97\%$$

By increasing the leverage once again exactly the opposite results have occurred from the previous example following the first increase. This time the total value of the firm has decreased and K_o has increased. Graphically

the traditional view is shown below.

Traditional Theory
Leverage and Cost of Capital

Traditional theory professes that an optimal capital structure definitely exists. In the graph above it would be anywhere in range 2 since K_o is lowest and constant in this area. Some traditional theorists show K_o bottoming out at one specific optimal point, in such a case range 2 would consist of this one optimal point. According to Solomon the

> . . . precise location of the optimal degree of leverage is the precise point where the rising marginal cost of borrowing is equal to the average overall cost of capital. For this purpose the marginal cost of a unit of debt capital must be measured as the sum of two things: (a) the increase in total interest payable

on debt capital when debt is increased from B [D] to
B' [D']; (b) the amount of extra net earnings required
to restore the value of the equity component to what
it would have been under the pre-existing capitaliza-
tion rate K_e which prevailed before debt is increased
from B [D] to B' [D'].[1]

Just where this optimal point is located, or even where the

boundries to each range are, is dependent to a large extent

upon the degree of risk the particular firm is associated

with and how investors and creditors react to this risk. As

a result there is no clear cut method of arriving at such

values or even proving in the real world that the traditional

method, or any of the other two for that matter, is repre-

sentative. Still real world evidence in relation to an

optimal structure is one argument supporters give in favor

of the traditional approach and will be dealt with more

closely in the section including empirical evidence.

The next two sections will compare and criticize

only the last two theories of leverage and cost of capital

presented. The reason for this being the general lack of

support by anyone for the NI theory. As stated previously,

this approach relies on the assumption that risk, as seen

by investors, does not increase in anyway as the degree of

leverage is increased. Since this assumption is rejected

by nearly all financial writers, disagreements center

around the two remaining theories, the NOI held by MM and

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[1]Ezra Solomon, The Theory of Financial Management
(New York: Columbia University Press, 1963), p. 97.

the theory of modern business finance backed by tradition-
alists.

COMPARISONS

It would seem that the question of which theory of
capital structure versus cost of capital is correct could
be answered by a look at real world financial practices.
This, however, is not the case since empirical results
of such probes have been interpreted in as many ways as
there are theories and then some. MM present tests that
favor their position.

> The results of these tests [an analysis of the relation
> between security yields and financial structures] are
> clearly favorable to our hypothesis . . . The data
> in short provide no evidence of any tendency for the
> cost of capital to fall as the debt ratio increases.
> [282-284]

In contrast Schwartz and Aronson feel there is some relation
between leverage and the cost of capital since an optimal
financial structure apparently exists in the market place.

> Our data . . . represent some surrogative evidence that
> in a capital market where sources of funds may be some-
> what segregated, the various classes of firms have
> developed typical financial structures that are optimal
> for their operational risks and asset structures.[1]

Barges, after studying three industries (railroads,
cement companies and department stores) in a test of the

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[1]Eli Schwartz and Richard Aronson, "Some Surrogate
Evidence in Support of the Concept of Optimal Financial
Structure," The Journal of Finance, Vol. XXII, No. 1,
(March, 1967), p. 10.

MM theory states that

> . . . the probability of all tests indicating the same
> thing because of chance is extremely small. Thus, on
> the basis of evidence presented herein, the hypothesis
> of independence between average cost and capital
> structure appears untenable.[1]

Though each side seems to prove their own theory
they can't disprove their opponents, ". . . the regression
studies undertaken are simply not precise enough to con-
stitute absolute refutation of the MM position."[2] "Those
who make sport with regressions have yet to show that the
traditional position . . . is either proven or contradicted
by the empirical evidence available."[3] Thus about the only
conclusion available after studying such evidence is no
conclusion at all due to the lack of positive and unre-
futable evidence.

When trying to compare the two views, it is helpful
to look at one portion of their differences as recognized
by Barges. He explains that the traditionalist views debt
costs plus the addition to equity costs caused by debt, as
less than the "real" cost of equity. Thus the average
cost of capital can decrease as cheap debt is substituted
for expensive equity. MM, however, feel that the "real"
cost of equity funds equal cost of debt funds plus the

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[1] Barges, The Effect of Capital Structure on the Cost
of Capital, p. 103.

[2] VanHorne, Financial Management and Policy, p. 167.

[3] Solomon, The Theory of Financial Management, p. 98.

addition to equity costs caused by debt (which is really a debt cost). Under this view average cost of capital can not be decreased through the substitution of debt for equity since the "real" cost of each are the same.[1]

In presenting the two theories it was noted that MM believed in no optimal capital structure until taxes were introduced and then only to the extent of tax savings on interest and the lower risk due to the surity of the extra after-tax earnings. Traditional theory supports an optimal capital structure with or without tax bonuses because of the interaction of risks and capital costs resulting in a U-shaped cost of capital function. Even under tax conditions, however, when each theory predicts an optimal capital structure, the cost of capital as figured by MM and traditionalists differs. This difference is reflected in the after-tax average cost of capital or earnings yield for a firm as viewed by each position. As presented previously on page 51 MM represent the after-tax cost of capital as:

$$\frac{\overline{X}}{V} = p^t - T(p^t - r) \frac{D}{V}$$

The effect of income taxes on $\overline{X}/V$ is to decrease the cost of capital of an unlevered firm (p^t) by $T(p^t - r)D/V$. For example, given the following data:

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[1]Barges, op. cit., pp. 4-5.

$$p^t = 10\%$$
$$T = 50\%$$
$$r = 6\%$$
$$D = \$ 3,000$$
$$V = \$10,000$$

$$\frac{X}{V} = p^t - T(p^t - r)\frac{D}{V}$$

$$\frac{X}{V} = .10 - .50(.10 - .06)\frac{3}{10}$$

$$\frac{X}{V} = .098$$

The cost of capital of an unlevered firm (p^t) has been lowered from 10% to the after-tax average cost of capital of 9.8% due to the introduction of income tax as figured via the MM method.

In comparison the traditional theory, according to MM, views the equation for after-tax cost of capital in tax situations as:

$$X = p^t - (p^t - r)\frac{D}{V}^1$$

Under this equation cost of capital will decrease as a result of leverage (up to the critical point) whether income taxes are present or not and any such decrease will be larger than that figured by the MM method. For example, using the same figures as in the previous case, the average after-tax cost of capital for an unlevered firm (p^t) will

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[1] Modigliani-Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," p. 439.

fall from 10% to 8.8%.

$$\frac{X}{V} = p^r - (p^t - r)\frac{D}{V}$$

$$\frac{X}{V} = .10 - (.10 - .06)\frac{3}{10}$$

$$\frac{X}{V} = .088$$

Thus MM say as an interpretation of their results that, "The predicted rate of decrease of X^t/V with D/V . . . is still considerably smaller than under the naive traditional view . . ."[1]

Since each theory is supported by strong arguments several writers have tried to accept both by offering explanations of possible reconciliation between them. Such explanations have generally been rejected by strong supporters of each theory since they usually include some tampering with theoretical structures or assumptions.

Robichek and Myers have postulated this sort of explanation under dynamic conditions. They claim that their hypothesises present, ". . . a plausible reconciliation of the MM logic and traditional conclusions."[2] To begin with optimal is defined as that degree of leverage where the overall value of the firm is highest, rather than

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[1]Ibid., p. 439.

[2]Alexander A. Robichek and Stewart C. Myers, Optimal Financing Decisions (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965), p. 44.

that point where overall cost of capital is lowest (these
two points need not be located at the same leverage ratio).
Next Robichek and Myers allow expected income ($\overline{X}$) to vary
inversely with the amount of leverage. This relation,
which is contrary to MM implications, is caused by the fact
that, ". . . stockholders' estimates of Y [X] may decline
because of the possibility of the firm's having to inter-
rupt future investments in order to meet interest payments."[1]
By replacing X with X (λ) (representing the tendency of
X to decrease with an increase in leverage) and using it
in MM's own equations, typical traditional results are
obtained. A graph of this hypothesis is located below.

Robichek's & Myers' Reconciliation
Expected Income & Leverage
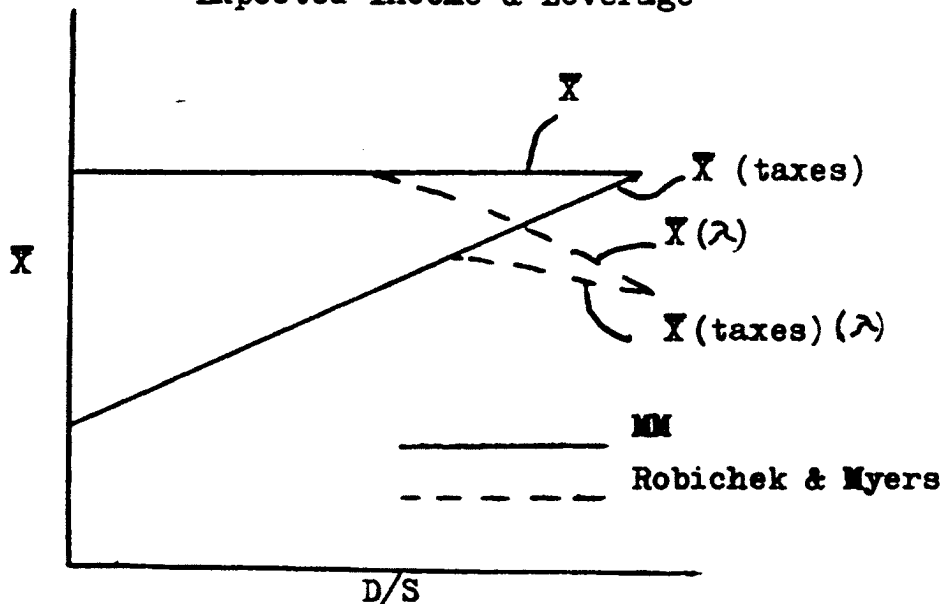
[1]Ibid., p. 43.

Thus it is shown that if X is allowed to vary in relation

to leverage both the X and X after taxes will have optimum

positions (O_1, O_2 on graph) much the same as predicted by

traditional theory. Robichek and Myers are unable to

prove this hypothesis and offer it only as a plausible

explanation until such time when more general dynamic

theories are developed.

Weston, using a different approach, also attempts

to reconcile MM with traditional theory.

> . . . in testing Proposition 2, Modigliani and Miller
> define leverage as the ratio of debt to debt-plus-stock.
> I have found that, by taking the regression equations
> obtained by Modigliani and Miller for Proposition 2
> and changing the measure of leverage from debt to stock
> to debt-plus-stock, the regression relation becomes
> curved upward as suggested by traditional theory.
> Thus, Proposition 2 of Modigliani and Miller reinforces
> traditional theory rather than controverts it.[1]

In other words MM's yield relation will change from a

straight line function to a curved function as shown in

graphs 1 and 2 below as D/V is substituted for D/S as a

measure of leverage.

Barges disagrees with Weston's analysis by

stating that

> The . . . conclusion by Professor Weston was apparently
> reached by him in an attempt to develop some kind of
> reconciliation between the MM and the traditional
> views. However, . . . it is the opinion of this
> writer that there is no basis for reconciling the two

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[1] J. Fred Weston, "The Management of Corporate Capital:
A Review Article," The Journal of Finance, Vol. XXXIV,
No. 2, (April, 1961), p. 135.

views. The two views are very much opposed: the one says that debt is cheap and the other says that it is not.[1]

Graph 1

K_e

% Yield

D/S

Graph 2

K_e

% Yield

D/V
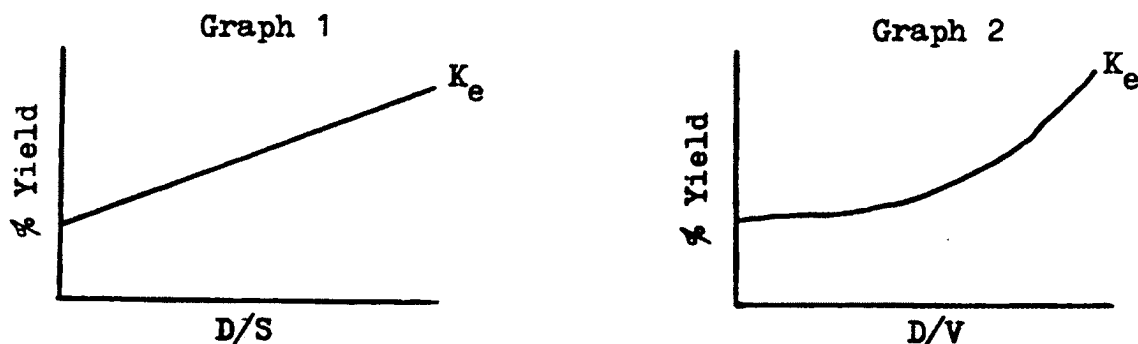
Another manner in which some writers try to reconcile the two diverse views is by creating two worlds for the theories to rule separately. Under this design King MM rules in the theoretical world and King Traditional reigns in the real world of imperfections.

Perhaps in fact such a reconciliation between existing theories, or the emergence of some new theory or additional proof for an old one may someday unify writers behind a single explanation of the relationship between capital structure and the cost of capital. A propitiation of theory at present, however, seems a long way off as is made readily apparent in the next section summarizing criticisms of existing theories.

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[1] Barges, The Effect of Capital Structure on the Cost of Capital, pp. 13-14.

CRITICISMS

Once there was a powerful king who ruled his world all alone until one day he was criticized and attacked by a new ideology which gained some support in his kingdom and strived for it all. The king, however, had many loyal followers of his own who fought back through numerous criticisms of their own directed toward the new ideology which was now forced onto the defensive.

This appears to be an accurate analogy of the present situation between MM theory and "King Traditional." The latter has many supporters with a great deal of criticism and as a result the former has mainly defended his original theory. Therefore, this section will consist mostly of criticism against MM theory.

Weston criticizes MM by charging them with the failure of taking into account the effects of growth in earnings per share. MM theory applies only to non-growth firms in the static sense. Based on regression analysis studies of data from the electric utility industry, Weston concludes that equity yields are negatively related to both leverage and growth in earnings per share. Therefore, as leverage increases growth decreases, causing a corresponding increase in equity yield which is interpreted by MM as being a positive function of leverage.

When the influence of growth is isolated, the net influence of leverage on the cost of capital is found to be consistent with traditional business finance theory rather than the Modigliani and Miller propositions.[1]

This of course is a dynamic type of test of a static theory. Although Robichek and Myers obtain results consistent with traditional theory also, they disagree with Weston's growth analysis.

. . . no restrictions concerning possible growth patterns in operating income or assumptions about whether or not investors agree in their estimates of the future performance of firms are necessary to prove Proposition I.[2]

Alexander Barges completed a full study of MM theory inclusive of empirical, theoretical, and procedural evaluations. The major criticisms that he found are:

(1) By using, as MM do, market valued debt-equity ratios as a measure of financial risk, many variations in a heterogeneous sample will not be cancelled out as they would if book valued debt-equity ratios were employed. This situation will be further agitated if there is a lack of observations in the sample having little or no debt.

(2) MM's arbitrage process and their Proposition I

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[1] J. Fred Weston, "A Test of Cost of Capital Propositions," The Southern Economic Journal, Vol. XXX, No. 2, (October, 1963), p. 112.

[2] Alexander A. Robichek and Stewart C. Myers, "Problems in the Theory of Optimal Capital Structure," Journal of Finance and Quantitative Analysis, (June, 1963), p. 13.

are in fact dependent upon certain assumptions relating to investor risk, in spite of their statements to the contrary. In addition MM have ignored induced reactions which may even cause their arbitrage process to work in reverse.

(3) In defining expected rate of return on common stock in order to derive Proposition II from Proposition I (page 46) MM were inconsistent in that they subtracted current interest (definite known value) from expected future profits (unknown value). For theoretical purposes they should have used some expected average future interest charges consistent with $\overline{X}$ rather than the current know charges, $K_d(D)$.

(4) MM's Proposition II assumes that the shapes of investor probability distributions are not skewed. Barges feels they may be due to the limited liability feature of many shares of common stock.[1]

Bodenhorn is critical of MM's handling of risk.

> Modigliani and Miller assume that the total amount of risk associated with the net operating income is independent of the financing, since risk is a function only of the variability of the earnings stream and this variability is unaffected by financial structure. Financial writers, however, frequently use the ratio of debt charges to net operating income as a measure of risk, because they are interested in the probability

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[1] Barges, The Effect of Capital Structure on the Cost of Capital.

that the net income will be negative and that the
firm cannot meet its fixed charges.[1]

In other words just because the variability of earnings

risk is not dependent upon capital structure does not mean

that total risk is independent because total risk may include

risks in addition to the variability of earnings risks.

Another writer who has generated extensive criticism

of the MM theory is David Durand, who's main denouncements

are as follows:

(1) MM state that no additional risk is incurred by

individual investors when they engage in the arbitrage

process, but this is not the case due to the limited liabil-

ity a creditor in a corporation enjoys in relation to the

unlimited liability of an individual engaged in marginal

purchases.

(2) The establishment of separate classes of firms

according to business risk, as determined in a static sense,

is not adaptable to the dynamic economy in the real world

where stocks do not sell at book value.

(3) Realistically the arbitrage process of corpora-

tions and individuals is subject to many restrictions in the

everyday market place.[2]

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[1]Diran Bodenhorn, "On the Problem of Capital Budget-
ing," The Journal of Finance, Vol. XIV, No. 4, (Dec., 1959),
p. 485

[2]William J. Baumol and Burton G. Malkiel, "The Firm's
Optimal Debt-Equity Combination and the Cost of Capital," The
Quarterly Journal of Economics, Vol. LXXXI, No. 4, (November,
1967), pp. 547-578.

The effect of one of these limitations, trans-
action costs, is studied by Baumol and Malkiel. They argue
that a levered firm may have a higher value than an unlevered
firm in the same risk class simply due to the effect of
these costs. This results since an individual investor
must, if he wishes to take part in the MM arbitrage process,
incur transaction costs. For example, if an individual
investor holds shares in a firm which provides him with more
than the desired degree of leverage he will sell them and
purchase shares in a less levered firm. If the value of
the first firm were equal to the second, transaction costs
would stop the process and therefore the unlevered firm
must be valued lower than the levered firm.[1]

Solomon, as well as criticizing MM for many of the
issues already listed, opposes MM's explanation of the
behavior of K_e under conditions of extreme leverage. As
was described earlier MM state that K_e, under high leverage,
will begin to increase at a decreasing rate and actually
decrease if necessary in order to keep K_o constant. Such
a situation according to Solomon is both inconsistent with
MM's own assumptions and with the actions of normal, rational
investors.[2]

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[1]William J. Baumol and Burton G. Malkiel, "The
Firm's Optimal Debt-Equity Combination and the Cost of
Capital," The Quarterly Journal of Economics, Vol. LXXXI,
No. 4, (November, 1967), pp. 547-578.

[2]Solomon, The Theory of Financial Management, pp.
110-111.

Eli Schwartz differs with MM in that he feels there is a definite optimum capital structure for any amount of equity. Given some fixed amount of ownership capital, equal absolute amounts of debt will result in lower debt-equity ratios for firms with the largest amount of equity. For example:

	Co. A (S = 1,000)	Co. B (S = 2,000)
add D = 200	D/S = 1/5	D/S = 1/10
add D = 400	D/S = 2/5	D/S = 1/5

The addition of debt of equal amounts results in a lower D/S for firm B, with the largest amount of S, than for firm A. Since a lower debt-equity ratio signifies a lower interest rate, the debt can be secured cheaper by a firm with S = $2,000 than one with S = $1,000. Thus, for each level of S and D there exists an earnings amount and this amount can be optimized by arriving at the correct amount of S.[1]

In contrast to these criticisms a recent article by Stiglitz is, for the most part, consistent with MM theory but in a more general sense. Stiglitz first high-lights the assumptions or limitations under which the original MM theory operates. Next he is able to eliminate many of them by formulating a generalized MM theorm whose validity, ". . . does not depend on the existence of risk

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[1]Schwartz, "Theory of the Capital Structure of the Firm," pp. 18-39.

classes, on the competitiveness of the capital market, or
on the agreement of individuals about the probability
distribution of outcomes."[1] Two assumptions that Stiglitz
does make are (1) that individuals and firms can borrow
at the same market rate of interest, and that (2) no firms
go bankrupt. In later portions of the article, however,
he does show that MM results may be valid even with certain
limitations on individual borrowing and under certain
specific conditions where the probability of bankruptcy
is finite.

In disagreeing with the traditional stance, Stiglitz
states that

> If there are three or more firms in the same risk
> class, and the firms with the highest and lowest
> debt-equity ratios have the same value, then the
> value of all other firms must be the same. . . .
> This result rules out the possibility of a U-shaped
> curve relating to the value of the firm to the debt-
> equity ratio.[2]

Even if this statement is correct, however, it does not
mean that it is representative of the actual market place.

CONCLUSIONS

The majority of criticism against MM has taken
place outside of the arena fenced in by their assumptions

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[1]Joseph E. Stiglitz, "A Re-Examination of the
Modigliani-Miller Theorm," The American Economic Review,
Vol. LIX, No. 5, (December, 1969), p. 784.

[2]Ibid., p. 788.

and in most cases this outer area has been the realistic
and imperfect world not dependent upon theoretical fences.
There are two possible ways of looking at this situation.

One is that both opponents are correct and in fact
not even opponents in the sense of facing each other since
they are standing on two different battlefields, "Realistic
Ridge" and the "Little-Big Theory." Under this view the
rule exists that one side cannot be attacked on the other's
battlefield.

The other view holds that, neither a real battle
nor a real winner is possible until the fences can be
broken down and all the invading variables correctly
related to theory. Theory per se is not advantageous un-
less it can be utilized within the everyday world to solve
problems; it must be representative and capable of useful
predictions and applications.

If the objective is progress there is no question
but that the latter view is superior. Under this framework
MM theory must be criticized and tested in the real world
if it is to be of any use to the manager of Joe's Drive-
In, who cannot always conform his environment to MM
assumptions.

Research into the leverage cost of capital dispute
has in general been of this preferred type but more is
necessary since a dispute still exists.

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Appendix 1

LISTING OF LEVERAGE MEASURES

1. $L = \dfrac{D}{S}$ (book value)

2. $L = \dfrac{D}{S}$ (market value)

3. $L = \dfrac{D}{D + S}$ (book value)

4. $L = \dfrac{D}{D + S}$ (market value)

5. $L = \dfrac{K_d}{K_d + K_e}$ (flow measure)

6. $L = \dfrac{D/\text{annuity factor for X years}}{\text{cash flow} - \text{annuity factor for X years}}$

7. $L = \dfrac{K_d + \text{yearly principle repaid}}{\text{total cash flow}}$

8. $L = \dfrac{K_d}{\text{income before } K_d}$

9. $L = 1 + \dfrac{S}{D}\left(1 - \dfrac{K_d}{e_1}\right)$

10. $L = 1 + \dfrac{S}{D}\left(1 - \dfrac{xK_d}{e_1}\right)$

11. $L = 1 + \dfrac{x(e_2 - f_1)}{e}$

12. $L = 1 + \dfrac{xe_2 - f_1}{e}$

13. $L = \dfrac{\text{net equity earnings}/S}{\text{EBI after T}/V}$

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14. $L = \dfrac{\text{new equity earnings/S}}{\text{old equity earnings/S}}$

15. $L = \dfrac{\text{net equity earnings/S}}{\text{EBIT/V}}$

16. $L = \dfrac{\dfrac{\text{change in net equity earnings caused by change in earnings before interest after tax}}{\text{net equity earnings}}}{\dfrac{\text{change in earnings before interest after taxes}}{\text{earnings before interest after taxes}}}$

17. $L = \dfrac{\dfrac{\text{change in earnings after interest before tax caused by change in earnings before interest \& tax}}{\text{earnings after interest before tax}}}{\dfrac{\text{change in earnings before interest \& taxes}}{\text{earnings before interest and tax}}}$

18. $L = \dfrac{\text{debt + preferred stock}}{\text{equity}}$ (book value)

19. $L = \dfrac{\text{debt + preferred stock}}{\text{equity}}$ (market value)

20. $L = \dfrac{\text{debt + preferred stock}}{\text{debt + preferred stock + equity}}$ (book value)

21. $L = \dfrac{\text{debt + preferred stock}}{\text{debt + preferred stock + equity}}$ (market value)

22. $L = \dfrac{\text{total interest + dividends on preferred}}{\text{EBIT}}$

23. $L = \dfrac{K_d + \text{div. on pref.}}{\text{EBIT} - (K_d + \text{div. on pref.})}$

24. $L = \dfrac{K_d + \text{div. on pref.}}{\left(\text{EBIT} - K_d - \dfrac{\text{div. on pref.}}{1-T}\right)(1-T)}$

25. $L = \dfrac{\text{EBIT}}{\text{EBIT} - K_d - \text{div. on pref.}}$

26. $L = \dfrac{\text{EBIT} - K_d - \text{div. on pref.}}{\text{equity}} \bigg/ \dfrac{\text{EBIT}}{\text{equity} + D + \text{preferred stock}}$

$$27. \quad L = \left(\frac{EBIT - K_d - \frac{\text{div. on pref.}}{1-T}}{\text{equity}} \right)(1-T) \Big/ \frac{EBIT(1-T)}{\text{equity + debt + preferred stock}}$$

Appendix 2
Calculations for Graph 1 page 35
$$(S = \bar{X} = \$10,000)$$

A	B	C	D	E	F	G	H	I
Debt (D)	$(K_d)(D)$ $K_d = 4\%$	$(K_d)(D)$ $K_d = 8\%$	$\dfrac{10,000}{-B}$	$\dfrac{10,000}{-C}$	$\dfrac{\sigma=3,000}{D}$	$\dfrac{\sigma=3,000}{E}$	$\dfrac{\sigma=6,000}{D}$	$\dfrac{\sigma=6,000}{E}$
0	0	0	10,000	10,000	.3000	.3000	.6000	.6000
1,000	40	80	9,960	9,920	.3012	.3024	.6024	.6048
2,000	80	160	9,920	9,840	.3024	.3048	.6048	.6099
3,000	120	240	9,880	9,760	.3036	.3073	.6073	.6148
4,000	160	320	9,840	9,680	.3049	.3099	.6099	.6198
5,000	200	400	9,800	9,600	.3061	.3125	.6122	.6250
6,000	240	480	9,760	9,520	.3073	.3151	.6148	.6302
7,000	280	560	9,720	9,440	.3086	.3178	.6173	.6356
8,000	320	640	9,680	9,360	.3099	.3205	.6198	.6410
9,000	360	720	9,640	9,280	.3112	.3233	.6224	.6466
10,000	400	800	9,600	9,200	.3125	.3261	.6250	.6522
11,000	440	880	9,560	9,120	.3138	.3289	.6276	.6578
12,000	480	960	9,520	9,040	.3151	.3319	.6302	.6623